EE 5323 Homework 1

State Variable Systems, Computer Simulation

1. Simulate the van der Pol oscillator $y''+\alpha(y^2-1)y'+y=0$ using MATLAB for various ICs. Plot y(t) vs. t and also the phase plane plot y'(t) vs. y(t). Use y(0)=0.1, y'(0)= 0.1
   a. For α= 0.04.
   b. For α= 0.85.

2. Do MATLAB simulation of the Lorenz Attractor chaotic system. Run for 150 sec. with all initial states equal to 0.4. Plot states versus time, and also make 3-D plot of x_1, x_2, x_3 using PLOT3(x1,x2,x3).

$$\dot{x}_1 = -\sigma(x_1 - x_2)$$
$$\dot{x}_2 = rx_1 - x_2 - x_1x_3$$
$$\dot{x}_3 = -bx_3 + x_1x_2$$

use σ= 10, r= 28, b= 8/3.

3. Consider the Voltera predator-prey system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_1x_2\\ \dot{x}_2 &= x_2 - x_1x_2\end{aligned}.$$

Simulate the system using MATLAB for various initial conditions. Take ICs spaced in a uniform mesh in the box x1=[-2,2], x2=[-2,2]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-5,5]x[-5,5].

EE 5323 Homework 2

Nonlinear Systems Simulation

1. Duffing's equation is interesting in that it exhibits bifurcation, or dependence of stability properties and number of equilibrium points on a parameter. The undamped Duffing equation is

$$\ddot{x} + \alpha x + x^3 = 0$$

   a. Find the equilibrium points. Show that for $\alpha > 0$ there is only one e.p.
   b. For $\alpha < 0$ there are 3 eps. Linearize the system and study the nature of these 3 e.p.s
   c. Simulate the Duffing oscillator and make time plot and phase plane plot. Do for 3 cases:
      a. $\alpha = -1$
      b. $\alpha = -0.1$
      c. $\alpha = 1$

For each case, take ICs spaced in a uniform mesh in a suitable box to show the behavior. Pick the box size. Make one phase plane plot for each case showing all trajectories for that case.

2. Consider the system

$$\dot{x} = y(1 + x - y^2)$$
$$\dot{y} = x(1 + y - x^2).$$

Simulate the system using MATLAB for various initial conditions for the two cases:

   a. Take ICs spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-15,15]x[-15,15].
   b. Take ICs spaced in a uniform mesh in the box x1=[-3,3], x2=[-3,3]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-5,5]x[-5,5].

3. The system of equations

$$\dot{x}_1 = ax_1 - bx_1x_2 - cx_1^2$$
$$\dot{x}_2 = dx_2 - ex_1x_2 - fx_2^2$$

describes the growth of two competing species that prey on each other. The constants are positive parameters. Pick *a=c=d=f=2, b=e=3.*

Simulate the system using MATLAB for various initial conditions. Take ICs spaced in a uniform mesh in the box x1=[-2,2], x2=[-2,2]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-5,5]x[-5,5].

EE 5323 Homework 3

Nonlinear Systems and Equilibrium Points

1. Consider the Voltera predator-prey system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_1 x_2 \\ \dot{x}_2 &= x_2 - x_1 x_2\end{aligned}.$$

Find the equilibrium points and their nature.

2. Equilibrium points and linearization

System is

$$\begin{aligned}\dot{x}_1 &= x_2(-x_1 + x_2 - 1) \\ \dot{x}_2 &= x_1(x_1 + x_2 + 1)\end{aligned}$$

   a. Find all equilibrium points
   b. Find Jacobian
   c. Find the nature of all e.p.s

3. Simulate the system

$$\begin{aligned}\dot{x}_1 &= x_2(-x_1 + x_2 - 1) \\ \dot{x}_2 &= x_1(x_1 + x_2 + 1)\end{aligned}$$

using MATLAB for various initial conditions for the two cases:

   a. Take ICs spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-15,15]x[-15,15].
   b. Take ICs spaced in a uniform mesh in the box x1=[-3,3], x2=[-3,3]. Make one phase plane plot with all the trajectories on it. Plot phase plane on square [-5,5]x[-5,5].

EE 5323 Homework 4

Vector Fields, Flows, First Integrals

1. Consider the undamped oscillator

$$\ddot{x} + x = 0$$

   a. Write position-velocity state space form $\dot{X} = f(X)$.
   b. Plot the trajectories $x(t), \dot{x}(t)$ vs. time. Use initial conditions of $x(0) = 0.1,\ \dot{x}(t) = 0$
   c. Plot the vector field $f(X) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$ in the phase plane $(x_1, x_2) = (x, \dot{x})$. Plot for points spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   d. Plot the system trajectories (flows or orbits) in the phase plane. Take ICs spaced in a uniform mesh in the box x1=[-10,10], x2=[-10,10].
   e. Derive the First Integral of Motion $F(x_1, x_2)$ as done in class. Plot the FIM as a 3-D surface over the phase plane on the x1=[-10,10], x2=[-10,10].

2. Repeat for the unstable system

$$\ddot{x} - x = 0$$

EE 5323 Homework 5

Lyapunov Stability Analysis, LaSalle

1. Use Lyapunov to examine the stability of these systems. Simulate time histories from many uniformly spaced ICs to verify your results.

   a. $$\begin{aligned}\dot{x}_1 &= x_1 x_2^2 - x_1 \\ \dot{x}_2 &= -x_1^2 x_2 - x_2\end{aligned}$$

   b. $$\begin{aligned}\dot{x}_1 &= x_2 + x_1(x_1^2 - 2) \\ \dot{x}_2 &= -x_1\end{aligned}$$

2. Use Lyapunov to show that the system

$$\begin{aligned}\dot{x}_1 &= x_1 x_2^2 + x_1(x_1^2 + x_2^2 - 3) \\ \dot{x}_2 &= -x_1^2 x_2 + x_2(x_1^2 + x_2^2 - 3)\end{aligned}$$

is locally asymptotically stable. Find the Region of Asymptotic Stability. Simulate the system from many uniformly spaced ICs.

3. LaSalle extension.

   a. Use quadratic Lyapunov Function to show this system is locally SISL

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1(x_1^2 - 2) \\ \dot{x}_2 &= -x_1\end{aligned}$$

Find a region within which $\dot{V} \le 0$.

   b. Use LaSalle's extension to verify that the system is actually AS. Find the equilibrium point.

   c. Simulate the system from many uniformly spaced ICs.

EE 5323 Homework 6

UUB and Lyapunov Equations

1. UUB of system with disturbance.
   Consider the system on S&L p. 66 with a disturbance d

$$\dot{x} + c(x) + d = 0$$

Assume that $xc(x) > ax^2$ with $a > 0$ a known positive constant

   a. Assume that d is unknown but is bounded by $\|d\| < D$ with D a known positive constant. Prove that the system is UUB and find the bound on *x(t)*.
   b. Assume that d is unknown but is bounded by $\|d\| < D\|x\|$ with D a known positive constant. Prove that the system is UUB and find the bound on *x(t)*.

2. UUB

   Use Lyapunov to show that the system

$$\dot{x}_1 = x_1 x_2^{\ 2} - x_1(x_1^{\ 2} + x_2^{\ 2} - 3)$$
$$\dot{x}_2 = -x_1^{\ 2} x_2 - x_2(x_1^{\ 2} + x_2^{\ 2} - 3)$$

is uniformly ultimately bounded UUB. That is, show that the Lyapunov derivative is NEGATIVE OUTSIDE A BOUNDED REGION. Find the radius of the bounded region outside which $\dot{V}$ <0. Any states outside this region are attracted towards the origin.

3. Use Lyapunov Equation to check the stability of the linear systems

   a. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x$

   b. $\dot{x} = Ax = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix} x$

   c. $\dot{x} = Ax = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} x$

EE 5323 Homework 7

Lyapunov Controls Design, Feedback Linearization

1. Limit Cycle. Use Lyapunov to show that the system

$$\dot{x}_1 = x_2 - x_1(x_1^2 + x_2^2 - 1)$$
$$\dot{x}_2 = -x_1 - x_2(x_1^2 + x_2^2 - 1)$$

has a stable limit cycle.

2. **Controls Design.** A system is given by

$$\dot{x}_1 = x_2 \operatorname{sgn}(x_1)$$
$$\dot{x}_2 = x_1 x_2 + u$$

Select Lyapunov function candidate

$$V(x) = \tfrac{1}{2}(x_1^2 + x_2^2)$$

Use Lyapunov to design a controller *u(x)* to make system SISL.

3. **Multi-input Control.** Use Lyapunov to design controls u_1, u_2 to make this system *i.* SISL, and then *ii.* AS

$$\dot{x}_1 = x_1 x_2^2 + u_1$$
$$\dot{x}_2 = x_1^3 x_2^7 + u_2$$

EE 5323 Homework 8

Feedback Linearization, Backstepping

1. Effect of Output Choice in i/o FB Linearization

It is desired to stabilize a system given by

$$\dot{x}_1 = x_2 \sin x_1 - x_1 + u$$
$$\dot{x}_2 = -x_1 + x_2^2$$

a. Select the output as $y = x_1$ and use FB lin. design to select the control *u(t)* to follow the desired trajectory $y_d(t)$. Check the internal dynamics. Set *y=0* to get the zero dynamics. Is the system minimum phase?

b. Select the new output $y = x_2$. Find the FB lin. controller u(t). Does this work? What about the internal dynamics?

2. Backstepping. Slotine and Li Problem 6.11. 'Do backstepping to stabilize the system

$$\dot{y} + zy^4 = u$$
$$\ddot{z} + (y-1)\dot{z}^2 + z^5 = 0$$

Show the full proof of stability.

3. Backstepping. Slotine and Li problem 6.8. Do backstepping to stabilize the system

$$\dot{y} + y^2 e^{yz} = 5u$$
$$\ddot{z} + \dot{z}^3 - z^7 + yz^2 = 0$$

Show the full proof of stability.