COM SCI 161 : HOMEWORK ~~8~~ 7

1. Prove the identity : $Pr(\alpha_1 \ldots, \alpha_n | \beta) = Pr(\alpha_1 | \alpha_2 \ldots \alpha_n, \beta) \cdots Pr(\alpha_n | \beta)$

Looking at the RHS

$Pr(\alpha_1 | \alpha_2 \ldots \alpha_n, \beta) = Pr(\alpha_1, \alpha_2 \ldots, \beta) / Pr(\alpha_2, \alpha_3 \ldots \alpha_n, \beta)$ {chain rule}

$Pr(\alpha_2 | \alpha_3 \ldots \alpha_n, \beta) = Pr(\alpha_2 \ldots \beta) / Pr(\alpha_3 \ldots \alpha_n \beta)$

$\vdots$

$Pr(\alpha_n | \beta) = Pr(\alpha_n, \beta) / Pr(\beta)$

{Based on conditional probability formula : $Pr(A|B) = Pr(A, B)/Pr(B)$}

Using these in the RHS,

$$\left(\frac{Pr(\alpha_1, \alpha_2 \ldots \beta)}{\cancel{Pr(\alpha_2 \ldots \beta)}}\right) \cdot \left(\frac{\cancel{Pr(\alpha_2, \alpha_3 \ldots \alpha_n, \beta)}}{\cancel{Pr(\alpha_3, \ldots \alpha_n, \beta)}}\right) \cdots \cdot \left(\frac{\cancel{Pr(\alpha_n, \beta)}}{Pr(\beta)}\right)$$

Notice, that every denominator gets cancelled by the next numerator in this proof.

So we are finally left :

$$RHS = \frac{Pr(\alpha_1, \alpha_2 \ldots \beta)}{Pr(\beta)} = Pr(\alpha_1, \alpha_2 \ldots \alpha_n | \beta) = LHS$$

Hence proved.

2. O: oil , NG: natural gas , N: neither , P: positive test

$Pr(O) = 0.5$ $Pr(P|O) = 0.9$

$Pr(NG) = 0.2$ $Pr(P|NG) = 0.3$

$Pr(N) = 0.3$ $Pr(P|N) = 0.1$

$Pr(O|P) = ?$

Applying Bayes' Theorem,

$$Pr(O|P) = \frac{Pr(P|O) \times Pr(O)}{Pr(P)}$$

$$= \frac{Pr(P|O) \times Pr(O)}{Pr(P|O) P(O) + Pr(P|NG) Pr(NG) + Pr(P|N) Pr(N)}$$

{Applying case analysis to denominator}

$$= \frac{(0.9)(0.5)}{(0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)}$$

$$= 0.8333 = 83.33\%$$

Thus, probability of finding oil when test returns positive i.e $Pr(O|P) = 83.33\%$.

3. $P(H|a) = 0.2$ $\quad$ $P(H|b) = 0.4$ $\quad$ $P(H|c) = 0.8$

Variables: C: coin ~~bag~~ $\in \{a, b, c\}$

X_1, X_2, X_3 : outcome $\in \{H, T\}$

B : bell $\in \{on, off\}$

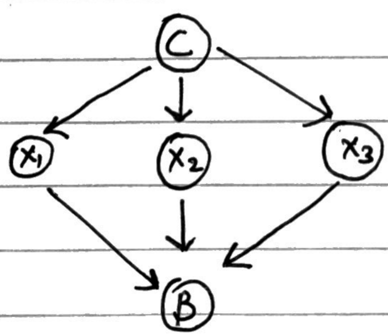
C	Pr
a	1/3
b	1/3
c	1/3

C	X	Pr
a	H	0.2
b	H	0.4
c	H	0.8
a	T	0.8
b	T	0.6
c	T	0.2

X_1	X_2	X_3	B	Pr
H	H	H	on	1
			off	0
H	H	T	on	0
			off	1
H	T	H	on	0
			off	1
H	T	T	on	0
			off	1
T	H	H	on	0
			off	1
T	H	T	on	0
			off	1
T	T	H	on	0
			off	1
T	T	T	on	1
			off	0

on only when $x_1 = x_2 = x_3$ otherwise off.

Network:



4. (a) Assumptions (Markov):

$I(A, \emptyset, BE)$

$I(B, \emptyset, AC)$

$I(C, A, BDE)$

$I(D, AB, CE)$

$I(E, B, ACDFG)$

$I(F, CD, ABE)$

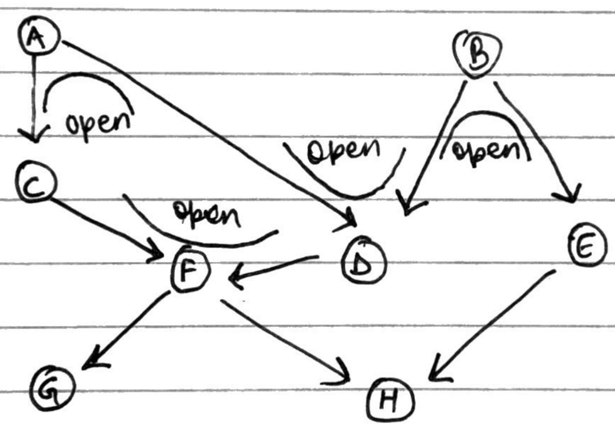
$I(G, F, ABCDEH)$

$I(H, EF, ABCDG)$

They are in the form
$\boxed{I(\text{Node}(v), \text{Parent}(v), \text{Non-descendants}(v))}$

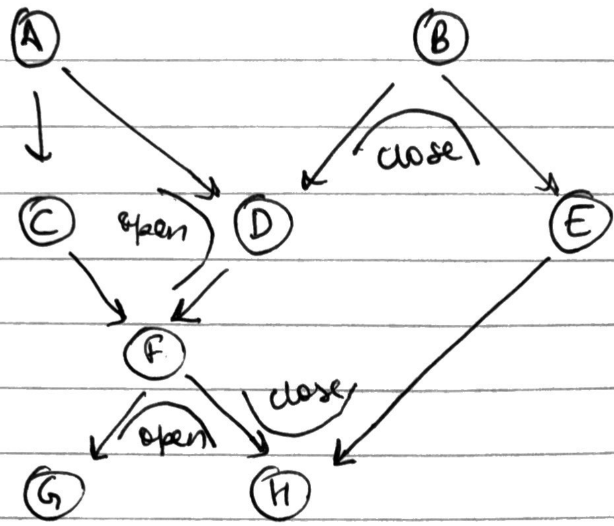
(b) d-seperated (A, F, E)

False. F does not block path ~~bee~~ between A and E. There exists a path through D.



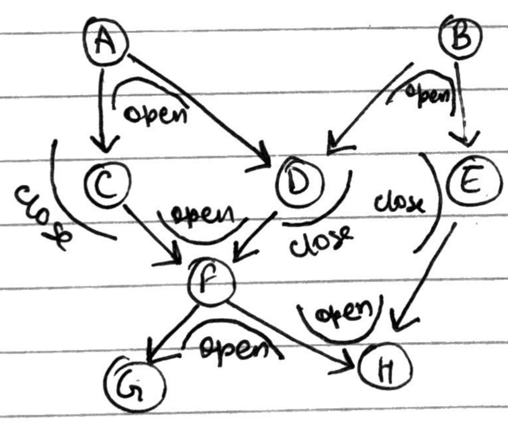
~~do~~ d-seperated (G, B, E)

True, There exists no path from G to E which isn't blocked by B. This means that ~~rem~~ recursively removing outgoing edges from B, and recursively removing leaves gives disconnection between G and E.



d-seperated (AB, CDE, GH)

True. There is no path possible between {A,B} and {G,H} which isn't blocked by at least one of the nodes from {C,D,E}. Using the same logic as above gives this result.



(c) Using chain rule for Bayesian networks.

$$Pr(a,b,c,d,e,f,g,h) = Pr(a) \cdot Pr(b) \cdot Pr(c|a) \cdot Pr(d|a,b) \cdot Pr(e|b) \cdot Pr(f|c,d) \cdot Pr(g|f) \cdot Pr(h|e,f)$$

Thus, this covers every node in the graph and considers child-parent pairs.

(d) As A and B are independent statements, with no parent each,

$$Pr(A=1, B=1) = Pr(A=1) \cdot Pr(B=1)$$
$$= (0.2) \cdot (0.7)$$
$$= (0.14)$$

As E is dependent on B but independent of A, we compute

$$Pr(A=0 \mid E=0) = Pr(A=0) \cdot [Pr(B=0) \cdot Pr(E=0 \mid B=0) + Pr(B=1) \cdot Pr(E=0 \mid B=1)]$$
$$= (0.8) \cdot [(0.3)(0.1) + (0.7)(0.9)]$$
$$= (0.8)[0.66]$$
$$= 0.528.$$

5. (a) $\alpha : A \Rightarrow B$

The world satisfying α are w_0, w_2, w_3

(b) $Pr(\alpha) = Pr(w_0) + Pr(w_2) + Pr(w_3)$
$= 0.3 + 0.1 + 0.4 = 0.8$

(c) $Pr(A, B \mid \alpha)$: That means given α is true so we will consider w_0, w_2, w_3.

w_0 : $Pr(A=T, B=T \mid \alpha) = P(w_0 \mid \alpha) = \frac{Pr(w_0)}{P(\alpha)} = 0.385$

w_2 : $Pr(A=F, B=T \mid \alpha) = Pr(w_2 \mid \alpha) = \frac{Pr(w_2)}{Pr(\alpha)} = 0.125$

w_3 : $Pr(A=F, B=F \mid \alpha) = Pr(w_3 \mid \alpha) = \frac{Pr(w_3)}{Pr(\alpha)} = 0.5$

For $Pr(A, B \mid \alpha)$ we consider $A=T, B=T$ given α.

$$P(A, B \mid \alpha) = Pr(w_0 \mid \alpha) = \frac{Pr(w_0)}{Pr(\alpha)} = \frac{0.3}{0.8} = 0.375$$

(d) $Pr(A \Rightarrow \neg B \mid \alpha)$

Since α is True, let's consider $A \Rightarrow \neg B$.

$A \Rightarrow \neg B$ is True in w_1, w_2, w_3, and since α is true in w_0, w_2, w_3, thus we only consider w_2, w_3.

$$Pr(A \Rightarrow \neg B \mid \alpha) = \frac{Pr(w_2)}{Pr(\alpha)} + \frac{Pr(w_3)}{Pr(\alpha)} = \frac{0.1}{0.8} + \frac{0.4}{0.8}$$

$$= 0.625$$