CS 161: FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE

ASSIGNMENT 5.

1. (a) $P \Rightarrow \neg Q$ is equivalent to $Q \Rightarrow \neg P$?

P	Q	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

As $P \Rightarrow \neg Q$ and $Q \Rightarrow \neg P$ have identical truth tables, thus $P \Rightarrow \neg Q$ is equivalents to $Q \Rightarrow \neg P$.

(b) $(P \Leftrightarrow \neg Q)$ is equivalent to $((P \wedge \neg Q) \vee (\neg P \wedge Q))$?

P	Q	$P \Leftrightarrow \neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$[(P \wedge \neg Q) \vee (\neg P \wedge Q)]$
F	F	F	F	F	F
F	T	T	F	T	T
T	F	T	T	F	T
T	T	F	F	F	F

As $(P \Leftrightarrow \neg Q)$ and $((P \wedge \neg Q) \vee (\neg P \vee Q))$ have ~~equivalent~~ identical truth tables, thus, $P \Leftrightarrow \neg Q$ is equivalent to $((P \wedge \neg Q) \vee (\neg P \wedge Q))$.

2. Let, S = Smoke, Fr = Fire, H = Heat.

(a)

S	Fr	$S \Rightarrow Fr$	$\neg S \Rightarrow \neg Fr$	$[(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg Fr)]$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

As there is one ~~Fr and thus T~~ world where the statement is false, and three ~~wold~~ worlds where it is True, thus the statement is neither valid or unsatisfiable.

(b)

S	Fr	H	$S \Rightarrow Fr$	$S \vee H$	$(S \vee H) \Rightarrow Fr$	$[(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow Fr)]$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

As there is one world where the statement is false while there are seven worlds where the statement is True. Thus, the statement is neither valid or unsatisfiable.

(c)

S	Fr	H	$S \wedge H$	$S \wedge H \Rightarrow Fr$	$S \Rightarrow Fr$	$H \Rightarrow Fr$	$(S \Rightarrow Fr) \vee (H \Rightarrow Fr)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	F	F	F
T	F	F	F	T	F	T	T
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	T	T

$(S \wedge H \Rightarrow Fr) \Leftrightarrow [(S \Rightarrow Fr) \vee (H \Rightarrow Fr)]$
T
T
T
T
T
T
T
T

As the sentence is valid in all worlds, it is valid.

3. Let, M: mythical, I: immortal, A: mammal, H: horned, Mg: magical

(a) Creating propositional ~~knowledge~~ logic knowledge base:

$\Delta = \{M \Rightarrow I,\ \neg M \Rightarrow (\neg I \wedge A),\ (I \vee A) \Rightarrow H,\ H \Rightarrow Mg\}$

(b) Converting each clause to CNF

$M \Rightarrow I \quad = \quad \neg M \vee I \quad \cdots (1)$

$\neg M \Rightarrow (\neg I \wedge A) \quad = \quad M \vee (\neg I \wedge A)$

$(M \vee \neg I) \wedge (M \vee A) \quad \cdots (2)$

$(I \vee A) \Rightarrow H \quad = \quad \neg (I \vee A) \vee H$

$= (\neg I \wedge \neg A) \vee H$

$= (\neg I \vee H) \wedge (\neg A \vee H) \quad \cdots (3)$

$H \Rightarrow Mg \quad = \quad \neg H \vee Mg \quad \cdots (4)$

Thus the CNF of the knowledge base:

$(\neg M \vee I) \wedge (M \vee \neg I) \wedge (M \vee A) \wedge (\neg I \vee H) \wedge (\neg A \vee H) \wedge (\neg H \vee Mg)$

(c) From the above CNF, I have the following:

Δ: $\neg M \vee I \quad \cdots (1)$

$M \vee \neg I \quad \cdots (2)$

$M \vee A \quad \cdots (3)$

$\neg I \vee H \quad \cdots (4)$

$\neg A \vee H \quad \cdots (5)$

$\neg H \vee Mg \quad \cdots (6)$

α_1: Mythical (M)? $\qquad \neg\alpha_1 : \neg M \quad \cdots (7)$

~~Through no combinations of resolution can I prove that $\Delta \wedge \neg\alpha_1$ is inconsistent, and thus we cannot say that the unicorn is mythical.~~

~~α_2: Magical? (Mg) $\qquad \neg\alpha_2 : \neg Mg \quad \cdots (8)$~~

~~(9) $\neg H \qquad (6, 8)$~~

$\Delta \models \alpha_1$ iff $\Delta \wedge \neg\alpha_1$ is inconsistent. By resolution,

(8) $\neg I \qquad (2, 7)$

(9) $\neg M \qquad (8, 1)$

(10) $A \qquad (9, 3)$

(11) $H \qquad (10, 5)$

(12) $Mg \qquad (11, 6)$ — Terminal

As we explored all permutations before terminating, thus, there is no possible inconsistency with $\neg\alpha_1$. Thus, unicorn is not always mythical.

α_2: Magical? (Mg) $\neg\alpha_2 = \neg Mg$ ... (7)

(8) $\neg H$ (7,6)

(9) $\neg I$ (8,4)

(10) $\neg M$ (9,1)

(11) $\neg A$ (8,5)

(12) M (11,3)

(13) $\{\}$ (12,10)

As $\Delta \wedge \neg\alpha_2$ is inconsistent, thus $\Delta \models \alpha_2$, and thus unicorn is magical.

α_3: Horned? (H) $\neg\alpha_3$: $\neg H$ ... (7)

(8) $\neg I$ (7,4)

(9) $\neg M$ (8,1)

(10) $\neg A$ (7,5)

(11) M (10,3)

(12) $\{\}$ (9,11)

As $\Delta \neg\alpha_3$ is inconsistent, thus $\Delta \models \alpha_3$ and unicorn is horned.

4. FIGURE 1:

Decomposable : Yes, as for every conjunct, there are no common variables on both sides of the conjunct.
[$vars(\alpha) = A, B$ and $vars(\beta) = C, D$ in every conjunct]

Deterministic : Yes, as all the disjuncts' $\alpha \wedge \beta$ is always inconsistent i.e $\alpha \wedge \beta = \emptyset$

Smooth : No, as $vars(\alpha) \neq vars(\beta)$ for all disjuncts while for smoothness we require $vars(\alpha) = vars(\beta)$

FIGURE 2:

Decomposable: Yes, as for every conjuncts, there are no common variables on both sides of the conjunct.
[$vars(\alpha) = A, B$ and $vars(\beta) = C, D$ for every conjunct]

Deterministic: No, as for every disjunct, $\alpha \wedge \beta$ is not always inconsistent. For example, look at the leftmost level 3, $\alpha = \neg A \wedge B$, $\beta = \neg A \wedge B$ and thus evidently $\alpha \wedge \beta = \emptyset$

Smooth : Yes, as all the disjuncts have the same variables on both sides i.e $vars(\alpha) = vars(\beta)$

5. (a) To evaluate weighted model count of $(\neg A \wedge B) \vee (\neg B \wedge A)$

A	B	$\neg A \wedge B$	$\neg B \wedge A$	$(\neg A \wedge B) \vee (\neg B \wedge A)$
F	F	F	F	F
F	T	T	F	T
T	F	F	T	T
T	T	F	F	F

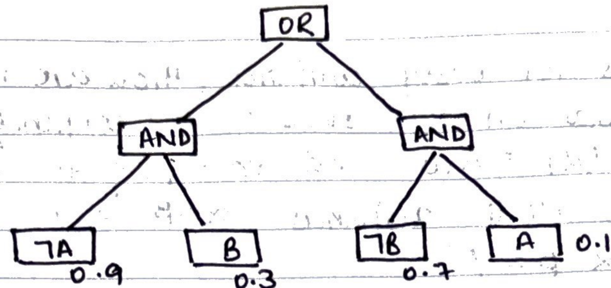
Thus, the condition is satisfied only when $\{A = F, B = T\}$, $\{B = F, A = T\}$.

$= \omega(\neg A) \cdot \omega(B) + \omega(A) \cdot \omega(\neg B)$

$= (0.9)(0.3) + (0.1)(0.7)$

$= 0.34$

(b)

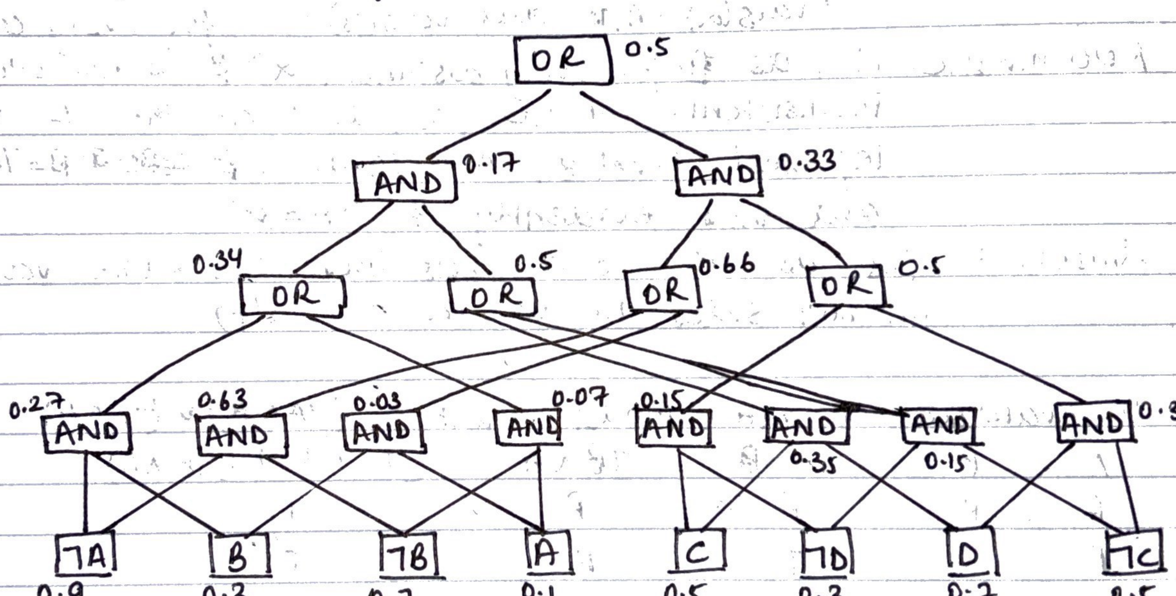


Evaluating ANDs : $(0.9)(0.3) = 0.27$
$(0.7)(0.1) = 0.07$

Evaluating OR : $0.27 + 0.07 = 0.34$.

Notice, that the result from this calculation is the same as what we got in (a) for weighted model count. The tree is the representation of the sentence. $(\neg A \wedge B) \vee (\neg B \wedge A)$

(c)



Weighted model count = count on root = 0.5