MBRTC1

November 19, 2019

1 Model-Based Real-Time Control

1.1 Les 1:

1. Introductie
2. Bemonsteren, reconstructie en aliasing
3. State-space modellen
4. Overzicht en leerdoelen
5. Huiswerk week 1

2 1. Computer-controlled systems

2.1 Sampled-data control

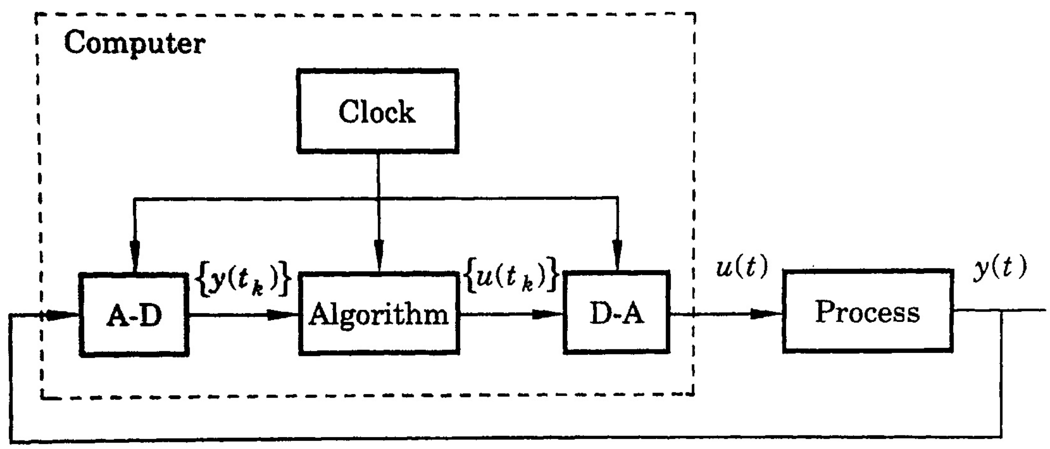


Figure 1.1 Schematic diagram of a computer-controlled system.

2.2 Modern industrial control (Industry 4.0 / Smart Industry)

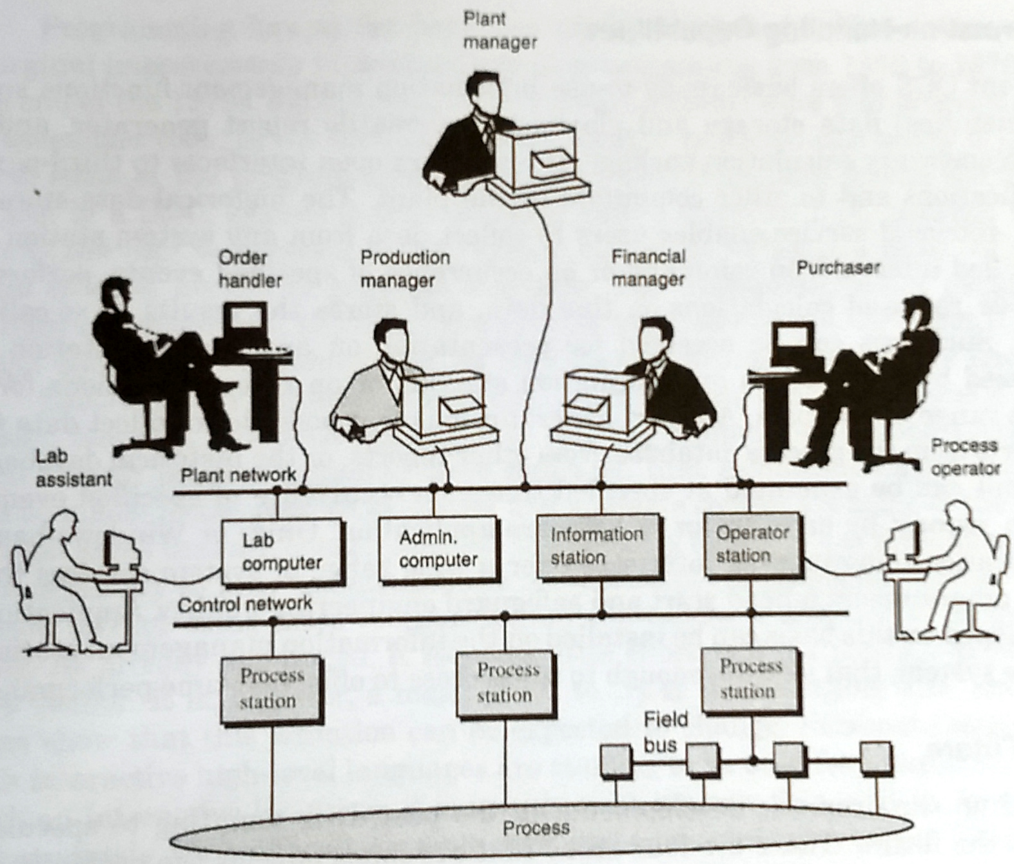


Figure 1.3 Modern industrial control systems like the Advant OCS tie computers together and help create a common uniform computer environment supporting all industrial activities, from input to output, from top to bottom. (By courtesy of ABB Industrial System, Västerås, Sweden.)

2.3 Effects of sampling

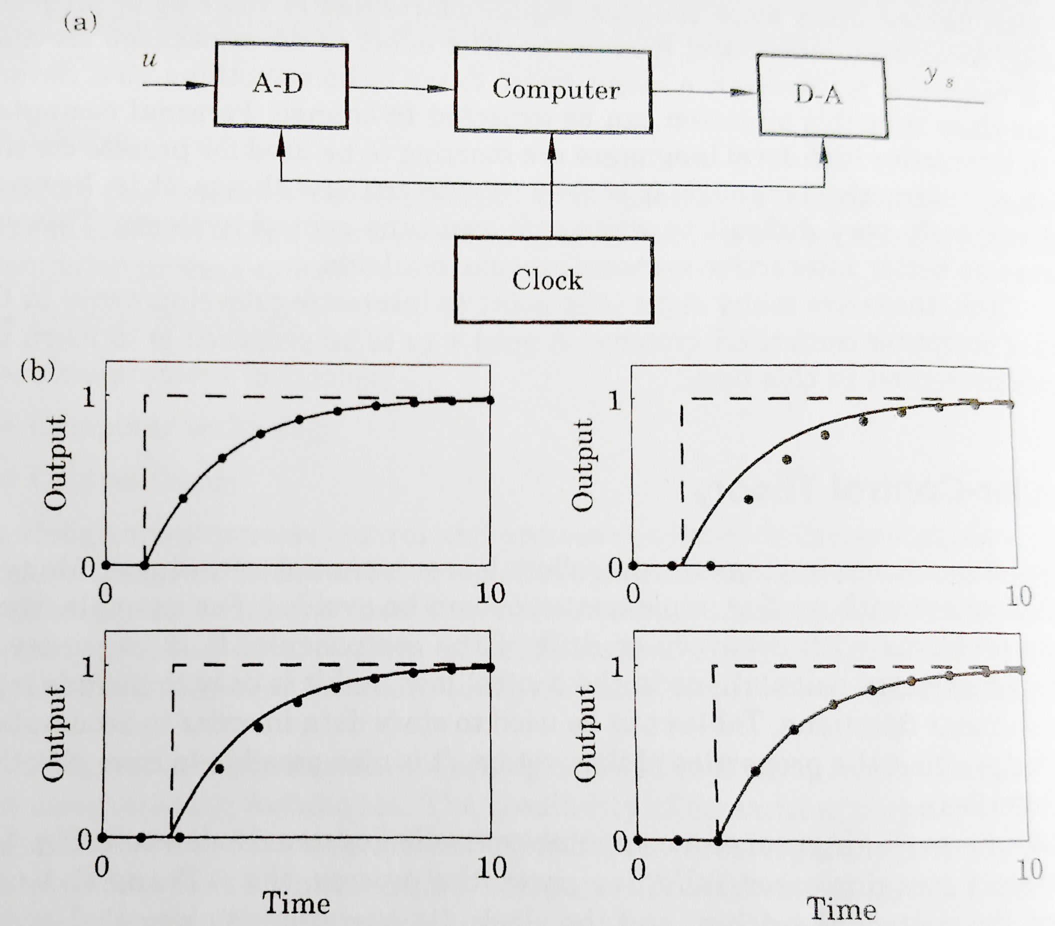


Figure 1.4 (a) Block diagram of a digital filter. (b) Step responses (dots) of a digital computer implementation of a first-order lag for different delays in the input step (dashed) compared with the first sampling instant. For comparison the response of the corresponding continuous-time system (solid) is also shown.

2.4 Example 1.2 Controlling the arm of a disk drive

Disk-drive assembly with J moment of inertia and k the motor constant:

Torque: $m(t) = ku(t)$ Angular-acceleration: $\alpha(t) = \frac{1}{J}m(t)$

Oefening: bepaal DV van systeem met ingang $u(t)$ en uitgang $\theta(t)$ (hoek) en bepaal overbrengingsfunctie.

Oplossing: DV:

$$J\ddot{\theta}(t) = ku(t)$$

Overbrengingsfunctie:

$$\frac{\Theta(s)}{U(s)} = \frac{k}{Js^2}$$

2.5 Example 1.2 Controlling the arm of a disk drive

Disk-drive assembly with J moment of inertia and k the motor constant:

$$G(s) \;=\; \frac{k}{Js^2}$$

Servo control law:

$$U(s) \;=\; \frac{bK}{a}U_c(s) - K\frac{s+b}{s+a}Y(s)$$

Choose control parameters:

$$a = 2\omega_0, \qquad b = \omega_0/2, \qquad K = 2\frac{J\omega_0^2}{k}$$

[2]:
```
import numpy as np
from matplotlib import pylab as plt
import control as ctrl
%matplotlib notebook
```

[3]:
```
%matplotlib notebook
k = 1.0
J = 1.0

G = ctrl.tf(k,[J,0.,0.])
omega_0 = 1.0
a = 2*omega_0
b = omega_0/2
K = 2*J*omega_0**2/k
C = ctrl.tf([1,b],[1,a])
Gcl_y = (K*b/a)*ctrl.feedback(G,K*C)
Gcl_u = (K*b/a)*ctrl.feedback(1,G*K*C)

t_cont = np.linspace(0,10,10000)
uc = 0*t_cont; uc[t_cont>1] = 1
_,y_cont,_ = ctrl.forced_response(Gcl_y,t_cont,uc)
_,u_cont,_ = ctrl.forced_response(Gcl_u,t_cont,uc)
plt.subplot(211)
plt.plot(t_cont,y_cont)
plt.ylabel('Output y'); plt.grid()
plt.subplot(212)
plt.plot(t_cont,u_cont)
plt.ylabel('Input u'); plt.xlabel('Time [s]'); plt.grid()
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

[9]:
```
%matplotlib notebook
h = .5# alleen waarden die >= t[1]-t[0] zijn invoeren
x = np.zeros((1))
u = np.zeros((0))
index = np.zeros((0),dtype=np.uint)

y_discr = 0*y_cont
state_init = np.zeros((2))
k = 0
while True:
    index = np.append(index,np.uint(np.argmin(np.abs(t_cont-k*h))))
    # simulate continuous system from tc[index[k-1]] until tc[index[k]]
    if k>0: # wait one step, so index[k-1] can be evaluated
        _,y_discr[np.uint(index[k-1]+1):np.uint(index[k]+1)],states = ctrl.
 ↪forced_response(G,t_cont[np.uint(index[k-1]+1):np.
 ↪uint(index[k]+1)],u[k-1]*np.ones((index[k]-index[k-1])),state_init)
        state_init = states[:,-1] # remember state for next step
    # discrete time controller:
    u = np.append(u,K*((b/a)*uc[index[k]]-y_discr[index[k]]+x[k]))
    x = np.append(x,x[k]+h*((a-b)*y_discr[index[k]]-a*x[k]))
    k += 1
    if k*h>t_cont[-1]:
        break

plt.subplot(211)
plt.plot(t_cont,y_cont)
plt.plot(t_cont,y_discr)
plt.ylabel('Output y'); plt.grid()
plt.subplot(212)
plt.plot(t_cont,u_cont)
plt.step(t_cont[index],u,where='post')
plt.ylabel('Input u'); plt.xlabel('Time [s]'); plt.grid()
```

<IPython.core.display.Javascript object>

<IPython.core.display.HTML object>

2.6 Deadbeat control:

[4]:
```
# deadbeat controller parameters
%matplotlib notebook

h = .1
s0=2.5/(h*h);
s1=-1.5/(h*h);
r1=0.75;
```

```
t0=1/(h*h);
t1=0;

k = 0
x = np.zeros((1))
u = np.zeros((0))
index = np.zeros((0),dtype=np.uint)

y_discr = 0*y_cont
state_init = np.zeros((2))
k = 0
while True:
    index = np.append(index,np.uint(np.argmin(np.abs(t_cont-k*h))))
    # simulate continuous system from tc[index[k-1]] until tc[index[k]]
    if k>0: # wait one step, so index[k-1] can be evaluated
        _,y_discr[np.uint(index[k-1]+1):np.uint(index[k]+1)],states = ctrl.
 ↪forced_response(G,t_cont[np.uint(index[k-1]+1):np.
 ↪uint(index[k]+1)],u[k-1]*np.ones((index[k]-index[k-1])),state_init)
        state_init = states[:,-1] # remember state for next step
    # discrete time controller:
    if k==0:
        u = np.append(u,0.0)
    else:
        unew = t0*uc[index[k]] + t1*uc[index[k-1]] - s0*y_discr[index[k]] -␣
 ↪s1*y_discr[index[k-1]] -r1*u[k-1]
        u = np.append(u,unew)
    k += 1
    if k*h>t_cont[-1]:
        break

plt.subplot(211)
plt.plot(t_cont,y_cont)
plt.plot(t_cont,y_discr)
plt.ylabel('Output y'); plt.grid()
plt.subplot(212)
plt.plot(t_cont,u_cont)
plt.step(t_cont[index],u,where='post')
plt.ylabel('Input u'); plt.xlabel('Time [s]'); plt.grid()
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

2.7 2. Bemonsteren, reconstructie en aliasing

[18]:
```
# Author:  Rufus Fraanje
# Email:   p.r.fraanje@hhs.nl
# Licence: GNU General Public License (GNU GPLv3)
# Creation date: 2019-03-07
# Last modified: 2019-03-12

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import Slider, TextBox, Button
#%matplotlib qt

def calc_plot_data(duration,f,phase,fsample,number):

    def calc_alias(f,phase,fsample,number,t):
        #adjust this function:
        x_alias = 0*t
        f_alias = 0
        return x_alias,f_alias

    tc = np.arange(0,duration,2/f*1/1000.)  # continuous time, 2 periods 1000␣
↪samples
    xc = np.sin(2*np.pi*f*tc + phase) # (approximated) continuous time signal
    td = np.arange(0,duration,1/fsample)    # time axis of samples
    xd = np.sin(2*np.pi*f*td + phase) # sampled signal
    xalias,falias = calc_alias(f,phase,fsample,number,tc)

    return tc,xc,td,xd,xalias,falias


f = 2.  # frequency of signal in Hz
phase = 0*np.pi/4 # phase of signal in radians
fsample = 6.    # sampling rate in Hz
number = 0 # number of alias
duration = 1. # simulation time

tc,xc,td,xd,xalias,falias = calc_plot_data(duration,f,phase,fsample,number)


fig, ax = plt.subplots()
plt.subplots_adjust(left=0.1,bottom=0.25)
ax.plot(tc,xc,'b',label=f'Signal')
ax.stem(td,xd,linefmt='g-',basefmt='g-',label='Sampled')
ax.plot(tc,xalias,'r-.',label=f'Alias')
ax.set_title(f'Sine wave with sampled and alias version')
ax.set_xlabel('Time [s]')
```

```python
ax.set_ylabel('Signal value [V]')
ax.legend(loc='upper right')
ax.axis([0,duration,-1.05,1.05])


axcolor='lightgoldenrodyellow'
axf        = plt.axes([0.2,0.15,0.65,0.03],facecolor=axcolor)
axfsample = plt.axes([0.2,0.11,0.65,0.03],facecolor=axcolor)
axphase = plt.axes([0.2,0.07,0.65,0.03],facecolor=axcolor)
axnumber    = plt.axes([0.2,0.03,0.05,0.03],facecolor=axcolor)
axfalias = plt.axes([0.35,0.03,0.07,0.03],facecolor=axcolor)
axduration = plt.axes([0.5,0.03,0.05,0.03],facecolor=axcolor)

sf       = Slider(axf,'Freq.',0.1,40,valinit=f,valstep=0.1)
sfsample = Slider(axfsample,'Fsample',0.2,40,valinit=fsample,valstep=0.2)
sphase = Slider(axphase,'phase',-3,3,valinit=phase,valstep=0.1)
tnumber = TextBox(axnumber,'number',initial=str(number),color=axcolor,hovercolor='0.975')
tduration = TextBox(axduration,'duration',initial=str(duration),color=axcolor,hovercolor='0.975')
tfalias = TextBox(axfalias,'falias',initial=f'{falias:.2f}',color='lightgray',hovercolor='lightgray')

def update_plot(val):
    f = sf.val
    fsample = sfsample.val
    phase = sphase.val
    number = float(tnumber.text)
    print(f"number = {number}")
    duration = float(tduration.text)
    tc,xc,td,xd,xalias,falias = calc_plot_data(duration,f,phase,fsample,number)
    tfalias.set_val(f'{falias:.2f}')
    tfalias.stop_typing() # remove cursor

    ax.cla()
    ax.plot(tc,xc,'b',label=f'Signal')
    ax.stem(td,xd,linefmt='g-',basefmt='g-',label='Sampled')
    ax.plot(tc,xalias,'r-.',label=f'Alias')
    ax.set_title(f'Sine wave with sampled and alias version')
    ax.set_xlabel('Time [s]')
    ax.set_ylabel('Signal value [V]')
    ax.legend(loc='upper right')
    ax.axis([0,duration,-1.05,1.05])

    fig.canvas.draw_idle()
```

```
    return True

sf.on_changed(update_plot)
sfsample.on_changed(update_plot)
sphase.on_changed(update_plot)

tnumber.on_submit(update_plot)
tduration.on_submit(update_plot)
tfalias.stop_typing() # remove cursor

axreset = plt.axes([0.75, 0.03, 0.1, 0.03])
button = Button(axreset, 'Reset', color=axcolor, hovercolor='0.975')

def reset(event):
    sf.reset()
    sfsample.reset()
    sphase.reset()
    tnumber.set_val(str(number))
    tduration.set_val(str(duration))
    return True

button.on_clicked(reset)

plt.show();
```

```
<IPython.core.display.Javascript object>
```

```
<IPython.core.display.HTML object>
```

[22]:
```
%matplotlib notebook
f   = 2 # Hz
fs_high = 1e3 # Hz

resample_factor = 50  # dus
f_sample  = fs_high/resample_factor
print("f_sample = ",f_sample)

t = np.arange(0,10,1/fs_high)
N = t.shape[-1]
Nhalf = np.uint(np.ceil(N/2))
y = np.sin(2*np.pi*f*t)

y_pulse = np.kron(y[0::resample_factor],np.hstack((np.array([1]),np.
 ↪zeros((resample_factor-1)))))
y_zoh = np.repeat(y[0::resample_factor],resample_factor)
```

```
plt.plot(t,y,'b')
plt.plot(t,y_pulse,'ro-')
plt.plot(t,y_zoh,'g')
```

```
f_sample =  20.0

<IPython.core.display.Javascript object>

<IPython.core.display.HTML object>
```

```
[22]: [<matplotlib.lines.Line2D at 0x7f4ab9145e48>]
```

```
[7]: %matplotlib notebook
Y = np.fft.fft(y)
Y_pulse = np.fft.fft(y_pulse)
Y_zoh = np.fft.fft(y_zoh)
freq = np.fft.fftfreq(N)*fs_high
plt.plot(freq[:Nhalf],abs(Y[:Nhalf]),'b')
plt.plot(freq[:Nhalf],abs(Y_pulse[:Nhalf]),'ro-')
plt.plot(freq[:Nhalf],abs(Y_zoh[:Nhalf]),'g')
```

```
<IPython.core.display.Javascript object>

<IPython.core.display.HTML object>
```

```
[7]: [<matplotlib.lines.Line2D at 0x7f4ab93575c0>]
```

2.8 3. State-space modellen (toestandsruimte modelen)

(Zie ook: Wikipedia: State-space representation (!))

State-space model beschrijving:

Continue tijd:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) + Du(t) \tag{2}$$

Discrete tijd:

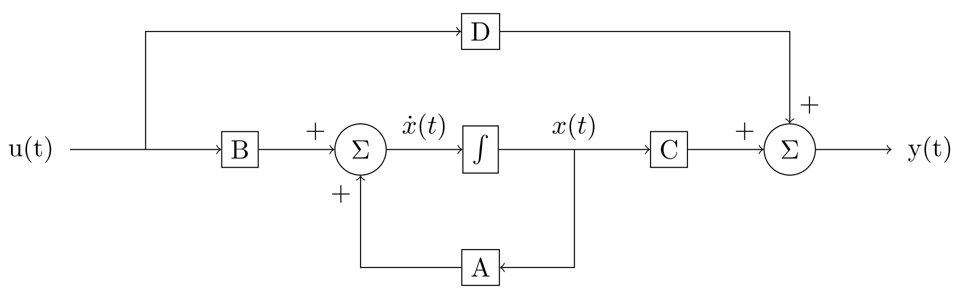
$$x_{k+1} = Ax_k + Bu_k \tag{3}$$
$$y_k = Cx_k + Du_k \tag{4}$$

```
[8]: %load_ext itikz
```

[2]:
```
%%itikz --temp-dir --file-prefix mbrtc1-tikz- --implicit-pic␣
↪--tex-packages=tikz --tikz-libraries=arrows,positioning
\node [circle] (u) {u(t)};
\coordinate [right=of u] (dot1) {};
\node [rectangle,draw,right=of dot1] (B) {B};
\node [circle,draw,right=of B] (sum1) {$\Sigma$};
\node [rectangle,draw,right=of sum1] (integ) {$\int$};
\coordinate [right=of integ] (dot2) {};
\node [rectangle,draw,below=of integ] (A) {A};
\node [rectangle,draw,right=of dot2] (C) {C};
\node [rectangle,draw,above=of integ] (D) {D};
\node [circle,draw,right=of C] (sum2) {$\Sigma$};
\node [circle,right=of sum2] (y) {y(t)};

\draw[->] (u) -- (B);
\draw[->] (dot1) |- (D);
\draw[->] (D) -| (sum2) node[pos=0.9,anchor=west] {+};
\draw[->] (B) -- (sum1) node[near end,anchor=south] {+};
\draw[->] (sum1) -- (integ) node[midway,anchor=south] {$\dot{x}(t)$};
\draw[->] (integ) -- (C)  node[midway,anchor=south] {${x}(t)$};
\draw[->] (dot2) |- (A);
\draw[->] (A) -| (sum1) node[pos=0.9,anchor=east] {+};
\draw[->] (C) -- (sum2) node[near end,anchor=south] {+};
\draw[->] (sum2) -- (y);
```

[2]:



2.9 4. Overzicht en leerdoelen

- wk 1: Inleiding computer-control theory, aliasing en state-space modellen
- wk 2: zoh-sampling van state-space modellen en andere transformaties
- wk 3: Input-output modellen (overbrengingsfuncties) en state-space modellen, polen en nulpunten
- wk 4: Systeem eigenschappen: stabiliteit, controllability, reachability en observability
- wk 5: State-feedback regelingen
- wk 6: State-observers en output feedback
- wk 7: Servo-control

2.10 Boek:

Karl J. Åström and Björn Wittenmark, Computer-Controlled Systems -- Theory and Design, 3rd edition, Dover, 2011, ISBN-13: 978-0-486-48613-0, ISBN-10: 0-486-48613-3

2.11 5. Huiswerk week 1

(zie studiewijzer!)

- Bestuderen:
  - CCS Hst 1 (§1.2 aangeraden maar geen tentamenstof; §1.5 alleen verrijking, geen tentamenstof)
  - Bijlage A (zero-order hold discretisatie stappen komen in college 2 aan bod, nu alleen lezen)
  - Slides(!)
  - Zo nodig: ophalen matrix rekenen (matrix-vector en matrix-matrix vermenigvuldiging, inverse matrix, determinant, eigenwaarden en eigenvectoren, karakteristieke vergelijking)
- Oefeningen:
  1. Voor een reëel getal a, los de volgende DV voor $x(t)$ op voor $t \geq 0$:
     $$\dot{x} + ax = -1, \qquad x(0) = 1$$
  2. Reken de determinant uit van de matrix $\begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$.
  3. Bereken de inverse van de matrix $\begin{bmatrix} 8 & 2 \\ 2 & 5 \end{bmatrix}$.
  4. Bereken de eigenwaarden van de matrix:
  5. $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
  6. $$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
  7. $$A = \begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix}$$
  8. Bestudeer de numpy-functie np.linalg.eig en controlleer met deze functie de uitkomsten bij 4.
- Oefeningen (vervolg): 6) De Fibonacci getallen zijn de getallen in de rij, beginnend met 0 en 1, en vervolgens is het volgende getal in de reeks de som van de twee voorafgaande: $0, 1, 1, 2, 3, 5, 8, \cdots$. Construeer een state-space model zonder ingang u:

$$x_{k+1} = Ax_k \tag{5}$$
$$y_k = Cx_k \tag{6}$$

en begin-toestand x_0 zodat y_k het k^e Fibonacci getal levert. Optie: maak een python programma om dit systeem te simuleren. 7) Een discrete-tijd systeem met ingang $u(k)$ en uitgang $y(k)$ wordt beschreven door de volgende differentie-vergelijking:

$$y(k) + 0.5y(k-1) \quad = \quad u(k)$$

Bereken $y(k)$ voor $k = 0, 1, 2, \cdots$ gegeven $y(-1) = 0$ en $u(0) = 1$ en $u(k) = 0$ voor $k \geq 1$ (ofwel de ingang is een impulse en de uitgang is de impulse-responsie van het systeem in discrete tijd).

- Bestudeer en maak practicum opdracht 1

[ ]:

[ ]: