[1] A machine contains two belts of different lengths. These have times to failure which are exponentially distributed, with means α and 2α. The machine will stop if either belt fails. The failures of the belts are assumed to be independent. What is the probability that the system performs after time α from the start?

[2] Let X be a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute (a) $P(X > 5)$, (b) $P(4 < X < 16)$, (c) $P(X < 8)$.

[3] Let $X \sim N(\mu, \sigma^2)$. If $P(X \le 0) = 0.5$ and $P(-1.96 \le X \le 1.96) = 0.95$, find μ and σ^2.

[4] It is assumed that the lifetime of computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ and $\sigma^2 = 3 \times 10^5$ hours. What is the approximate probability that a batch of 10 chips will contain at least 2 chips whose lifetime are less than 1.8×10^6 hours?

[5] Let X be a Normal random variable with mean 0 and variance 1, i.e. $N(0,1)$. Prove that

$$P(|X| \ge t) \le \sqrt{\frac{2}{\pi}} \frac{e^{-t^2/2}}{t}; \forall\, t > 0.$$

[6] Show that if X is a discrete random variable with values $0,1,2,\ldots.$ then $E(X) = \sum_{k=0}^{\infty} (1 - F(k))$, where $F(x)$ is the distribution function of the random variable X.

[7] The cumulative distribution function of a random variable *X* defined over $0 \le x < \infty$ is $F(x) = 1 - e^{-\beta x^2}$, where $\beta > 0$. Find the mean, median and variance of *X*.

[8] Show that for any $x > 0$, $1 - \Phi(x) \le \frac{\phi(x)}{x}$, where $\Phi(x)$ is the c.d.f. and $\phi(x)$ is the p.d.f. of standard normal distribution.

[9] A point m_0 is said to be mode of a random variable *X*, if the p.m.f. or the p.d.f. of *X* has a maximum at m_0. For the distribution given in problem [7], if m_0 denotes the mode; μ, the mean and σ^2, the variance of the corresponding random variable, then show that $m_0 = \mu\sqrt{2/\pi}$ and $2m_0^2 - \mu^2 = \sigma^2$.

[10] Let X be a Poisson random variable with parameter λ. Find the probability mass function of $Y = X^2 - 5$.

[11] Let X be Binomial random variable with parameters n and p. Find the probability mass function of $Y = n - X$.

[12] Consider the discrete random variable X with the probability mass function

$$P(X=-2)=\frac{1}{5},\quad P(X=-1)=\frac{1}{6},\quad P(X=0)=\frac{1}{5},$$
$$P(X=1)=\frac{1}{15},\quad P(X=2)=\frac{10}{30},\quad P(X=3)=\frac{1}{30}.$$

Find the probability mass function of $Y = X^2$.

[13] The probability mass function of the random variable X is given by

$$P(X=x)=\begin{cases}\frac{1}{3}\left(\frac{2}{3}\right)^x & x=0,1,2,\ldots\\ 0 & \text{otherwise.}\end{cases}$$

Find the distribution of $Y = X/(X+1)$.

[14] Let X be a random variable with probability mass function

$$P(X=x)=\begin{cases}e^{-1}, & x=0\\ \dfrac{e^{-1}}{2\,(|x|)!}, & x\in\{\pm1,\pm2,\ldots\}\\ 0 & \text{otherwise.}\end{cases}$$

Find the p.m.f. and distribution of the random variable $Y = |X|$.

Useful data

$\Phi(1/3)=0.6293, \Phi(5/6)=0.7967, \Phi(1)=0.8413, \Phi(4/3)=0.918$
