MSO201A (Quiz 2)
Time: 40 minutes.
Maximum Points = 13.

Name:
Roll Number:

1. Let X and Y be independently and identically distributed standard normal random variables ($f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $x \in \mathbb{R}$ is the probability density function of the standard normal random variable). Find the probability density function of $\frac{X}{|Y|}$. (6 points)

Solution :-

3 points

Let us denote $Z = \frac{X}{|Y|}$

Note that $Z = \frac{X}{|Y|}$

$= \frac{X}{\sqrt{Y^2/1}}$

Since $Y \sim N(0,1)$, this implies that $Y^2/1 \sim \chi^2$ distribution with 1 degree of freedom.

3 points

So, using this fact along with that X is indep. of Y, by definition, $Z \sim t$ distribution with 1 degree of freedom.

Hence, $f_Z(z) = \frac{1}{\pi(1+z^2)}$, $z \in \mathbb{R}$.

Probability density fn. of Z

2. Let $\{X_n\}_{n\geq 1}$ be i.i.d sequence of random variables with normal distribution having mean $= \mu$ and variance $= 1$. Let c be a function of μ such that $\frac{1}{(n-113)}\sum_{i=1}^{n} X_i^2$ converges to c in probability as $n \to \infty$. Find c. (7 points)

Solution :-

2 points

Note that

$$\frac{1}{n-113}\sum_{i=1}^{n} X_i^2$$

$$= \left(\frac{1}{n}\sum_{i=1}^{n} X_i^2\right) \times \left(\frac{n}{n-113}\right) \text{ —— } (*)$$

2 points

Now, observe that

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} E(X_i^2) = \mu^2+1$$

as $n \to \infty$ by weak law of large number.

1 point Further, observe that $\frac{n}{n-113} \to 1$ as $n \to \infty$

2 points

Hence, $(*) \to (\mu^2+1) \times 1 = \mu^2+1$ as $n \to \infty$ in probability.

Therefore, $c = \mu^2+1$.