MSO201A (Quiz 1)
Time: 40 minutes.
Maximum Points = 12.

Name:
Roll Number:

1. Let X be a continuous random with cumulative distribution function F, whose probability density function is strictly positive everywhere in $\mathbb{R}$. Is it true that $F^{-1}(\alpha)$ is an increasing function of α, where $\alpha \in (0,1)$? Justify your answer. (6 points)

Yes, $F^{-1}(\alpha)$ will be an increasing function of $\alpha \in (0,1)$. } 2 points even if someone does NOT provide any argument

Justification:-

2 points

F has a positive density function (say, f) on $\mathbb{R}$.

$\Rightarrow$ F is a strictly increasing function on $\mathbb{R}$.

Now, for ~~any~~ any $\alpha \in (0,1)$, we have

$$F(F^{-1}(\alpha)) = \alpha$$

2 points

D.w.r.t. α, we have

$$\Rightarrow f(F^{-1}(\alpha)) \frac{dF^{-1}(\alpha)}{d\alpha} = 1$$

$$\Leftrightarrow \frac{dF^{-1}(\alpha)}{d\alpha} = \frac{1}{f(F^{-1}(\alpha))} > 0 \text{ since } f(y) > 0 \; \forall y \in \mathbb{R}$$

Hence, $F^{-1}(\alpha)$ is an increasing function of α.

Remark:- If someone gives correct arguments in any other way give full mark

2. Let $A_1, A_2, \ldots, A_n$ be n mutually independent events.

Is it true that $\left\{1 - e^{-\sum_{i=1}^{n} P(A_i)}\right\} \leq P\left(\bigcup_{i=1}^{n} A_i\right)$? Justify your answer. (6 points)

Yes, it is true $\longrightarrow$ 2 points even if someone does NOT give any argument.

Reason:-

$$P\left(\bigcup_{i=1}^{n} A_i\right)$$

$$= 1 - P\left[\left(\bigcup_{i=1}^{n} A_i\right)^c\right]$$

$$= 1 - P\left[\bigcap_{i=1}^{n} A_i^c\right]$$

$$= 1 - \prod_{i=1}^{n} P(A_i^c) \quad \text{since } A_i\text{'s are indep.}$$

$$= 1 - \prod_{i=1}^{n} \{1 - P(A_i)\} \quad \text{——— } (*)$$

2 points

Now, $1 - P(A_i) \leq e^{-P(A_i)}$ since $1 - x \leq e^{-x}$ if $x \in [0,1]$

$$\Leftrightarrow \prod_{i=1}^{n} \{1 - P(A_i)\} \leq e^{-\sum_{i=1}^{n} P(A_i)}$$

$$\Leftrightarrow -\prod_{i=1}^{n} \{1 - P(A_i)\} \geq -e^{-\sum_{i=1}^{n} P(A_i)} \quad \text{——— } (**)$$

Using $(**)$ in $(*)$, we have

$$P\left(\bigcup_{i=1}^{n} A_i\right) \geq 1 - e^{-\sum_{i=1}^{n} P(A_i)}.$$

Hence, the statement is true.

2 points