

CS109 - lecture 2

x

We can use both methods for counting from last lesson

1. Product rule of counting - If experiment has 2 indep parts
2. Sum rule of counting - If experiment can be either from A or B and they may overlap then

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= N^2 \text{ of Outc.}$$

Example:

6 bit String over network recognized only if start/end 01/10

Start: 01 Step based Approach - Method 1

Step 1: 01

Step 2: 01 □

Step 3: 01 □ □

4: 01 □ □ □

5: 01 □ □ □ □ (6 bit)

$$1 \times 2 \times 2 \times 2 \times 2 = 16 \text{ outcomes}$$

End 01: 5 Steps as well = 16 outcomes

But there are overlaps!

So if we count the overlaps

Same Outcomes

01 □ □ 10
4 outcomes

A or B use method 2

$$\text{Total} = 16 + 16 - 4$$
$$= 28$$

Permutations

A permutation is an ordered arrangement of objects
 The No of unique orderings (permutations) of a
 distinct objects is : $n! = n \times (n-1) \times \dots \times 2 \times 1$

Problem : With a 6 digit password how many
 permutations ?

1 2 3

4 5 6

7 8 9

0

Not
 unique
 numbers

$$= 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

$$= 10^6$$

Unique

Numbers

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5$$

$$= \frac{10!}{4!} = 151200 \text{ passcodes}$$

6
 Sort set of objects but some are not distinct
 First we imagine them distinct

Example : 5 cans of coke : $5!$

But we have overcounted because i are not distinct
 How many did we overcounted

If : $i = 2$ then we have counted ce and y
 distinct

$$2 \times \left\{ \begin{array}{c} \boxed{u} \boxed{y} \boxed{} \boxed{} \boxed{} \\ \boxed{y} \boxed{u} \boxed{} \boxed{} \boxed{} \end{array} \right. = \underbrace{\quad \quad \quad}_{\text{overcounting}}$$

as since
but they were
not

$$3! = 3 \times 2 \times 1 = 6$$

$$\text{Total overcounting} = 2 \cdot 3!$$

General rule for counting permutations

If n objects and m_1, m_2, \dots, m_r are indistinct
then the number of permutations is

$$\frac{n!}{m_1! \cdot m_2! \cdot \dots \cdot m_r!}$$

Problem: How many orderings of the word

MISSISSIPPI

1 2 3 4 5 6 7 8 9 10 11

1 M

4 I

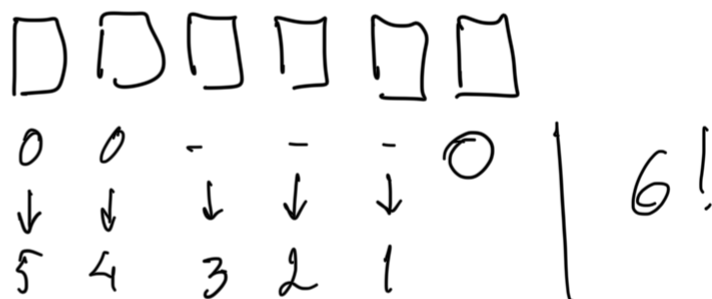
4 S

2 P

$$\frac{11!}{4! \cdot 4! \cdot 2! \cdot 1!} = 34650$$

5 Smudges & digit passwords (unique passwords)

If it was 6×6 then $6!$



But with 5 smudges then a number is being repeated
It could be repeated in how many ways?

ABCDE

If A is repeated

If B

C

D

E

5 ways

$$5 \times \frac{n! \text{ total distinct arrangements}}{m! \text{ total indistinct arrangements}} = 5 \times \frac{6!}{2!}$$

account for duplicates
(remove them)

$$= 1800$$

Combinations with cake

$n = 20$ people $k = 5$ get cake

Order

Cake Room

Not cake room

1 2 3 4 5

6 7 8 ... 20

Min 10

Max 20

- Steps:
1. n people in line : $n!$ ways to order them
 2. k people go into cake room : 1 way
 3. Cake group mingle : $k!$ ways
 4. Allow non cake people to mingle : $(n-k)!$

So a Combination is an unordered selection of k objects from a set of n distinct objects

$$\binom{n}{k} = \frac{n!}{k! (n-k)!} = n! \times 1 \times \underbrace{\frac{1}{k!} \times \frac{1}{(n-k)!}}_{\text{are not worried about ordering}}$$

Binomial
Coefficient

Steps: $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
1 2 3 4

Combination does not care about ordering

$$\binom{6}{3} = \frac{6!}{3! 3!} = 20 \text{ ways}$$

Problem:

How many unique hands of 5 cards in a 52 card deck

$$\binom{52}{5} = \frac{52!}{5! (52-5)!} = 2,598,960$$

Put N objects in R buckets $\begin{cases} \rightarrow \text{distinct case} \\ \rightarrow \text{indistinct case} \end{cases}$

$$\text{Distinct} = r^n$$

$$\text{Indistinct} = \frac{(N+R-1)!}{N! \cdot (R-1)!}$$