CS109 - lecture 2

x

We can use both methods for counting from last lesson

1. Product rule of counting - If experiment has 2 indep parts
   $|A||B| = m \cdot n = N^{o}$ of Outc.
2. Sum rule of counting - If experiment can be either from A or B and they may overlap then

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= N^{o} \text{ of Outc.}$$

Example:

6 bit string over network recognized only if start/end 01/10
or

Start: 01 Step based Approach - Method 1

Step 1: 01
Step 2: 01 $\square$
Step 3: 01 $\square\square$
4: 01 $\square\square\square$
5: 01 $\square\square\square\square$ (6 bit)

$1 \times 2 \times 2 \times 2 \times 2 = 16$ outcomes

End 01: 5 Steps as well $= 16$ outcomes

But there are overlaps!

So if we count the overlaps $A \cap B$ we use method 2

Same Outcomes $01\underbrace{\square\square}10$ → 4 outcomes

Total $= 16 + 16 - 4$
$= 28$

Permutations

A permutation is an <u>ordered</u> arrangement of objects

The Nr of unique orderings (permutations) of n distinct objects is : $n! = n \times (n-1) \times \dots \times 2 \times 1$

<u>Problem :</u> With a 6 digit password how many permutations ?

1 2 3
4 5 6
7 8 9
0

$\square_1 \square_2 \square_3 \square_4 \square_5 \square_6$

Not unique numbers: $= 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^6$

Unique Numbers: $= 10 \times 9 \times 8 \times 7 \times 6 \times 5$

$= \frac{10!}{4!} = 151\,200$ passcodes

Sort set of objects but some are not distinct

First we imagine them distinct

Example : 5 cans of coke : $5!$

But we have overcounted because i are not distinct

How many did we overcounted

If : $i = 2$ then We have counted x and y distinct

distinct

but they were not

$$2\times\left\{\begin{array}{l} \boxed{u}\ \boxed{y}\ \square\ \square\ \square \\ \qquad\quad\ W\ \ T\ \ S \\ \quad = \\ \boxed{y}\ \boxed{u}\ \underbrace{\square\ \square\ \square}_{\text{overcounting } 3! = 3\times 2\times 1 = 6} \end{array}\right.$$

Total overcounting $= 2 \cdot 3!$

<u>General rule</u> for counting permutations

If n objects and $n_1, n_2, \dots n_r$ are indistinct

then the number of permutations is

$$\frac{n!}{n_1! \cdot n_2! \cdots n_r!}$$

Problem: How many orderings of the word

MISSISSIPPI
1 2 3 4 5 6 7 8 9 10 11

1 M
4 I
4 S
2 P

$$\frac{11!}{4! \cdot 4! \cdot 2! \cdot 1!} = 34\,650$$

5 Smudges 6 digit passwords (unique passwords)

If it was 6×6 then 6!

□ □ □ □ □ □

0 0 - - - O | 6!

↓ ↓ ↓ ↓ ↓

5 4 3 2 1

But with 5 smudges then a number is being repeated

It could be repeated in how many ways?

A B C D E

If A is repeated ↳ A A B C D E

If B ↳ A B B C D E

C → A B C C D E

D ↓ A B C D D E

E ↳ A B C D E E

5 ways

$$5 \times \frac{n!\ \text{total distinct arrangements}}{m!\ \text{total indistinct arrangements}} = 5 \times \frac{6!}{2!}$$

$$= 1800$$

account for duplicates / remove them

Combinations with cake

order ↳ | Cake Room: ① ② ③ ④ ⑤ | Not cake room: 6 7 8 … 20 |

n = 20 people k = 5 get cake

Steps:

1. n people in line : $n!$ ways to order them
2. k people go into cake room : 1 way
3. Cake group mingle : $k!$ ways
4. Allow non cake people to mingle : $(n-k)!$

So a Combination is an unordered selection of k objects from a set of n distinct objects

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = n! \times 1 \times \underbrace{\frac{1}{k!} \times \frac{1}{(n-k)!}}_{\text{are not worried about ordering}}$$

Binomial coeficient

Steps: 1 2 3 4

Combination does not care about ordering

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

Problem:

How many unique hands of 5 cards in a 52 card deck

$$\binom{52}{5} = \frac{52!}{5!\,(52-5)!} = 2598960$$

Put N objects in R buckets → distinct case
→ indistinct case

Distinct = r^n

Indistinct = $\dfrac{(N+R-1)!}{N!\cdot(R-1)!}$