Cs109 - lecture 2

X

We can use both methods for counting from last lesson

1. Product rule of counting - If experiment has 2 indep parts $|A||B| = m \cdot n = N^o$ of Outc.
2. Sum rule of counting - If experiment can be either from A or B and they may overlap then

$$|A \cup B| = |A| + |B| - |A \cap B| = N^o \text{ of Outc.}$$

Example:

6 bit String over network recognized only if start/end 01/10 or

Start: 01 Step based Approach - Method 1

Step 1: 01
Step 2: 01 $\square$
Step 3: 01 $\square\square$
4: 01 $\square\square\square$
5: 01 $\square\square\square\square$ (6 bit)

$1 \times 2 \times 2 \times 2 \times 2 = 16$ outcomes

End 01: 5 Steps as well = 16 outcomes

But there are overlaps!

So if we count the overlaps $A \cap B$ we use method 2

Same Outcomes 01 $\underbrace{\square\square}$ 10 → 4 outcomes

Total = 16 + 16 - 4
= 28

Permutations

A permutation is an <u>ordered</u> arrangement of objects

The Nr of unique orderings (permutations) of a distinct objects is : $n! = n \times (n-1) \times \dots \times 2 \times 1$

<u>Problem :</u> With a 6 digit password how many permutations ?

1 2 3

4 5 6

7 8 9

0

$\square_1 \; \square_2 \; \square_3 \; \square_4 \; \square_5 \; \square_6$

Not unique numbers $= 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= 10^6$

Unique Numbers $= 10 \times 9 \times 8 \times 7 \times 6 \times 5$

$= \frac{10!}{4!} = 151\,200$ passcodes

Sort set of objects but some are not distinct

First we imagine them distinct

Example : 5 cans of coke : $5!$

But we have overcounted because i are not distinct

How many did we overcounted

If : $i = 2$ then we have counted x and y distinct

distinct

but they were not

$$2 \times \left\{ \begin{array}{c} \boxed{u}\ \boxed{y}\ \square\ \square\ \square \\ \phantom{\boxed{u}\ \boxed{y}\ } W\ \ T\ \ S \\ = \\ \boxed{y}\ \boxed{u}\ \underbrace{\square\ \square\ \square}_{\text{overcounting } 3! = 3\times 2\times 1 = 6} \end{array} \right.$$

Total overcounting $= 2 \cdot 3!$

<u>General rule</u> for counting permutations

If n objects and $n_1, n_2, \cdots n_r$ are indistinct

then the number of permutations is

$$\frac{n!}{n_1! \cdot n_2! \cdots n_r!}$$

Problem: How many orderings of the word

MISSISSIPPI

1 2 3 4 5 6 7 8 9 10 11

1 M

4 I

4 S

2 P

$$\frac{11!}{4! \cdot 4! \cdot 2! \cdot 1!} = 34650$$

---

5 Smudges 6 digit passwords (unique passwords)

If it was 6×6 then $6!$

$$\begin{array}{cccccc|c} \square & \square & \square & \square & \square & \square & \\ 0 & 0 & - & - & - & 0 & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & 6! \\ 5 & 4 & 3 & 2 & 1 & & \end{array}$$

But with 5 smudges then a number is being repeated

It could be repeated in how many ways?

A B C D E

If A is repeated → A A B C D E

If B → A B B C D E

C → A B C C D E

D → A B C D D E

E → A B C D E E

5 ways

$$5 \times \frac{n!\ \text{total distinct arrangements}}{m!\ \text{total indistinct arrangements}} = 5 \times \frac{6!}{2!}$$

$$= 1800$$

account for duplicates / remove thems

Combinations with cake

Cake Room: order → ① ② ③ ④ ⑤ | Not cake room: 6 7 8 … 20

$n = 20$ people $k = 5$ get cake

Steps:

1. n people in line : $n!$ ways to order them
2. k people go into cake room : 1 way
3. Cake group mingle : $k!$ ways
4. Allow non cake people to mingle : $(n-k)!$

So a Combination is an unordered selection of k objects from a set of n distinct objects

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = n! \times 1 \times \underbrace{\frac{1}{k!} \times \frac{1}{(n-k)!}}_{\text{are not worried about ordering}}$$

Binomial coeficient

Steps: 1 2 3 4

Combination does not care about ordering

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20 \text{ ways}$$

Problem:

How many unique hands of 5 cards in a 52 card deck

$$\binom{52}{5} = \frac{52!}{5!\,(52-5)!} = 2598960$$

Put N objects in R buckets → distinct case
→ indistinct case

$$\text{Distinct} = r^n$$

$$\text{Indistinct} = \frac{(N+R-1)!}{N! \cdot (R-1)!}$$