Doppler-broadened atomic sample – 2D grid

Doppler line profile: $\vartheta(\omega-\omega_0) \equiv \frac{1}{\sqrt{2\pi}}\frac{m}{k_B T} e^{-\frac{(\omega-\omega_0)^2}{2\sigma^2}} d\omega$

let $\omega - \omega_0 = \vec{k}\cdot\vec{v}$

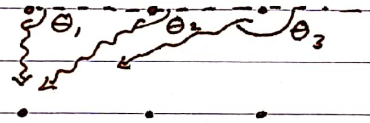
Average ρ over velocities:

$\rho(\Delta)$ can be found numerically; time-dep. suppressed here

$$\bar{\rho}(\Delta) = \frac{1}{\sqrt{2\pi}\sigma}\int e^{-\frac{(\vec{k}\cdot\vec{v})^2}{2\sigma^2}} \rho(\Delta - \vec{k}\cdot\vec{v})\, d(\vec{k}\cdot\vec{v})$$

$$= (\vartheta * \rho)(\Delta)$$

For 2D grid, $\vec{k}$ has multiple possible ~~values~~ directions:



$$\bar{\rho} \sim \sum_i \int e^{\frac{-(\vec{k}_i\cdot\vec{v})^2}{2\sigma^2}} \rho(\Delta - \vec{k}_i\cdot\vec{v})\, d(\vec{k}_i\cdot\vec{v})$$

$\vec{k}_i = |\vec{k}|\hat{e}_i$, $\vec{k}_i\cdot\vec{v} = kv\cos\theta_i$, θ_i in y-z plane

$k\cos\theta_i\, dv$

$$\sim \sum_{\theta_i}\int e^{\frac{-(kv\cos\theta_i)^2}{2\sigma^2}} f(v)\, \rho(\Delta - kv\cos\theta_i)\, ~~kv\, d(\cos\theta_i)~~$$

Need to acct. for speed distribution (Maxwellian) } Does this double count? $e^{-(\,)^2}$ is already acct. for $f(v)$?

Assume finite sampling of Maxwell distribution:

$$\bar{\rho} \sim \sum_{v_j}\sum_{\theta_i} e^{\frac{-(kv_j\cos\theta_i)^2}{2\sigma^2}} f(v_j)\, \rho(\cancel{\Delta} - kv_j\cos\theta_i)\, k\cos\theta_i\, v_j$$

Δ Take to be 0

↳ Compute this sum for each element of ρ after $\rho(t)$ found numerically.

☆ Not every atom sees all k orientations, so limit values of θ_i by atom location