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Experimental study of heat transfer in rarefied gas flow in a circular tube with constant wall temperature

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ABSTRACT

This paper presents an experimental study of heat transfer in a slightly rarefied gas flowing in a circular tube with constant wall temperature boundary condition. Local temperature measurements are carried out for the first time in rarefied gas flows to investigate into the anomalous values of Nusselt number (*Nu*) obtained by a previous experimental study (Demsis et al., 2009). The present measurements agree well with the low *Nu* reported by Demsis et al. (2009) when the *Nu* is obtained using their procedure; additionally, the measurements reveal the importance of end effects in determining the Nusselt number in rarefied gases. The Nusselt number obtained in the present experiments tends to zero with increasing axial conduction. *Nu* shows a weak dependence on Knudsen number for the range investigated here ($0.001 < Kn < 0.012$).

1. Introduction

Heat transfer in an incompressible fluid flowing in a circular tube with constant wall temperature has been well documented in the laminar continuum regime. It is well known that under such conditions, for a fully developed flow, the Nusselt number is a constant ($Nu = 3.66$). However, the heat transfer in rarefied gas is not very well understood. In such flows, the Knudsen number (Kn) becomes significant and temperature and velocity jumps at the wall become important. Although there are a lot of theoretical and simulation studies concerning heat transfer in rarefied gas flows, there is considerable lack of experimental data on the subject.

Ameel et al. [1] analytically studied laminar gas flow in a microtube with constant heat flux boundary condition. They assumed hydrodynamically fully developed flow condition and incorporated both velocity and temperature jump conditions at the boundary. They found significant reduction in Nusselt number with increase in gas rarefaction. Kavehpour et al. [22] numerically studied the effect of gas compressibility and rarefaction inmicrochannel gas flow in slip regime. They found that both the friction factor and the Nusselt number decrease from their continuum values, as rarefaction is increased. Larrode et al. [23] analytically studied the effect of rarefaction on heat transfer in circular tubes with the usual assumptions associated with the classical Graetz problem i.e. hydrodynamically fully developed flow, constant properties of fluid, high Peclet number and negligible energy dissipation. They found that the amount of heat transfer depends both on the ratio of accommodation coefficients and also on the amount of rarefaction. Hadjiconstantinou and Simek [14] studied the constant wall temperature convective heat transfer characteristics in gaseous flow in two dimensional micro and nano channels under hydrodynamically and thermally fully developed flow conditions. They used the slip theory to obtain the Nusselt number in the slip regime and Direct Simulation Monte Carlo (DSMC) for obtaining *Nu* in the transition regime. They found that *Nu* decreases monotonously with increase in *Kn*. Yan and Farouk [35] used DSMC simulations to study heat transfer in ducts under low pressure conditions. They proposed a correlation connecting the Nusselt number to Knudsen and Peclet numbers.

It should be noted that the above studies neglect axial conduction, viscous dissipation, and pressure work. The influence of these effects on Nusselt number in rarefied gas flows is significant and needs to be included in the analysis. Boundary shear work, which is due to the combined effect of viscous dissipation and pressure work, also becomes important during heat transfer in rarefied gas flows [28]. Tunc and Bayazitoglu [30] studied the effect of rarefaction and viscous dissipation for gas flow in microtubes by integral transform technique. They reported that accommodating temperature jump results in the reduction in Nusselt number while viscous dissipation effects lead to an increase in the value of the fully developed Nusselt number. Cetin et al. [7] studied the Graetz problem with additional effects of rarefaction, viscous dissipation and axial conduction for uniform wall temperature

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in the slip flow regime. They found that fully developed Nusselt number and thermal entrance length both increase with finite axial conduction (i.e. low Peclet numbers).

Kushwaha et al. [19] studied the effect of second order velocity and temperature slip boundary conditions for rarefied gas flow in a micropipe with isoflux boundary condition. They reported an increase in Nusselt number for the wall heating case ($T_{wall} > T_{fluid}$), when viscous dissipation effects were included in the analysis. Kushwaha et al. [20] studied the effect of shear work on heat transfer in gas flow in microchannels. They found that neglecting the shear work underpredicts the Nusselt number. A few recent works highlight the effects of shear work and viscous dissipation in rarefied gas flow in microchannels using DSMC with uniform wall heat flux boundary condition [2–5]. These studies are able to capture the singularity in Nusselt number, where the viscous dissipation effects are able to overcome the cooling wall heat flux, leading to heating of the entire flow field.

Among the experimental works reported in the literature, one of the foremost is the study of heat transfer characteristics of gas flows in fine channel heat exchangers by Wu and Little [34]. They reported the dependence of Nusselt number on the Reynolds number for fully developed laminar flow. Choi et al. [9] measured the friction factor and heat transfer coefficient for microtubes with inside diameter of 3 and 81 μm, with nitrogen as the working fluid. Due to very large inlet pressures (5–10 MPa), the Knudsen number was relatively small in their experimental runs and hence rarefaction effects were negligible. They also found the Nusselt number to be dependent on the Reynolds number in both the laminar and turbulent regimes.

Demsis et al. [10] conducted experiments explicitly to study the effect of Knudsen number on the Nusselt number. Although, there is a near consensus among the research community in the reduction of Nusselt number with increase in *Kn*, there are still differences in the magnitude of this reduction. Demsis et al. [10] attempted to provide experimental data that could consolidate the theoretical and numerical studies and thus narrow down the uncertainty in the magnitude of reduction of *Nu* with increasing *Kn*. These measurements were among the first of heat transfer measurements in the slip flow regime presenting the heat transfer coefficient in rarefied gases. They found two to three orders of magnitude difference in the values of *Nu* obtained experimentally and the numerical and theoretical results available in the literature. The calculation of Nusselt number in their study was based on the log mean temperature difference approach, involving temperature measurements made at the inlet and outlet of the test section.

In a technical note by Herwig and Hausner [16], the authors argued that calculations based on the global temperature measurements could be misleading, as the temperature distribution across the channel might not be linear because of various scaling effects. They compared two different Nusselt numbers obtained under similar flow and heat transfer conditions. The interpolated *Nu* (Nu_{int}) was obtained by using the experimental bulk fluid temperature. The experimental bulk fluid temperature was obtained by measuring the inlet and outlet temperature of the fluid and then performing a linear interpolation of these values. This was compared with the Nusselt number obtained by simulations. Herwig and Hausner [16] observed that due to the strong conjugate effects (high relative wall thickness) and axial fluid conduction (high Peclet number), the bulk temperature of the fluid is not linear near the inlet and the outlet. By calculating the *Nu* by using the actual bulk temperature obtained from the simulations, it was seen that the Nusselt number is very close to 4.36 (i.e. the standard value for laminar, fully developed heat transfer with constant wall flux boundary condition).

From the literature it is evident that the heat transfer in the slip regime has been studied analytically and numerically for different flow geometries and boundary conditions. However, little experimental data exists to verify the accuracy of these predictions or to test the limits of the velocity jump and temperature jump boundary conditions for heat transfer in rarefied gas flows. The verification of predicted results has become all the more necessary with a few experimental works giving anomalous behaviour of Nusselt number [9–11]. The lack of experimental data is due to extreme sensitivity of the measurements to intrusion in rarefied flows, especially in microchannels due to the microdimensions involved. Following Demsis et al. [10], measurements are performed in conventional sized tube and the rarefaction is obtained by reducing the working pressure of the flow. The significant difference between the present work and the earlier measurements is the provision for making local axial temperature measurements along the length of the circular tube. As pointed out by Herwig and Hausner [16], obtaining the fluid bulk temperature by local temperature measurements might be necessary in the case of rarefied gas flows to isolate the end-effects. The primary motivation of this work is to ascertain the Nusselt number in rarefied gas flow in circular tube of conventional size by making local temperature measurements and to investigate the reason for the anomalously low Nusselt number obtained in the previous study of Demsis et al. [10].

2. Experimental setup

The objective of achieving a high Knudsen number is usually accomplished by reducing either the characteristic dimension of the test section or the operating pressure. In the present experiments, a relatively large test section (22.2 mm ID) is used for the convenience of making local temperature measurements. Hence a trade-off exists between obtaining accurate local temperature without discernable intrusion (which necessitates large tube diameter) and the degree of rarefaction that can be achieved (smaller tube diameter). In the current experiments, the Knudsen numbers correspond to the early slip regime.

Fig. 1 shows the schematic of the experimental set-up, which consists of vacuum pumping system, nitrogen cylinder, particle filter (25 μm size), mass flow controller and constant temperature water circulator (Julabo F-25). The test-section is connected between two large reservoirs in order to minimize pressure fluctuations and achieve uniform flow at the inlet and outlet. After achieving ultimate vacuum (10^{-3} mbar) in the system, mass driven flow is established by allowing nitrogen to flow through the test-section. The amount of nitrogen is regulated via mass flow controller (MKS Mass Flo 1179A) of three different full scale ranges 5000 SCCM, 200 SCCM and 20 SCCM (1 SCCM = 1.87×10^{-8} kg/s). The pressure drop across the test section is measured using high accuracy capacitance based pressure transducers (MKS Baratron 626C). Leakage tests were conducted (as presented in [11]) and it was observed that the leakage is lower than 2% of the lowest mass flow rate encountered in the experiments.

Fig. 2 shows the schematic of the test section used for the experiments. The test section comprises counter-flow tube-in-tube heat exchanger. Hot water is circulated in the outside jacket that maintains a constant temperature boundary condition at the inner surface of the tube through which the nitrogen gas is allowed to flow. The heat exchanger is connected to two end units (made of PTFE) which act as insulators and check the axial wall heat conduction on both ends. These end units also have provisions for connecting pressure gauges and insertion of thermocouples. The end connector is sufficiently long to allow the flow to be hydrodynamically fully developed before entering the heat exchanger. Axial temperature measurements are accomplished by fastening a thin string along the axis of the tube (indicated in Fig. 2 by the line in the center). K-type thermocouples are attached to this string at different axial locations for measuring centerline temperatures. The temperature and pressure drop across the heat exchanger are measured with and without this arrangement to estimate any errors induced by this slightly intrusive measurement technique. It was observed that these errors were within the range of experimental uncertainties (not shown). The whole heat exchanger assembly is wrapped in glass wool insulation to minimize the heat loss to the surrounding atmosphere.

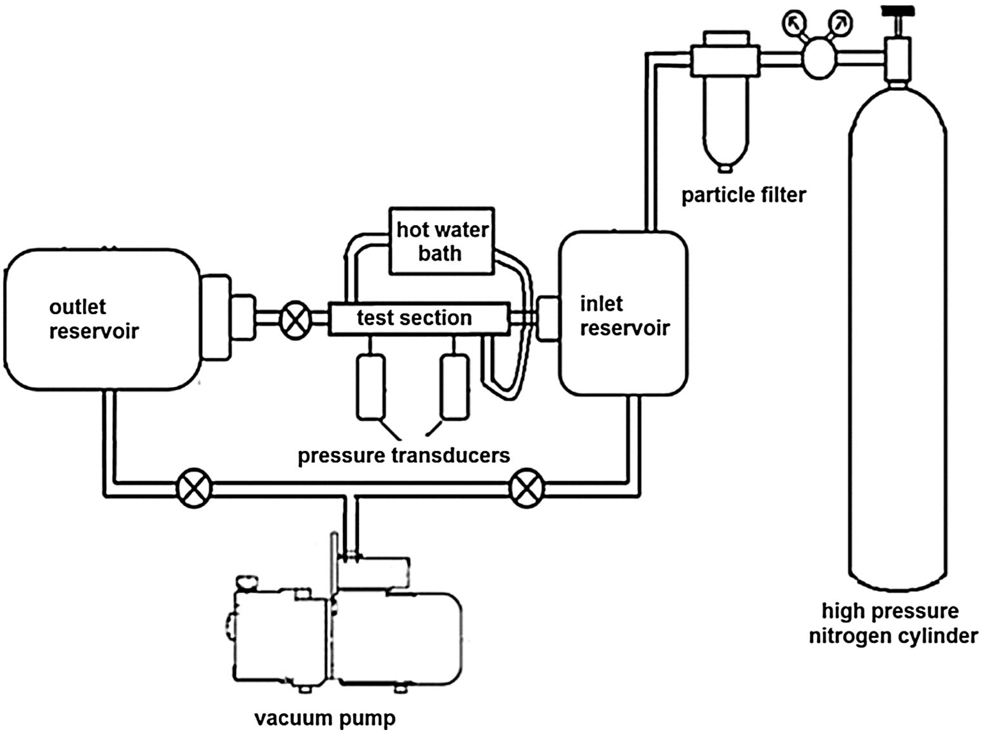

Fig. 1. Schematic of the experimental set-up.

3. Data reduction

For a given mass flow rate Q, the Reynolds number Re is given by

$$Re = \frac{QD}{A\mu} \tag{1}$$

Here, μ is the kinematic viscosity of the fluid, D is the diameter of the pipe and A is the cross sectional area of the pipe. The mean free path, λ of the gas is calculated as

$$\lambda = \frac{\mu}{P_m}\sqrt{\frac{\pi RT}{2}} \tag{2}$$

Here, R is the specific gas constant, P_m is the mean of inlet and outlet pressure and T is the film temperature. The mean Knudsen number is given by

$$Kn = \frac{\mu\sqrt{\pi RT/2}}{P_m D} \tag{3}$$

The friction factor (f) is calculated using the following relation as suggested by Wu and Little [33]

$$\frac{P_i - P_o}{P_i} = \frac{1}{2}G^2\frac{RT_m}{P_i^2}\left[f\frac{L}{D}\frac{P_i}{P_m} + 2\left(\frac{P_i}{P_o} - 1\right)\right] \tag{4}$$

Here, P_i and P_o are the pressures at inlet and outlet of the pipe, P_m and T_m are the mean of the inlet and outlet pressure and temperature, respectively, L is the total length of the pipe, and G is the mass flux. The friction factor is then used to obtain the Poiseuille number (Po) as:

$$Po = fRe \tag{5}$$

For calculating the Nusselt number on the gas side, the following procedure is adopted. The logarithmic mean temperature difference (LMTD) is calculated across the heat exchanger by knowing the inlet and outlet temperatures of both hot and cold fluids:

$$\Delta T_{LMTD} = \frac{(T_{ci} - T_{ho}) - (T_{co} - T_{hi})}{\ln\frac{(T_{ci} - T_{ho})}{(T_{co} - T_{hi})}} \tag{6}$$

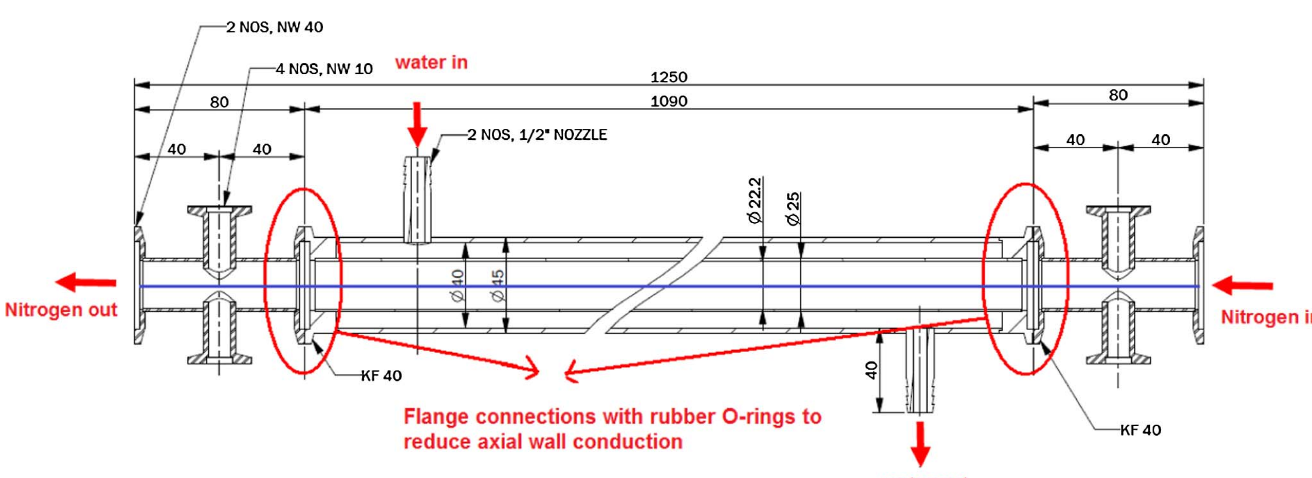

Fig. 2. Schematic of the test section highlighting the end connectors used to check axial wall conduction.

The overall heat transfer coefficient across the heat exchanger (U) is obtained by performing the energy balance

$$(UA)\Delta T_{LMTD} = Qc_p(T_{co}-T_{ci}) \tag{7}$$

where c_p is the specific heat at constant pressure. The overall heat transfer coefficient can also be written as

$$\frac{1}{UA} = \frac{1}{h_iA_i} + \frac{\ln(r_o/r_i)}{2\pi k_t L} + \frac{1}{h_oA_o} \tag{8}$$

Here r_i is inner radius of the inner tube, r_o is outer radius of the inner tube, k_t is the thermal conductivity of the tube material and h is the heat transfer coefficient. The subscripts 'i' and 'o' refer to the inner (nitrogen) and outer (water) side of the tube. An order of magnitude analysis shows that the second and third terms in the right side of Eq. (8) can be neglected [10], and h_i can be calculated from the following expression.

$$UA = h_iA_i \tag{9}$$

Knowing h_i, the Nusselt number can then be obtained as

$$Nu = \frac{h_iD}{k} \tag{10}$$

The significance of axial fluid conduction and viscous dissipation in heat transfer is characterized by Peclet number (Pe) and Brinkman number (Br), respectively. They are defined as follows.

$$Pe = \frac{u_{avg}D}{\alpha} \tag{11}$$

$$Br = \frac{\mu u_{avg}^2}{k(T_{ci}-T_w)} \tag{12}$$

where u_{avg} is the average velocity and k is the thermal conductivity of the fluid. Here, a negative Br implies wall heating case ($T_w > T_{ci}$) and a positive Br is representative of the cooling case ($T_w < T_{ci}$).

The maximum uncertainty in various parameters involved in the study of heat transfer coefficient is given in Table 1. These uncertainties tend to reduce with increasing mass flow rate.

4. Validation of the experimental setup

Before starting the heat transfer experiments, the pressure measurements and temperature measurements were validated by comparing the experimentally obtained Poiseuille number and Nusselt number with the theoretical expressions reported in the literature. The pressure measurements were validated by comparing the Po with the theoretical expression provided by Valougeorgis [31] for a rarefied gas flowing in a circular tube (Fig. 3). The semi-empirical correlation obtained by Verma et al. [32] is also included for comparison. Fig. 3 shows a very good agreement between the experimental and theoretical values of Po.

The temperature measurements are validated by carrying out experiments in the continuum regime, since the experimental data available for heat transfer in rarefied regime is not well established. Fig. 4(a) shows the comparison of centerline temperature obtained in the experiments with that obtained by using the series solution provided in Kakac et al. [21]. This solution is given as

Table 1
Maximum uncertainty in various parameters of interest.

Parameter	Maximum uncertainty
Mass flow rate	± 1% of full scale
Pressure	± 0.15%
Temperature	± 0.5°K
Knudsen number	± 0.5%
Reynolds number	± 4%
Poiseuille number	± 6%
Nusselt number	± 10%

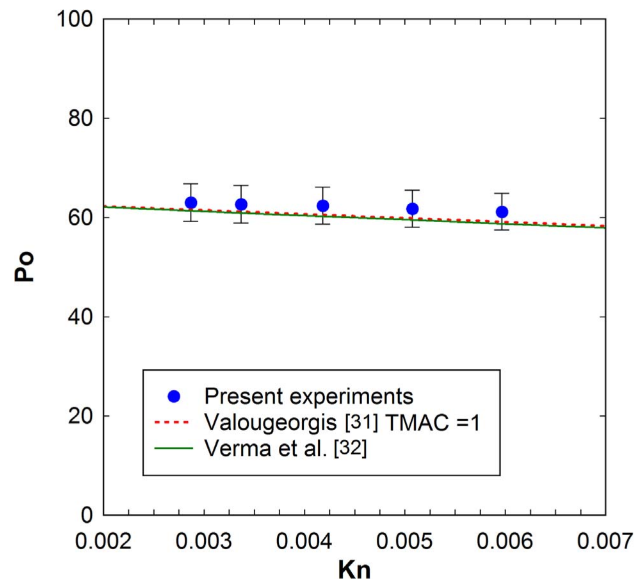

Fig. 3. Validation of Poiseuille number.

$$\frac{T_w-T}{T_w-T_m} = \sum_{n=0}^{\infty} C_{2n}\left(\frac{r}{a}\right)^{2n} \tag{13}$$

where, T_m is obtained as [36]

$$T_w-T_m = (T_w-T_{ci})\exp\left[\frac{\alpha N_u}{r_i^2 u_{avg}}x\right]$$

where α is the thermal diffusivity ($\alpha = \rho c_p/k$), and x is the axial distance from the inlet.

$$C_0 = 1;\ C_2 = -\frac{\lambda_0^2}{2};$$

$$C_{2n} = \frac{\lambda_0^2}{2n^2}(C_{2n-4}-C_{2n-2});$$

$$\lambda_0 = 2.704364419$$

It is seen that the centerline temperature measurements in the current experiments agree well with the theoretical solution of Kakac et al. [21]. The assumption of fully developed flow condition made in the theoretical analysis is ensured by carrying out measurements sufficiently far downstream from the inlet. Thin inner tube wall and insulated outer wall, respectively ensures that the wall conduction is negligible and convection losses to the atmosphere are minimized.

Fig. 4(b) shows the Nusselt number obtained from the local axial temperature measurements. The experimentally obtained Nusselt number is seen to match the constant value of 3.66 corresponding to the fully developed Nu for the classical Graetz solution. Due to the low heat transfer rates involved in the present experiments ($\sim 10^{-3}$ W), the drop in the temperature of the hot fluid (water) across the heat exchanger is rather low and is within the uncertainty of the thermocouples ($\pm$ 0.5 °C). Hence a heat balance exercise is not feasible and only the rise in temperature of the cold fluid (nitrogen) is considered for determining the amount of heat transfer (Eq. (7)).

These comparisons confirm that the experimental setup is capable of making accurate temperature and pressure measurements and forms the baseline for conducting measurements under rarefied gas conditions.

5. Axial temperature variation

A typical variation of centreline temperature along the length of the heat exchanger is shown in Fig. 5. The tube has an unheated length of

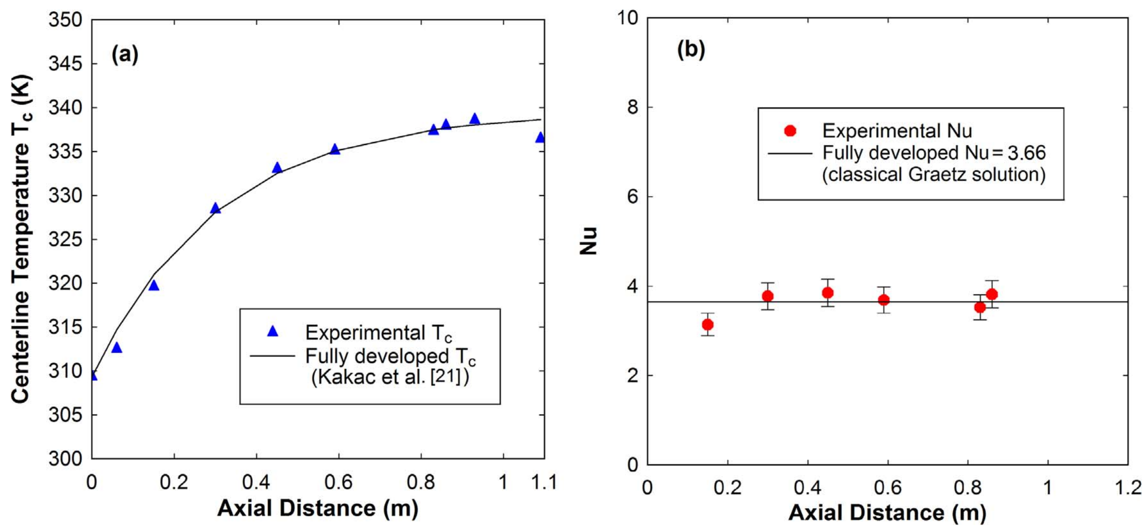

Fig. 4. Validation of temperature measurements (a) Comparison of centerline fluid temperature and (b) Nusselt number for near continuum regime $Kn < 0.0001$)

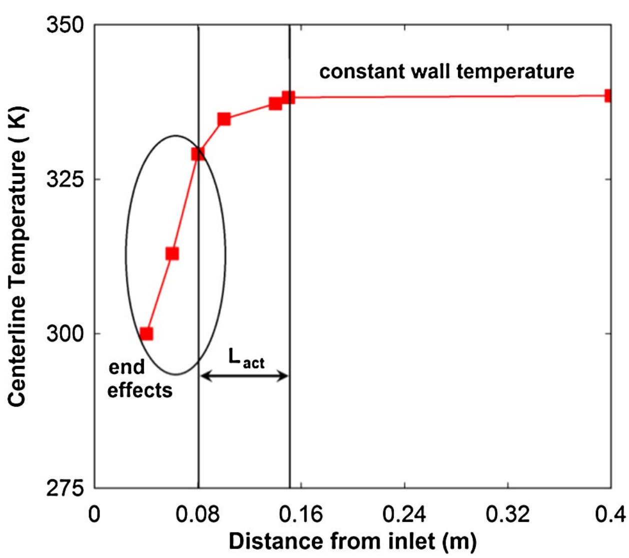

Fig. 5. Typical axial variation of centreline fluid temperature along the length of the tube ($Re = 5.95$, $Pe = 4.11$, $Kn = 0.004$).

80 mm on both ends to ensure that the flow is hydrodynamically fully developed before entering the heat exchanger. The graph shows that the gas temperature at the entry to the heat exchanger is approximately 328 K, indicating that there is a considerable amount of pre-heating of gas taking place prior to the entry into the heat exchanger (the room temperature is maintained at 300 K). The pre-heating of gas can be associated with the presence of end effects, i.e., axial conduction along the tube taking place at both the ends of the test section. Generally, in addition to the axial conduction heat transfer in the wall, axial conduction in the fluid is also significant due to the low Peclet numbers ($0.3 < Pe < 17$). However, since the axial wall conduction is restricted by end connectors as described in Section 2, it is believed that the temperature-rise at the entrance and the temperature-drop at the exit are largely due to axial conduction in the fluid.

The centreline temperature of the fluid attains the constant wall temperature (339 K) at a very short distance downstream of the inlet section. The Nusselt numbers in the case of local temperature measurements are obtained from the measurements made in this region (i.e. L_{act}: length from the start of the heat exchanger to the point where the fluid attains the constant wall temperature). After the fluid attains the wall temperature, the flow is essentially isothermal with no heat transfer occurring for the remaining major portion of the tube (axial distance > 0.15 m in Fig. 5). It is deduced from the axial temperature variation that the anomalously low Nusselt number values reported by Demsis et al. [10] was due to neglecting the end effects and obtaining the *Nu* from the inlet and outlet temperature measurements.

This observation is verified by calculating the global *Nu* by using the inlet and outlet temperatures only. These Nusselt numbers are compared with the data obtained by Demsis et al. [10] which report extremely low values of *Nu* (Fig. 6). The local *Nu* obtained by making local temperature measurements is also included for comparison. (Note: The error bars are of the size of the marker and hence not included.) It is seen from the figure that when the Nusselt number is calculated based on the inlet and outlet fluid temperature, the presently obtained values match with those reported by Demsis et al. [10]. When the Nusselt number is based on the actual axial local temperatures of the flow, the Nusselt number comes out considerably higher than the experimental values reported earlier (Fig. 6).

6. Nusselt number

The Nusselt number obtained by local temperature measurements is

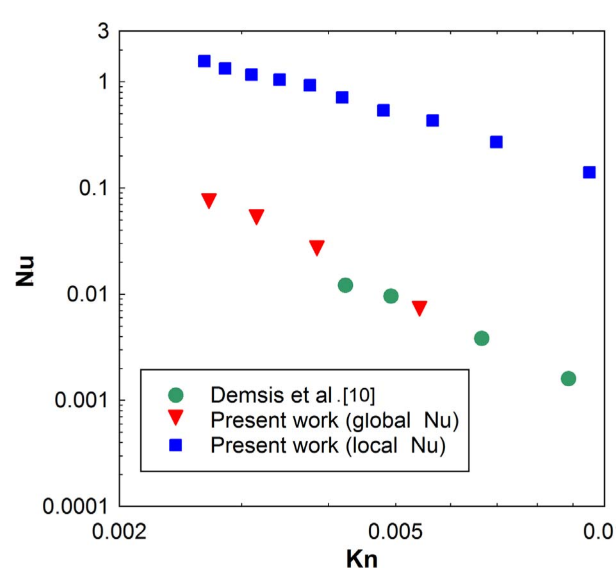

Fig. 6. Comparison of Nusselt number obtained using axial local temperature measurement (local *Nu*) and inlet and exit temperature measurement (global *Nu*).

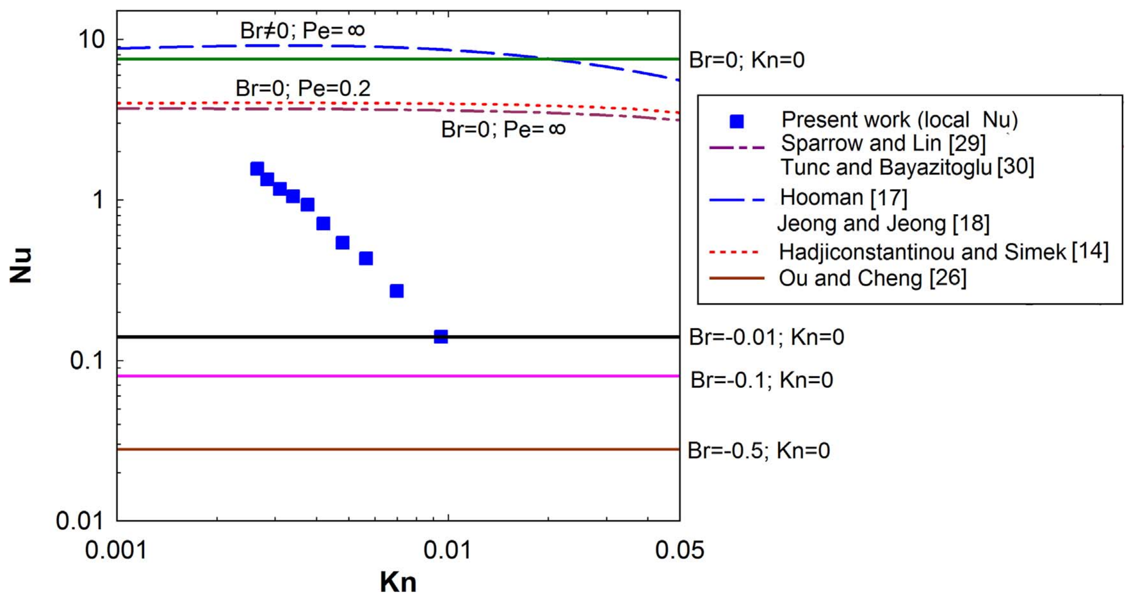

Fig. 7. Comparison of Nusselt number obtained in various studies.

further compared with other theoretical studies that consider heat transfer in a circular tube for rarefied gas flows (Fig. 7). There are quite a few theoretical studies available in literature that study the classical Graetz problem with additional effects like viscous dissipation, shear work, pressure work, and axial conduction. The solutions of Tunc and Bayazitoglu [30] and Sparrow and Lin [29] correspond to fully developed conditions for flow in a microtube with constant wall temperature boundary condition with velocity slip and temperature jump at the wall. In both these studies viscous dissipation effect is neglected and the continuum fully developed *Nu* is obtained as 3.66 (Tunc and Bayazitoglu [30] have included viscous dissipation only for the cooling case for constant wall temperature condition, i.e. for $T_{wall} < T_{fluid}$). Jeong and Jeong [18] and Hooman [17] showed that for the constant wall temperature boundary condition, the Nusselt number is independent of the Brinkman number but has a different asymptotic value ($Nu = 9.6$) when viscous dissipation is included. Hadjiconstantinou and Simek [14] have considered axial fluid conduction (but neglected viscous dissipation) in their analysis, and showed that the Peclet number has negligible influence on the fully developed Nusselt number. All the above studies neglected the pressure work term in the energy equation. Ou and Cheng [26] studied heat transfer between flow through parallel plates by including both pressure work and viscous dissipation. It was seen that the fully developed Nusselt number asymptotically approaches zero for heating case ($T_{wall} > T_{fluid}$) as opposed to the conventionally accepted value of 7.54. The Nusselt numbers of Ou and Cheng [26] for different Brinkman numbers (obtained by averaging over the developing length before *Nu* reaches the fully developed asymptotic value of zero) are close to those obtained in the present work.

In Fig. 7, the Reynolds number is also varying along with the Knudsen number. Further experiments were carried out to isolate the effect of *Kn* on *Nu*. Fig. 8 presents the variation of Nusselt number with Knudsen number when the Reynolds number is maintained constant. Table 2 lists the corresponding values of *Re*, *Pe* and *Br* for these runs. Fig. 8 shows that rarefaction has negligible effect on the Nusselt number for the range considered in the present experiments. The effect of axial fluid conduction (*Pe*) is important ($Pe < 20$), while viscous dissipation and pressure work effects (*Br*) are low as apparent from Table 2. The rise and fall of *Nu* at $Re = 1.19$ is not very significant and is within the uncertainty of the measurements (± 10%). Further, the last data point for $Re = 0.3$ is considered as an outlier. That point corresponds to the lowest mass flow rate and highest *Kn*. It is seen that the Nusselt number

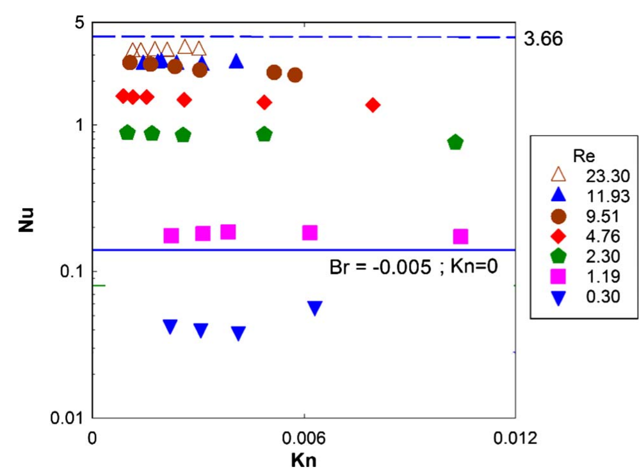

Fig. 8. Variation of *Nu* with rarefaction (*Kn*) at different Reynolds numbers. The solid line corresponds to Ou and Cheng [26] and the dashed line is the fully developed *Nu* for the classical Graetz solution.

Table 2
List of values of relevant non-dimensional parameters.

Re	*Pe*	*Br*
23.30	16.08	−0.0026
11.93	8.21	−0.0032
9.51	6.57	−0.0030
4.76	3.28	−0.0020
2.30	1.59	−0.0030
1.19	0.82	−0.0005
0.33	0.23	−0.0005

decreases with increase in axial conduction (decreasing *Pe*) in contrast to the reports of some earlier theoretical works [27,7] that consider the effect of *Pe* on *Nu* in the thermal entrance region with uniform inlet fluid temperature. Therefore, the present experimental results show two important deviations from the theoretical predictions considering extended Graetz problem: first, the Nusselt number is seen to decrease with decreasing Peclet number; second, Nusselt number tends to zero when bulk fluid temperature approaches the wall temperature. An overview of the theoretical studies on extended Graetz problem available in the literature is provided in the next section in order to seek an explanation for these deviations.

7. Discussion

The classical Graetz problem or the Graetz-Nusselt problem was studied separately by Graetz [13] and Nusselt [25] and is concerned with incompressible, laminar, hydrodynamically fully developed and thermally developing fluid flow in a tube with constant wall temperature boundary condition. Although the model is able to predict the Nusselt number in the continuum regime, it neglects non-continuum effects like rarefaction and viscous dissipation which appear in many real world applications, especially in the case of microflows. In these flows, the effects of viscous dissipation, axial conduction, pressure work and boundary shear are significant and need to be included in the analysis. These various additional effects are accounted for in theoretical studies by including additional terms in the energy equation. Although, such exercises provide information about the significance of the isolated effects of various terms, they may not actually correspond to practical engineering scenarios where most of these effects occur in combination.

The effect of viscous dissipation has been traditionally related to the Brinkman number. The study of Ou and Cheng [26] showed that the exclusion of viscous dissipation and pressure work terms in the energy equation on the basis of order of magnitude analysis is not appropriate for the constant wall temperature boundary condition case. In these conditions it was shown that even for a small value of Br (~ 0.005), the analysis provides an asymptotic Nusselt number tending to zero (as opposed to the conventionally accepted value of 7.53, when viscous dissipation is neglected). However, Nield et al. [24] questioned the validity of the solution of Ou and Cheng [26] as it is seen to violate the first law of thermodynamics far downstream. As discussed in Section 6, there are a lot of variance in the results as to the effect of viscous dissipation on the value of fully developed Nusselt number.

Another distinction between the theoretical extended Graetz solution with axial fluid conduction and the present experiments is the assumption of uniform inlet fluid temperature in most of the theoretical studies. As discussed by some authors [6,12,15], the presence of axial conduction causes heat to be carried upstream of the heated section into the unheated portion of the tube. Therefore, the fluid temperature at the inlet of the heat exchanger is not uniform. Bilir et al [6] showed that the Nusselt number curves for different Peclet numbers cross over one another under such conditions. Thus, while the Nusselt number decreases with Peclet number at the entrance of the test section, it increases with decreasing Peclet number further downstream. This could be the reason for the trend shown by Nu with Pe in the present experiments, since our measurements are restricted to near the entrance region. Under these conditions, Bilir et al. [6] reported the wall heat transfer to be zero ($q_w = 0$) when the fluid temperature approaches the wall temperature. However, the Nusselt number (which is defined as $Nu = 2q_w/(T_w - T_f)$) is initially seen to approach the classical Graetz solution without wall conduction when $T_w \approx T_f$. The Nusselt number is seen to have a minimum value further downstream of the test section.

Although there are considerable number of theoretical studies on extended Graetz problem, the present experiments show that they might not correspond to real world applications. More experimental data is necessary to obtain a clear perspective on the behaviour of Nusselt number and to logically connect the seemingly contradictory predictions obtained in various studies regarding the effect of viscous dissipation and axial conduction. As noted in Colin [8], providing accurate experimental data is a challenge in the coming years and is crucial to carry out a meaningful discussion on slip flow heat transfer.

8. Conclusions

In this paper, experimental study of heat transfer in a slightly rarefied gas flowing in a circular tube with constant wall temperature boundary condition is carried out. Local temperature measurements were made to investigate into the anomalous values of Nusselt number obtained in previous experimental studies [10,11]. The study involved inserting thermocouples along the axis of the tube at different locations. The local measurements revealed that end effects are significant for rarefied gas flows and neglecting them can lead to erroneous conclusions.

The low Nusselt numbers obtained in the present experiments and also that of Demsis et al. [10] are determined to be the effect of axial conduction (Peclet number) rather than rarefaction (Knudsen number). The Nusselt number is seen to be weakly dependent on the Knudsen number for the range investigated here ($0.001 < Kn < 0.012$). It is seen that the Nusselt number in rarefied gas flowing in a tube with constant wall temperature boundary condition asymptotically tends to zero when fluid temperature approaches the constant wall temperature. The difficulties in direct comparison of experimental and theoretical Nu are discussed. It is a challenging problem to experimentally isolate the various effects and to carry out local heat transfer measurements, especially at higher Knudsen numbers. But considering the total lack of experimental data in this area it is worthy to take up the challenge and come up with new ideas that bring out the effects of various parameters that are relevant in heat transfer study of rarefied gas flows.

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