Assignment 1

Friday, February 8, 2019 5:06 PM

Integrate and Fire Model:

$$C_m \frac{\mathrm{d}V\_m}{\mathrm{d}t} = I(t)$$

Where, with a constant capacitance C_m, the change in the membrane voltage (or membrane potential) over time increases with an input current until it reaches a constant threshold V_{th}, at which point a delta function spike occurs and the voltage is set back to the resting potential, after which the model continues to run. The firing frequency thus increases linearly without bound as input current increases.

Leaky Integrate and Fire Model:

$$C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} = I(t) - \frac{V_m(t)}{R_m}$$

Where, with a constant capacitance C_m, the change in the membrane voltage (or membrane potential) over time increases. However, a "leak" term has been added. R_m is the membrane resistance to leaks. The input current has to reach the threshold $I_{\mathrm{th}} = \frac{V_{th}}{R_m}$ for the neuron to fire.

Izhikevich model

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 0.04v^2 + 5v + 140 - u + I$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = a(bv - u)$$

$$\text{if } v \geq 30, \text{then } \begin{cases} v = c \\ u = u + d \end{cases}.$$

- *V* is the membrane voltage
- U is the membrane recovery variable
  - Parameter a = time scale of recovery variable u. Smaller values means slower recovery. Typically, a = 0.02
  - Parameter b = sensitivity of recovery variable u to fluctuations of membrane potential v. Typically, b = 0.2
  - Parameter c = after spike reset value of membrane potential v. Typically, c = -65 mV
  - Parameter d = after-spike reset of recovery variable u. Typically d = 2

Hodgkin-Huxley

$$C \frac{\mathrm{d}v}{\mathrm{d}t} = I - g_{Na} m^3 h(v - v_{Na}) - g_K n^4 (v - v_K) - g_L(v - v_L)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\left(I - g_{Na} m^3 h(v - v_{Na}) - g_K n^4 (v - v_K) - g_L(v - v_L)\right)}{C}$$

- g_{Na}, g_K, g_L are conductances to K, Na, and Cl
- *n, m, and h* are dimensionless quantities whose variation with time after a change of membrane potential is determined by:
  - $\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(1 - n) - \beta_n n$
    - $\alpha_n = \frac{0.01(v+10)}{e^{\frac{v+10}{10}} - 1}$ and $\beta_n = 0.125 e^{v/80}$
  - $\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(1 - m) - \beta_m m$
    - $\alpha_m = \frac{0.1(v+25)}{e^{\frac{v+25}{10}} - 1}$ and $\beta_m = 4e^{v/18}$
  - $\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(1 - h) - \beta_h h$
    - $\alpha_h = \frac{1}{e^{\frac{v+30}{10}} + 1}$ and $\alpha_h = 0.07 e^{v/20}$

All constant parameters from the paper:

TABLE 3

Constant (1)	Value chosen (2)	Experimental values: Mean (3)	Experimental values: Range (4)	Reference (5)
C_M (μF/cm^2)	1·0	0·91	0·8 to 1·5	Table 1, Hodgkin *et al.* (1952)
V_{Na} (mV)	−115	−109	−95 to −119	p. 455, Hodgkin & Huxley (1952*a*)
V_K (mV)	+ 12	+ 11	+ 9 to + 14	Table 3, values for low temperature in sea water, Hodgkin & Huxley (1952*b*)
V_l (mV)	−10·613*	− 11	− 4 to − 22	Table 5, Hodgkin & Huxley (1952*b*)
$\bar{g}_{Na}$ (m.mho/cm^2)	120	80	65 to 90	Fully analysed results, Table 2† — Hodgkin & Huxley (1952*a*)
		160	120 to 260	Fresh fibres, p. 465† — Hodgkin & Huxley (1952*a*)
$\bar{g}_K$ (m.mho/cm^2)	36	34	26 to 49	p. 463, Hodgkin & Huxley (1952*a*)
$\bar{g}_l$ (m.mho/cm^2)	0·3	0·26	0·13 to 0·50	Table 5, Hodgkin & Huxley (1952*b*)

* Exact value chosen to make the total ionic current zero at the resting potential ($V = 0$).

† The experimental values for $\bar{g}_{Na}$ were obtained by multiplying the peak sodium conductances by factors derived from the values chosen for α_m, β_m, α_h, and β_h.

(From A quantitative description of membrane current and its application to conduction and excitation in nerve.)

Questions

1. What do you expect to happen if an IF neuron is fed a very low input current? An LIF neuron?

- If an IF neuron is fed a very low input current over time, the membrane voltage also increases in proportion to the input current over time. The membrane potential increases over time until it reaches a threshold V_{th}, at which point the spike occurs and the voltage is set back to the resting potential.
- If an LIF neuron is fed a very low input current the membrane potential does not change significantly over time because of the "leak term". As the input current is being fed over a certain period of time, the membrane potential "leaks out" at the rate of $\frac{V_m(t)}{R_m}$ and the threshold input current $I_{\mathrm{th}} = \frac{V_{th}}{R_m}$ will never reach and the neuron will never be able to fire.

2. What do you expect to happen if an IF neuron is fed a larger input current? An LIF neuron?

- When a large current Is fed to an IF model, the membrane potential will reach the threshold membrane potential V_{th} at which point, the neuron will spike and the voltage will be set back to its resting potential.
- If a LIF neuron is fed a large current, as long as the input current reaches the threshold input current, the neuron will fire and the voltage will be set back to its resting potential
- Compared to the LIF neuron, the IF neuron will spike faster or sooner than the LIF neuron given the same, large input current

3. What are the limitations of an LIF neuron?

- The LIF neuron does not take into consideration a membrane recovery variable that accounts for the activation and inactivation of ionic currents. Also, the LIF neuron's "leak term" simplifies the behavior of opening and closing of ion channels by generalizing it to $\frac{V_m(t)}{R_m}$ rather than taking into account the variety and differences throughout the different types of ionic channels.

Programming

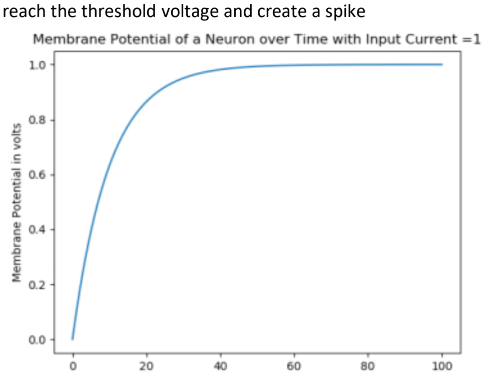
1. Simulate an LIF neuron with different input currents and plot the membrane potential, showing (a) potential decay over time and (b) spiking behavior.

- In order to simulate an LIF neuron, we first start by looking at the equation:
  $$\tau_m \frac{\mathrm{d}V_m}{\mathrm{d}t} = -V_m(t) + R_m I(t)$$
  This equation can be re-written as:
  $$\frac{\mathrm{d}V_m}{\mathrm{d}t} = \frac{-V_m(t) + R_m I(t)}{\tau_m}$$

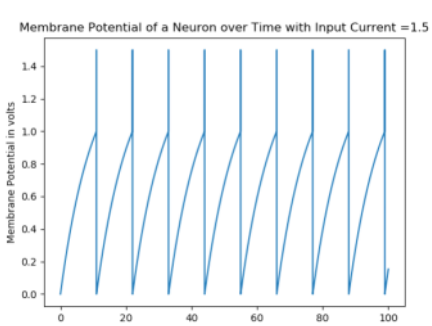
  We use the Euler method as follows to approximate V_{m_n} at time t based on the previous value of the membrane potential, $V_{m_{n-1}}$

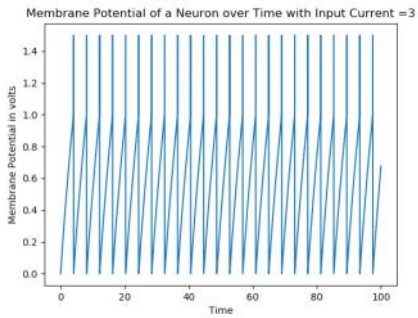
  LIF model:
  $$\frac{dV_m}{dt} = \frac{-V_m(t) + R_m I(t)}{\tau_m}$$
  $$dV_m \approx \frac{-V_m(t) + R_m I(t)}{\tau_m} dt$$
  $$\Rightarrow dV_m \approx \frac{-V_m(t) + R_m I(t)}{\tau_m} (t_n - t_{n-1})$$
  $$\Rightarrow V_{m_n} - V_{m_{n-1}} = \frac{-V_m(t) + R_m I(t)}{\tau_m} (t_n - t_{n-1})$$
  $$\Rightarrow V_{m_n} = V_{m_{n-1}} + (t_n - t_{n-1})\left(\frac{-V_{m_{n-1}}(t_{n-1}) + R_m I(t_{n-1})}{\tau_m}\right)$$

- The figure below shows the decay of the membrane potential over time. As time increases the change in the membrane potential decreases. The input current is not high enough to ever reach the threshold voltage and create a spike

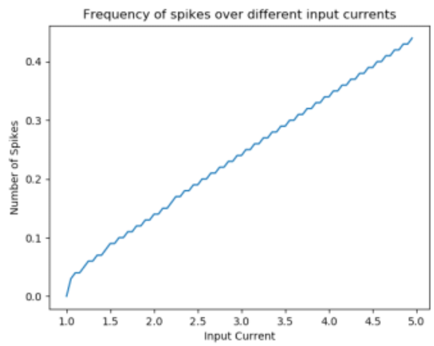


- The figure below shows spiking behavior of the neuron. After each spike, the membrane potential is set to 0 again. As seen from the two figures below, as the input current increases, the number of spikes also increases





2. Plot the firing rate as a function of the input current.

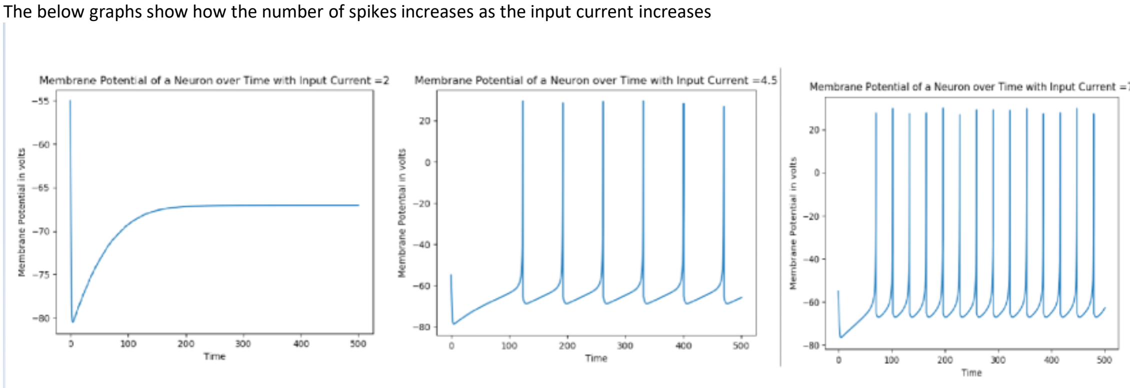
- The below graph shows the change in the number of spikes as a function of the input current
- 

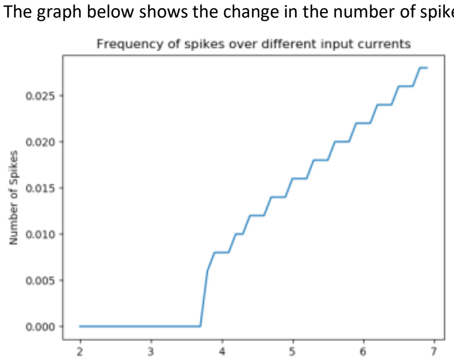
3. What happens to the firing rate as you continue to increase the input current? Why?

- The firing rate increases linearly after a sharp increase in the beginning as we increase the input current. This is because according to the integrate and fire model, the number of spikes scales from the input current linearly since *I* is simply multiplied to a constant term, if we look at the Euler approximation equation above. Also, the LIF model is too simple and does not include enough parameters to consider oversaturation of the membrane as input current become extremely high like an actual biological neuron would.

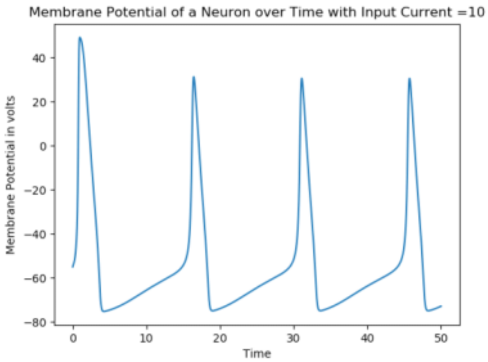
4. Simulate a neuron using the Izhikevich model.

- The Euler approximation method is used to program the Izhikevich model, similar to how the LIF model was approximated above
- The below graphs show how the number of spikes increases as the input current increases



- The graph below shows the change in the number of spikes as a function of the input current
- 

5. Simulate a neuron using the Hodgkin-Huxley model.



<u>References</u>

[1] Wulfram Gerstner. Time structure of the activity in neural network models. Physical review E, 51(1):738, 1995.
[2] Alan L Hodgkin and Andrew F Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. The Journal of physiology, 117(4):500–544, 1952.
[3] Eugene M Izhikevich. Simple model of spiking neurons. IEEE Transactions on neural networks, 14(6):1569–1572, 2003.