CM 1110 Fundamentals of Mathematics and Statistics

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Contents

Course Syllabus

Pre-requisites

None

Learning Outcomes

On successful completion of this module, students will be able to apply fundamental concepts in Mathematics and Statistics for real world problem solving.

Outline Syllabus

- Number Systems
- Sequence and Series
- Introduction to Logic
- Boolean Algebra
- Differentiation and Integration
- Descriptive Statistics
- Sets and Relations
- Probability
- Correlation and Regression

Method of Assessment

- Mid-semester examination
- End-semester examination

Lecturer

Dr. Priyanga Dilini Talagala

Schedule

Lectures: TBA

Consultation times: TBA

Chapter 1

Number Systems

Chapter 2

Sequence and Series

Chapter 3

Introduction to Logic

Chapter 4

Boolean Algebra

Chapter 5

Differentiation and Integration

Chapter 6

Descriptive Statistics

6.1 Introduction to Statistics

6.1.1 Some Basic Terminologies Used in Statistics

i Population

- The set of **all** possible elements in the universe of interest to the researcher

ii Sample

- A Sample is a **subset** (a portion or part) of the population of interest
- The sample must be a representative of the population of interest

iii Element

- Element is an **entity or object** which the information is collected.
- *Eg: Student, household, farm, company, tomato plant*

iv Variable

- A variable is **a feature characteristic which has different 'values' or categories for different elements** (items/subjects/individuals)
- *Eg: Gender of client, brand of mobile phones, risk level, number of emails received per day, age of client, income of client*

v Data

- Data are **measurements or facts** that are collected from a statistical unit/entity of interest
- We collect data on variables
- Data are raw numbers or facts that must be processed (analysed) to get useful information.
- We get information from data.
- *Eg:*

Variable: *Age (in years) of client*

Data: *21, 45, 18, 32, 30, 22, 23, 27*

Information:

The mean age is 27.25 years

The minimum age is 18 years

The range of ages is 18-45

The percentage of clients below 25 years of age: 50%

vi Statistic

- **Characteristic** of a **sample**
- The value which calculated based on sample data

vii Parameter

- **Characteristic** of a **population**
- The value which calculated based on population data

viii Census

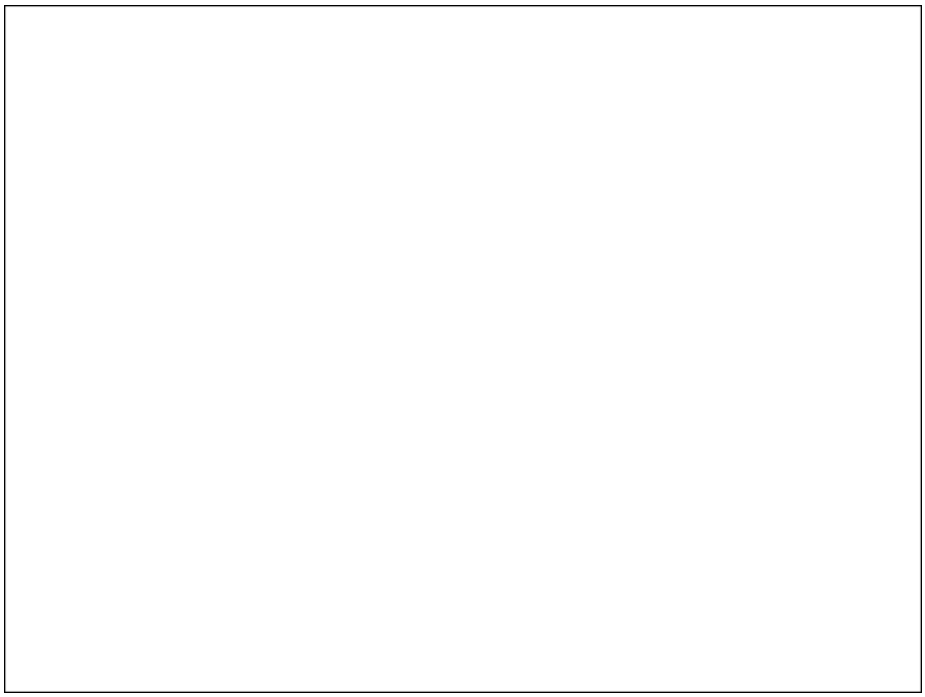
- When a researcher **gathers data from the whole population for a given measurement,** it is called a census

ix Sampling

- When a researcher **gathers data from a sample of the population for a given measurement,** it is called sampling
- The process of selecting a sample is also called sampling

Why take a sample instead of studying every member of the population ?

- Prohibitive cost of census
- Destruction of item being studied may be required
- Not possible to test or inspect all members of a population being studied.



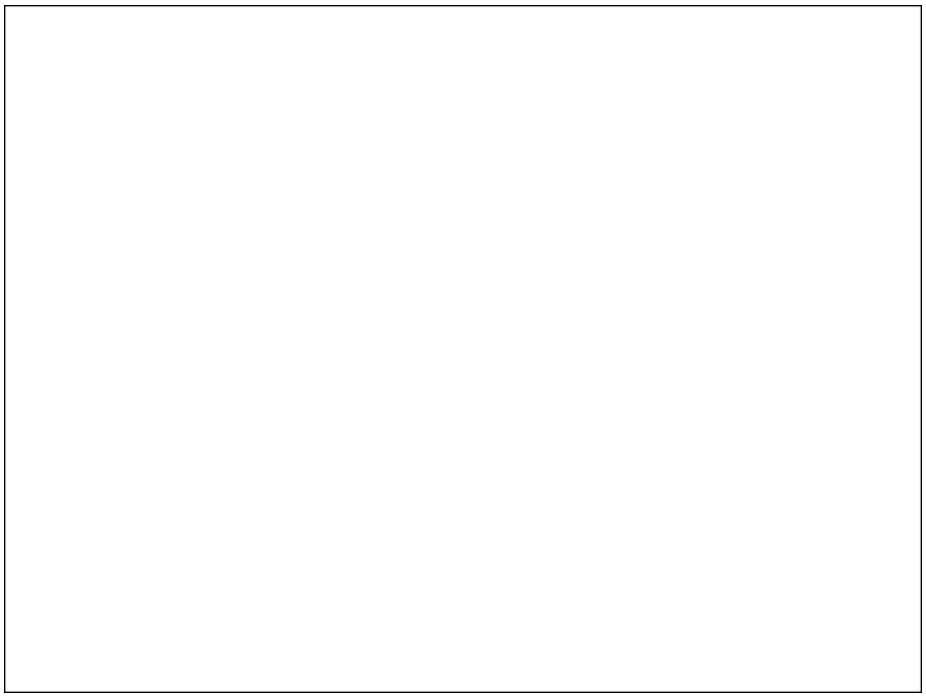
6.1.2 Branches of Statistics

i Descriptive Statistics

- Descriptive statistics consists of organizing, summarizing and presenting data in an informative way.
- The main purpose of descriptive statistics is to provide an overview of the data collected.
- Descriptive statistics describes the data collected through frequency tables, graphs and summary measures (mean, variance, quartiles, etc.).

ii Inferential Statistics

- In inferential statistics sample data are used to draw inferences (i.e. derive conclusions) or make predictions about the populations from which the sample has been taken.
- This includes methods used to make decisions, estimates, predictions or generalizations about a population based on a sample.
- This includes point estimations, interval estimation, test of hypotheses, regression analysis, time series analysis, multivariate analysis, etc.



6.1.3 Types of Variables

6.1.3.1 Qualitative / Quantitative Variables

i Qualitative variable (Categorical variable)

- The characteristic is a quality.
- The data are categories.
- They cannot be given numerical values.
- However, it may be given a numerical label
- Qualitative variables are sometimes referred as categorical variables.
- *Eg:*

Gender:

Age group:

Education level:

A/L stream:

Degree type:

Hair colour:

FIT student batch:

Undergraduate level:

Grade that you can obtain for CM 1110/ CM1130

ii Quantitative variable

- The characteristic is a quantity
- The data are numbers
- Quantitative data require numeric values that indicate how much or how many.
- They are obtained by counting or measuring with some scale
- *Eg:*

Number of family members:

Number of emails received per day:

Weight of a student:

Age:

Credit balance in the SIM card:

Time remaining in class:

Temperature:

Marks

6.1.3.2 Discrete/ Continuous Variables

- Quantitative variables can be classified as either discrete or continuous.

i Discrete Variables

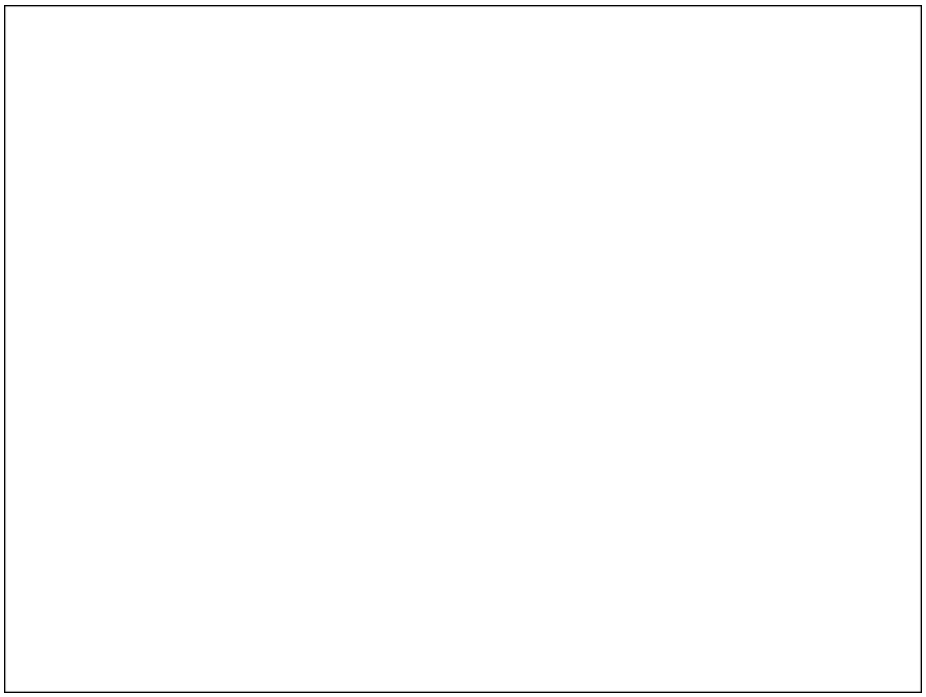
- Quantitative
- Usually the data are obtained by counting
- There are impossible values between any two possible values
- *Eg:*

Number of family members:

Number of emails received per day:

ii Continuous Variables

- Quantitative
- Usually, the data are obtained by measuring with a scale



- There are no impossible values between any two possible values.(any value between any two possible values is also a possible value)
- i.e a continuous variable can take any value within a specified range.
- *Eg:*

Weight of a student:

Age:

Credit balance in the SIM card:

Time remaining in class:

Temperature:

Marks

6.1.4 Scales of Measurements

- There are four levels of measurements called, **nominal, ordinal, interval and ratio.**
- Each levels has its own rules and restrictions
- Different levels of measurement contains different amount of information with respect to whatever the data are measuring

i Nominal Scale

- Qualitative
- No order or ranking in categories.
- These categories have to be mutually exclusive, i.e. it should not be possible to place an individual or object in more than one category
- A name of a category can be substituted by a number, but it will be mere label and have no numerical meaning

ii Ordinal Scale

- Qualitative
- Categories can be ordered or ranked
- A name of a category can be substituted by a number, but such a sequence does not indicate absolute quantities.
- Difference between any two numbers on the scale does not have a numerical meaningful.
- It cannot be assumed that the differences between adjacent numbers on the scale are equal.

iii Interval Scale

- Quantitative
- Data can be ordered or ranked
- There is no absolute zero point. Zero is only an arbitrary point with which other values can compare
- Difference between two numbers is a meaningful numerical value
- Ration of two numbers is not a meaningful numerical value.

iv Ratio Scale

- Quantitative
- Highest level of measurement
- There exist an absolute zero point (It has a true zero point)
- Ratio between different measurements is meaningful

6.2 Presentation of Data

The sinking of the Titanic is one of the most infamous shipwrecks in history.

On April 15, 1912, during her maiden voyage, the widely considered “unsinkable” RMS Titanic sank after colliding with an iceberg. Unfortunately, there weren’t

enough lifeboats for everyone onboard, resulting in the death of 1502 out of 2224 passengers and crew

[1]

Here's a quick summary of our variables:

Variable Name	Description
PassengerID	Passenger ID (just a row number, so obviously not useful for prediction)
Survived	Survived (1) or died (0)
Pclass	Passenger class (first, second or third)
Name	Passenger name
Gender	Passenger Gender
Age	Passenger age
SibSp	Number of siblings/spouses aboard
Parch	Number of parents/children aboard
Ticket	Ticket number
Fare	Fare
Cabin	Cabin
Embarked	Port of embarkation (S = Southampton, C = Cherbourg, Q = Queenstown)

6.2.1 Tabular Presentations of Data

Raw Data

- Raw data are collected data that have not been organized numerically
- Eg: Passenger age

```
##   PassengerId Survived Pclass
## 1           1        0      3
## 2           2        1      1
## 3           3        1      3
## 4           4        1      1
## 5           5        0      3
## 6           6        0      3
##                                                  Name    Sex Age SibSp Parch
## 1                             Braund, Mr. Owen Harris   male  22     1     0
```

[1] Data source: https://www.kaggle.com/varimp/a-mostly-tidyverse-tour-of-the-titanic

```
## 2 Cumings, Mrs. John Bradley (Florence Briggs Thayer) female  38     1     0
## 3                              Heikkinen, Miss. Laina female  26     0     0
## 4        Futrelle, Mrs. Jacques Heath (Lily May Peel) female  35     1     0
## 5                            Allen, Mr. William Henry   male  35     0     0
## 6                                    Moran, Mr. James   male  NA     0     0
##             Ticket    Fare Cabin Embarked
## 1        A/5 21171  7.2500              S
## 2         PC 17599 71.2833   C85        C
## 3 STON/O2. 3101282  7.9250              S
## 4           113803 53.1000  C123        S
## 5           373450  8.0500              S
## 6           330877  8.4583              Q
```

```
##  [1] 22 38 26 35 35 NA 54  2 27 14  4 58 20 39 14 55  2 NA 31 NA 35 34 15 28  8
## [26] 38 NA 19 NA NA 40 NA NA 66 28 42 NA 21 18 14
```

An array

- An array is an arrangement of raw numerical data in ascending or descending order of magnitude.
- Eg: Passenger age

```
##  [1]  2  2  4  8 14 14 14 15 18 19 20 21 22 26 27 28 28 31 34 35 35 35 38 38 39
## [26] 40 42 54 55 58 66
```

Frequency Table (Frequency Distributions)

- A frequency table (frequency distribution) is a listing of the values a variable takes in a data set, along with how often (frequently) each value occurs
- frequency can be recorded as a
  - **frequency or count:** the number of times a value occurs, or
  - **percentage frequency:** the percentage of times a value occurs
- Percentage frequency can be calculated as,

$$Percentage frequency = \frac{a}{b} \times 100\%$$

- The objective of constructing a frequency table are as follows
  - to organize the data in a meaningful manner
  - to determine the nature or shape of the distribution
  - to draw charts and graphs for the presentation of data

  - to facilitate computational procedures for measures of average and spread
  - to make comparisons between different data sets

- - There are two basic types of frequency tables
    1. Simple frequency tables (Ungrouped frequency distribution)
    2. Grouped frequency distribution

6.2.1.1 Simple frequency table (Ungrouped frequency distribution)

- Each possible value or category is taken as a class
- More suitable for
  - Qualitative variables
  - Discrete variables
- Sometimes construct for continuous variables when there is a small number of possible values between the minimum and maximum.

Examples:

CASE I:

Example 1

The native countries of 56 students from a certain education institute are as follows:

```
##  [1] "SL" "BD" "SL" "SL" "SL" "SL" "IN" "SL" "SL" "SL" "BD" "SL" "SL" "SL" "IN"
## [16] "SL" "SL" "BD" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "MD" "SL" "SL"
## [31] "SL" "SL" "SL" "SL" "PK" "MD" "PK" "SL" "SL" "SL" "SL" "SL" "PK" "MD" "SL"
## [46] "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "MD" "MD"
```

BD- Bangladesh, IN-India, MD-Maldives, PK-Pakistan, SL- Sri Lanka

Construct a frequency table

The frequency distribution of native countries

Native Country

Count

Percentage (%)

Bangladesh

3

5.357

India

2

3.571

Maldives

5

8.929

Pakistan

3

5.357

Sri Lanka

43

76.786

Total

56

100.000

CASE II:

Example 2

The grades of 30 students for Statistics are as follows:

```
##  [1] "B" "C" "B" "D" "B" "C" "C" "A" "B" "C" "C" "B" "E" "B" "B" "D" "D" "F" "B"
## [20] "D" "D" "A" "B" "A" "B" "C" "E" "A" "A"
```

Construct a frequency table

The frequency distribution of grades for Statistics

Grade

Count

Percentage (%)

A

5

17.241

B

10

34.483

C

6

20.690

D

5

17.241

E

2

6.897

F

1

3.448

Total

29

100.000

CASE III:

Example 3

The number of family members of a sample of undergraduates of Batch 19 are as follows:

```
##  [1] 7 5 3 4 5 4 3 6 4 4 5 2 7 4 5 6 4 4 3 5
```

Construct a frequency table

The frequency distribution of the number of family members

Number of family members

Count

Percentage (%)

A

5

17.241

B

10

34.483

C

6

20.690

D

5

17.241

E

2

6.897

F

1

3.448

Total

29

100.000

CASE IV:

Example 4

The ages (in years) of a sample of undergraduates of Batch 19 are as follows:

```
##  [1] 21 22 22 23 22 24 24 23 21 22 23 22 22 23 21 21 22 23 22 23
```

Construct a frequency table

The frequency distribution of ages of undergraduates of Batch 19

Age (years)

Count

Percentage (%)

A

5

17.241

B

10

34.483

C

6

20.690

D

5

17.241

E

2

6.897

F

1

3.448

Total

29

100.000

6.2.1.2 Grouped frequency distribution

- A grouped frequency distribution (table) is obtained by constructing classes (or intervals) for the data and then listing the corresponding number of values in each interval.
- Suitable for quantitative variables with large number of possible values in the range of data.
- Note that when items have been grouped in this way, their individual values are lost.
- When studying about frequency distributions it is very important to know the meaning of the following terms

i Class intervals

- In a frequency distribution the total range of the observations are divided into a number of classes. Those are called *class intervals*
- Eg: Class intervals: 10-14, 15-19, 20-24, ..., 40-44

ii Class limits

- Class limits are the smallest and largest piece of data value that can fall into a given class.
- In the class interval 10-14, the end numbers, 10 and 14, are called class limits
- The smaller number (10) is the *lower class limit*
- The larger number (14) is the *upper class limit*

iii Class boundaries

- Class boundaries are obtained by adding the upper limit of one class interval to the lower limit of the next-higher class interval and dividing by 2.
- Class boundaries are also called **True class limits**
- Class boundaries **should not** *coincide with actual observations*

Class interval	Class boundaries
10 - 14	9.5 – 14.5
15 - 19	14.5 – 19.5
20 - 24	19.5 – 24.5
25 - 29	24.5 – 29.5
30 - 34	29.5 – 34.5
35 - 39	34.5 – 39.5
40 - 44	39.5 – 44.5

iv The size or width of a class interval

- The size or width of a class interval is the difference between the *lower and upper class boundaries*
- It is also referred to as the *class width, class size, or class length*
- Eg: The class width for the class 10-14 is = 14.5-9.5 = 5

v The class mark (Midpoint of the class)

- Midpoint of the class
- Also called as *class midpoint*
- Midpoint of the class = $\frac{\text{Lower limit+Upper limit}}{2}$

or

- Midpoint of the class = $\frac{\text{Lower boundary+Upper boundary}}{2}$

vi Open class intervals

- A class interval that, at least theoretically, has either no upper class limit or no lower class limit indicated is called an *open class interval*
- For example, referring to age groups of individuals, the class interval "65 year and over" is an open class interval

Rules and Practices for constructing grouped frequency tables

- Every data value should be in an interval
- The intervals should be mutually exclusive
- The classes of the distribution must be arrayed in size order.
- The number of classes not less than 5 or not greater than 15 is recommended.
- The following formula is often used to determine the number of classes: If n is the number of observations, then

$$\text{Number of classes} = \sqrt{n}$$

$$\text{Width of the class interval} = \frac{Range}{\sqrt{n}} = \frac{Min - Max}{\sqrt{n}}$$

- Data should be represented within classes having limits which the data can attain
- Classes should be continuous
- By convention, the beginning of the interval is given the appropriate exact value, rather than the end.
  Eg: intervals of 0-49, 50-99,100-149 would be preferred over the intervals 1-50, 51-100, 101-150 etc.
- The number f observations falling into each category or class interval (class frequency) can be easily found using *tally marks*.

Examples:

In a grouped frequency distribution, class intervals can be constructed in different ways

Example 1

Class interval	Number of students
10 - 14	4
15 - 19	5
20 - 24	11
25 - 29	9
30 - 34	6
35 - 39	3
40 - 44	2

Example 2

Salary	Number of employees
0 – 1999	1
2000 – 3999	31
4000 – 5999	18
6000 – 7999	4
8000 – 9999	2
10000 - 11999	1
12000 – 13999	0
14000 – 15999	0
16000 – 17999	1
18000 -19999	1
20000-21999	1

Salary	Number of employees
0 – 1999	1
2000 – 3999	31
4000 – 5999	18
6000 – 7999	4
8000 – 9999	2
10000 - 15999	1
16000 – 21999	3
Total	60

Example 3

Salary	Number of employees
Less than 2000	1
2000 – 2999	11
3000 – 3999	20
4000 – 5999	18
6000 – 9999	6
Greater than or equal to 10000	4
Total	60

6.2.1.3 Two-way frequency table

- Cross tabulation, Cross classification table, Contingency table, Two-way table

- Display the relationship between two or more qualitative variables (categorical variables (nominal or ordinal))

```
## # A tibble: 2 x 4
##   Survived First Second Third
##   <chr>    <dbl>  <dbl> <dbl>
## 1 died        80     97   372
## 2 Survived   136     87   119
```

```
## # A tibble: 2 x 4
##   Survived First Second Third
##   <chr>    <dbl>  <dbl> <dbl>
## 1 died      0.37   0.53  0.76
## 2 Survived  0.63   0.47  0.24
```

6.2.2 Graphic Presentations of Data

- A diagram is a visual form for presentation of statistical data.
- The form of the diagram varies according to the nature of the data

6.2.2.1 Describing Qualitative Data

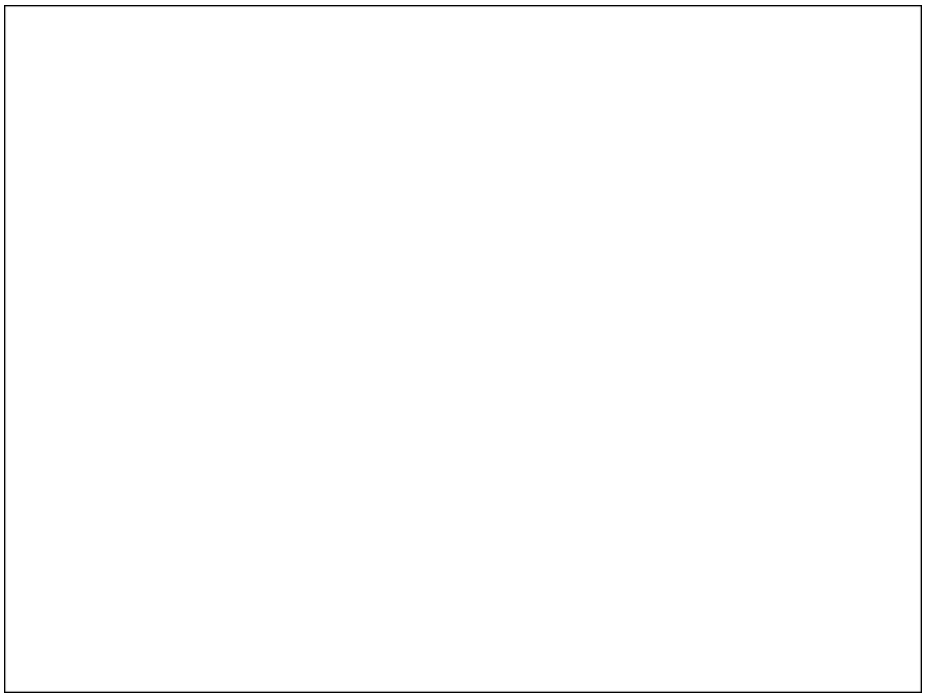
- Bar chart / Pie chart
- Suitable for
  - Qualitative variables (nominal or ordinal)
  - Discrete variables (when the number of bars or number of different values is small)

I Bar Chart

- A bar graph uses bars to represent discrete categories of data
- It can be drawn either on horizontal (more common) or vertical base
- A rectangle of equal width is drawn for each category
- The height (in vertical bar chart) or the length (in horizontal bar chart) of the rectangle is equal to the category's **frequency** or **percentage**.

i Simple Bar Chart

- Only one categorical variable can be presented
- Often used in conjunction with simple frequency tables
- The bars do not touch each other
- The gaps between adjacent bars are same in length



The frequency distribution of the Passenger class

Passenger class

Count

Percentage

First

216

24.242

Second

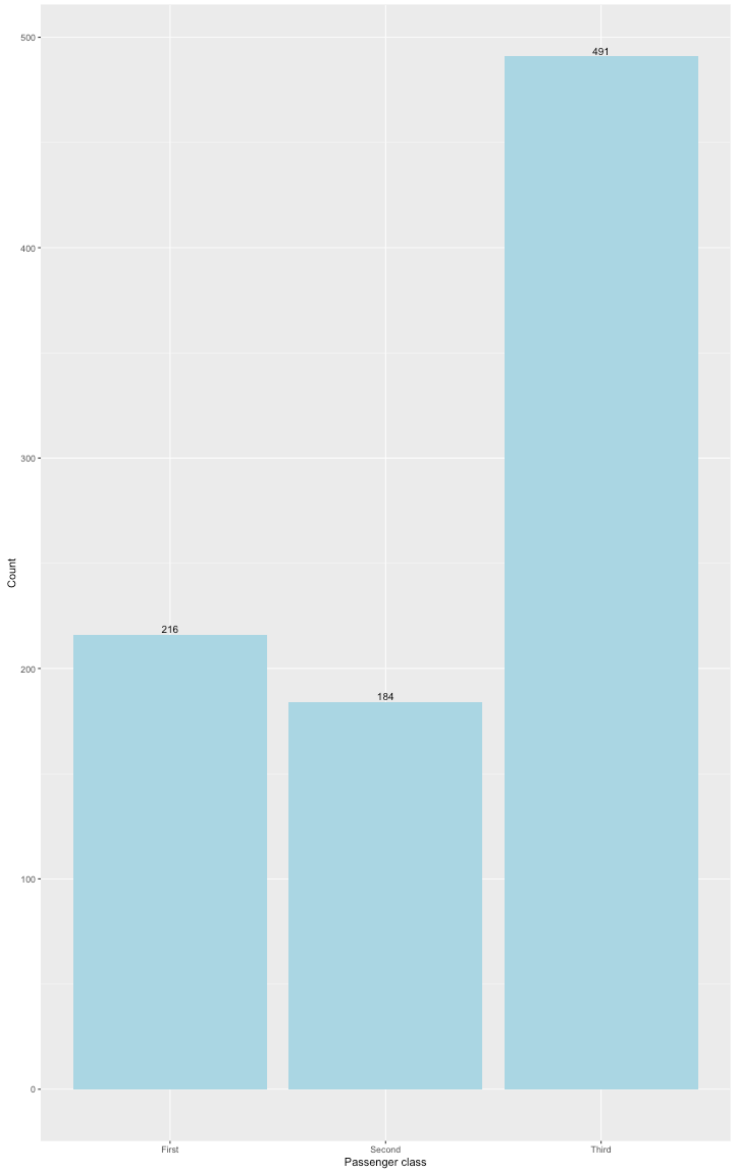
184

20.651

Third

491

55.107



ii Component Bar Chart

- **Sub divided bar chart/ Stacked bar chart**
- Use to compare two or more qualitative variables (nominal or ordinal)
- Often used in conjunction with two way tables
- Start by drawing a simple bar chart with the total figures.
- The bars are then divided into the component parts

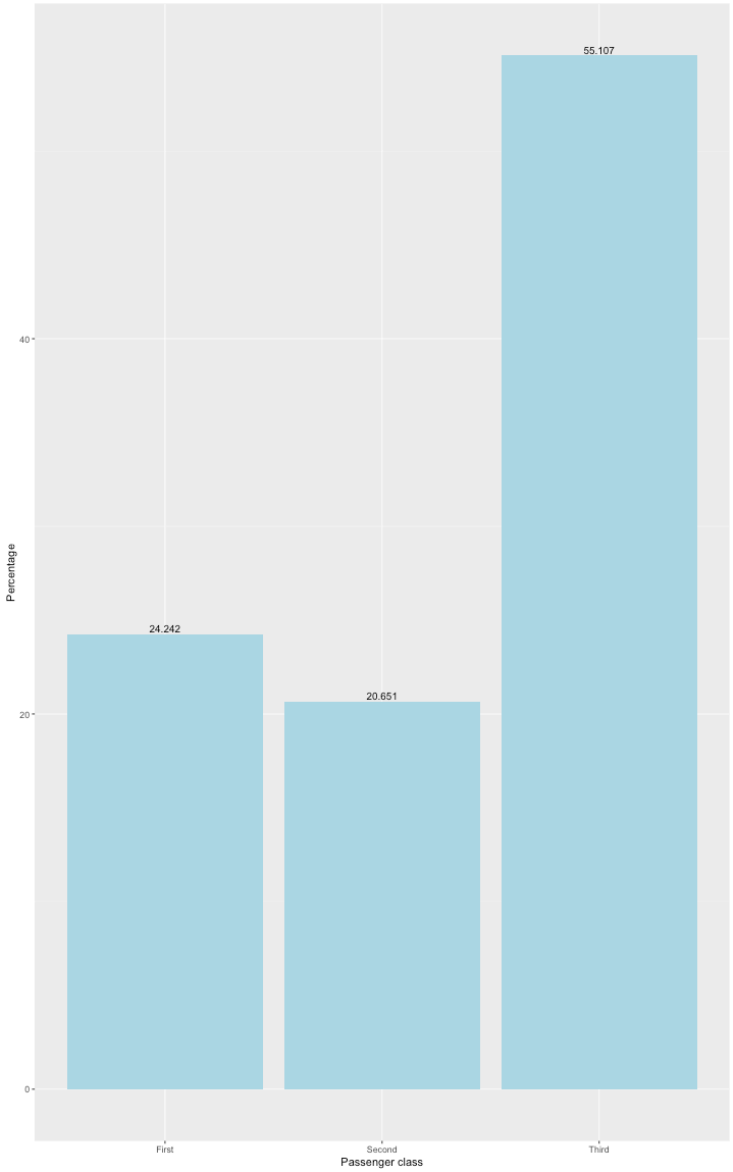
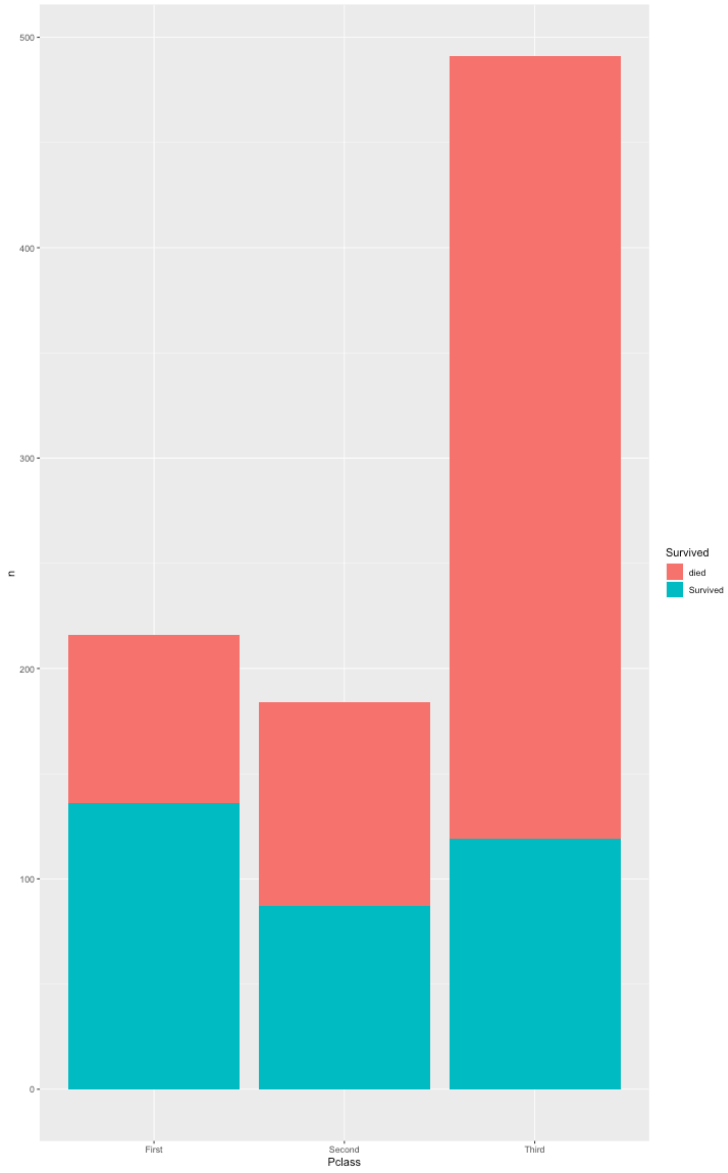


Figure 6.1: Composition of the passengers by passenger class (Sample size = 891)
(#fig:passengerP)

- Can be drawn on absolute figures or percentages
- The various components should be kept in the same order in each bar
- To distinguish different components from one another, different colours or shades can be used

```
## # A tibble: 2 x 4
##   Survived First Second Third
##   <chr>    <dbl>  <dbl> <dbl>
## 1 died        80     97   372
## 2 Survived   136     87   119
```



**Percentage component bar chart - When sub-divided bar chart is drawn on percentage basis it is called percentage bar chart - The various components are expressed as percentage to the total - All bars are equal in height

Chapter 7

Sets and Relations

Chapter 8

Probability

Chapter 9

Correlation and Regression