CM 1110 Fundamentals of Mathematics and Statistics

Dr. Priyanga D. Talagala

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Contents

Course Syllabus

Pre-requisites

None

Learning Outcomes

On successful completion of this module, students will be able to apply fundamental concepts in Mathematics and Statistics for real world problem solving.

Outline Syllabus

- Number Systems
- Sequences and Series
- Introduction to Logic
- Boolean Algebra
- Differentiation and Integration
- Descriptive Statistics
- Sets and Relations
- Probability
- Correlation and Regression

Method of Assessment

- Mid-semester examination
- End-semester examination

Lecturer

Dr. Priyanga D. Talagala

Schedule

Lectures:

- Monday [1.15 pm - 4.30 pm]

Tutorial:

- Thursday [1.15 pm - 4.30 pm]

Consultation time:

- Tuesday [11.30 am to 12.30 pm]

Chapter 1

Number Systems

Numbers can be classified according to how they are represented or according to the properties that they have.

1.1 Main types

1.1.1 Complex numbers

- Every number in number system is considered as a complex number

- A number of the form $a + ib$ is called a complex number when a and b are real numbers and $i = \sqrt{-1}$.
- For a given complex number, $a + ib$, 'a' is known as the real part and 'b' is known as the imaginary part.
- If $a = 0$, the number ib is said to be purely imaginary, if $b = 0$ the number a is real.
- A pair of complex number $a + ib$ and $a - ib$ are said to be conjugate of each other.

Show that the sum and product of a complex number and its conjugate complex are both real.

- Let $a + ib$ and $c + id$ be two complex numbers. Then

Addition. $(a + ib) + (c + id) = (a + c) + i(b + d)$

Subtraction $(a + ib) - (c + id) = (a - c) + i(b - d)$

Multiplication $(a + ib) \times (c + id) = ac - bd + i(ad + bc)$

Addition. $\frac{a+ib}{c+id} = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$

- Complex numbers are denoted by $\mathbb{C}$.

1.1.2   Imaginary numbers

- A number that does not exist in the number line is known as imaginary number.
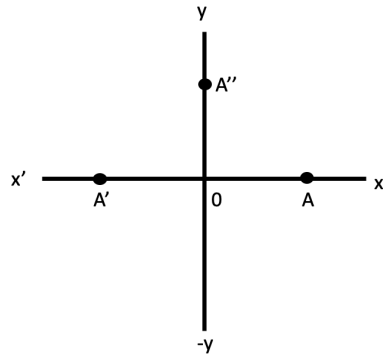
- For example, square root of negative numbers are imaginary numbers. It is denoted by i. i.e

$$\sqrt{-1} = i$$

$$i^2 = 1$$

- So there is no real number i that satisfies the above equation.
- The quantity, i is called the unit imaginary number.

Geometrical Representation of imaginary numbers

- Let OA be positive numbers which is represented by x and OA' by $-x$.
- And $-x = (i)^2 x = i(ix)$ is on OX'.
- According to the above expression, the multiplication of the real number x by i twice amounts is equivalent to the rotation of OA through two right angles to reach OA'.
- Therefore, the multiplication of x by i is equivalent to the rotation of x through one right angle to reach OA''.
- Therefore, y-axis is known as imaginary axis.
- Multiplication by i rotates its direction through right angle.

1.1.3 Real numbers

- All numbers that can be represented on the number line are known as real numbers.
- The real numbers is the set of numbers containing all of the rational numbers and all of the irrational numbers.
- Real Numbers are denoted by $\mathbb{R}$.

1.1.4   Rational numbers

- Rational Numbers are denoted by $\mathbb{Q}$

- A rational number is defined as number of the form x/y where x and y are integers and $y \neq 0$.

- The set of rational numbers encloses the set of integers and fractions.

- The rational numbers that are not integral will have decimal values. These values can be of two types

    - Terminating decimal fractions (finite decimal factors): For example $1/5 = 0.5$, $13/5 = 2.6$.
    - Non Terminating decimal fractions. The non terminating decimal fractions having two types:
        - i) Non terminating periodic fractions
        - ii) Non terminating non periodic fractions

i) Non terminating periodic fractions

a. These are non terminating decimal fractions of the type $a.b1b2b3b4b5.....bmb1b2b3b4b5.....bm$

b. Examples

$19/6 = 3.16666666.....$

$18/7 = 2.57142857142857.......$

$21/9 = 2.3333.......$

ii) Non terminating non periodic fractions

a. These are non terminating and there is no periodic decimal places for that number.
b. i.e $a.b1b2b3b4b5.....bmc1c2.........$
c. for example $6.789542587436512..........$

- **So from above terminating and non terminating periodic fraction numbers belongs to rational numbers.**

1.1.5   Irrational numbers

- Irrational numbers are denoted by $\mathbb{I}$
- Irrational numbers are consisted with **non terminating and non periodic fractions**.

- i.e irrational number is a number that cannot be written as a ratio x/y form (or fraction).
- In decimal form, it never ends or repeats.
- Examples for irrational numbers are $\sqrt{2}$ = 1.414213......, π = 3.14159265......., $\sqrt{3}$, $\sqrt{5}$ etc.

1.1.6 Integers

- All numbers that do not have the decimal places in them are called integers.
- $\mathbb{Z} = \{..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...\}$
- i.e it may be positive or negative or zero.
- Integers are denoted by $\mathbb{Z}$.
- Any integers are added, subtracted, or multiplied the result is always is an integer.
- When any integers multiplied , each of the multiplied integer is called a factor or divisor of the resulting product.

1.1.7 Whole numbers

- The set of whole numbers means narrator numbers and 0
- Whole numbers = $\mathbb{W} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, ...\}$

1.1.8 Natural numbers

- The counting numbers start with 1 and their end is not defined.
- i.e $\mathbb{N} = \{1, 2, 3, 4, ...\}$

Reading :

Dass, H. K. (2008). 'Complex Numbers', *Advanced Engineering Mathematics.* S. Chand Publishing. pp. 474-520.

1.2   Number representations

1.2.1   Glossary of terms used in the positional numeral systems

- There are various types of the number system in mathematics.

- The four most common number system types are:

    - Decimal number system (Base- 10)
    - Binary number system (Base- 2)
    - Octal number system (Base-8)
    - Hexadecimal number system (Base- 16)

System	Radix/ Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

1.2.2   Decimal Number System

- The Decimal number system is a number of base or radix equal to 10.

- To determine the actual number in each position, take the number that appears in the position and multiply it by 10^x, where x is the power representation.

Example 1: The value of the combination of symbols 453 is determined by adding the weights of each position as

$$4 \times 10^2 + 5 \times 10^1 + 3 \times 10^0$$

$$= 4 \times 100 + 5 \times 10 + 3 \times 1$$

Or

$$= 400 + 50 + 3 = 453$$

Example 2: The value of the combination of symbols 369.54 is determined by adding the weights of each position as

$$3 \times 10^2 + 6 \times 10^1 + 9 \times 10^0 + 5 \times 10^{-1} + 4 \times 10^{-2}$$

Or

$$= 300 + 60 + 9 + \frac{5}{10} + \frac{4}{100}$$

Or

$$= 300 + 60 + 9 + 0.5 + 0.04 = 369.54$$

1.2.3 Binary Number System

- The binary number system is a number system of base or radix equal to 2.
- In the binary number system, there are two symbols to represent number: 0 and 1
- When the symbols 0 an 1 are used to represent binary number, each symbol is called a binary digit or a bit.
- Therefore, the binary number 1011 is a four-digit number or a 4-bit binary number.

1.2.3.1 Binary-to-Decimal Conversion

- Multiply binary digit (1 or 0) in each position by the weight of the position and add the results.

Example 1: Convert the binary number 11010 to its decimal equivalent

Example 2: Convert the binary number 0.011 to its decimal equivalent

Example 3: Convert the binary number 110.011 to its decimal equivalent

1.2.3.2   Decimal-to-Binary Conversion

- To convert decimal numbers to their binary equivalent, the following procedures are employed:

1.2.3.2.1   Whole number conversion: Repeated division by 2

- The remainder resulting from each division forms binary number.
- The first remainder to be produced is called the least significant bit (LSB) and the last remainder is called most significant bit (MSB).

Example 1: Convert the decimal number 17 to binary

1.2.3.2.2   Fractional number conversion: Repeated multiplication by 2

- Multiply any factional part repeatedly by 2.
- The equivalent binary number is formed from the 1 or 0 in the units position (10^0 position).

Example 1: Convert the decimal number 0.625 to binary

- Sometimes it will be necessary to terminate the multiplication when an acceptable degree of accuracy is obtained. Then the resulted binary number will be an approximation.

Example 2: Convert the decimal number 0.6375 to binary

- We see that it continues in this way and does not terminate.

- So, $0.6375_{10} = (0.1010001101....)_2$.

- If we discard all the bits after the 7th bit, then we get the approximate representation $0.6375_{10} \approx (0.1010001)_2$, committing an error amounting to

$$0.6375_{10} - (0.1010001)_2 = 0.6375_{10} - 0.6328125_{10} = (.0046875)_{10},$$

which is known as ***round-off error***.

- Although, strictly, this is *chopping-off error*, it is generally, termed as *round-off error*.

Example 3: Convert the decimal number 49.683 to binary

1.2.4 Octal Number System

- The octal system is a number system of base or radix equal to 8.

1.2.4.1 Octal-to-Decimal Conversion

Example 1: Convert the following octal numbers to their decimal equivalent

(a) 35_8
(b) 100_8
(c) 0.24_8

1.2.4.2   Decimal-to-Octal Conversion

- To convert decimal numbers to their octal equivalent, the following procedures are employed:

1.2.4.2.1   Whole number conversion: Repeated division by 8

Example 1: Convert the decimal number 245 to octal equivalent

1.2.4.2.2   Fractional number conversion: Repeated multiplication by 8

Example 2: Convert the decimal fraction 0.432 to octal equivalent

- This conversion to octal is not precise as there is a remainder.
- If greater accuracy is required, continue multiplying by 8 to obtain more octal digits.

Example 3: Convert the decimal number 419.95 to octal equivalent

1.2.4.3   Octal-to-Binary Conversion

Octal and binary number correspondence

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

- To convert from octal to binary, simply replace each octal digit with the corresponding three-digit binary number.

Example 1: Convert the following octal numbers to their binary equivalent

(a) 247_8

(b) 124.375_8

1.2.4.4 Binary-to-Octal Conversion

- To convert from binary to octal, subdivide the number into groups of three bits, proceeding both left and right from the binary point, and if necessary padding the last group with zero. The octal representation of each group gives the required octal number.

Example 1: Convert the binary number 11010101.01101 to its octal equivalent

1.2.5   Hexadecimal Number System

- The Hexadecimal system is a number system of base or radix equal to 16.
- Hexadecimal is similar to the octal numeral system because each can be easily compared to the binary numeral system.
- Example values of hexadecimal numbers converted into binary, octal and decimal.

Hexadecimal	Binary	Octal	Decimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
A	1010	12	10
B	1011	13	11
C	1100	14	12
D	1101	15	13
E	1110	16	14
F	1111	17	15

1.2.5.1   Hexadecimal-to-Decimal Conversion

Example 1: Convert the following octal numbers to their decimal equivalent

(a) $C7_{16}$
(b) $2F_{16}$

1.2.5.2   Decimal-to-Hexadecimal Conversion

- To convert decimal numbers to their hexadecimal equivalent, the following procedures are employed:

1.2.5.2.1   Whole number conversion: Repeated division by 16

Example 1: Convert the decimal number 245 to Hexadecimal equivalent

1.2.5.2.2 Fractional number conversion: Repeated multiplication by 16

Example 2: Convert the decimal fraction 0.0738 to Hexadecimal equivalent

Example 3: Convert the decimal number 420.0095 to Hexadecimal equivalent

1.2.5.3   Hexadecimal-to-Binary Conversion

- To convert from Hexadecimal to binary, simply replace each Hexadecimal digit with the corresponding four-digit binary number.

Example 1: Convert the following Hexadecimal numbers to their binary equivalent

(a) $7B_{16}$
(b) $6D.8C_{16}$

1.2.5.4 Binary-to-Hexadecimal Conversion

- To convert from binary to Hexadecimal, subdivide the number into groups of four bits. The hexadecimal representation of each group then gives the hexadecimal representation.

Example 1: Convert the binary number 11010101.01101 to its Hexadecimal equivalent

1.2.5.5 Octal-to-Hexadecimal Conversion

- First convert octal to binary, and then convert binary to hexadecimal.

Example 1: Convert the octal number 365.52 to its Hexadecimal equivalent

1.2.5.6 Hexadecimal-to-octal conversion

- First convert Hexadecimal to binary, and then convert binary to octal.

Example 1: Convert the octal number D2B.284 to its Hexadecimal equivalent

1.2.6 Binary Arithmetic

The arithmetic operations: addition, subtraction, multiplication and division of binary numbers follow the rules as summarized below

Addition	Subtraction	Multiplication	Division
$0 + 0 = 0$	$0 - 0 = 0$	$0 \times 0 = 0$	$\frac{0}{1} = 0$
$0 + 1 = 1$	$1 - 0 = 1$	$0 \times 1 = 0$	$\frac{1}{1} = 1$
$1 + 0 = 1$	$1 - 1 = 0$	$1 \times 0 = 0$	$\frac{0}{0} = undefined$
$1 + 1 = 10$	$1 - 0 = 10 - 1 = 1$	$1 \times 1 = 1$	$\frac{1}{0} = undefined$

1.2.6.1 Binary Addition

Example 1: Add (a) 111 and 101 (b) 1010, 1001 and 1101

1.2.6.2 Binary Subtraction

Example 1: Perform the following subtractions

(a) $11 - 01$ (b) $11 - 10$ (c) $100 - 011$

- When subtracting a larger number from a smaller number, the output will be negative.

- To perform this subtraction, subtract the smaller number from the larger number and prefix the output with the sign of the larger number.

Example 2: Perform the following subtraction 101 - 111

1.2.6.3 Binary multiplication

Example 1: Multiply the following binary numbers

(a) 101×11 (b) 1101×10 (c) 1010×101
(b) 1011×1010

- Multiplication of factional numbers is perform in the same manner as with decimal factional numbers.

Example 2: Perform the binary multiplication 0.01×11

1.2.6.4   Binary Division

Example 1: Perform the following binary division

(a) $110 \div 11$ (b) $1100 \div 11$ (c) $1111 \div 110$ (d) $1100 \div 101$

Chapter 2

Sequences and Series

2.1 Sequences

- A **Sequence** can be thought of as a list of numbers written in a definite order:
$$a_1, a_2, a_3, a_4, ..., a_n, ...$$
- The number a_1 is called the *first term*, a_2 is the *second term*, and in general a_n is called the nth *term*.
- We will deal exclusively with infinite sequences and so each term a_n will have a successor a_{n+1}
- Notice that for every positive integer n there is a corresponding number a_n and so a sequence can be defined as a function whose domain is the set of positive integers.
- But we usually write a_n instead of the function notation $f(n)$ for the value of the function at the number n.
- **Notation:** The sequence $\{a_1, a_2, a_3, ...\}$ is also denoted by
$$\{a_n\}$$
  or
$$\{a_n\}_{n=1}^{\infty}$$
- Some sequences can be defined by giving a formula for the nth term.
- In the following examples we give three descriptions of the sequences:

  i. by using the preceding notation
  ii. by using the defining formula
  iii. by writing out the terms of the sequence

- Notice that n doesn't have to start at 1.

a) $\{\frac{n}{n+1}\}_{n=1}^{\infty}$ $\qquad\qquad a_n = \frac{n}{n+1}$ $\qquad\qquad \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ..., \frac{n}{n+1}, ...\}$

b) $\{\frac{(-1)^n(n+1)}{3^n}\}_{n=1}^{\infty}$ $a_n = \frac{(-1)^n(n+1)}{3^n}$ $\{-\frac{2}{3}, \frac{3}{9}, -\frac{4}{27}, \frac{5}{81}, ..., \frac{(-1)^n(n+1)}{3^n}, ...\}$

c) $\{\sqrt{n-3}\}_{n=3}^{\infty}$ $a_n = \sqrt{n-3}, n \geq 3$ $\{0, 1, \sqrt{2}, \sqrt{3}, ..., \sqrt{n-3}, ...\}$

d) $\{cos\frac{n\pi}{6}\}_{n=0}^{\infty}$ $a_n = cos\frac{n\pi}{6}, n \geq 0$ $\{1, \frac{\sqrt{3}}{2}, \frac{1}{2}, 0, ..., cos\frac{n\pi}{6}, ...\}$

Example

Find a formula for the general term a_n of the sequence

$$\{\frac{3}{5}, -\frac{4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, ...\}$$

assuming that the pattern of the first few terms continues.

SOLUTION

We are given that

2.2   Series

Reading:

Stewart, J., Clegg, D. K., & Watson, S. (2020). 'Infinite Sequences and Series', *Calculus: early transcendentals.* Cengage Learning.

Chapter 3

Introduction to Logic

Chapter 4

Boolean Algebra

Chapter 5

Differentiation and Integration

Chapter 6

Descriptive Statistics

6.1 Introduction to Statistics

6.1.1 Some Basic Terminologies Used in Statistics

i Population

- The set of **all** possible elements in the universe of interest to the researcher

ii Sample

- A Sample is a **subset** (a portion or part) of the population of interest
- The sample must be a representative of the population of interest

iii Element

- Element is an **entity or object** which the information is collected.
- *Eg: Student, household, farm, company, tomato plant*

iv Variable

- A variable is **a feature characteristic which has different 'values' or categories for different elements** (items/subjects/individuals)
- *Eg: Gender of client, brand of mobile phones, risk level, number of emails received per day, age of client, income of client*

v Data

- Data are **measurements or facts** that are collected from a statistical unit/entity of interest

- We collect data on variables

- Data are raw numbers or facts that must be processed (analysed) to get useful information.

- We get information from data.

- *Eg:*

Variable: *Age (in years) of client*

Data: *21, 45, 18, 32, 30, 22, 23, 27*

Information:

The mean age is 27.25 years

The minimum age is 18 years

The range of ages is 18-45

The percentage of clients below 25 years of age: 50%

vi Statistic

- **Characteristic** of a **sample**
- The value which calculated based on sample data

vii Parameter

- **Characteristic** of a **population**
- The value which calculated based on population data

viii Census

- When a researcher **gathers data from the whole population for a given measurement,** it is called a census

ix Sampling

- When a researcher **gathers data from a sample of the population for a given measurement,** it is called sampling
- The process of selecting a sample is also called sampling

Why take a sample instead of studying every member of the population ?

- Prohibitive cost of census
- Destruction of item being studied may be required
- Not possible to test or inspect all members of a population being studied.

6.1.2 Branches of Statistics

i Descriptive Statistics

- Descriptive statistics consists of organizing, summarizing and presenting data in an informative way.
- The main purpose of descriptive statistics is to provide an overview of the data collected.
- Descriptive statistics describes the data collected through frequency tables, graphs and summary measures (mean, variance, quartiles, etc.).

ii Inferential Statistics

- In inferential statistics sample data are used to draw inferences (i.e. derive conclusions) or make predictions about the populations from which the sample has been taken.
- This includes methods used to make decisions, estimates, predictions or generalizations about a population based on a sample.
- This includes point estimations, interval estimation, test of hypotheses, regression analysis, time series analysis, multivariate analysis, etc.

6.1.3   Types of Variables

6.1.3.1   Qualitative / Quantitative Variables

i Qualitative variable (Categorical variable)

- The characteristic is a quality.
- The data are categories.
- They cannot be given numerical values.
- However, it may be given a numerical label
- Qualitative variables are sometimes referred as categorical variables.
- *Eg:*

Gender:

Age group:

Education level:

A/L stream:

Degree type:

Hair colour:

FIT student batch:

Undergraduate level:

Grade that you can obtain for CM 1110/ CM1130

ii Quantitative variable

- The characteristic is a quantity
- The data are numbers
- Quantitative data require numeric values that indicate how much or how many.
- They are obtained by counting or measuring with some scale
- *Eg:*

Number of family members:

Number of emails received per day:

Weight of a student:

Age:

Credit balance in the SIM card:

Time remaining in class:

Temperature:

Marks

6.1.3.2 Discrete/ Continuous Variables

- Quantitative variables can be classified as either discrete or continuous.

i Discrete Variables

- Quantitative
- Usually the data are obtained by counting
- There are impossible values between any two possible values
- *Eg:*

Number of family members:

Number of emails received per day:

ii Continuous Variables

- Quantitative
- Usually, the data are obtained by measuring with a scale

- There are no impossible values between any two possible values.(any value between any two possible values is also a possible value)
- i.e a continuous variable can take any value within a specified range.
- *Eg:*

Weight of a student:

Age:

Credit balance in the SIM card:

Time remaining in class:

Temperature:

Marks

6.1.4   Scales of Measurements

- There are four levels of measurements called, **nominal, ordinal, interval and ratio.**
- Each levels has its own rules and restrictions
- Different levels of measurement contains different amount of information with respect to whatever the data are measuring

i Nominal Scale

- Qualitative
- No order or ranking in categories.
- These categories have to be mutually exclusive, i.e. it should not be possible to place an individual or object in more than one category
- A name of a category can be substituted by a number, but it will be mere label and have no numerical meaning

ii Ordinal Scale

- Qualitative
- Categories can be ordered or ranked
- A name of a category can be substituted by a number, but such a sequence does not indicate absolute quantities.
- Difference between any two numbers on the scale does not have a numerical meaningful.
- It cannot be assumed that the differences between adjacent numbers on the scale are equal.

iii Interval Scale

- Quantitative
- Data can be ordered or ranked
- There is no absolute zero point. Zero is only an arbitrary point with which other values can compare
- Difference between two numbers is a meaningful numerical value
- Ration of two numbers is not a meaningful numerical value.

iv Ratio Scale

- Quantitative
- Highest level of measurement
- There exist an absolute zero point (It has a true zero point)
- Ratio between different measurements is meaningful

6.2   Presentation of Data

The sinking of the Titanic is one of the most infamous shipwrecks in history.

On April 15, 1912, during her maiden voyage, the widely considered "unsinkable" RMS Titanic sank after colliding with an iceberg. Unfortunately, there weren't enough lifeboats for everyone onboard, resulting in the death of 1502 out of 2224 passengers and crew

[1]

Here's a quick summary of our variables:

Variable Name	Description
PassengerID	Passenger ID (just a row number, so obviously not useful for prediction)
Survived	Survived (1) or died (0)
Pclass	Passenger class (first, second or third)
Name	Passenger name
Gender	Passenger Gender
Age	Passenger age
SibSp	Number of siblings/spouses aboard
Parch	Number of parents/children aboard
Ticket	Ticket number
Fare	Fare
Cabin	Cabin
Embarked	Port of embarkation (S = Southampton, C = Cherbourg, Q = Queenstown)

6.2.1   Tabular Presentations of Data

Raw Data

- Raw data are collected data that have not been organized numerically
- Eg: Passenger age

```
##   PassengerId Survived Pclass
## 1           1        0      3
## 2           2        1      1
```

---

[1]Data source: https://www.kaggle.com/varimp/a-mostly-tidyverse-tour-of-the-titanic

```
## 3              3         1        3
## 4              4         1        1
## 5              5         0        3
## 6              6         0        3
##                                                           Name    Sex Age SibSp Parch
## 1                                   Braund, Mr. Owen Harris    male  22      1      0
## 2 Cumings, Mrs. John Bradley (Florence Briggs Thayer) female  38      1      0
## 3                               Heikkinen, Miss. Laina female  26      0      0
## 4       Futrelle, Mrs. Jacques Heath (Lily May Peel) female  35      1      0
## 5                             Allen, Mr. William Henry    male  35      0      0
## 6                                     Moran, Mr. James    male  NA      0      0
##             Ticket    Fare Cabin Embarked
## 1        A/5 21171  7.2500              S
## 2         PC 17599 71.2833   C85        C
## 3 STON/O2. 3101282  7.9250              S
## 4           113803 53.1000  C123        S
## 5           373450  8.0500              S
## 6           330877  8.4583              Q
```

```
##  [1] 22 38 26 35 35 NA 54  2 27 14  4 58 20 39 14 55  2 NA 31 NA 35 34 15 28  8
## [26] 38 NA 19 NA NA 40 NA NA 66 28 42 NA 21 18 14
```

An array

- An array is an arrangement of raw numerical data in ascending or descending order of magnitude.
- Eg: Passenger age

```
##  [1]  2  2  4  8 14 14 14 15 18 19 20 21 22 26 27 28 28 31 34 35 35 35 38 38 39
## [26] 40 42 54 55 58 66
```

Frequency Table (Frequency Distributions)

- A frequency table (frequency distribution) is a listing of the values a variable takes in a data set, along with how often (frequently) each value occurs
- frequency can be recorded as a

  - **frequency or count:** the number of times a value occurs, or
  - **percentage frequency:** the percentage of times a value occurs

- Percentage frequency can be calculated as,

$$Percentage frequency = \frac{a}{b} \times 100\%$$

- The objective of constructing a frequency table are as follows

  - to organize the data in a meaningful manner
  - to determine the nature or shape of the distribution
  - to draw charts and graphs for the presentation of data
  - to facilitate computational procedures for measures of average and spread
  - to make comparisons between different data sets

- There are two basic types of frequency tables

  1. Simple frequency tables (Ungrouped frequency distribution)
  2. Grouped frequency distribution

6.2.1.1   Simple frequency table (Ungrouped frequency distribution)

- Each possible value or category is taken as a class
- More suitable for

  - Qualitative variables
  - Discrete variables

- Sometimes construct for continuous variables when there is a small number of possible values between the minimum and maximum.

Examples:

CASE I:

Example 1

The native countries of 56 students from a certain education institute are as follows:

```
##  [1] "SL" "BD" "SL" "SL" "SL" "SL" "IN" "SL" "SL" "SL" "BD" "SL" "SL" "SL" "IN"
## [16] "SL" "SL" "BD" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "MD" "SL" "SL"
## [31] "SL" "SL" "SL" "SL" "PK" "MD" "PK" "SL" "SL" "SL" "SL" "SL" "PK" "MD" "SL"
## [46] "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "SL" "MD" "MD"
```

BD- Bangladesh, IN-India, MD-Maldives, PK-Pakistan, SL- Sri Lanka

Construct a frequency table

```
##   Native Country Count Percentage (%)
##        Bangladesh     3          5.357
##             India     2          3.571
##          Maldives     5          8.929
##          Pakistan     3          5.357
##         Sri Lanka    43         76.786
##             Total    56        100.000
```

CASE II:

Example 2

The grades of 30 students for Statistics are as follows:

```
##  [1] "B" "C" "B" "D" "B" "C" "C" "A" "B" "C" "C" "B" "E" "B" "B" "D" "D" "F" "B"
## [20] "D" "D" "A" "B" "A" "B" "C" "E" "A" "A"
```

Construct a frequency table

```
##   Grade Count Percentage (%)
##     A     5         17.241
##     B    10         34.483
##     C     6         20.690
##     D     5         17.241
##     E     2          6.897
##     F     1          3.448
## Total    29        100.000
```

CASE III:

Example 3

The number of family members of a sample of undergraduates of Batch 19 are as follows:

```
##  [1] 7 5 3 4 5 4 3 6 4 4 5 2 7 4 5 6 4 4 3 5
```

Construct a frequency table

```
##  Number of family members Count Percentage (%)
##                         2     1              5
##                         3     3             15
##                         4     7             35
##                         5     5             25
##                         6     2             10
##                         7     2             10
##                     Total    20            100
```

CASE IV:

Example 4

The ages (in years) of a sample of undergraduates of Batch 19 are as follows:

```
##  [1] 21 22 22 23 22 24 24 23 21 22 23 22 22 23 21 21 22 23 22 23
```

Construct a frequency table

```
##  Age (years) Count Percentage (%)
##           21     4             20
##           22     8             40
##           23     6             30
##           24     2             10
##        Total    20            100
```

6.2.1.2   Grouped frequency distribution

- A grouped frequency distribution (table) is obtained by constructing classes (or intervals) for the data and then listing the corresponding number of values in each interval.
- Suitable for quantitative variables with large number of possible values in the range of data.
- Note that when items have been grouped in this way, their individual values are lost.
- When studying about frequency distributions it is very important to know the meaning of the following terms

i Class intervals

- In a frequency distribution the total range of the observations are divided into a number of classes. Those are called *class intervals*
- Eg: Class intervals: 10-14, 15-19, 20-24, ..., 40-44

ii Class limits

- Class limits are the smallest and largest piece of data value that can fall into a given class.
- In the class interval 10-14, the end numbers, 10 and 14, are called class limits
- The smaller number (10) is the *lower class limit*
- The larger number (14) is the *upper class limit*

iii Class boundaries

- Class boundaries are obtained by adding the upper limit of one class interval to the lower limit of the next-higher class interval and dividing by 2.
- Class boundaries are also called **True class limits**
- Class boundaries **should not** *coincide with actual observations*

Class interval	Class boundaries
10 - 14	9.5 – 14.5
15 - 19	14.5 – 19.5
20 - 24	19.5 – 24.5
25 - 29	24.5 – 29.5
30 - 34	29.5 – 34.5
35 - 39	34.5 – 39.5
40 - 44	39.5 – 44.5

iv The size or width of a class interval

- The size or width of a class interval is the difference between the *lower and upper class boundaries*
- It is also referred to as the *class width, class size, or class length*
- Eg: The class width for the class 10-14 is = 14.5-9.5 = 5

v The class mark (Midpoint of the class)

- Midpoint of the class
- Also called as *class midpoint*
- Midpoint of the class $= \frac{\text{Lower limit}+\text{Upper limit}}{2}$

or

- Midpoint of the class $= \frac{\text{Lower boundary}+\text{Upper boundary}}{2}$

vi Open class intervals

- A class interval that, at least theoretically, has either no upper class limit or no lower class limit indicated is called an *open class interval*

- For example, referring to age groups of individuals, the class interval "65 year and over" is an open class interval

Rules and Practices for constructing grouped frequency tables

- Every data value should be in an interval
- The intervals should be mutually exclusive
- The classes of the distribution must be arrayed in size order.
- The number of classes not less than 5 or not greater than 15 is recommended.

- The following formula is often used to determine the number of classes: If n is the number of observations, then

$$\text{Number of classes} = \sqrt{n}$$

$$\text{Width of the class interval} = \frac{Range}{\sqrt{n}} = \frac{Min - Max}{\sqrt{n}}$$

- Data should be represented within classes having limits which the data can attain
- Classes should be continuous
- By convention, the beginning of the interval is given the appropriate exact value, rather than the end.
  Eg: intervals of 0-49, 50-99,100-149 would be preferred over the intervals 1-50, 51-100, 101-150 etc.
- The number f observations falling into each category or class interval (class frequency) can be easily found using *tally marks.*

Examples:

In a grouped frequency distribution, class intervals can be constructed in different ways

Example 1

Class interval	Number of students
10 - 14	4
15 - 19	5
20 - 24	11
25 - 29	9
30 - 34	6
35 - 39	3
40 - 44	2

Example 2

Salary	Number of employees
$0 - 1999$	1
$2000 - 3999$	31
$4000 - 5999$	18
$6000 - 7999$	4
$8000 - 9999$	2

Salary	Number of employees
10000 - 11999	1
12000 − 13999	0
14000 − 15999	0
16000 − 17999	1
18000 -19999	1
20000-21999	1

Salary	Number of employees
0 − 1999	1
2000 − 3999	31
4000 − 5999	18
6000 − 7999	4
8000 − 9999	2
10000 - 15999	1
16000 − 21999	3
Total	60

Example 3

Salary	Number of employees
Less than 2000	1
2000 − 2999	11
3000 − 3999	20
4000 − 5999	18
6000 − 9999	6
Greater than or equal to 10000	4
Total	60

6.2.1.3 Two-way frequency table

- Cross tabulation, Cross classification table, Contingency table, Two-way table
- Display the relationship between two or more qualitative variables (categorical variables (nominal or ordinal))

```
## # A tibble: 2 x 4
##   Survived First Second Third
##   <chr>    <dbl>  <dbl> <dbl>
## 1 died        80     97   372
```

```
## 2 Survived   136     87    119
```

```
## # A tibble: 2 x 4
##   Survived First Second Third
##   <chr>    <dbl>  <dbl> <dbl>
## 1 died      0.37   0.53  0.76
## 2 Survived  0.63   0.47  0.24
```

6.2.2   Graphic Presentations of Data

- A diagram is a visual form for presentation of statistical data.
- The form of the diagram varies according to the nature of the data

6.2.2.1   Describing Qualitative Data

- Bar chart / Pie chart
- Suitable for

  - Qualitative variables (nominal or ordinal)
  - Discrete variables (when the number of bars or number of different values is small)

I Bar Chart

- A bar graph uses bars to represent discrete categories of data
- It can be drawn either on horizontal (more common) or vertical base
- A rectangle of equal width is drawn for each category
- The height (in vertical bar chart) or the length (in horizontal bar chart) of the rectangle is equal to the category's **frequency** or **percentage**.

i Simple Bar Chart

- Only one categorical variable can be presented
- Often used in conjunction with simple frequency tables
- The bars do not touch each other
- The gaps between adjacent bars are same in length

Passenger class	Count	Percentage
First	216	24.242
Second	184	20.651
Third	491	55.107

Composition of passengers by passenger class

Composition of passengers by passenger class
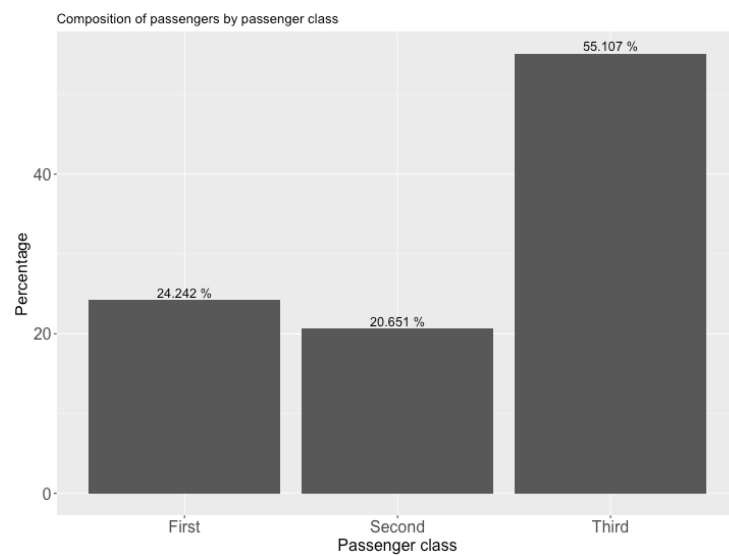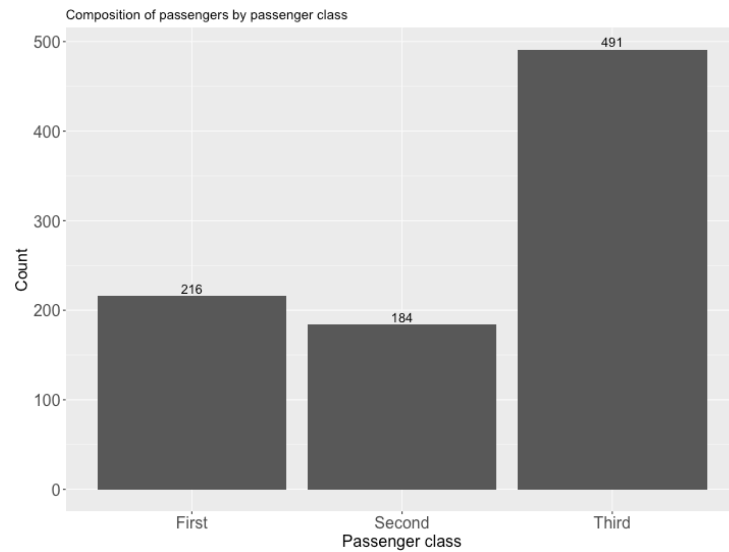
ii Component Bar Chart

- **Sub divided bar chart/ Stacked bar chart**
- Use to compare two or more qualitative variables (nominal or ordinal)
- Often used in conjunction with two way tables
- Start by drawing a simple bar chart with the total figures.
- The bars are then divided into the component parts
- Can be drawn on absolute figures or percentages

- The various components should be kept in the same order in each bar
- To distinguish different components from one another, different colours or shades can be used

Survived	First	Second	Third
died	80	97	372
Survived	136	87	119

Composition of the passengers by passenger class and survival condition
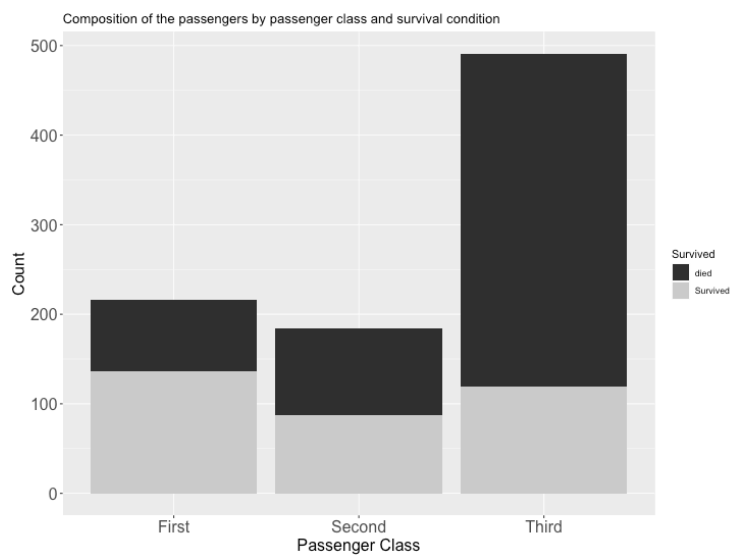
Percentage component bar chart

- When sub-divided bar chart is drawn on percentage basis it is called percentage bar chart
- The various components are expressed as percentage to the total
- All bars are equal in height

Survived	First	Second	Third
died	0.3703704	0.5271739	0.7576375
Survived	0.6296296	0.4728261	0.2423625

Composition of the passengers by passenger class and survival condition
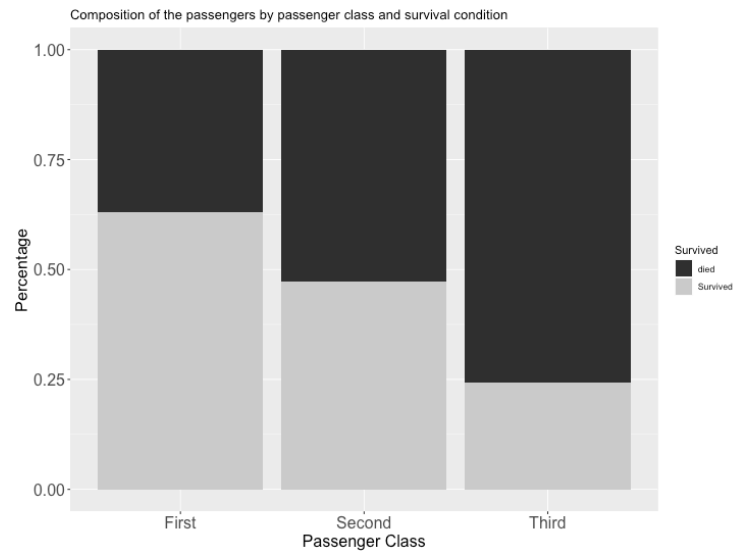
iii Multiple Bar Chart

- Compound bar chart/ Cluster bar chart
- Use to compare two or more qualitative variables (nominal or ordinal)
- Often used in conjunction with two way tables
- These bar charts are drawn side by side

Survived	First	Second	Third
died	37.04	52.72	75.76
Survived	62.96	47.28	24.24

Composition of the passengers by passenger class and survival condition
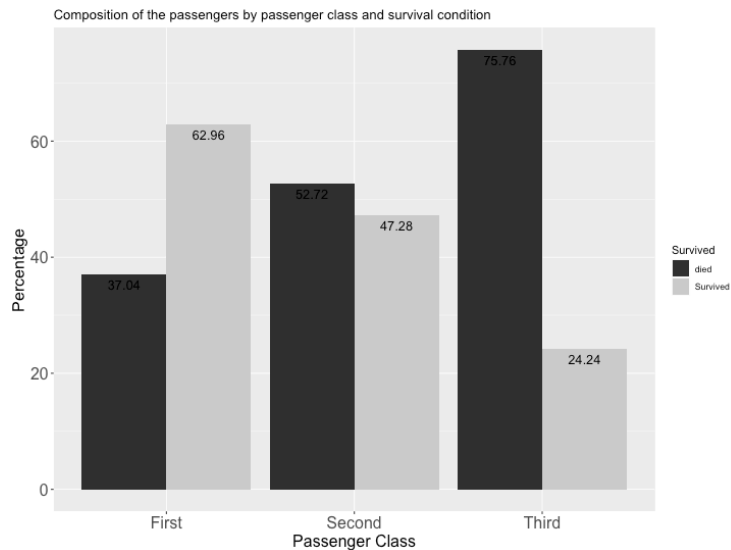
6.2.2.2 Describing Quantitative Data

- Histogram/ Dot plot / Box plot/ Scatter plot

II Histogram

- Histogram looks similar to bar chart since it also has bars.
- However, it is different from a bar chart in a number of aspects.
- One main difference is that in the histogram, the bars are drawn attached to each other; there are no gaps between bars like in a bar chart.
- Histogram is used to show the shape of the distribution of a **continuous variable**.
- However, the histogram is also used for discrete variables when the data are grouped in to class intervals.
- In a histogram, **the area of a bar should be proportional to the frequency of the corresponding class.**
- If all the bars have the same width, then the height of a bar can represent the frequency.
- The bar corresponding to a class interval should be drawn from the lower class boundary to the upper class boundary. In this way there will be no gaps between the bars.

Example: The marks(out of 50) of a group of students are recorded in the accompanying table. Draw a histogram for the data

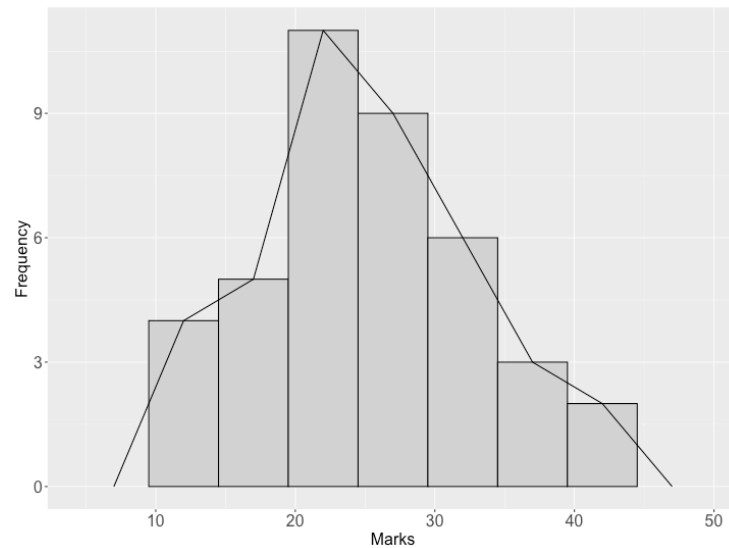| Marks	Number of students
10 - 14	4
15 - 19	5
20 - 24	11
25 - 29	9
30 - 34	6
35 - 39	3
40 - 44	2
Total	40
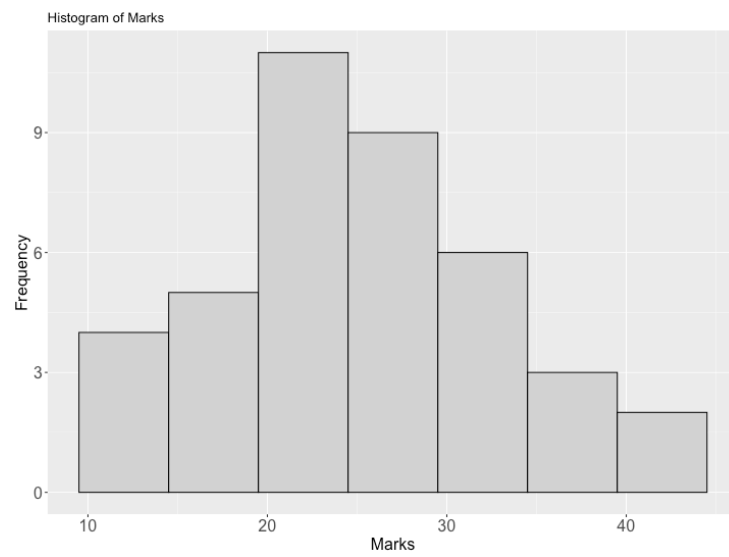
Example 2
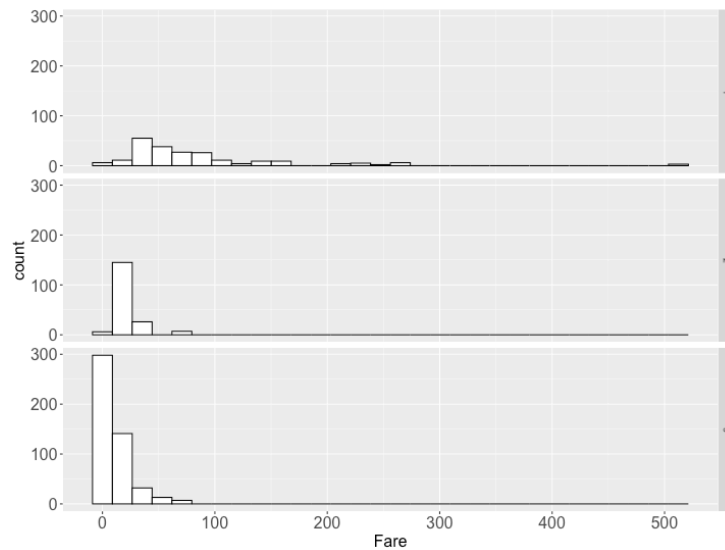
III Frequency polygon

- If the mid-point of the top of each block in a histogram is joined by a straight line, a frequency polygon is produced.
- This is done under the assumption that the frequencies in a class-interval are evenly distributed throughout the class

Example: The marks(out of 50) of a group of students are recorded in the accompanying table. Draw a frequency polygon for the data

| Marks	Number of students
10 - 14	4
15 - 19	5
20 - 24	11
25 - 29	9
30 - 34	6
35 - 39	3
40 - 44	2
Total	40

IV Frequency curve

- A frequency curve is drawn by smoothing the frequency polygon.
- It is smooth in such a way that the sharp turns are avoided

Example: The marks(out of 50) of a group of students are recorded in the accompanying table. Draw a frequency curve for the data

Marks	Number of students
10 - 14	4
15 - 19	5
20 - 24	11
25 - 29	9
30 - 34	6
35 - 39	3
40 - 44	2
Total	40
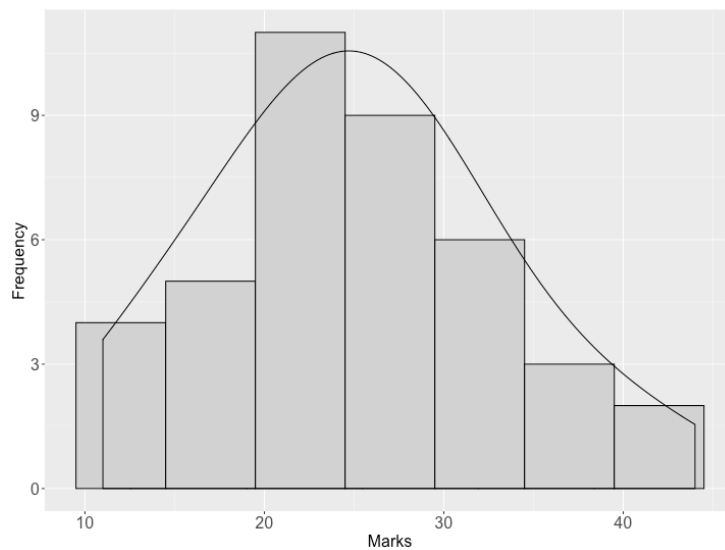
frequency curves arising in practice take on certain characteristics shapes as shown bellow
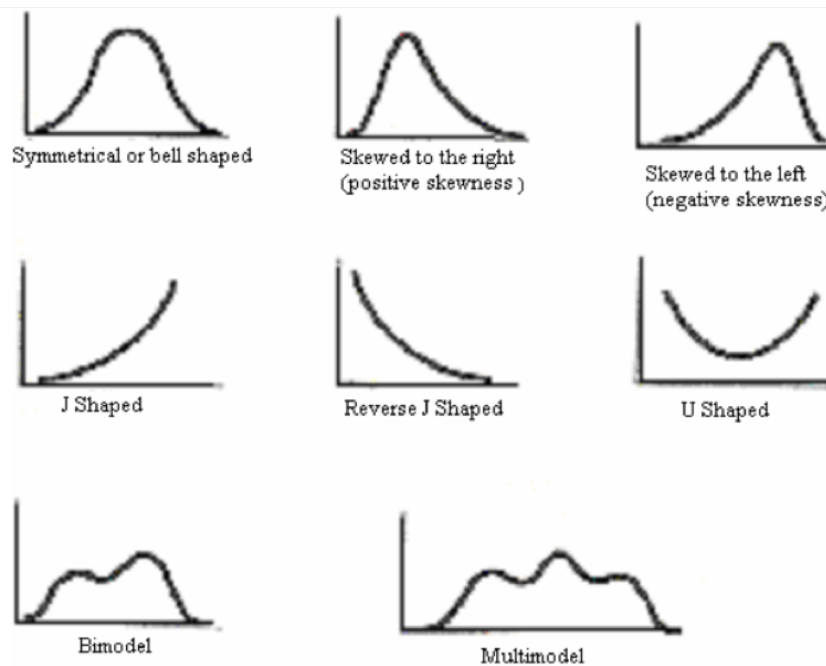
1. The **symmetrical** or **bell shaped** frequency curves are characterized by

the fact that observations equidistant from the central maximum have the same frequency. An important example is the normal curve.

2. In the **moderately asymmetrical** or **skewed** frequency curves the tail of the curve to one side of the central maximum is longer than that to the other. If the longer tail occurs to the right the curve is said to be **skewed to the right** or to have **positive skewness**.While if the reverse is true the curve is said to be **skewed to the left** or to have **negative skewness**.

3. In a **J shaped** or **reverse J shaped** curve a maximum occurs at one end.

4. A **U shaped** frequency curve has maxima at both ends.

5. A **bimodal** frequency curve has two maxima. These appear as two distinct peaks (local maxima) in the frequency curve.When the two modes are unequal the larger mode is known as the major mode and the other as the minor mode.

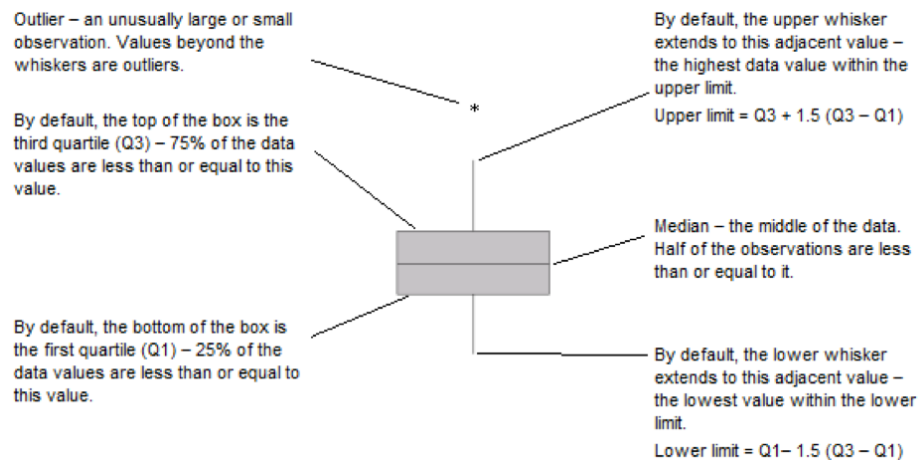6. A **multimodal** frequency curve has more than two maxima.

V Dot Plot

- A dot plot is a method of presenting data which gives a rough but rapid visual appreciation of the way in which the data are distributed

- It consists of a horizontal line marked out with divisions of the scale on which the variable is being measured -This graph can be used to represent only the numerical data.

Distribution of age of titanic passengers
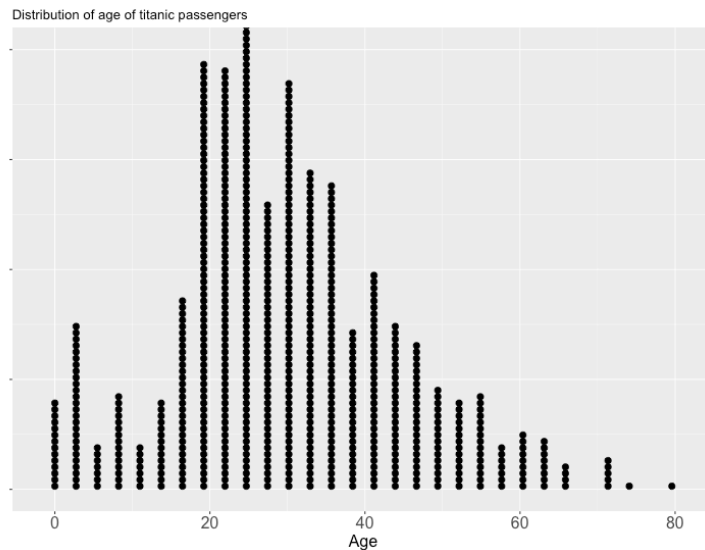
VI Box plot (Box and whisker plot)

- Box plot is also a useful method of representing the behavior of a data set or comparing two or more data sets.
- Box plot is constructed by identifying five statistics from the data set as largest value, smallest values, median, Q1 and Q3.

Outlier – an unusually large or small observation. Values beyond the whiskers are outliers.

By default, the top of the box is the third quartile (Q3) – 75% of the data values are less than or equal to this value.

By default, the bottom of the box is the first quartile (Q1) – 25% of the data values are less than or equal to this value.

By default, the upper whisker extends to this adjacent value – the highest data value within the upper limit.
Upper limit = Q3 + 1.5 (Q3 – Q1)

Median – the middle of the data. Half of the observations are less than or equal to it.

By default, the lower whisker extends to this adjacent value – the lowest value within the lower limit.
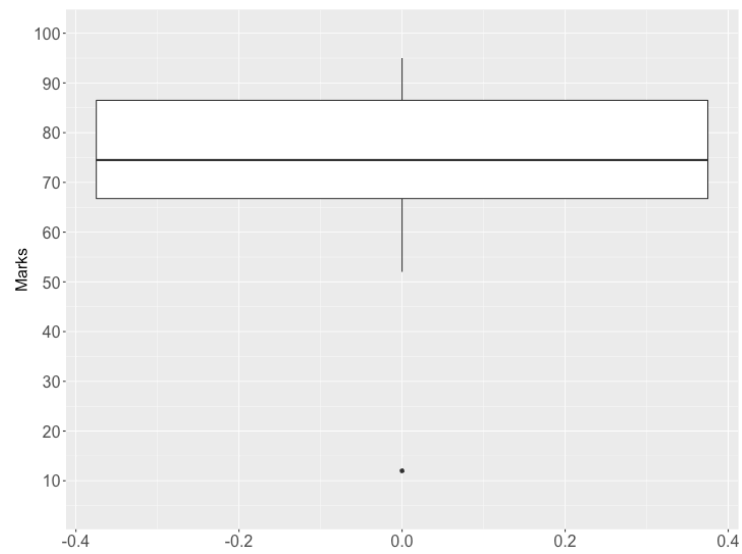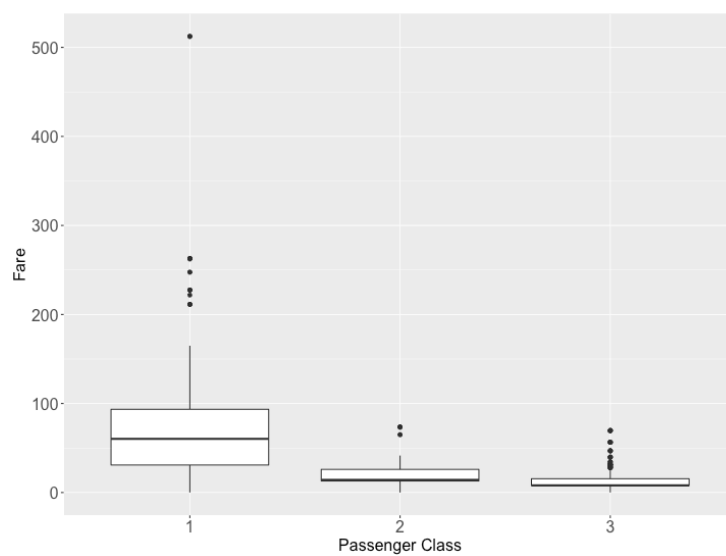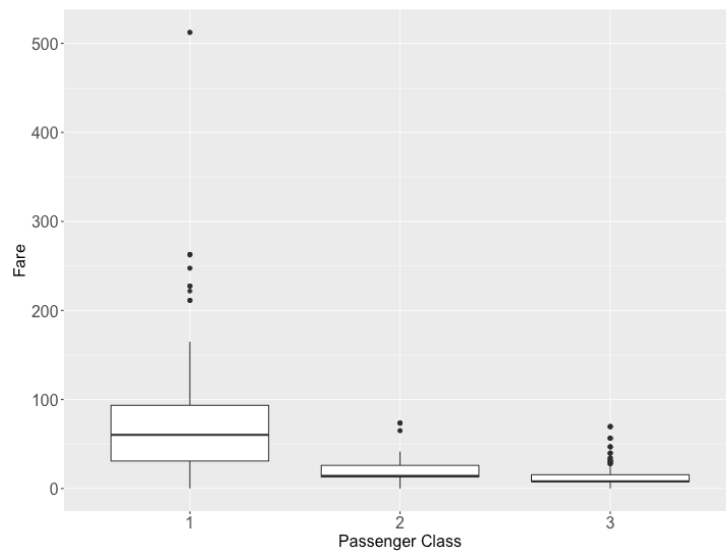Lower limit = Q1– 1.5 (Q3 – Q1)

Example:

Construct a box plot for the following data set (Marks of students)

52, 88, 56, 79, 72, 91, 85, 88, 68, 63, 76, 73, 86, 95, 12, 69

Xmin = 12 Xmax = 95 Q1 =64.25 Q2 = Median = 74.5 Q3 = 87.5

VII Violin plot

- A violin plot is a method of plotting quantitative data.
- It is similar to a box plot, with the addition of a rotated kernel density plot on each side.
- Violin plots are similar to box plots, except that they also show the probability density of the data at different values, usually smoothed by a kernel density estimator.
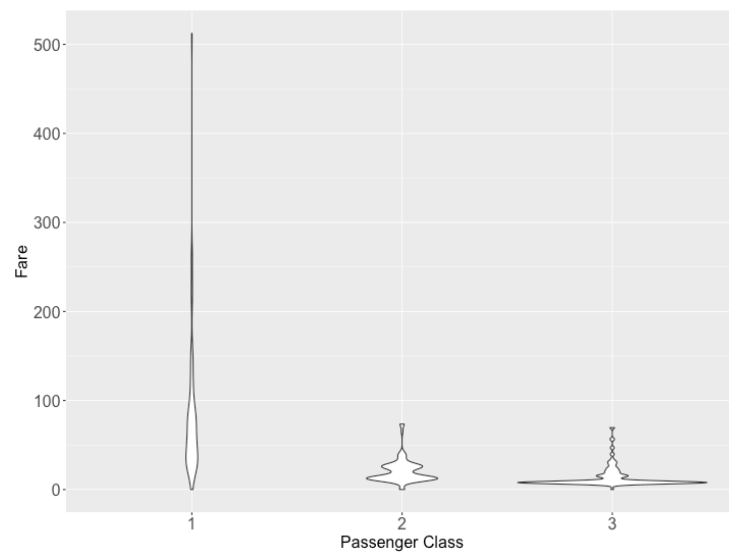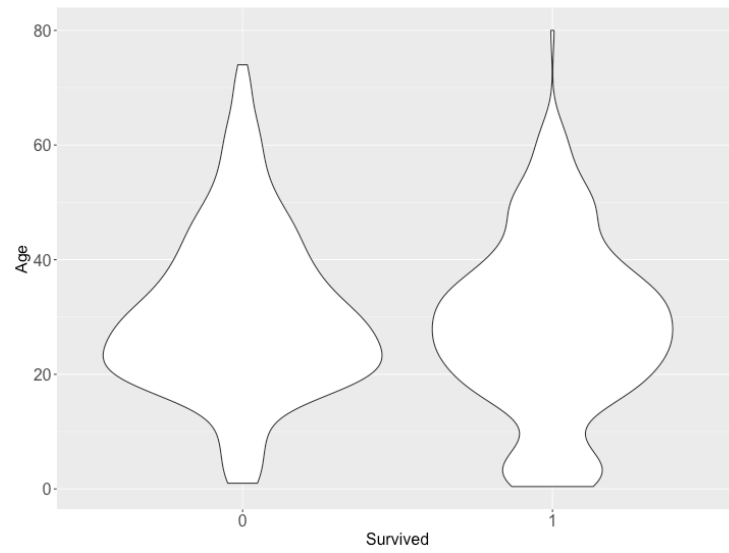
6.3 Summary Measures

- Although frequency distribution serves useful purpose, there are many situations that require other types of data summarization.

- What we need in many instances is the ability to summarize the data by means of a single number called a descriptive measure.

- Descriptive measures may be computed from the data of a sample or the data of a population. To distinguish between them we have the following definitions.

Definitions

- A descriptive measure computed from the data of a sample is called a **statistic**.

- A descriptive measure computed from the data of a population is called a **parameter**.

6.3.1 Measures of Central Tendency

- **Measure of central tendency** yield information about the center, or middle part, of a group of numbers.

- Eg: Mode, Median, Arithmetic Mean, Geometric mean, Harmonic Mean, Quadratic Mean, Quartiles, Deciles, and Percentiles

6.3.1.1 Mode

- The Mode is the most frequently occuring value in a set of data

- Organizing the data into an ordered array (an ordering of the numbers from smallest to largest) helps to locate the mode.

- A series having only one mode is called as **uni-modal**

- In the case of a tie for the most frequent occuring value, two modes are listed. Then the data are set to be **bimodal**

- If a set of data is not exactly bimodal but contains two values that are more dominant than others, some researchers take the liberty of referring to the data set as bimodal even without an *exact tie* for the mode.

- Data sets with more than two modes are referred to as **multimodal**.

- The mode is an appropriate measure of central tendency for nominal-level data.

- The mode can be used to determine which category occurs most frequently.

For ungrouped data

Example 01: Find mode of the following datasets

Dataset 1: 12, 14, 10, 8, 6, 8, 15, 8

Dataset 2: 40, 44, 57, 48, 78

Dataset 3: 42, 45, 55, 50, 45, 40, 55, 45, 52, 55, 54

For grouped frequency data

Example 02: Find mode of the following data

Marks	Number of students
20	8
30	10
40	16
50	8
60	5
70	3

- Advantages and disadvantages of mode

Advantages

- Easy to understand
- Easy to calculate
- Not affected by extreme values in the dataset
- Good for qualitative data

Disadvantages

- Not suitable for further mathematical calculations
- There may be more than one mode for a given dataset
- It is not based upon all the observations
- In some cases, we may not be able to find a mode for a given dataset

6.3.1.2   Median

- The median is the middle value in an ordered array of numbers.

- Median divides the series into equal parts

- The following steps are used to determine the median.

- STEP 1: Arrange the observations in an ordered data array.

- STEP 2: For an **odd number** of terms, find the middle term of the ordered array. It is the median.

- STEP 3: For an **even number** of terms, find the arithmetic mean of the middle two terms. This arithmetic mean is the median.

$$Median = \text{the } (\frac{n+1}{2})\text{th item in the data array}$$

- The level of data measurement must be at least ordinal for a median to be meaningful.

Example 1: Find the median of the dataset 1, 8, 6, 3, 2

Example 2: Find the median of the dataset 8, 9, 1, 2, 14, 12

- Advantages and disadvantages of median

Advantages

- Simple to understand
- Easy to calculate
- Not affected by extreme values in the dataset
- Can be calculated even for qualitative variables (ordinal scale data)

Disadvantages

- It is not based upon all the observations

6.3.1.3   Arithmetic Mean

- The arithmetic mean (usually called mean) is the sum of all observations divided by the total number of observations.

- Population Mean

     – The poupulation mean us represented by the Greek letter *mu* (μ).

     – Let, N is the number of terms in the population.

$$\mu = \frac{\sum x}{N} = \frac{x_1 + x_2 + x_3 + ... + x_N}{N}$$

- Sample Mean

     – The sample mean is represented by $\bar{x}$
     – Let, n is the number of terms in the sample

$$\bar{x} = \frac{\sum x}{n} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

- It is inappropriate to use the mean to analyse data that are not at least interval level in measurement.

Example 1: Calculate the mean from the following data

Student	1	2	3	4	5	6	7	8	9	10
Marks	40	50	53	78	58	60	73	35	43	48

- Advantages and disadvantages of arithmetic mean

Advantages

- Simple to understand
- Easy to calculate
- Based on all the observations
- Well defined
- Unique
- Can be used in further calculation

Disadvantages

- Can be affected by extreme values in the dataset
- May lead to false conclusions
- Only applicable to quantitative data (not applicable to qualitative data)

Empirical relationship between mean, mode, median

- In case of symmetrical distribution, mean, median and mode coincide ($mean = meadian = mode$)

- For a moderately asymmetrical distribution, the following relationship exists $MeanMode = 3(Mean - Median)$

Choice between mean and median

- Mean is very sensitive to outliers.Median is not sensitive to outliers
- When there are outliers in a data set, median is more appropriate than mean

6.3.1.4 Quartiles, Deciles and Percentiles

- Median divides the data set into two equal parts.
- There are other values which divide the data set into a number of equal parts
- Those are Quartiles, Deciles and Percentiles

(a) Quartiles (Q) – Quartiles divide an array into four equal parts

$$Q_i = \text{the } \frac{i}{4}(n+1)\text{th item in the data array}$$

(b) Deciles (D) – Deciles divide an array into ten equal parts

$$D_i = \text{the } \frac{i}{10}(n+1)\text{th item in the data array}$$

(c) Percentiles (P) – Percentiles divide an array into 100 equal parts

$$P_i = \text{the } \frac{i}{100}(n+1)\text{th item in the data array}$$

6.3.2 Measures of Variability

- Measure of central tendency yield information about particular points of a data set.

- However, business researchers can use another group of analytic tools to describe a set of data.

- These tools are measures of variability, which describe the spread or the dispersion of a set of data.

- Using measures of variability in conjunction with measures of central tendency makes possible a more complete numerical description of the data.

- This section focuses on seven measures of variability for ungrouped data: range, interquartile range, variance, standard deviation, z score and coefficient of variation.

6.3.2.1   Range

- The range is the difference between the largest value of a data set and the smallest value.

$$Range = Maximum - Minimum$$

- One important use of the range is in quality assurance, where the range is used to construct control charts

- Advantages and disadvantages of range

Advantages

- Easy to understand and calculate

Disadvantages

- Consider only the highest and lowest values of the data and fails to take account of any other observations in the dataset
- Heavily influenced by extreme values

6.3.2.2   Interquartile Range (IQR)

- We use the interquartile range (IQR) to measure the spread of a data around the median (M).

- The interquartile range is the range of values between the first and third quartile.

- Essentially it is the range of the middle 50% of the data and is determined by computing the value of $Q_3 - Q_1$.

- The interquartile range is especially useful in situations where data users are more interested in values towards the middle and less interested in extremes.

- The interquartile range is used in the construction of box and whisker plots.

- By eliminating the lowest 25% and the highest 25% of the items in a series, we are left with the central 50% , which are ordinarily free of extreme values.

Advantages

- Easy to understand and calculate
- Not influenced by extreme values

Disadvantages

- Ignore the first 25% and the last 25% in the dataset

6.3.2.3 Variance and Standard Deviation

- To measure the spread of data around the mean, we use the standard deviation (S).
- The variance and standard deviation are two very popular measures of dispersion.
- These measures are not meaningful unless the data are at least interval-level data.
- Their formulations are categorized into whether to evaluate from a population or from a sample.

NOTE

- Sum of deviations from the arithmetic mean is always zero.

$$\sum (x - \mu) = 0$$

- This property requires considering the alternative ways to obtain measure of variability.

6.3.2.3.1 Variance

- The **variance** is the average of the squared deviations about the mean for a set of numbers.

- The population variance is denoted by σ^2

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$$

- *The sum of the squared deviations about the mean of a set of values* - called the **sum of squares of** x and sometimes abbreviated as SS_x

- Because the variance is computed from squared deviations, the final result is expressed in terms of squared units of measurements.

- Statistics measured in squared units are problematic to interpret.

6.3.2.3.2   Standard Deviation

- The standard deviation is the square root of the variance.

- The population standard deviation is denoted by σ

$$\sigma = \sqrt{\frac{\sum(x - \mu)^2}{N}} = \sqrt{\sigma^2}$$

- One feature of standard deviation that distinguishes it from a variance is that the standard deviation is expressed in the same units as the raw data, whereas the variance is expressed in those units squared.

Advantages

- Based on all the observations
- Since this is based on arithmetic mean, it has all the merits of it
- The most important and widely used measure of dispersion

Disadvantages

- Not easy to understand and difficult to calculate
- Gives more weight to extreme values, because the values are squared up

6.3.2.4   Empirical Rule

- The empirical rule is an important rule of thumb that is used to state the approximate percentage of values that lie within a given number of standard deviations from the mean of a set of data **if the data are normally distributed**

- The empirical rule is used only for three numbers of standard deviations: 1σ, 2σ, 3σ

Distance from the mean	Values within distance
$\mu \pm 1\sigma$	68%
$\mu \pm 2\sigma$	95%
$\mu \pm 3\sigma$	99.7%

- If a set of data is normally distributed, or bell shaped, approximately 68% of the data values are within one standard deviation of the mean, 95% are within two standard deviations, and almost 100% are within three standard deviations.

6.3.2.5 Population versus sample variance and standard deviations

- The sample variance is denoted by s^2 and the sample standard deviation by s.

- The main use for sample variances and standard deviations is as estimators of population variances and standard deviations.

- Thus, computation of the sample variance and standard deviation differs slightly from computation of the population variance and standard deviation.

- Both the sample variance and sample standard deviation use $n-1$int he denominator instead of n because using n in the denominator of a sample variance results in a statistic that tends to underestimate the population variance.

- While discussion of the properties of *good estimator* is beyond the scope of this course, one of the properties of a good estimator is being *unbiased*.

- Whereas, using n in the denominator of the sample variance makes it a *biased* estimator, using $n-1$ allows it to be an *unbiased* estimator, which is a desirable property in inferential statistics.

Sample variance

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Sample standard deviation

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} = \sqrt{s^2}$$

6.3.2.6   Computational formulas for variance and standard deviation

- An alternative method of computing variance and standard deviation, sometimes referred to as the computational method or shortcut method, is available.

- Algebraically,

$$\sum (x - \mu)^2 = \sum x^2 - \frac{(\sum x)^2}{N}$$

and

$$\sum (x - \bar{x})^2 = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$$

- Substituting these equivalent expressions into the original formulas for variance and standard deviation yields the following computational formulas.

Computational formula for population variance and standard deviation

$$\sigma^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{N}}{N}$$

$$\sigma = \sqrt{\sigma^2}$$

Computational formula for sample variance and standard deviation

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}$$

$$s = \sqrt{s^2}$$

- For situations in which the mean is already computed or is given, alternative forms of these formulas are:

$$\sigma^2 = \frac{\sum x^2 - N\mu^2}{N}$$

$$s^2 = \frac{\sum x^2 - n(\bar{x})^2}{n - 1}$$

6.3.2.7   Coefficient of variation

- In general, for two variables measured with he same units (eg: two groups of people both weighed in kg), the group with the larger variance and standard deviation has n=more variability among their scores.

- The unit of measure affects the size od the variance.

- The same population weights, expressed in 'grams' rather than kg would have a larger variance and standard deviation.

- The *coefficient of variation*, a measure of relative variability gets around this difficulty and makes it possible to compare variability across variables measured in different units.

- The coefficient of variation is the ratio of the standard deviation to the mean, expressed as a percentage and is denoted CV.

$$CV = \frac{\sigma}{\mu}(100)$$

Using median and quartile deviation

$$CV = \frac{\frac{Q_3 - Q_1}{2}}{Median}(100)$$

- The coefficient of variation essentially is a relative comparison of a standard deviation to its mean.

- The coefficient of variation can be useful in comparing standard deviations that have been computed from data with different means.

- The choice of whether to use a coefficient of variation or raw standard deviations to compare multiple standard deviations is a matter of preference

- The coefficient of variation also provides and optional method of interpreting the value of a standard deviation.

6.3.3 Measures of Shape

- Measures of shape are tools that can be used to describe the shape of a distribution of data.

- In this section, we examine two measures of shapes: skewness and Kurtosis.

6.3.3.1 Skewness

- A distribution of data in which the right half is a mirror image of the left half is said to be *symmetrical.*

- One example of a symmetrical distribution is the normal distribution, or bell curve.

- **Skewness** is a measure of symmetry, or more precisely, the lack of symmetry

- The measures of asymmetry are called as measures of skewness.

- The skewed portion is the long, thin part of the curve

Skewness and the relationship of the mean, median and mode

- The concept of skewness helps us to understand the relationship between the mean, median and mode.

- In a unimodal distribution (distribution with a single peak or mode) that is skewed, the mode is the apex (high point) of the curve and the median is the middle value.

- The mean tends to be located toward the tail of the distribution, because the mean is affected by all values, including the extreme ones.

- A bell-shaped or normal distribution with the mean, median and mode all at the centre of the distribution has no skewness.
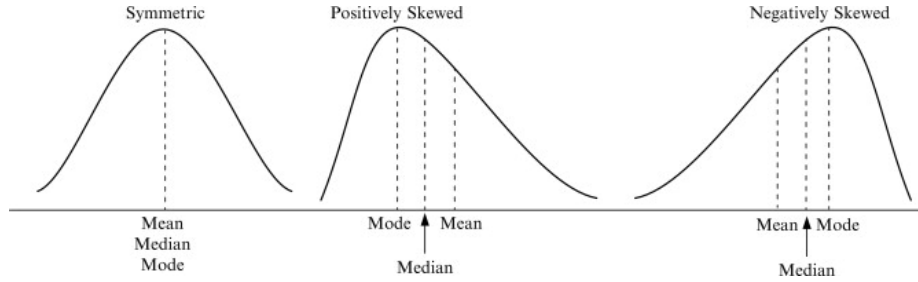
Figure 6.1: Relationship of mean, median and mode for different types of skewness

6.3.3.1.1   Pearsonian coefficient of skewness

- This coefficient compares the mean and median in light of the magnitude of the standard deviation

$$S_k = \frac{3(\mu - M_d)}{\sigma}$$

where S_k = coefficient of skewness, M_d = median

- Note that if the distribution is symmetrical, the mean and median are the same value and hence the coefficient of skewness is equal to zero.

- If the value of S_k is positive, the distribution is positively skewed.

- If the value of S_k is negative, the distribution is negatively skewed.

- The greater the magnitude of S_k, the more skewed is the distribution.

6.3.3.2   Kurtosis

- Kurtosis describes the amount of peakedness of a distribution.

- **Kurtosis** is a measure of whether the data are peaked or flat relative to a normal distribution

- Distributions that are high and thin are referred to as **leptokurtic** distributions.

- Distributions that are flat and spread out are referred to as **platykurtic** distributions.

- Between the above two types are distributions that are more 'normal' in shape, referred to as **mesokurtic** distributions
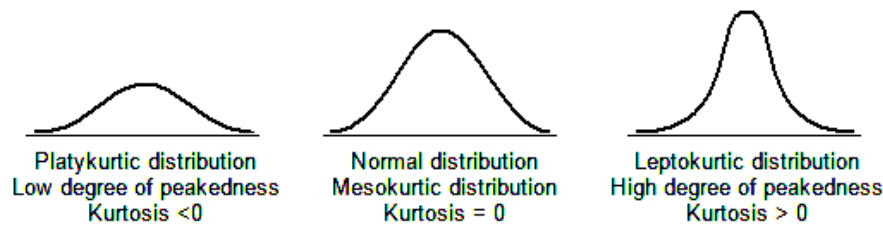
Platykurtic distribution
Low degree of peakedness
Kurtosis <0

Normal distribution
Mesokurtic distribution
Kurtosis = 0

Leptokurtic distribution
High degree of peakedness
Kurtosis > 0

Figure 6.2: Types of kurtosis

6.3.4 Measures of Association

- Many times in business it is important to explore the relationship between two numerical variables
- Measures of association are statistics that yeild information about the relatedness of numerical variables.
- In this section, we discuss only one measure of association, correlation, and d so only for two numerical variables.

6.3.4.1 Scatter plot

- A scatter plot is a two dimensional graph of pairs of points from **two numerical** variables
- In a quantitative bi-variate dataset, we have a (x, y) pair for each sampling unit, where x denotes the independent variable and y denotes the dependent variable.
- Each (x, y) pair can be considered as a point on the Cartesian plan.
- Scatter plot is a plot of all the (x, y) pairs in the dataset.
- The purpose of scatter plot is to illustrate any relationship between two quantitative variables.

  - If the variables are related, what kind of relationship it is, linear or nonlinear?
  - If the relationship is linear, the scatter plot will show whether it is negative or positive.

Example: *Edgar Anderson's Iris Data*

This famous (Fisher's or Anderson's) iris data set gives the measurements in centimeters of the variables sepal length and width and petal length and width, respectively, for 50 flowers from each of 3 species of iris. The species are Iris setosa, versicolor, and virginica.
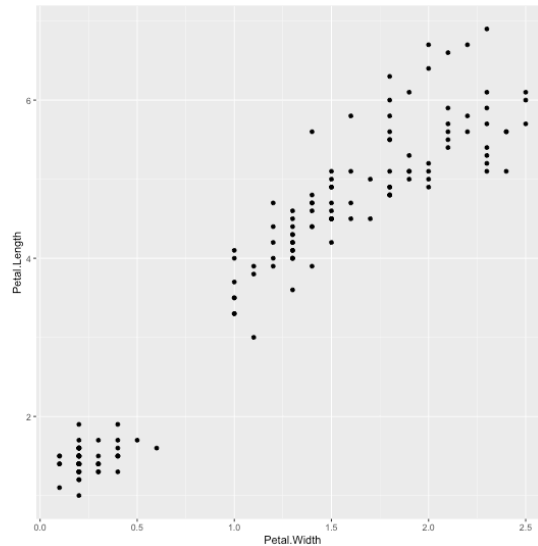
6.3.4.2   Correlation

- Correlation is a measure of the degree of relatedness of two or more variables.
- Several measures of correlation are available , the selection of which depends mostly on the level of data being analysed.
- Ideally, researchers would like to calculate ρ, the **population** coefficient of correlation.
- However, because researchers virtually always deal with sample data, this section introduces a widely used sample coefficient of correlation, r.
- This measure is applicable **only if both variables being analysed have at least an interval level of data**
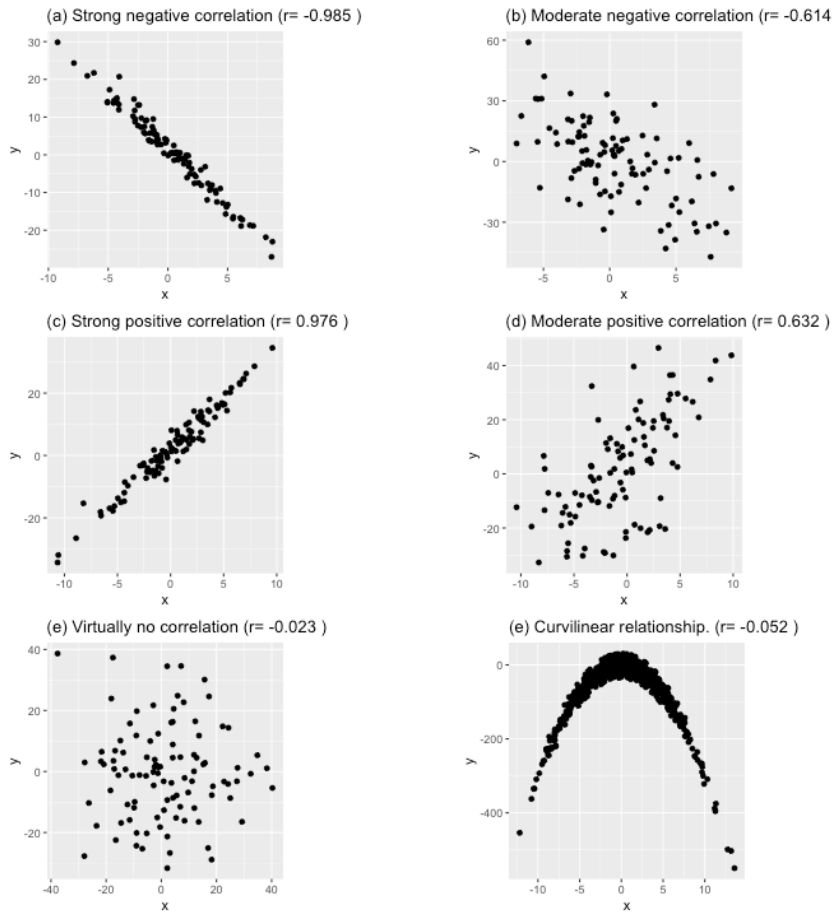
Pearson product-moment correlation coefficient (r)

- The statistic r is the Pearson product-moment correlation coefficient, named after Karl Pearson (1857 - 1936).
- The tern r is a measure of the **linear** correlation of two variables.
- It is a number that ranges from -1 to 0 to +1, representing th strength of the linear relationship between the variables.
- An r value of $+1$ denotes a perfect **linear** positive relationship between two variables.
- An r value of -1 denotes a perfect **linear** negative relationship between two variables, which indicates an inverse relationship between two variables: as one variable gets larger, the other gets smaller.
- An r value of 0 means no **linear** relationship is present between the two variables.

$$r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$$

$$r = \frac{\sum xy - \frac{(\sum x \sum y)}{n}}{\sqrt{[\sum x^2 - \frac{(\sum x)^2}{n}][\sum y^2 - \frac{(\sum y)^2}{n}]}}$$

- Examples: Following figure shows five different degrees of correlation:

(a) Strong negative correlation (r= -0.985)
(b) Moderate negative correlation (r= -0.614
(c) Strong positive correlation (r= 0.976)
(d) Moderate positive correlation (r= 0.632)
(e) Virtually no correlation (r= -0.023)
(e) Curvilinear relationship. (r= -0.052)

NOTE

- When $r = 0$, it signifies there is **no linear** relationship between the two variables. (There can be a non-linear relationship, Figure (e))
- Figure (e): There is a very strong curvilinear relationship. But there is **no linear** relationship.

References

Black, K., Asafu-Adjaye, J., Khan, N., Perera, N., Edwards, P., & Harris, M. (2007). *Australasian business statistics*. John Wiley & Sons.

6.4 Tutorial

Chapter 6: Descriptive Statistics

1. A power company in Sri Lanka designs and manufactures power distribution switchboards for hospitals, bridges, airports, highways and water treatment plants. Power company marketing director wants to determine client satisfaction with their products and services. He developed a questionnaire that yields a satisfaction score between 0 and 100 for participant responses. A random sample of 50 of the company's 1000 clients is asked to complete a satisfaction survey. The satisfaction scores for the 50 participants are averaged to produce a mean satisfaction score.

   a. What is the objective of this study?
   b. What is the population for this study?
   c. What is the sample for this study?
   d. What is the statistic for this study?
   e. What would be a parameter for this study?

2. Determine the type and the scale of measurement of the following variables

   a. The time required to produce an item on an assembly line
   b. the number of litres of milk a family drinks in a week
   c. The ranking of 50 students in your class after their overall performance have been designated as excellent, good, satisfactory or poor
   d. The telephone area code of clients in Australia
   e. The age of each of your batch mates
   f. The sales at the local pizza restaurant each month
   g. A student index number
   h. The response time of emergency services
   i. The number of tickets sold at a ticket counter on any given day
   j. Monthly maximum air temperature

3. The grades of 30 students for Statistics are as follows:

   B, C, B, D, B, C, C, A , B, C,

   C, B, E, B, B, D, D, F, B, D,

   D, A, B, A, B, C, E, A, A, E

   a. Construct a frequency table with suitable values (eg: absolute frequency, relative frequency, cumulative frequency, cumulative relative frequency)

4. The gender type of a group of 10 students are as follows:

   Male, Female, Female, Male, Female Female, Male, Male, Female, Female

a. Construct a frequency table with suitable values (eg: absolute frequency, relative frequency, cumulative frequency, cumulative relative frequency)

5. In a study conducted to investigate the relationship between delivery time and computer-assisted ordering, the following data were gathered from a sample of 40 firms, that 16 use computer-assisted ordering, while 24 do not. Furthermore, past data are used to categorize each firm's delivery times below the industry average, equal to the industry average, or above the industry average. The results obtained are given in the table below.

Computer Assisted Ordering	Delivery Time Below Industry Average	Delivery Time Equal to Industry Average	Delivery Time Above industry Average	Row Total
No	4	12	8	24
Yes	10	4	2	16
Total	14	16	10	40

a) For each row and column total, calculate the corresponding row or column percentage.
b) For each cell, calculate the corresponding cell, row, and column percentages.
c) Carry out graphical analysis to investigate the relationship between delivery-time performance and computer-assisted ordering.
d) What conclusions can be made about the nature of the relationship?

6.

a) Giving suitable examples distinguish between multiple bar chart and a component bar chart.
b) Write down advantages and disadvantages of each chart and when they should be used

7. In a study conducted to investigate the effect of wearing helmets in riding motorcycles to reduce the head injuries, the following data were gathered by investigating 151 motorcycle accidents reported during last 10 month period. Out of the total number of cases (151) investigated, only 102 people were wearing helmets properly at the time of the accidents. Out of them, only 9 got severe head injuries. The rest got either minor head injuries or no head injuries at all. Among 49 who were not wearing helmets at the time of the accident, 15 got server head injuries and the rest got minor head injuries or no head injuries at all.

a) Use an appropriate table to represent the above data and state what conclusions can be drawn from it

b) Carry out a suitable graphical analysis to investigate the effect of wearing helmets in riding motorcycles to reduce the head injuries.

8. The table below shows a frequency distribution of the weekly wages of 65 employees at the ABC Company. With reference to this table,

Wages (in $)	Number of Employees
250.00 - 259.99	8
260.00 - 269.99	10
270.00 - 279.99	16
280.00 - 289.99	14
290.00 - 299.99	10
300.00 - 309.99	5
310.00 - 319.99	2
Total	65

a) Construct a histogram
b) Construct a frequency polygon
c) Construct a frequency curve
d) State what conclusions can be drawn from the above graphical analysis

9. The contribution of the agriculture, industrial, and service sector to the Gross Domestic Product (GDP) for each province in Sri Lanka is given in the table below. It is required to investigate whether the contributions are varying from province to province. What kind of graph you would suggest for representing data to serve the purpose. Justify your answer. Sketch the proposed graph.

Sector	Western	Southern	Sabaragam uwa	Central	Uva	Eastern	North Western	North Central	Northern	Total
Agriculture	29767	58761	39209	60373	45689	27709	45118	40320	15851	362797
Industry	341636	41762	39163	34294	13183	25626	57483	10887	4986	569020
Services	695094	86593	54828	82575	33956	47122	84365	38485	43168	1166187
GDP	1066495	187115	133199	177241	92827	100457	186964	89691	64004	2098004

10. Marks of 16 students are given below.

    52, 88, 56, 79, 72, 91, 85, 88, 68, 63, 76, 73, 86, 95, 12, 69

(a) Find the quartile of the distribution and interpret the values

(b) Construct a box plot for the data set

(c) Are there outliers in the data set?

11. Listed below, ordered from smallest to largest, is the time in days the customers take to pay their invoices.

$13, 13, 13, 20, 26, 27, 31, 34, 34, 34, 35, 35, 36, 37, 38, 41, 41, 41, 45, 47, 47, 47, 50, 51, 53, 54, 56, 62, 67, 82$

(a) Determine the median

(b) Determine the first and third quartiles

(c) Determine the 2nd decile and the 8th decile

(d) Determine the 67th percentile

12.

(a) What are outliers?

(b) Which of the following is/are unaffected by outliers? Underline the correct answer/ answers.

a. Mean

b. Median

c. Mode

d. Range

e. Standard deviation

f. Inter-quartile range

(c) Some summary measures of a variable is given below. Descriptive Statistics:

N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum
50	5.389	0.997	3.150	4.658	5.350	6.020	8.600

(d) Are there outliers in this data set? YES/NO Justify your answer using a box plot

13. The mean and the standard deviation of 250 observations of a variable X are 62.1 and 4.3 respectively. However, in a re-scrutinizing process, it was found that the observations 72 and 81 were incorrectly recorded as 92 and 87. Find the correct mean and standard deviation of the data set.

14. The weekly sales from a sample of AB company were organized into a frequency distribution. The mean of weekly sales was computed to be $105,900, the median $105,000, and the mode $104,500

    (a) Sketch the sales in the form of a smoothed frequency polygon. Note the location of the mean, median and mode on the X-axis
    (b) Is the distribution symmetrical, positively skewed, or negatively skewed? Explain.

15. Compare the variation of the annual incomes of executives with the variation of the incomes of unskilled employees. The sample information is given below.

Type	$\bar{x}$	SD
Executives	500,000	50,000
Unskilled employees	22,000	2,200

16. Marks for the course module AB and the corresponding lecture attendance of 10 students are given below

Student Number	1	2	3	4	5	6	7	8	9	10
Marks in course module AB	84	51	91	60	68	89	98	58	53	47
Lecture attendance	13	6	15	4	12	14	15	10	11	6

    (a) Draw a scatter plot for the above data
    (b) Calculate the coefficient of correlation
    (c) Comment on the relationship between the marks and the lecture attendance

Chapter 7

Sets and Relations

7.1 Introduction

- Everyday we hear, or make statements to convey ideas about possibilities of occurring events.

  - Which lottery gives me the highest **chance** of winning?
  - What are **odds** of our cricket team winning the next world cup?
  - What is the **chance** of getting an A+ for CM1110?

- In order to make decisions or take actions, we need to **quantify the possibility** of occurrence of an event. For this, we use the term **probability**.
- Probability is a **quantitative measure** of uncertainty, a **number** that conveys the strength of our belief in the occurrence of an uncertain event.
- The likelihood of an outcome is quantified by assigning a number from the interval [0, 1] to the outcome (or a percentage from 0 to 100%).
- Higher numbers indicate that the outcome is more likely than lower numbers.
- A probability of 0 indicates an outcome will not occur.
- A probability of 1 indicates that an outcome will occur with certainty.

7.2 Sample Spaces and Events

7.2.1 Random Experiments

- Statisticians use the word **experiment** to describe any process that generates a set of data.
- An experiment usually involves observing or counting or measuring. For example;

1

- we might classify items coming off an assembly line as defective or non-defective
- we may record the number of accidents that occur monthly at the Katubedda junction, hoping to justify the installation of a traffic light;
- or we may be interested in the volume of gas released in a chemical reaction when the concentration of an acid is varied.

- An experiment that is repeatable in an identical fashion is called a **random experiment** or a **probability experiment**.
- Random experiment is an activity that leads to the occurrence of one of several possible outcomes which is not likely to be known until its completion, that is, the outcome is not perfectly predictable. This process has the properties that,

    - All possible outcomes can be specified in advance.
    - It can be repeated in an **identical manner.**
    - The same outcome may not occur on various repetitions so that the actual outcome is not known in advance.

Examples

- Tossing a coin.
- Throwing a die (or several dice)
- Drawing cards from a pack
- Launching of missile and observing its velocity at specified times.

7.2.2   Outcomes

- The result of a **single experiment** is called an outcome.
- It can be instrument reading, counts, "yes" or "no" answers, values obtained through extensive calculations etc.
- To model and analyze a random experiment, we must understand the set of possible outcomes from the experiment.

7.2.3   Sample space

- The set of all possible outcomes of a random experiment is called the **sample space** of the experiment.
- We denote the sample space by Ω.
- Each outcome in a sample space is called an **element** of the sample space or simply a **sample point**.

Activity 01 *Write down the corresponding sample space of the following experiments?*

Example I: A player rolls a die.

Example II: A player rolls two dice.

Example III: A quality control engineer of a certain company takes a random sample of 4 products as the products come off its production line and counts the number of defective products in the sample.

Example IV: A quality control engineer of a certain company takes a random sample of 4 products as products come off its production line and determines whether each product is defective or non-defective

Example V: A company produces a certain type of car batteries. If a battery has a voltage that is outside certain limits, that battery is characterized as bad (B); if the battery has a voltage within the prescribed limits it is characterized as good (G). A quality inspector selects batteries one by one as they come off the production line and tests them until he gets a bad battery. He uses this data to estimate the proportion of bad batteries produced by the company.

Example VI: An experiment consists of measuring the lifetime of a bulb.

- A sample space is often defined based on the objectives of the analysis.
- The following example illustrates several alternatives

Example:

Consider an experiment that selects a digital camera and records the recycle time of a flash (the time taken t ready the camera for another flash). The possible values for recycle time can vary according to the resolution of the timer and on the minimum and maximum recycle times.

However, since the time is positive we can define the sample space as simply the positive real line

Case I

Case II If it is known that all recycle times are between 1.0 and 5.5 seconds, the sample space can be written as

Case III If the objective of the analysis is to consider only whether the recycle time is low, medium or high, the sample space can be written as a set of three outcomes

Case IV If the objective is only to evaluate whether or not a particular camera confirms to a minimum recycle time specification, the sample space can be written as a set of two outcomes

- The number of outcomes in the sample space can be **finite** or **infinite**.
- Infinite sample spaces can be **countable** or **uncountable**.
- A sample space is countable if the outcomes can be associated with the integers 1, 2, ….

- If the sample space contains a finite number of elements or an infinite though countable number of elements, it is said to be **discrete**.
- The outcomes of some experiments are neither finite nor countably infinite, it is said to be **continuous**.

- In the above Example, the choice $\Omega = x|x \geq 0$ is an example of a continuous sample space,
- $\Omega = yes, no$ is a discrete sample space.
- As mentioned, the best choice of a sample space depends on the objectives of the study.

7.2.4   Events as subsets of sample spaces

- **An event is a subset of the sample space of a random experiment**
- Events are usually denoted by capital English letters A, B, C, D, E etc.
- An event containing only one outcome is called a **simple event**.
- An event containing more than one outcome is called a **compound event**.

Activity 02 *Express the following events as subsets of the sample spaces. Decide whether each event is simple or compound. (No need to calculate probabilities yet.)*

Example I: A player rolls a die.

a) A = the event of getting 3
b) B = the event of getting an even number
c) C = the event of getting a number less than 4
d) D = the event of getting 2 or 5

Example II: A player rolls two dice.

a) The event that one number is even.
b) The event that sum of the two numbers is odd
c) The event that the numbers are equal
d) The event of getting 2 and 5
e) The event of getting 2 or 5

Example III: Consider Example III in Activity 01,

a) E = the event that sample contains more than 2 defective products.
b) F = the event that the sample contains only two defective products.

Example IV: Consider Example IV in Activity 01

a) G = the event that sample contains more than 2 defective products.
b) H = the event that the sample contains two defective products.

Example V: Consider Example V in Activity 01,

a) I = the event that number of good batteries before a bad battery is 3.
b) J = the event that the number of good batteries before a bad battery is less than 3.

Example VI: Consider Example VI in Activity 01,

a) A = the event that bulb burns for at least 25 hours but burns out before 50 hours.

7.3 Event Operations

- We can describe new events from combinations of existing events.
- Because events are subsets, we can use basic set operations to form other events of interest.
- Some of the basic set operations are summarized here in terms of events:

Venn diagrams

- Diagrams are often used to portray relationships between sets, and these diagrams are also used to describe relationships between events.
- We can use Venn diagrams to represent a sample space and events in a sample space.

7.3.1 Union

- Let A and B be two events.
- The set of outcomes that belong to **either only A or only B or to both A and B,** is called the **union** of the two events.
- It is denoted by $A \cup B$.

$$A \cup B = \{\omega \in \Omega : \omega \in A \text{ or } \omega \in B\}$$
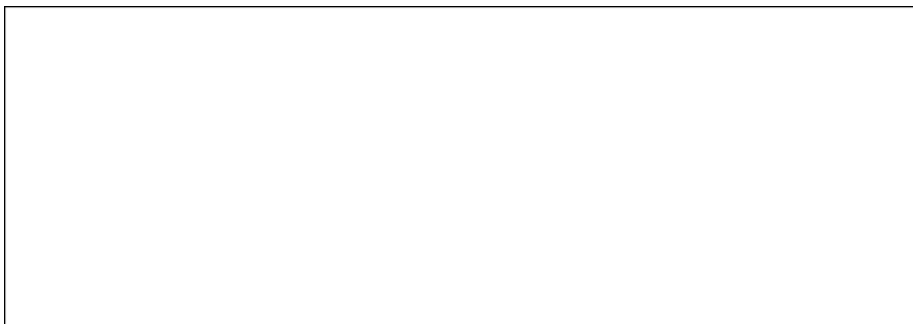
- Shaded set in the Venn diagram represents the event $A \cup B$

7.3.2   Intersection

- Let A and B be two sets.
- The set of outcomes that belong to **sets A and B both** is called the intersection of the two events.
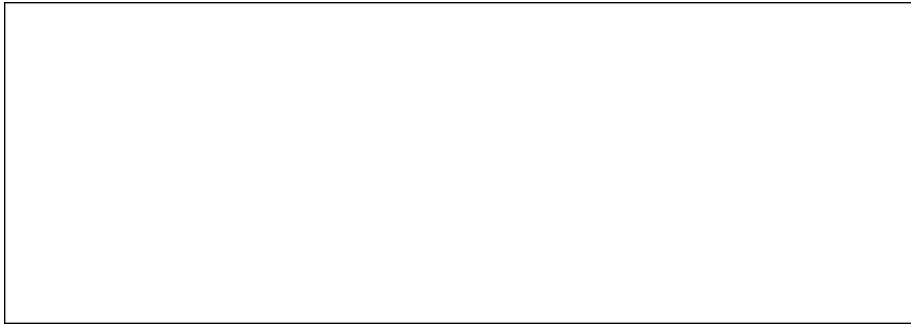- It is denoted by $A \cap B$.

$$A \cap B = \{\omega \in \Omega : \omega \in A \text{ and } \omega \in B\}$$

- Shaded set in the Venn diagram represents the event $A \cap B$

7.3.2.1   Mutually exclusive events

- If the events A and B cannot occur together, in other words if $A \cap B$ cannot occur, then the events A and B are said to be **disjoint** or **mutually exclusive**.
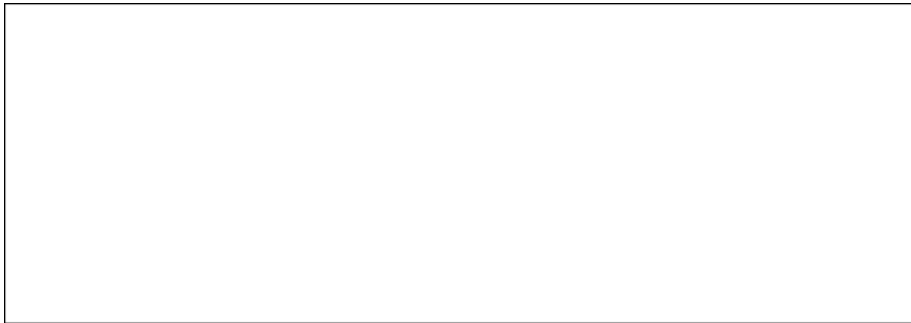
- If A and B are mutually exclusive, then we denote the intersection as

$$A \cap B = \{\} \text{ or } A \cap B = \Phi.$$

7.3.3 Complement

- The set of outcomes that are in sample space (Ω) but not in A is called the compliment of A.
- It is denoted by A' or A^c or $\bar{A}$

$$A' = \{\omega \in \Omega : \omega \notin A\}.$$

7.4 Tutorial

Chapter 7: Sets and Relations

1. A player rolls a die.

(a) What is the sample space of this experiment?
(b) Write down the following events as subsets of the sample space. Decide whether each event is simple or compound.

  i. A = the event of getting 3
  ii. B = the event of getting 7
  iii. C = the event of getting a number less than 4
  iv. D = the event of getting an even number
  v. E = the event of getting 2 or 5

2. A player rolls two dies

(a) What is the sample space of this experiment?
(b) Write down the following events as subsets of the sample space. Decide whether each event is simple or compound.

  i. A= the event that one number is even
  ii. B= the event that sum of the two numbers is odd
  iii. C= the event that the numbers are equal
  iv. D= the event of getting 2 and 5
  v. E= the event of getting 2 or 5

3. A player rolls three dies

(a) What is the sample space of this experiment? (Use set builder form of a set to represent the sample space)
(b) Write down the following events as subsets of the sample space. Decide whether each event is simple or compound

  i. A = the event of getting all the numbers greater than 4
  ii. B = the event that two numbers are equal

4. A company produces a certain type of car batteries. If a battery has a voltage that is outside certain limits, that battery is characterized as bad (B); if the battery has a voltage within the prescribed limits it is characterized as good (G). A quality inspector selects batteries one by one as they come off the production line and tests them until he gets a bad battery. He uses this data to estimate the proportion of bad batteries produced by the company.

  (a) What is the sample space of this experiment
  (b) Write down the following events as subsets of the sample space. Decide whether each event is simple or compound.

   i. I = the event that number of good batteries before a bad battery is 3.
   ii. J = the event that the number of good batteries before a bad battery is less than 3.

5. A professor is giving ten true-false questions (10 marks for a correct answer). If a student guesses on each question, how many different ways could the ten questions be answered?

6. A security supervisor of a large corporation wishes to issue each employee an identity card with one English letter followed by two digits.

  (a) How many different ID cards could be made
  (b) How many identity cards will have the same two numbers
  (c) How many identity cards will have first number greater than the second number

7. A doctor observes the lifetime of leukemia patients after being treated with a newly developed treatment.

  (a) What is the sample space of this experiment?
  (b) Write down the following event as subsets of the sample space. Decide whether it is simple or compound.

   i. K= the event that a randomly selected patient lives more than 10 years

8. Three students are selected at random from the class. Let P1 = the event that the first student pass P2 = the event that the second student pass P3 = the event that the third student pass Write the following events using the above events.

  (a) The event that all three students pass
  (b) The event that all three students fails
  (c) The event that only 2 of the three students pass after the treatments

9. Identify the events of interest as subsets of the sample space. A box contains one ticket with number 1 on it, 2 tickets with number 2, 3 tickets with number 3, 4 tickets with number 4, 5 tickets with number 5, and 6 tickets with number 6. A ticket is taken randomly from the box

  (a) A = The event of getting an even number?

(b) B = The event of getting a number less than 5?
(c) C = The event of getting an even number less than 5?
(d) Express C in terms of A and B using event operations
(e)) D = The event of getting an even number or a number less than 5?
(f) Express D in terms of A and B using event operations
(g) Express, E= the event of getting an odd number, using the above defined event or events

10. Three components are connected to form a system as shown in figure (a). Because the components in the 2-3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must be 2-3 subsystem.
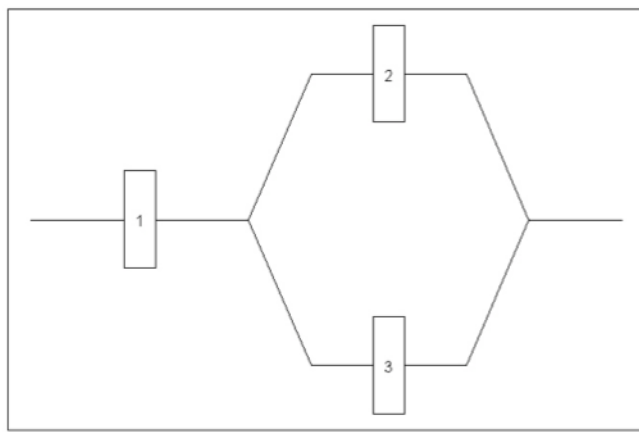
Each component is either in working condition (s) or not working condition (f) at any time. Let $W1$ = The event that component 1 is in working condition $W2$ = The event that component 2 is in working condition $W3$ = The event that component 3 is in working condition W = The event that the system is in working condition

(a) Write the above events as subsets of the sample space. (b) Write the following events.

  i. $W3'$
 ii. $W1 \cap W2$
iii. $W2 \cup W3$
 iv. $W1 \cap (W2 \cup W3)$

(c) Express W in terms of $W1, W2$ and $W3$ using event operations.

11. Three products are selected at random, from a production line. Each product is either defective or non defective. Let $D1 =$ The event that the first item is defective $D2 =$ The event that the second item is defective $D3 =$ The event that the third item is defective $E =$ The event that two of the three products are defective Express E in terms of $D1, D2$ and $D3$ using event operations

Chapter 8

Probability

Chapter 9

Correlation and Regression