UNIVERSITY OF MORATUWA

Department of Mathematics

M.Sc./Postgraduate Diploma in Financial Mathematics

MA 5124 Financial Time Series Analysis and Forecasting

Semester 2 Examination

THREE HOURS **May 2024**

INSTRUCTIONS TO CANDIDATES

Prior to the Assessment

1. This online examination will be held on **12th May 2024 from 2.00 PM to 5.30 PM.**
2. Only **handwritten answers** are allowed.
3. The total time allocated for the examination is **THREE hours for writing and extra 20 minutes for uploading answers**.
   - To **download** the question paper: 10 minutes
   - Writing time for the **five (05) questions**: 3 hours
   - To **upload** answers to the moodle: 20 minutes

During the Assessment

1. This paper contains 5 questions on 13 pages (including cover page).
2. This is an open book examination.
3. Answer **ALL** the questions.
4. You can attempt the questions in any order you like.
5. The total marks obtainable for the examination is 100.
6. All questions carry equal marks.
7. All symbols carry their usual meaning.
8. Write answers in papers.
9. Write your answers on A4 Sheets or full scape papers using a black or blue pen. **Don not use pencils.**

10. **Use separate paper for each question.**
11. **Clearly number the questions and subsections.** Neat and orderly presentation is important.
12. **All the necessary steps** for the answers should be **clearly indicated.**
13. If you have a doubt as to the interpretation of the wording of a question, make sure you take your own decision, but clearly state it on the answer script.
14. Number and write the index number on top of all pages. When numbering the pages, use the format: 1/5, 2/5,. . . ,5/5, if there are 5 pages in total.
15. **In the first page of your answer script** (top), write your name, index number, subject name with code and total number of pages.
16. **Write your index number on all the pages.**
17. Place your signature on each page.

Submission of Answers

1. You will get extra 20 minutes to upload your **handwritten answer** sheets in **.pdf formats** at the end of the examination.
2. Use Cam Scanner/Doc Scanner or any other scanning app to merge the images and **produce a single pdf document**.
3. Make sure the quality of the images are good enough (clear and not blurred) for evaluation and that the document is properly arranged and all the pages are attached.
4. Use **IndexNo_Coursecode_Yourname** to name the document.
5. It is the student's responsibility to ensure that the document submitted is legible and complete.
6. All submissions will be checked for plagiarism, and you are strongly advised not to copy from the web resources or from any other resource or receive help from anyone in person.
7. All examinations are conducted under the rules and regulations of the University.

-End of Instructions-

Question 1 *[Total marks allocated: 20 Marks]*

(a) Are the following statements true or false? Briefly explain your answers.

i. Good forecast methods should have normally distributed residuals.

ii. The cyclical component of time-series data is usually estimated using SLT decomposition method

iii. Assume that we have data from the following process

$$y_t = 0.4 - 0.8\epsilon_{t-1} + \epsilon_t,$$

where disturbances have a zero mean and unit variance. Then a positive linear trend will be visible in the scatterplot of Y_t versus Y_{t-1}, while a random scatter of points will be visible in the scatterplot of Y_t versus Y_{t-2}.

iv. High correlation always implies high cointegration and vice versa.

v. The pacf is necessary for distinguishing between an MA and an ARMA model

[*10 Marks*]

(b) The time series plots (i-v) and ACF plots (A-E) given in Figure 1 correspond to five different time series. Match each time series plot in the first column with one of the ACF plots in the second column. Briefly Justify your answer for each selection.

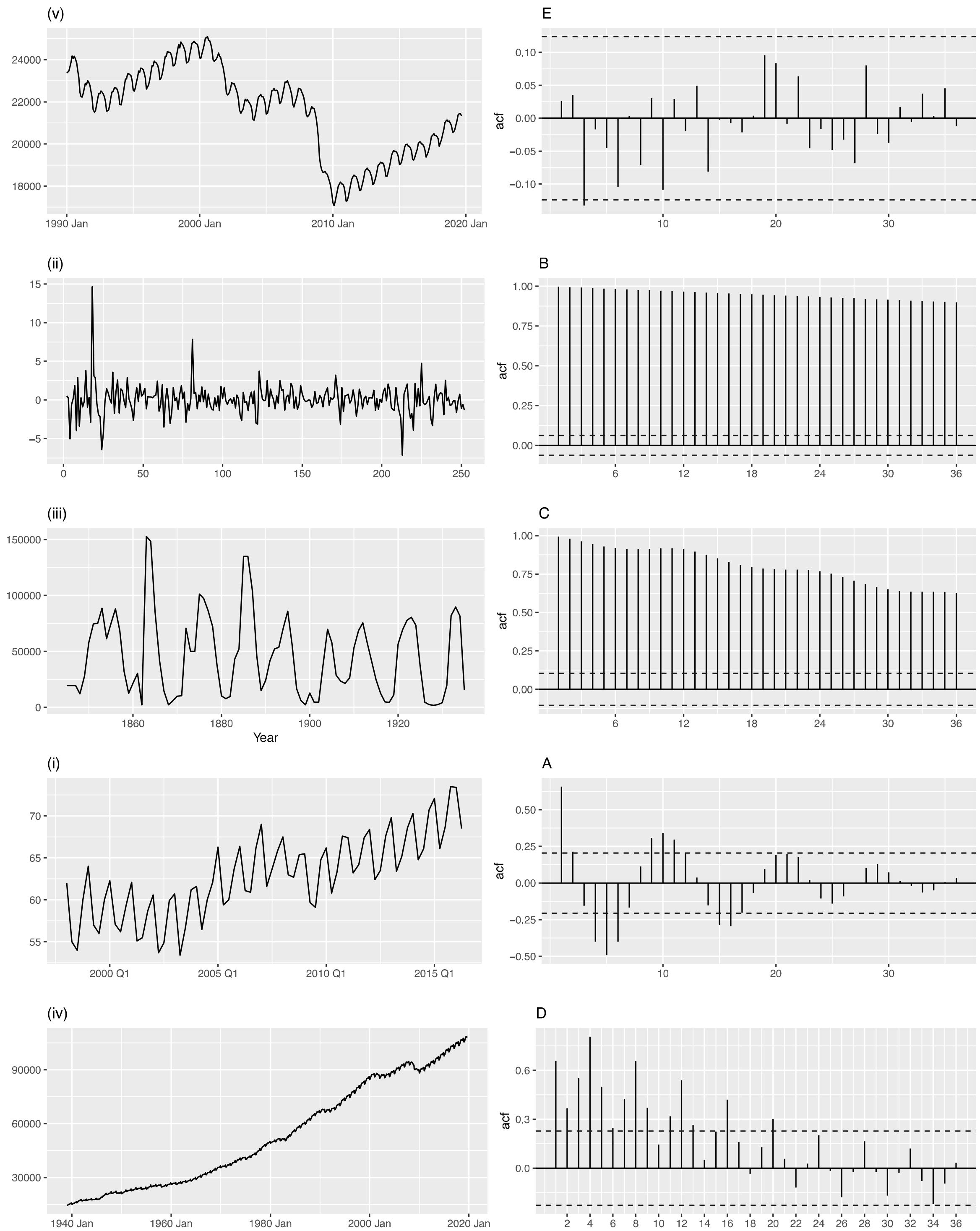

Figure 1: Time series plots and ACF plots

[5 Marks]

Question 1 continued ...

(c) Show that the following two processes

$$X_t = Z_t + \theta Z_{t-1}, \quad \{Z_t\} \sim WN(0, \sigma^2)$$

$$Y_t = \tilde{Z}_t + \frac{1}{\theta}\tilde{Z}_{t-1}, \quad \{\tilde{Z}_t\} \sim WN(0, \sigma^2\theta^2),$$

where $0 < |\theta| < 1$, have the same autocovariance functions.

[*5 Marks*]

Question 2 *[Total marks allocated: 20 Marks]*

(a) Monthly gas production (in billions of cubic metres) of a company from January 1960 to February 2005 is shown in Figure 2. The decomposition of the data is shown in Figure 3.

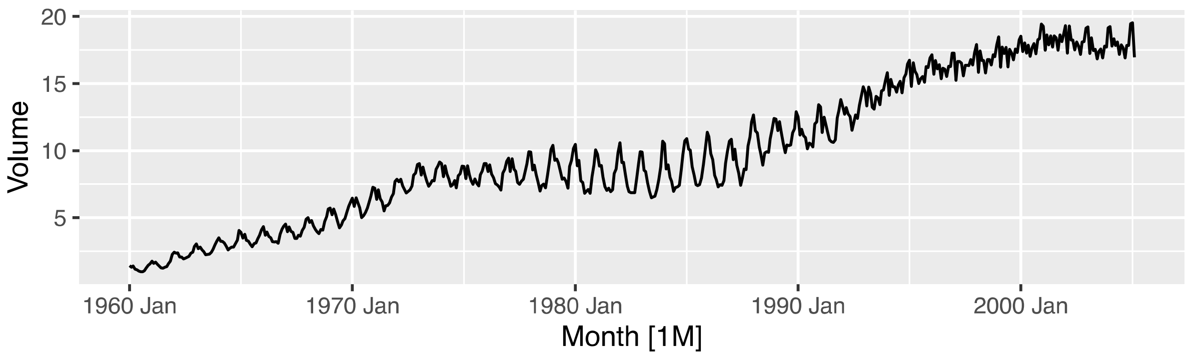

Figure 2: Monthly gas production from anuary 1960 to February 2005

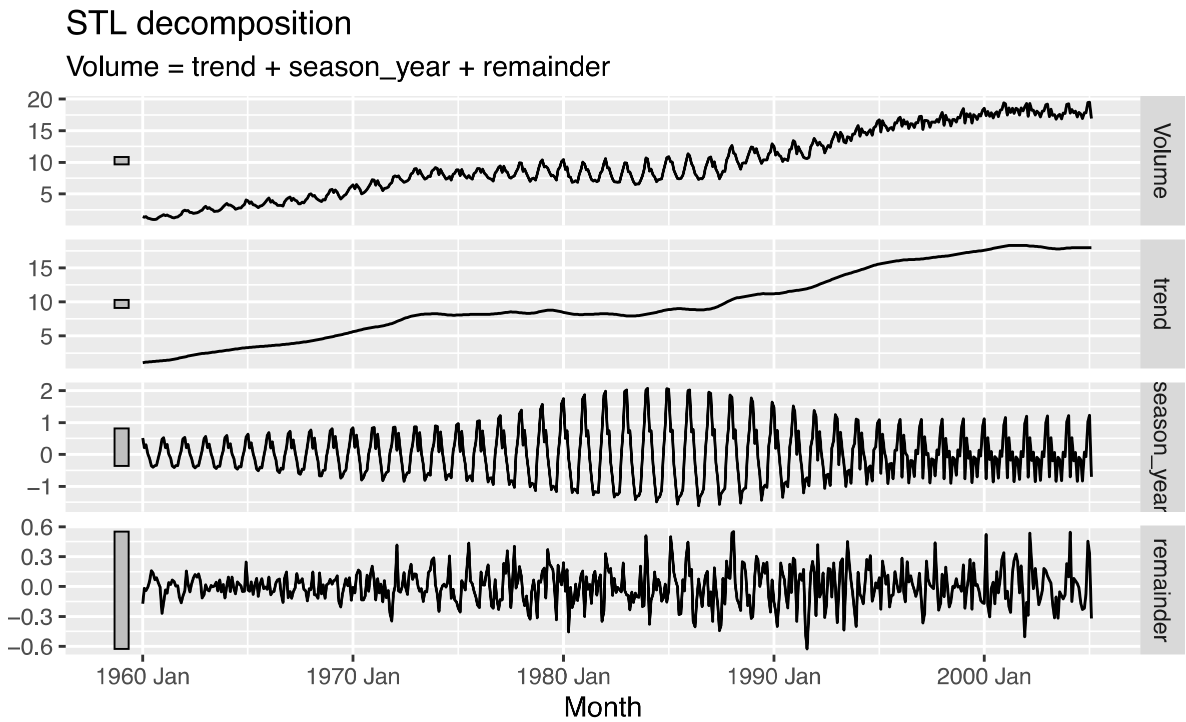

Figure 3: Decomposition of the data

i. Describe the series in 3 sentences including the time series patterns evident in the data. Your sentences should explain what is plotted in each of Figures 2 and 3. Pay particular attention to the scales of the graphs in Figure 3 in making your interpretation.

[*3 Marks*]

ii. Briefly explain why a Box-Cox transformation is unhelpful for the data presented in Figure 2?

[*2 Marks*]

(b) You are asked to provide forecasts of the data shown in Figure 2 for the next two years. Consider each of the methods below. Assume the methods will be applied to the data shown in Figure 2. Comment, in a few words each, on whether the methods listed might be appropriate for this data. The visual representation given in Figure 4 can also be used to justify your answers.

```
data %>%
  gg_tsdisplay(difference(Value, 12),
  plot_type='partial')
```

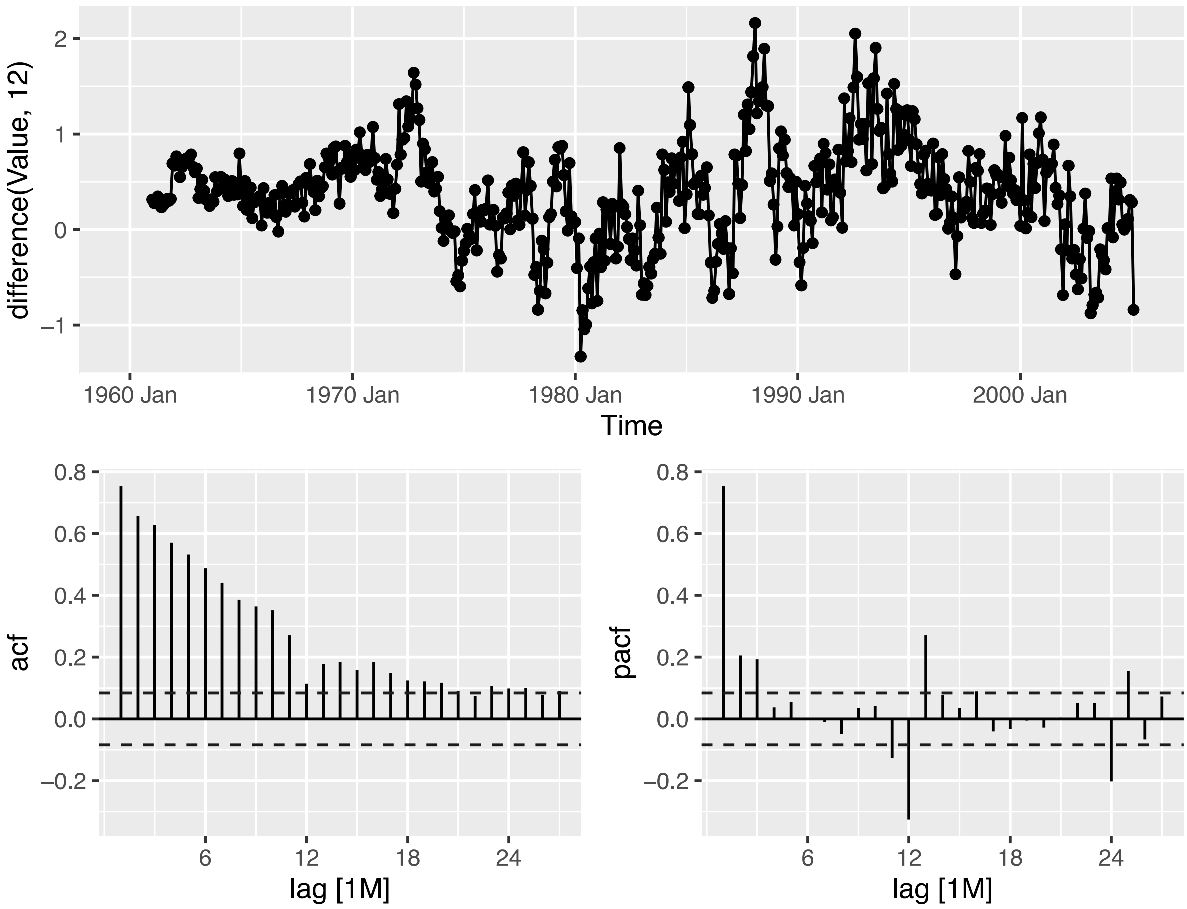

Figure 4: Seasonally differenced series

i. AR(3)
ii. ARIMA$(3, 1, 0)$
iii. ARIMA$(0, 1, 1)(0, 0, 0)_{12}$
iv. ARIMA$(2, 0, 1)(0, 1, 1)_{12}$

[*8 Marks*]

(c) Write the selected model in part (b) in terms of the backshift operator.

[*2 Marks*]

(d) Consider the model in the form

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t$$

where ϕ_0 is a constant and $\epsilon_t \sim WN(0, \sigma^2)$. Show that the (unconditional) variance of the above process is given by

$$Var(Y_t) = \frac{(1-\phi_2)\sigma^2}{(1+\phi_2)[(1-\phi_2^2)-\phi_1^2]}.$$

Provide all the necessary steps and assumptions behind your calculations.

[*5 Marks*]

Question 3 *[Total marks allocated: 20 Marks]*

(a) Given below are the results of a GARCH model fitting procedure for daily returns of the adjusted closing prices of the FB stock from January 1, 1990 to May 01, 2022 (Figure 5).

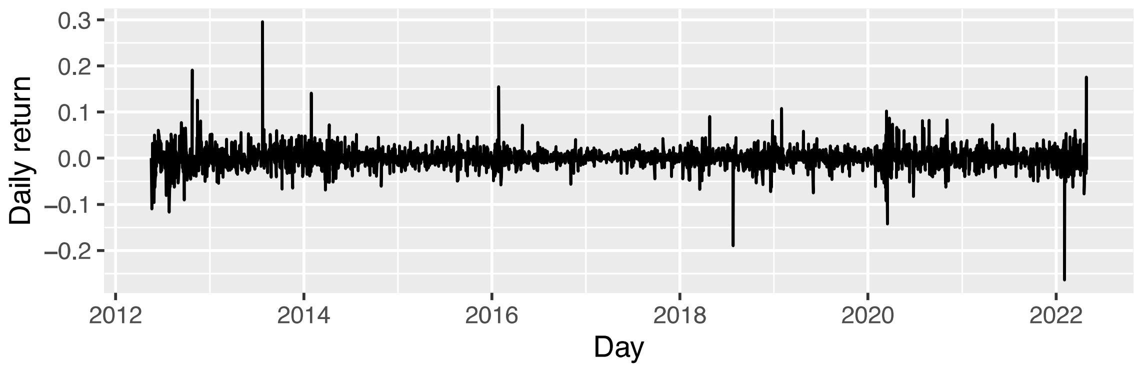

Figure 5: Daily returns of the adjusted closing prices of the FB stock from January 1, 1990 to May 01, 2022

Consider the two models below.

Model I

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(3, 0), data = fb_return$daily.returns)

Mean and Variance Equation:
 data ~ garch(3, 0)
<environment: 0x000001ff39e82e70>
 [data = fb_return$daily.returns]

Conditional Distribution:
```

```
 norm

Coefficient(s):
        mu       omega      alpha1      alpha2      alpha3
0.00046165  0.00035604  0.22162901  0.17261561  0.11723059

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.617e-04   4.368e-04    1.057    0.291
omega  3.560e-04   2.015e-05   17.666  < 2e-16 ***
alpha1 2.216e-01   5.074e-02    4.368 1.25e-05 ***
alpha2 1.726e-01   4.012e-02    4.303 1.69e-05 ***
alpha3 1.172e-01   3.002e-02    3.904 9.44e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 5859.81    normalized:  2.34018

Description:
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Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  100717.4  0
 Shapiro-Wilk Test  R    W      0.8564137 0
 Ljung-Box Test     R    Q(10)  10.79142  0.3739976
 Ljung-Box Test     R    Q(15)  11.76041  0.697066
 Ljung-Box Test     R    Q(20)  22.9577   0.2908762
 Ljung-Box Test     R^2  Q(10)  0.9502902 0.9998639
 Ljung-Box Test     R^2  Q(15)  2.720776  0.9997819
 Ljung-Box Test     R^2  Q(20)  3.103615  0.9999945
 LM Arch Test       R    TR^2   1.100507  0.9999759

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.676366 -4.664733 -4.676374 -4.672143
```

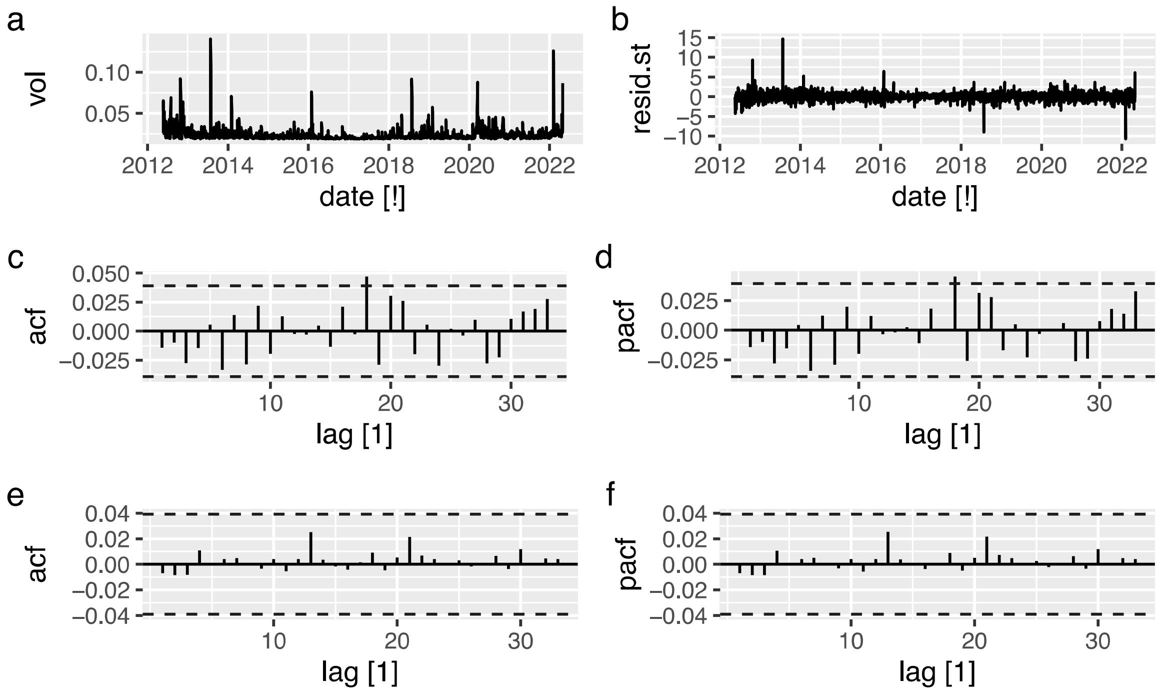

Figure 6: (a) Estimated volatility $\hat{\sigma_t}$, (b) the standardized residual $\hat{\epsilon_t}$, (c) sample ACF of $\hat{\epsilon_t}$, (d) sample PACF of $\hat{\epsilon_t}$, (e) sample ACF of $\hat{\epsilon_t^2}$, (d) sample PACF of $\hat{\epsilon_t^2}$, in **Model I** for daily returns of adjusted closing prices of the FB stock from January 1, 1990 to May 01, 2022

Model II

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 1) + garch(1, 1), data = fb_return$daily.returns)

Mean and Variance Equation:
 data ~ arma(1, 1) + garch(1, 1)
<environment: 0x000001ff34d15a38>
 [data = fb_return$daily.returns]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1           ma1        omega       alpha1        beta1
 4.8149e-04   7.0430e-01  -7.4740e-01   1.1693e-06   2.0943e-02   9.7929e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      4.815e-04   6.103e-04    0.789  0.43018
ar1     7.043e-01   2.679e-01    2.629  0.00857 **
```

```
ma1    -7.474e-01   2.610e-01   -2.863  0.00419 **
omega   1.169e-06   4.024e-07    2.906  0.00366 **
alpha1  2.094e-02   2.538e-03    8.252 2.22e-16 ***
beta1   9.793e-01   2.113e-03  463.411  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 5942.76    normalized:  2.373307

Description:
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Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  92211.3   0
 Shapiro-Wilk Test  R    W      0.8599461 0
 Ljung-Box Test     R    Q(10)  10.81585  0.3720436
 Ljung-Box Test     R    Q(15)  12.69254  0.6260326
 Ljung-Box Test     R    Q(20)  20.6899   0.4155828
 Ljung-Box Test     R^2  Q(10)  0.3828824 0.9999982
 Ljung-Box Test     R^2  Q(15)  0.7342845 1
 Ljung-Box Test     R^2  Q(20)  1.147417  1
 LM Arch Test       R    TR^2   0.6283532 0.999999

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-4.741821 -4.727862 -4.741832 -4.736753
```

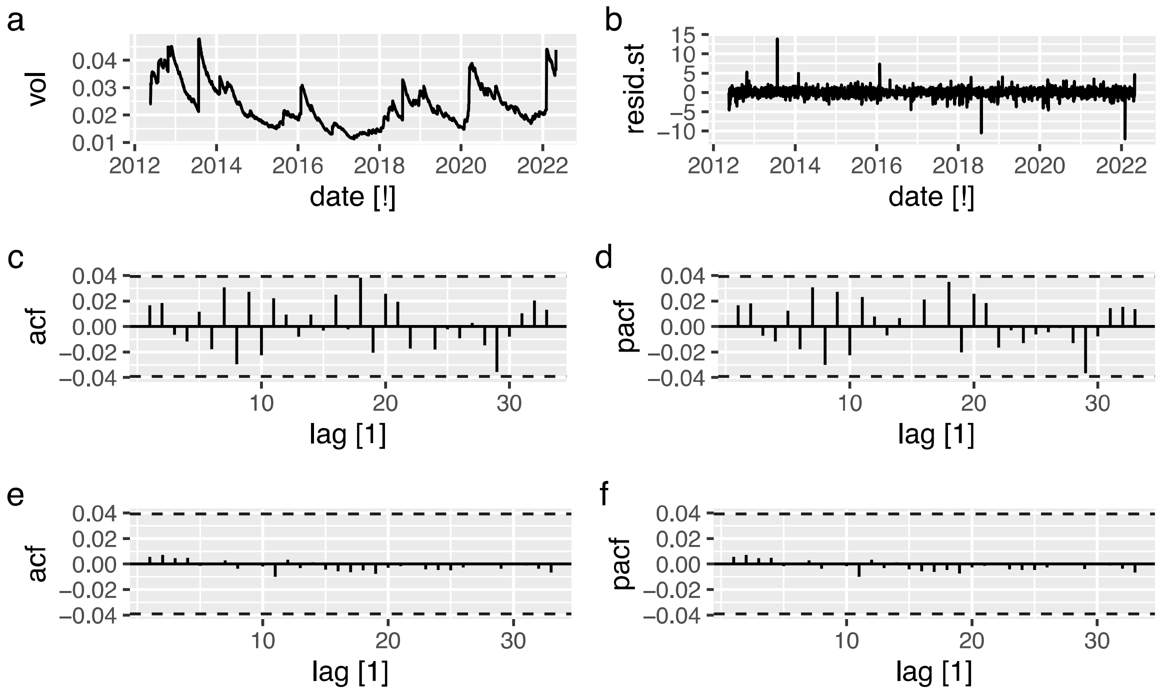

Figure 7: (a) Estimated volatility $\hat{\sigma_t}$, (b) the standardized residual $\hat{\epsilon_t}$, (c) sample ACF of $\hat{\epsilon_t}$, (d) sample PACF of $\hat{\epsilon_t}$, (e) sample ACF of $\hat{\epsilon_t^2}$, (d) sample PACF of $\hat{\epsilon_t^2}$, in **Model II** for daily returns of adjusted closing prices of the FB stock from January 1, 1990 to May 01, 2022.

i. Which model would you select (Model I or Model II) for daily returns of adjusted closing prices of the FB stock from January 1, 1990 to May 01, 2022? Justify your answer using all the information available in the computers outputs and the Figures 5 - 7. Use 5% significance level to perform the test.
ii. Write down the equations of the selected model.
iii. Discuss the suitability of IGARCH model for the above data scenario.

[*10 Marks*]

(b) "A GARCH model can be regarded as an application of the ARMA idea". Is this true or false? Briefly explain your answer.

[*5 Marks*]

(c) The basic $GARCH(1,1)$ model is given by $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ where σ_t^2 is the conditional variance of the error u_t in the mean equation of the time series model for $\{y_t\}$. The mean equation for $\{y_t\}$ is given by $y_t = \mu + u_t$ where μ is a constant.

Suppose now that the researcher had estimated the above GARCH model for a series of returns on a stock index and obtained the following parameter estimates: $\hat{\mu} = 0.0023$, $\hat{\alpha_0} = 0.0172$, $\hat{\alpha_1} = 0.1251$, $\hat{\beta}_1 = 0.8611$. If the researcher has data available up to and including time T , write down a set of equations in σ_T^2 and u_T^2 and their lagged values, which could be employed to produce one-, two-, and three-step-ahead forecasts for the conditional variance of u_t. Derive equations of the one-, two-, and three-step-ahead forecasts for σ_T^2.

[*5 Marks*]

Question 4 *[Total marks allocated: 20 Marks]*

(a) Consider the following bivariate VAR(2) model:

$$y_{1t} = a_0 + a_1 y_{1t-1} + a_2 y_{1t-2} + b_1 y_{2t-1} + b_2 y_{2t-2} + u_{1t}$$

$$y_{2t} = c_0 + c_1 y_{2t-1} + c_2 y_{2t-2} + d_1 y_{1t-1} + d_2 y_{1t-2} + u_{2t}$$

What conditions the above system must hold for it to be said that Granger causality runs from y_1 to y_2 only?

[*3 Marks*]

(b) Consider Sri Lanka's annual gross domestic product (gdp), and imports of goods and services (% of GDP) (base year 2010) from 1960 to 2017. Following computer outputs were obtained as a result of an analysis performed using the two series.

Output I

```
## Granger causality test
##
## Model 1: gdprate ~ Lags(gdprate, 1:3) + Lags(importsrate, 1:3)
## Model 2: gdprate ~ Lags(gdprate, 1:3)
##   Res.Df Df      F Pr(>F)
## 1     47
## 2     50 -3 0.0233 0.9951
```

Output II

```
## Granger causality test
##
## Model 1: importsrate ~ Lags(importsrate, 1:3) + Lags(gdprate, 1:3)
## Model 2: importsrate ~ Lags(importsrate, 1:3)
##   Res.Df Df      F   Pr(>F)
## 1     47
## 2     50 -3 4.4339 0.007974 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Stating any assumptions you made, test the following hypothesis. Use 5% significance level to perform the test. **Provide all the necessary steps to support your answer.**

i. GDP Granger causes Imports
ii. Imports Granger causes GDP
iii. Does the result indicate a presence of an instantaneous granger causality? Justify your answer.

[*9 Marks*]

(c) An analysis was performed to investigate whether two time series, X_t and Y_t, are cointegrated. The results of the different steps of the testing procedure can be summarised as below.

Test statistics and p value of Augmented Dickey Fuller (ADF) test performed on series X_t and Y_t

Series	Test statistic	p value
X_t	-0.093	0.9261
Y_t	0.257	0.7981

R output of the Least square regression for X_t and Y_t

```
dynlm(formula = y ~ x)

(Intercept)           x
  0.3547610   0.9764282
```

Test statistics and p value of the Augmented Dickey Fuller (ADF) test applied to the residual series of the previous model

Series	Test statistic	p value
Residual	-2.372	0.0204

i. Using the given information, test whether the two series, X_t and Y_t, are cointegrated. Provide all the necessary steps and assumptions to support your answer. Use 5% significance level to perform the test.
ii. Suppose that the two series are cointegrated. Briefly explain how to improve a VAR(1) of ΔX_t and ΔY_t.

[*8 Marks*]

Question 5 *[Total marks allocated: 20 Marks]*

(a) Consider the following two-variable VAR model with one lag and no intercept:

$$Y_t = \beta_{11} Y_{t-1} + \gamma_{11} X_{t-1} + u_{1t}$$
$$X_t = \beta_{21} Y_{t-1} + \gamma_{21} X_{t-1} + u_{2t}$$

Show that the iterated two-period ahead forecast for Y can be written as $Y_{t|t-2} = \delta_1 Y_{t-2} + \delta_2 X_{t-2}$, and derive values for δ_1 and δ_2 in terms of the coefficients in the VAR.

[*6 Marks*]

(b) Consider two variables, X_t and Y_t that are cointegrated. Are the following statements true or false? Justify your answers.

i. Y_t can be an $I(0)$.
ii. The cointegrating equation for X_t and Y_t describes the short-run relationship between the two series.
iii. When the Engle-Granger test is applied to the residuals of a potentially cointegrating regression, the interpretation of the null hypothesis is "The variables are cointegrated".

[*6 Marks*]

(c) Let Rk_t denote a k-period interest rate, let $R1_t$ denote a one-period interest rate, and let e_t denote an $I(0)$ term premium. Then $Rk_t = \frac{1}{k}\sum_{i=0}^{k-1} R1_{t+i|t} + e_t$ where $R1_{t+i|t}$ is the forecast made at date t of the value of $R1$ at date $t+i$. Suppose that $R1_t$ follows a random walk, so that $R1_t = R1_{t-1} + u_t$.

i. Show that $Rk_t = R1_t + e_t$.
ii. Show that Rk_t and $R1_t$ are cointegrated.
iii. What is the cointegrating coefficient?

[*8 Marks*]

-End of the Question Paper-