MA 5124 Financial Time Series Analysis and Forecasting

Chapter 1: Introduction to Time Series & Forecasting
Lesson 3

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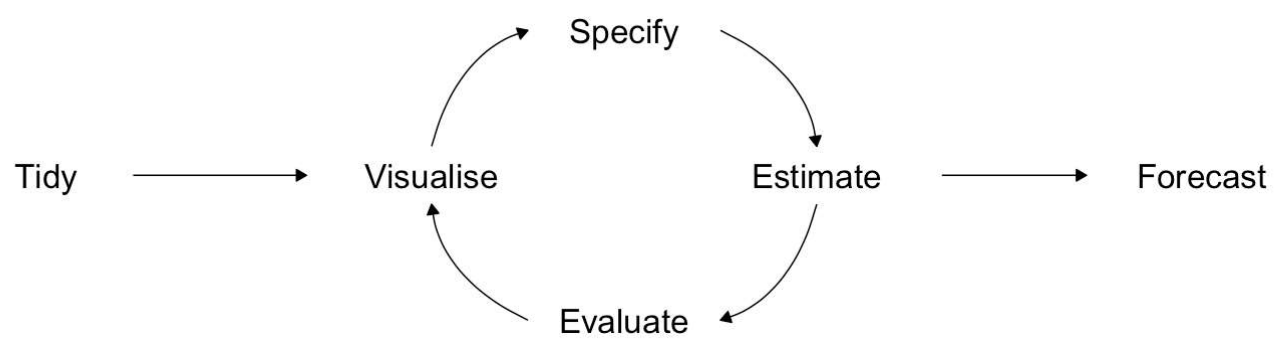
Useful tools for different forecasting situations

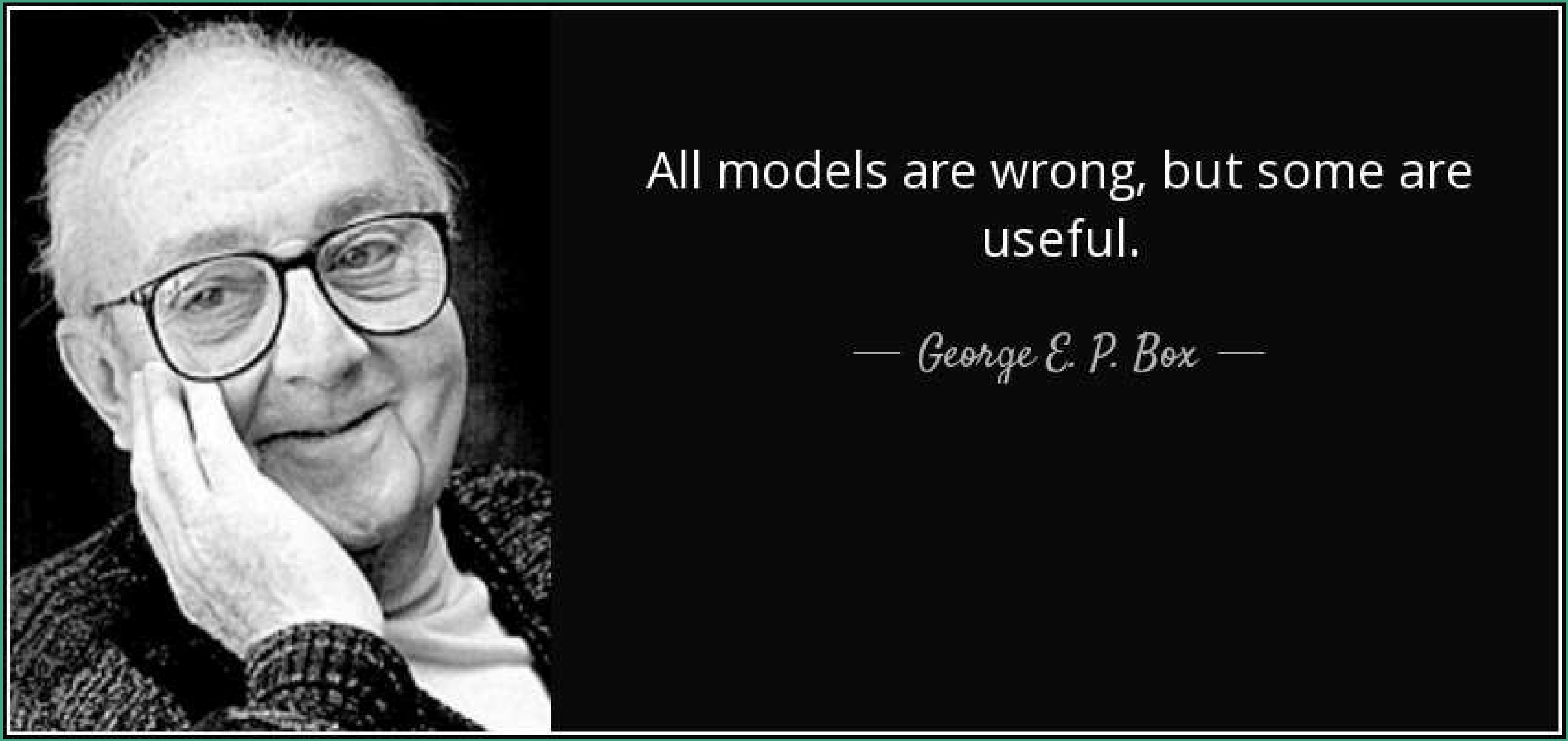
A tidy forecasting workflow

The process of producing forecasts can be split up into a few fundamental steps.

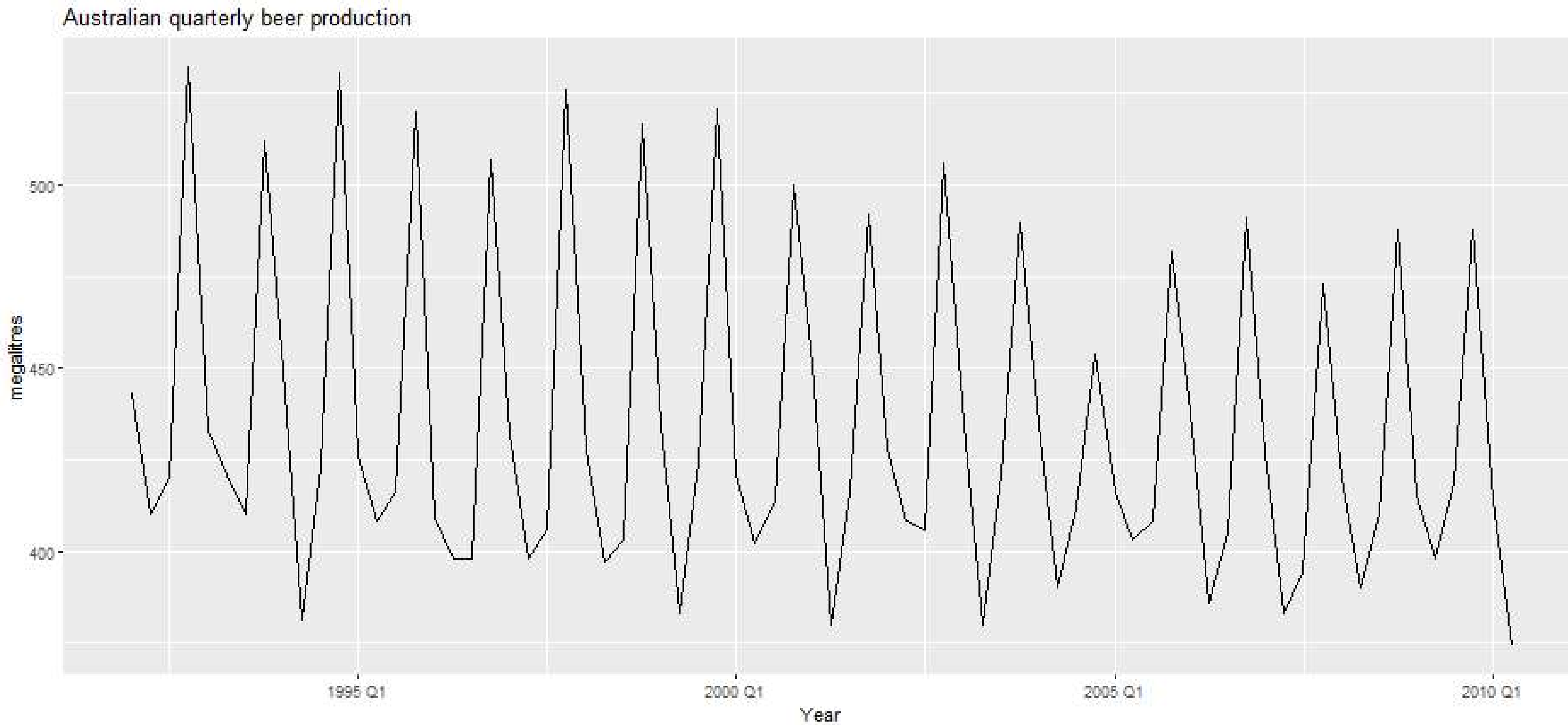


1 Preparing data
2 Data visualisation
3 Specifying a model
4 Model estimation
5 Accuracy and performance evaluation
6 Producing forecasts

Some simple forecasting methods

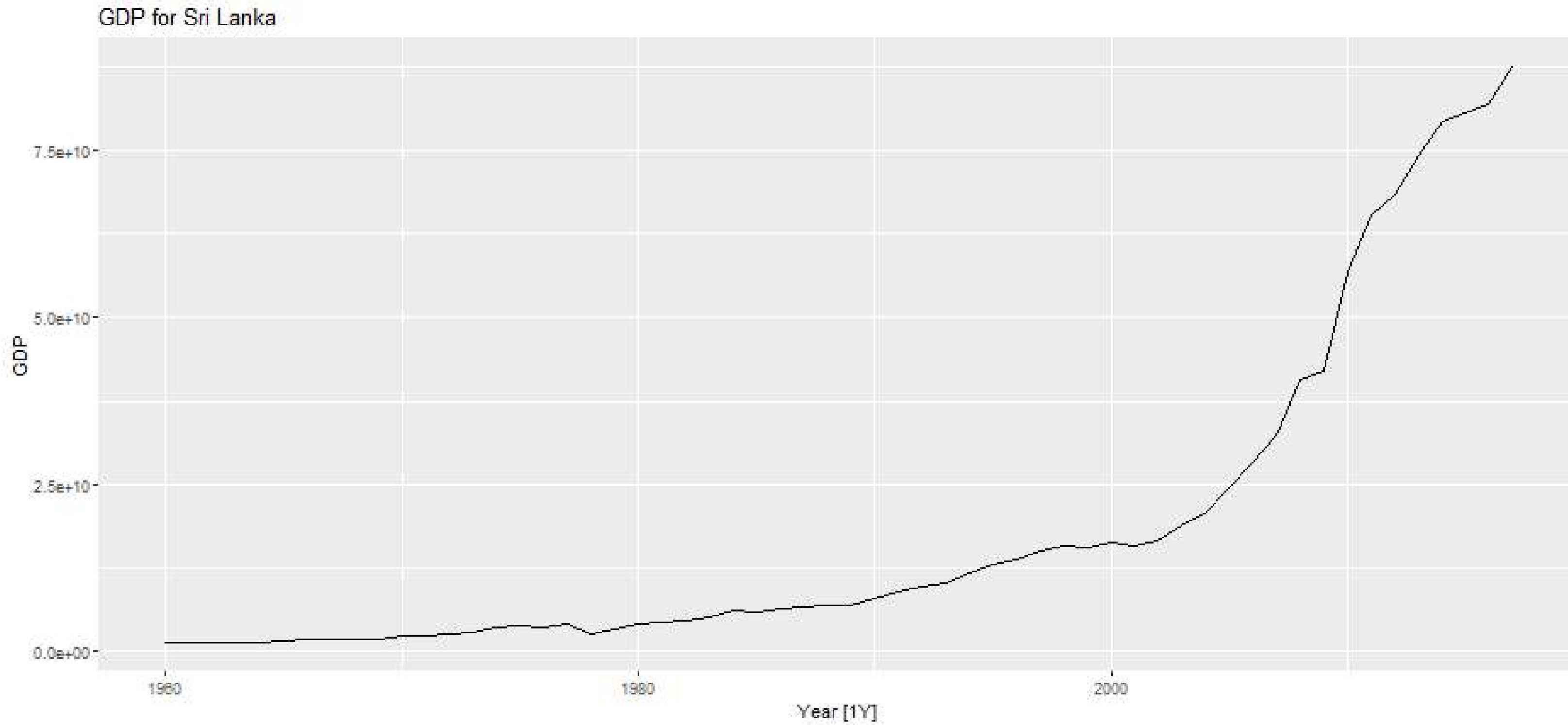


Some simple forecasting methods



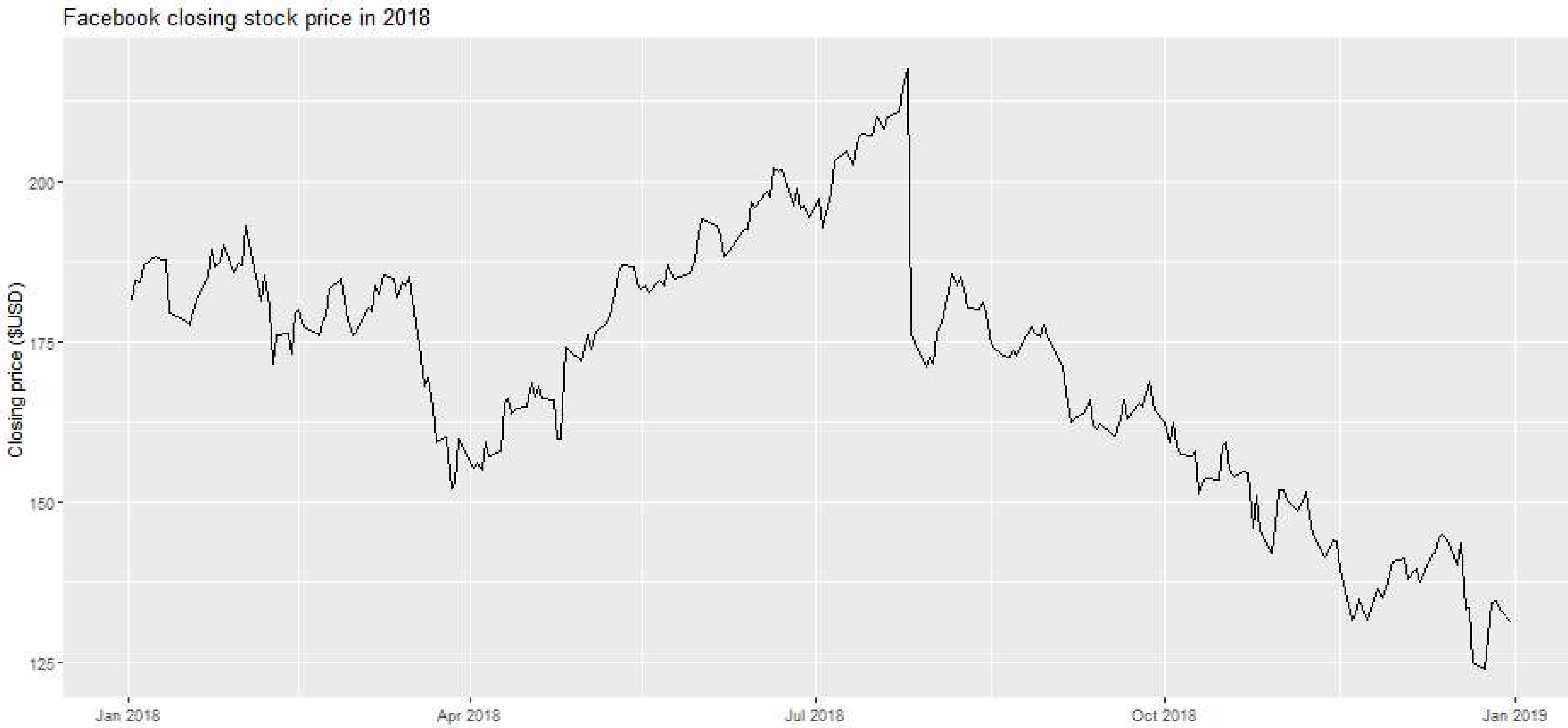
How would you forecast these series?

Some simple forecasting methods



How would you forecast these series?

Some simple forecasting methods



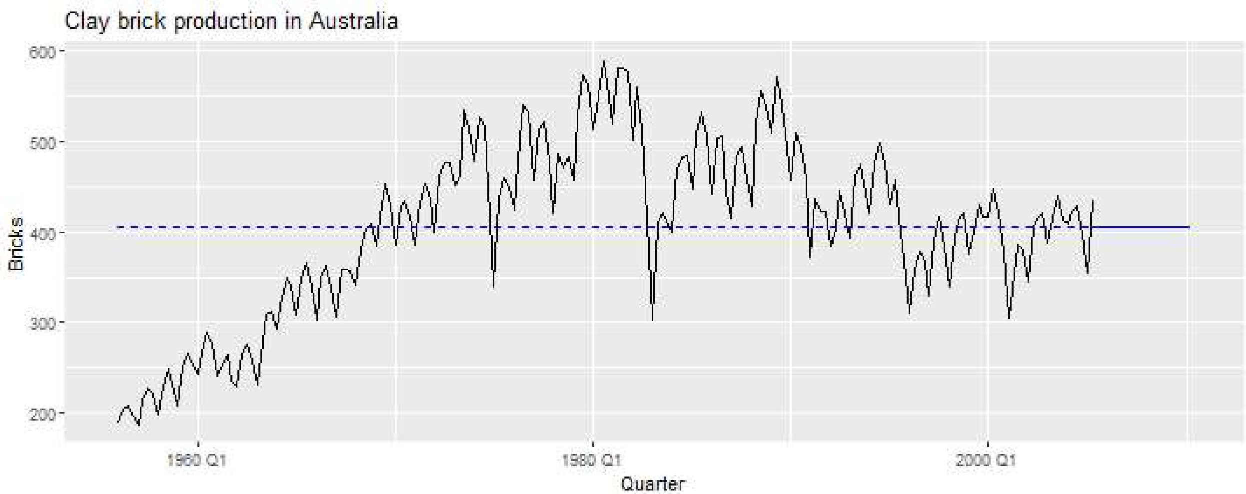
How would you forecast these series?

Some simple forecasting methods

`MEAN(y)`: Average method

- Forecasts of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

```
fit <- bricks |> model(MEAN(Bricks))
```

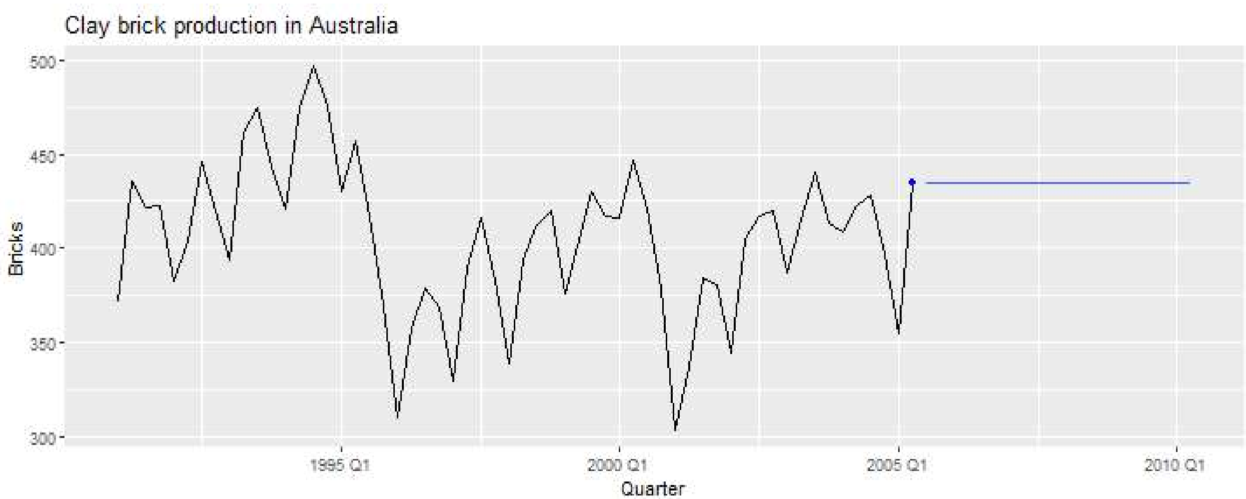


Some simple forecasting methods

`NAIVE(y)`: Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.

```
fit <- bricks |> model(NAIVE(Bricks))
```



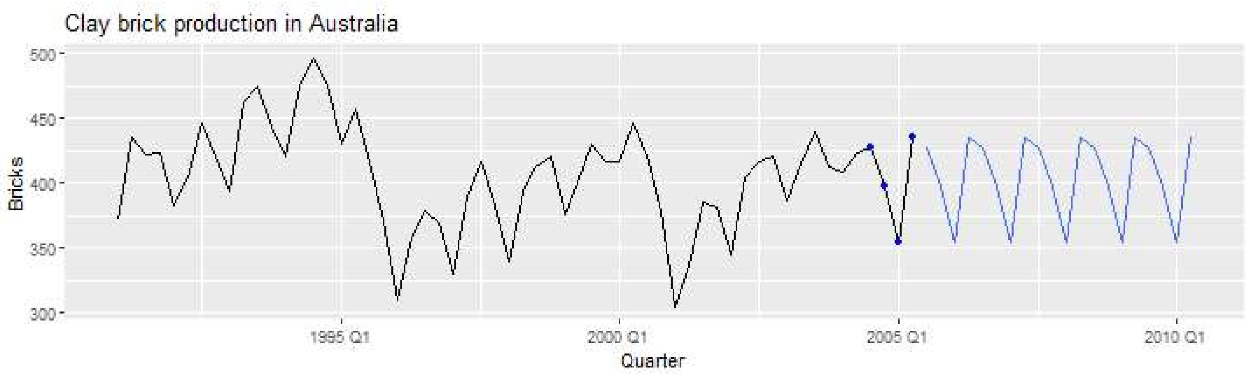
Some simple forecasting methods

`SNAIVE(y ~ lag(m))`: Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where $m =$ seasonal period and k is the integer part of $(h-1)/m$

.

```
fit <- bricks |> model(SNAIVE(Bricks~lag("year")))
```



Some simple forecasting methods

`RW(y ~ drift())`: Drift method

- Forecasts equal to last value plus average change.
- Forecasts:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1})\\ &= y_T + \frac{h}{T-1}(y_T - y_1)\end{aligned}$$

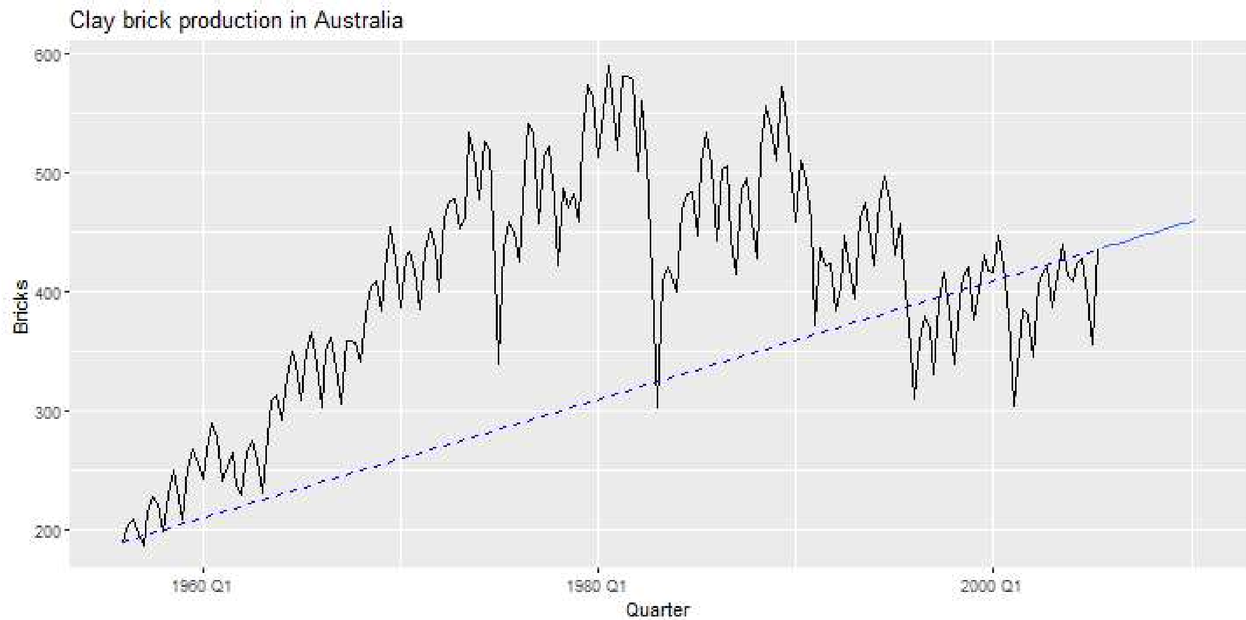
.

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods

Drift method

```
bricks |> model(RW(Bricks ~ drift()))
```



Model fitting

The `model()` function trains models to data.

```
brick_fit <-  aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )

brick_fit
```

```
## # A mable: 1 x 4
##   Seasonal_naive   Naive         Drift    Mean
##          <model> <model>       <model> <model>
## 1       <SNAIVE> <NAIVE> <RW w/ drift>  <MEAN>
```

A `mable` is a model table, each cell corresponds to a fitted model.

Producing forecasts

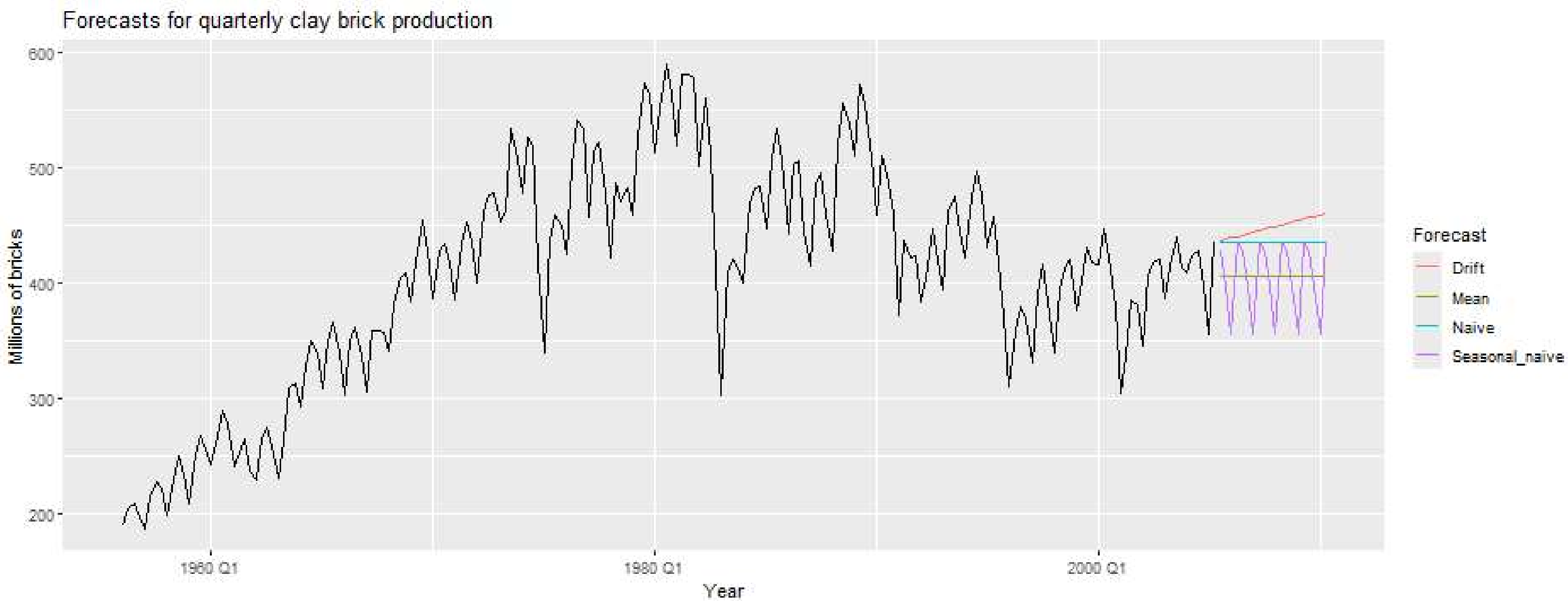
```
brick_fc <- brick_fit |>
  forecast(h = "5 years")

print(brick_fc)
```

```
## # A fable: 80 x 4 [1Q]
## # Key:     .model [4]
##    .model         Quarter       Bricks .mean
##    <chr>            <qtr>       <dist> <dbl>
##  1 Seasonal_naive 2005 Q3 N(428, 2336)   428
##  2 Seasonal_naive 2005 Q4 N(397, 2336)   397
##  3 Seasonal_naive 2006 Q1 N(355, 2336)   355
##  4 Seasonal_naive 2006 Q2 N(435, 2336)   435
##  5 Seasonal_naive 2006 Q3 N(428, 4672)   428
##  6 Seasonal_naive 2006 Q4 N(397, 4672)   397
##  7 Seasonal_naive 2007 Q1 N(355, 4672)   355
##  8 Seasonal_naive 2007 Q2 N(435, 4672)   435
##  9 Seasonal_naive 2007 Q3 N(428, 7008)   428
## 10 Seasonal_naive 2007 Q4 N(397, 7008)   397
## # ℹ 70 more rows
```

A `fable` is a forecast table with point forecasts and distributions.

Visualising forecasts

```
brick_fc |>
  autoplot(aus_production, level = NULL) +
  ggtitle("Forecasts for quarterly clay brick production") +
  xlab("Year") + ylab("Millions of bricks") +
  guides(colour = guide_legend(title = "Forecast"))
```



Residual diagnostics

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_{t-1}$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T-1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.

2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

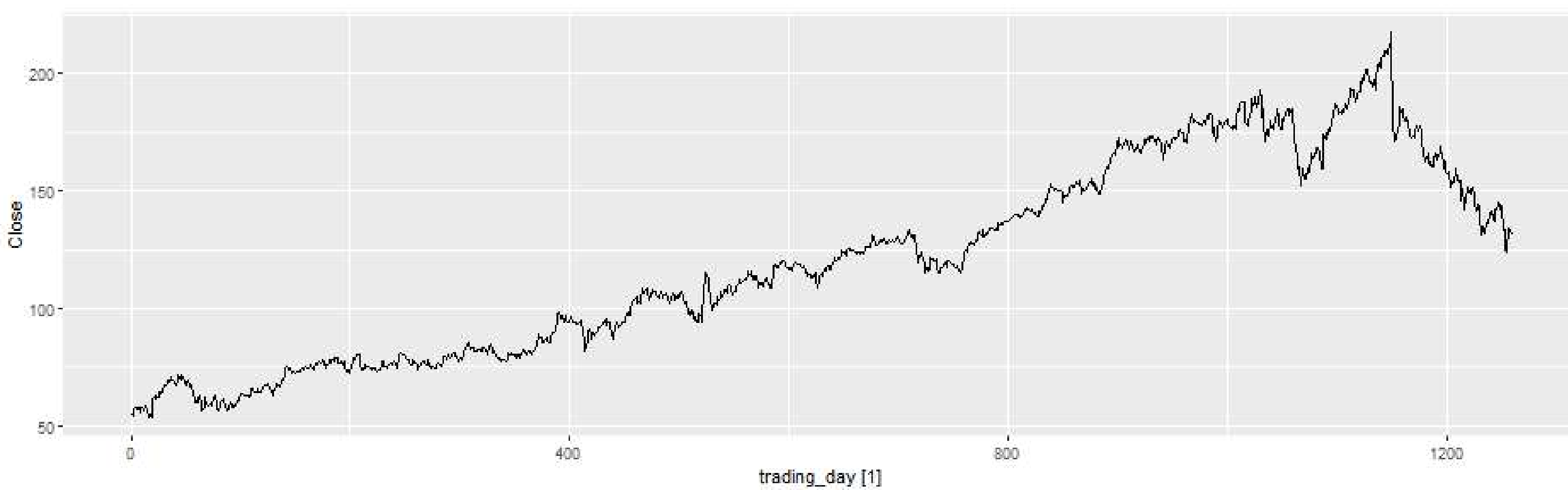
Useful properties (for distributions and prediction intervals)

3 $\{e_t\}$ have constant variance.

4 $\{e_t\}$ are normally distributed.

Facebook closing stock price

```
fb_stock <- gafa_stock |>
  filter(Symbol == "FB") |>
  mutate(trading_day = row_number()) |>
  update_tsibble(index = trading_day, regular = TRUE)
fb_stock |> autoplot(Close)
```



Facebook closing stock price

```
fit <- fb_stock |> model(NAIVE(Close))
fit
```

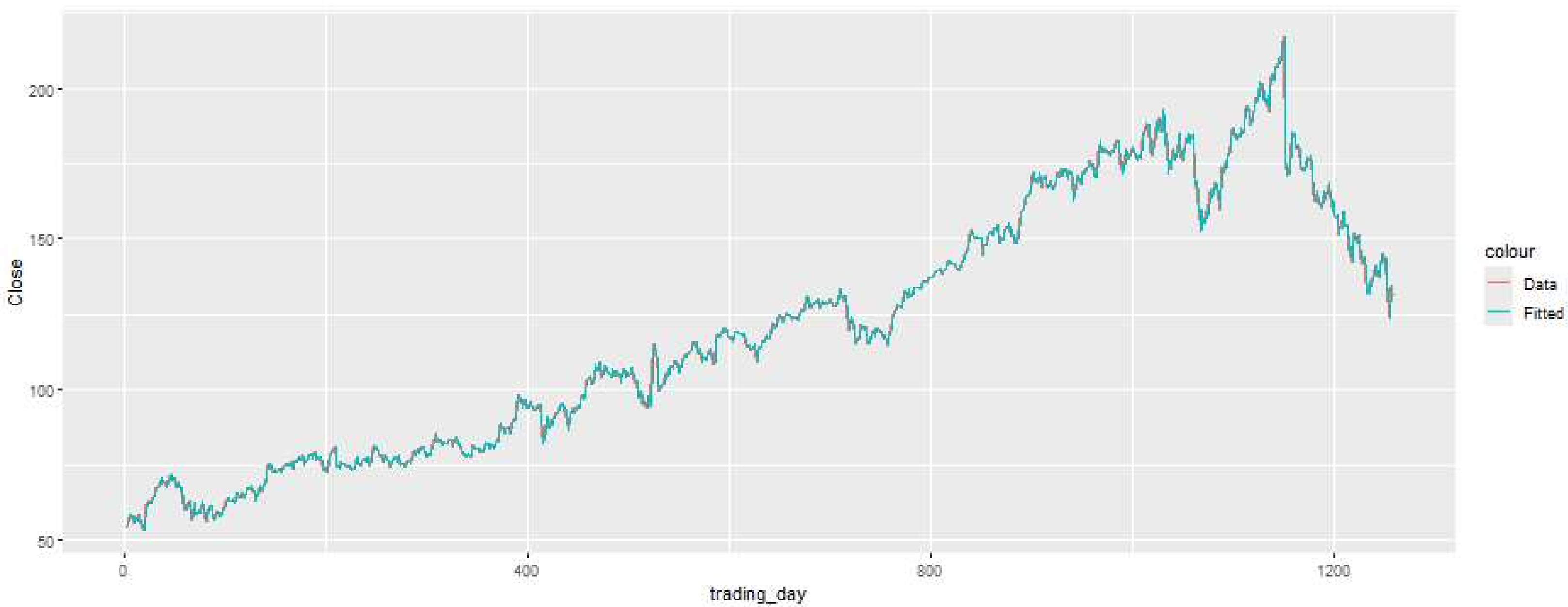
```
# A mable: 1 x 2
# Key:     Symbol [1]
  Symbol `NAIVE(Close)`
  <chr>         <model>
1 FB            <NAIVE>
```

```
augment(fit)
```

```
# A tsibble: 1,258 x 7 [1]
# Key:       Symbol, .model [1]
   Symbol .model       trading_day Close .fitted .resid .innov
   <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
 1 FB     NAIVE(Close)           1  54.7    NA   NA     NA    
 2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
 3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64 
 4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
 5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
 6 FB     NAIVE(Close)           6  57.2    58.2 -1.01  -1.01 
 7 FB     NAIVE(Close)           7  57.9    57.2  0.720  0.720
 8 FB     NAIVE(Close)           8  55.9    57.9 -2.03  -2.03 
```

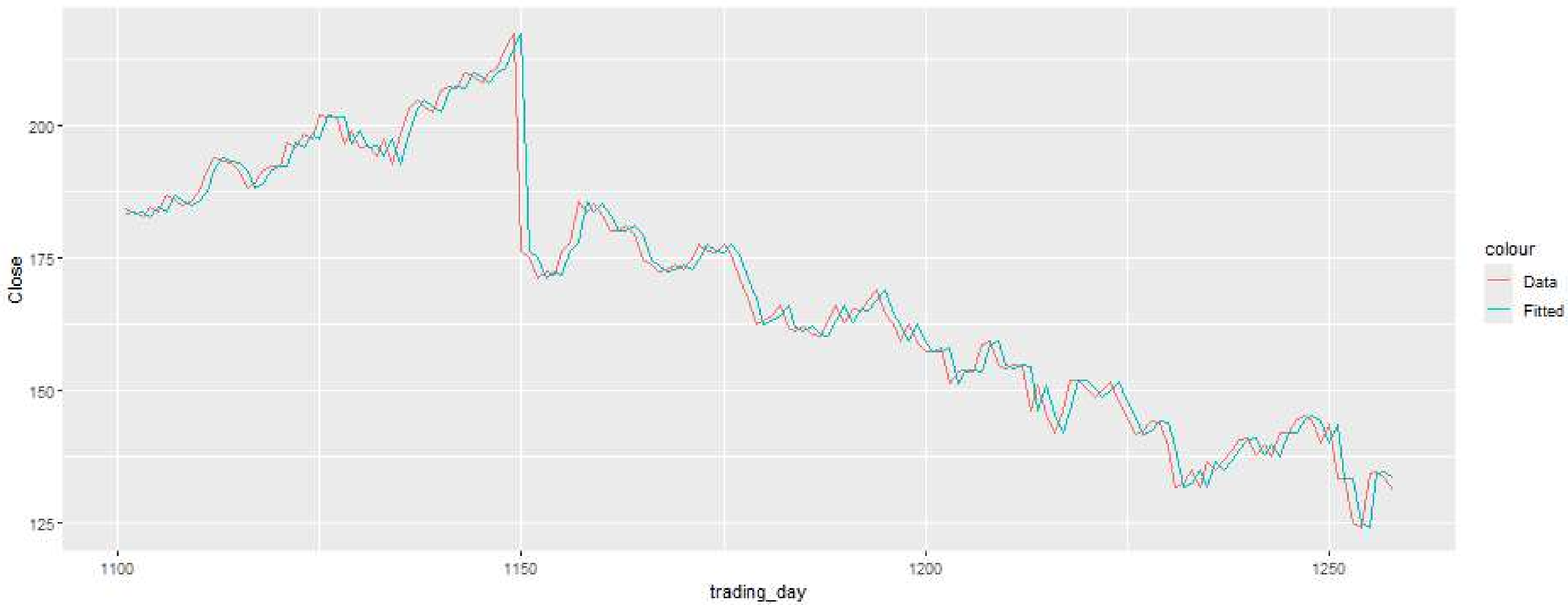
Facebook closing stock price

```
augment(fit) |>
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



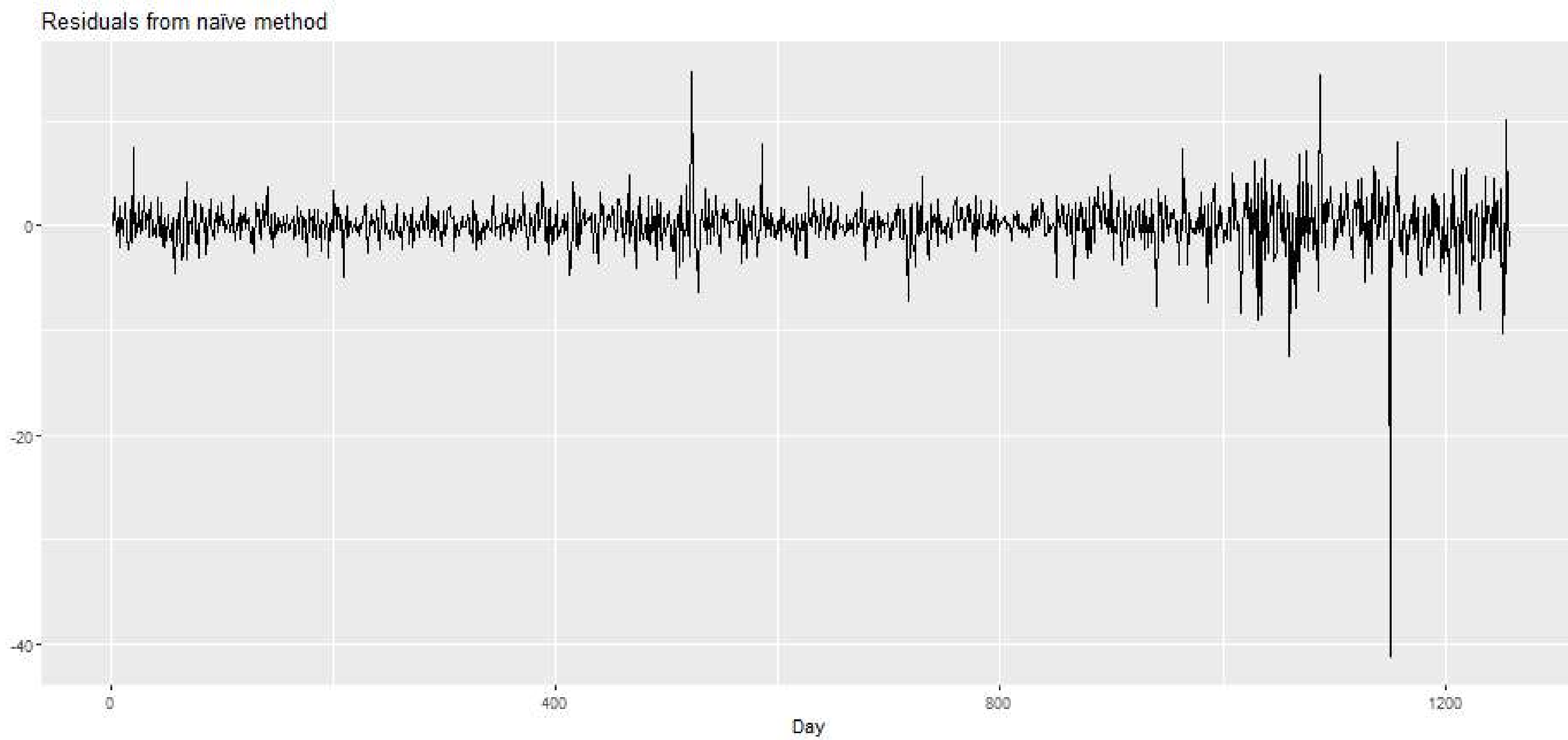
Facebook closing stock price

```
augment(fit) |>
  filter(trading_day > 1100) |>
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



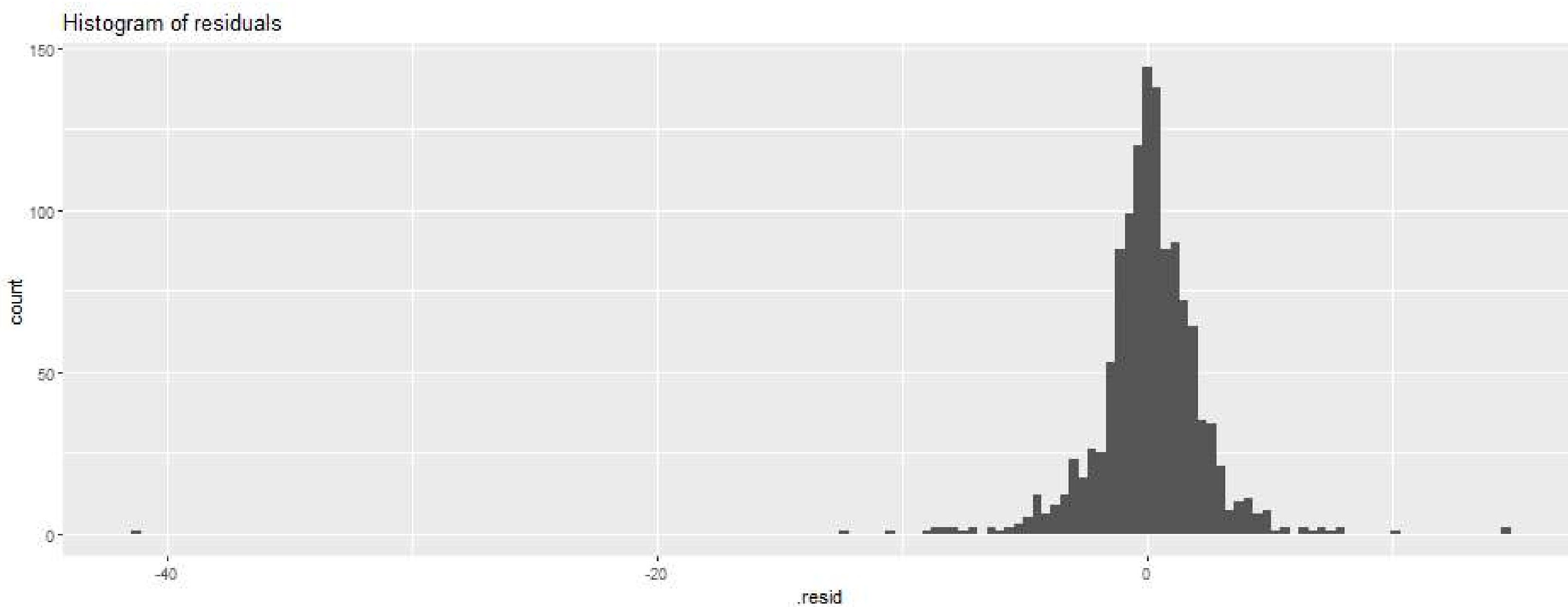
Facebook closing stock price

```
augment(fit) |>
  autoplot(.resid) + xlab("Day") + ylab("") +
  ggtitle("Residuals from naïve method")
```



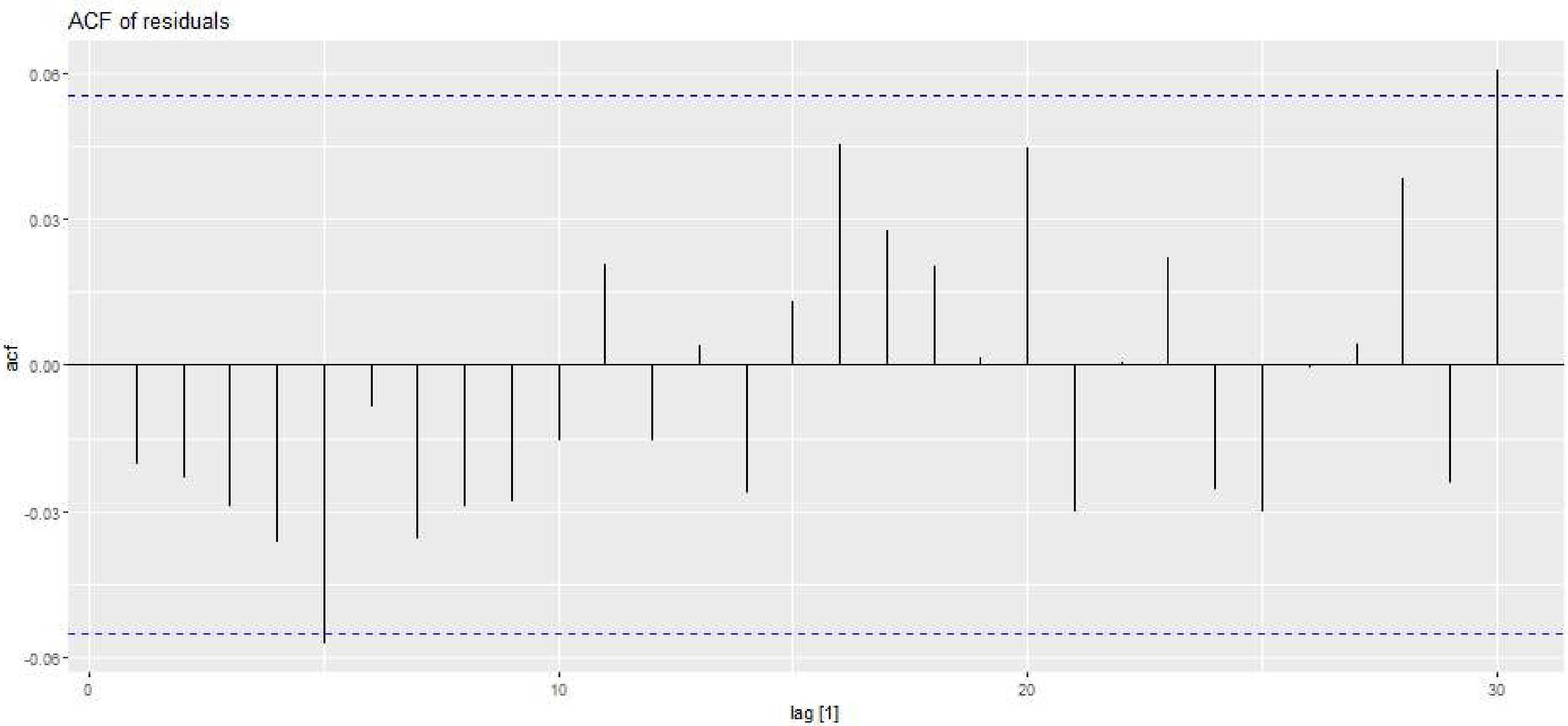
Facebook closing stock price

```
augment(fit) |>
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 150) +
  ggtitle("Histogram of residuals")
```



Facebook closing stock price

```
augment(fit) |>
  ACF(.resid) |>
  autoplot() + ggtitle("ACF of residuals")
```

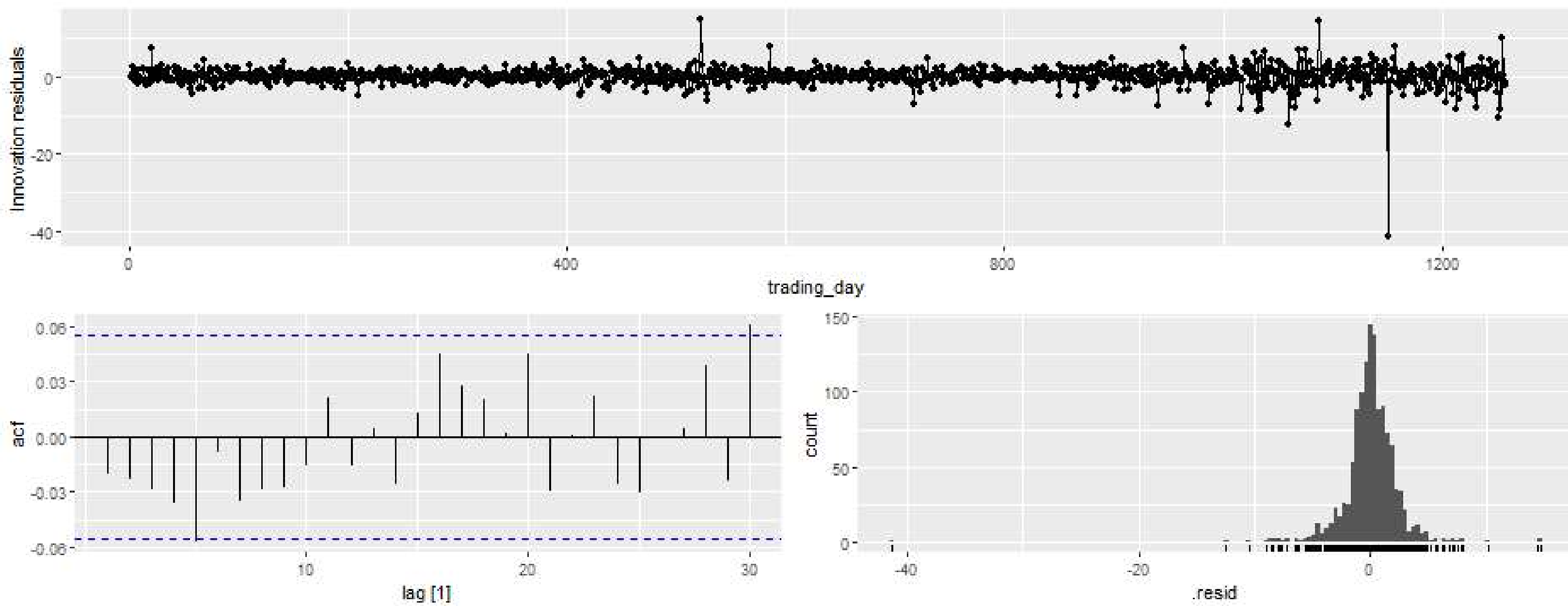


ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

`gg_tsresiduals` function

```
gg_tsresiduals(fit)
```



Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T\sum_{k=1}^{h} r_k^2$$

where h is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{h}(T-k)^{-1}r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where $K =$ no. parameters in model.
- When applied to raw data, set $K = 0$.

```
augment(fit) |>
  features(.resid, ljung_box, lag=10, dof=0)
```

```
## # A tibble: 1 × 4
##   Symbol .model       lb_stat lb_pvalue
##   <chr>  <chr>          <dbl>     <dbl>
## 1 FB     NAIVE(Close)    12.1     0.276
```

Distributional forecasts and prediction intervals

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

- **Mean:** $\hat{y}_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
- **Naïve:** $\hat{y}_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$
- **Seasonal naïve** $\hat{y}_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$
- **Drift:** $\hat{y}_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h-1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h-step distribution.

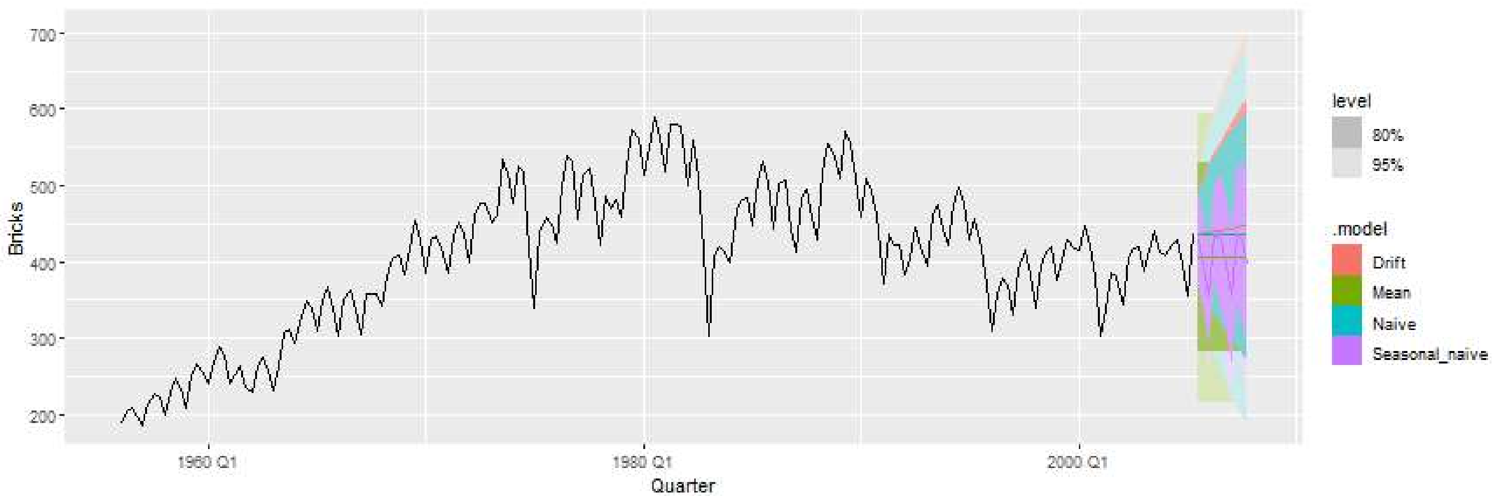
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```r
brick_fc |> hilo(level = 95)
```

```
## # A tsibble: 80 x 5 [1Q]
## # Key:       .model [4]
##    .model         Quarter       Bricks .mean                    `95%`
##    <chr>            <qtr>       <dist> <dbl>                   <hilo>
##  1 Seasonal_naive 2005 Q3 N(428, 2336)   428 [333.2737, 522.7263]95
##  2 Seasonal_naive 2005 Q4 N(397, 2336)   397 [302.2737, 491.7263]95
##  3 Seasonal_naive 2006 Q1 N(355, 2336)   355 [260.2737, 449.7263]95
##  4 Seasonal_naive 2006 Q2 N(435, 2336)   435 [340.2737, 529.7263]95
##  5 Seasonal_naive 2006 Q3 N(428, 4672)   428 [294.0368, 561.9632]95
##  6 Seasonal_naive 2006 Q4 N(397, 4672)   397 [263.0368, 530.9632]95
##  7 Seasonal_naive 2007 Q1 N(355, 4672)   355 [221.0368, 488.9632]95
##  8 Seasonal_naive 2007 Q2 N(435, 4672)   435 [301.0368, 568.9632]95
##  9 Seasonal_naive 2007 Q3 N(428, 7008)   428 [263.9292, 592.0708]95
## 10 Seasonal_naive 2007 Q4 N(397, 7008)   397 [232.9292, 561.0708]95
## # ℹ 70 more rows
```

```
bricks |>
  filter(!is.na(Bricks)) |>
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  ) |>
  forecast(h = 10) |> autoplot(bricks)
```



Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- Multi-step forecasts for time series require a more sophisticated approach (with PI getting wider as the forecast horizon increases).

Prediction intervals

- Computed automatically from the forecast distribution.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them (we will see this next class).
- Usually too narrow due to unaccounted uncertainty.

Forecasting and decomposition

Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method.
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

US Retail Employment

```r
us_retail_employment <- us_employment |>
  filter(year(Month) >= 1990, Title == "Retail Trade") |>
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##       Month Title        Employed
##       <mth> <chr>           <dbl>
##  1 1990 Jan Retail Trade   13256.
##  2 1990 Feb Retail Trade   12966.
##  3 1990 Mar Retail Trade   12938.
##  4 1990 Apr Retail Trade   13012.
##  5 1990 May Retail Trade   13108.
##  6 1990 Jun Retail Trade   13183.
##  7 1990 Jul Retail Trade   13170.
##  8 1990 Aug Retail Trade   13160.
##  9 1990 Sep Retail Trade   13113.
## 10 1990 Oct Retail Trade   13185.
## # ℹ 347 more rows
```

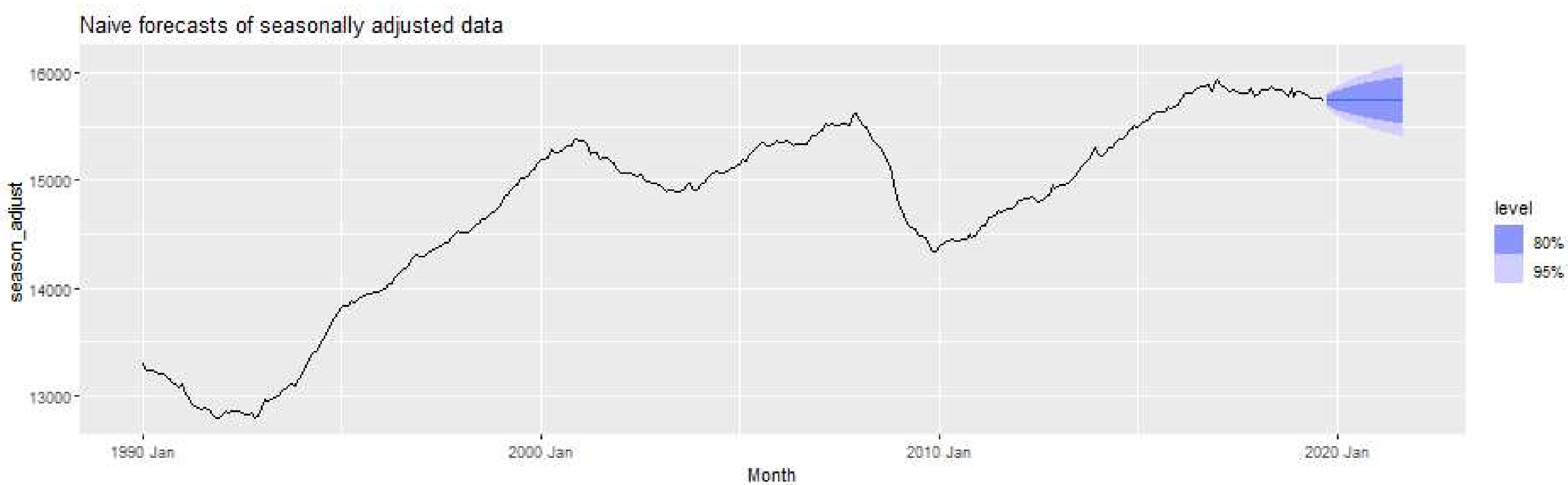
US Retail Employment

```
dcmp <- us_retail_employment |>
  model(STL(Employed)) |>
  components() |>
  select(-.model)
dcmp
```

```
## # A tsibble: 357 x 6 [1M]
##       Month Employed  trend season_year remainder season_adjust
##       <mth>    <dbl>  <dbl>       <dbl>     <dbl>         <dbl>
##  1 1990 Jan   13256. 13288.       -33.0     0.836        13289.
##  2 1990 Feb   12966. 13269.      -258.    -44.6          13224.
##  3 1990 Mar   12938. 13250.      -290.    -22.1          13228.
##  4 1990 Apr   13012. 13231.      -220.      1.05         13232.
##  5 1990 May   13108. 13211.      -114.     11.3          13223.
##  6 1990 Jun   13183. 13192.       -24.3    15.5          13207.
##  7 1990 Jul   13170. 13172.       -23.2    21.6          13193.
##  8 1990 Aug   13160. 13151.        -9.52   17.8          13169.
##  9 1990 Sep   13113. 13131.       -39.5    22.0          13153.
## 10 1990 Oct   13185. 13110.        61.6    13.2          13124.
## # ℹ 347 more rows
```

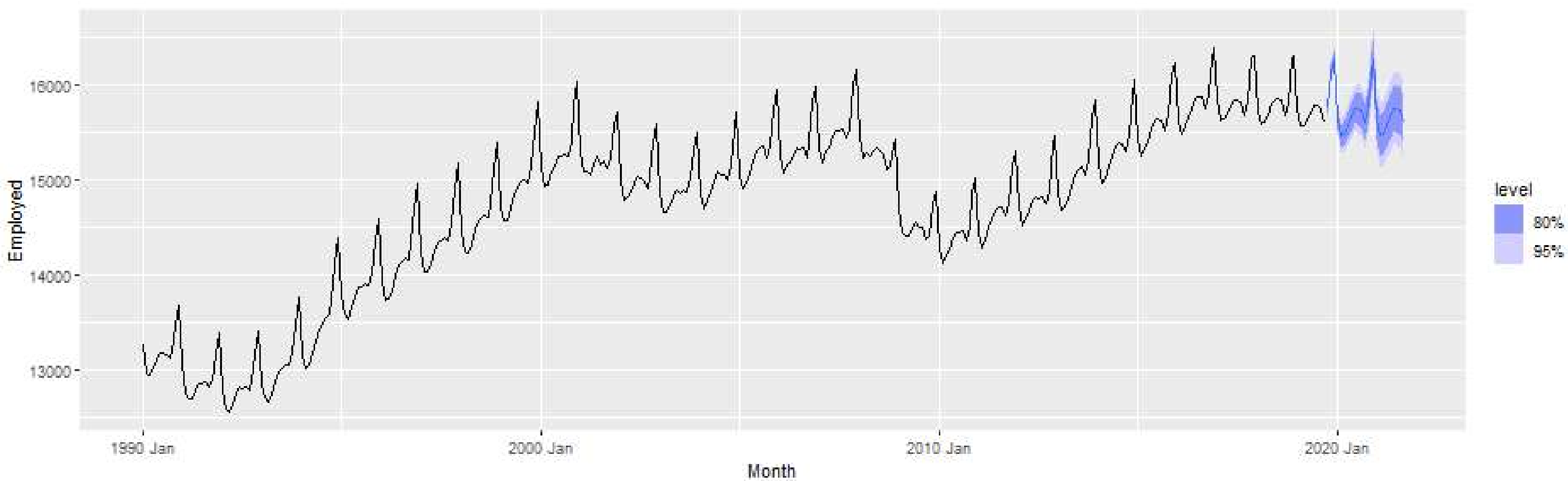
US Retail Employment

```
dcmp |>
  model(NAIVE(season_adjust)) |>
  forecast() |>
  autoplot(dcmp) +
  ggtitle("Naive forecasts of seasonally adjusted data")
```



US Retail Employment

```
fit_dcmp <- us_retail_employment |>
  model(stlf = decomposition_model(
    STL(Employed ~ trend(window = 7), robust = TRUE),
    NAIVE(season_adjust)
  ))

fit_dcmp |> forecast() |>
  autoplot(us_retail_employment)
```

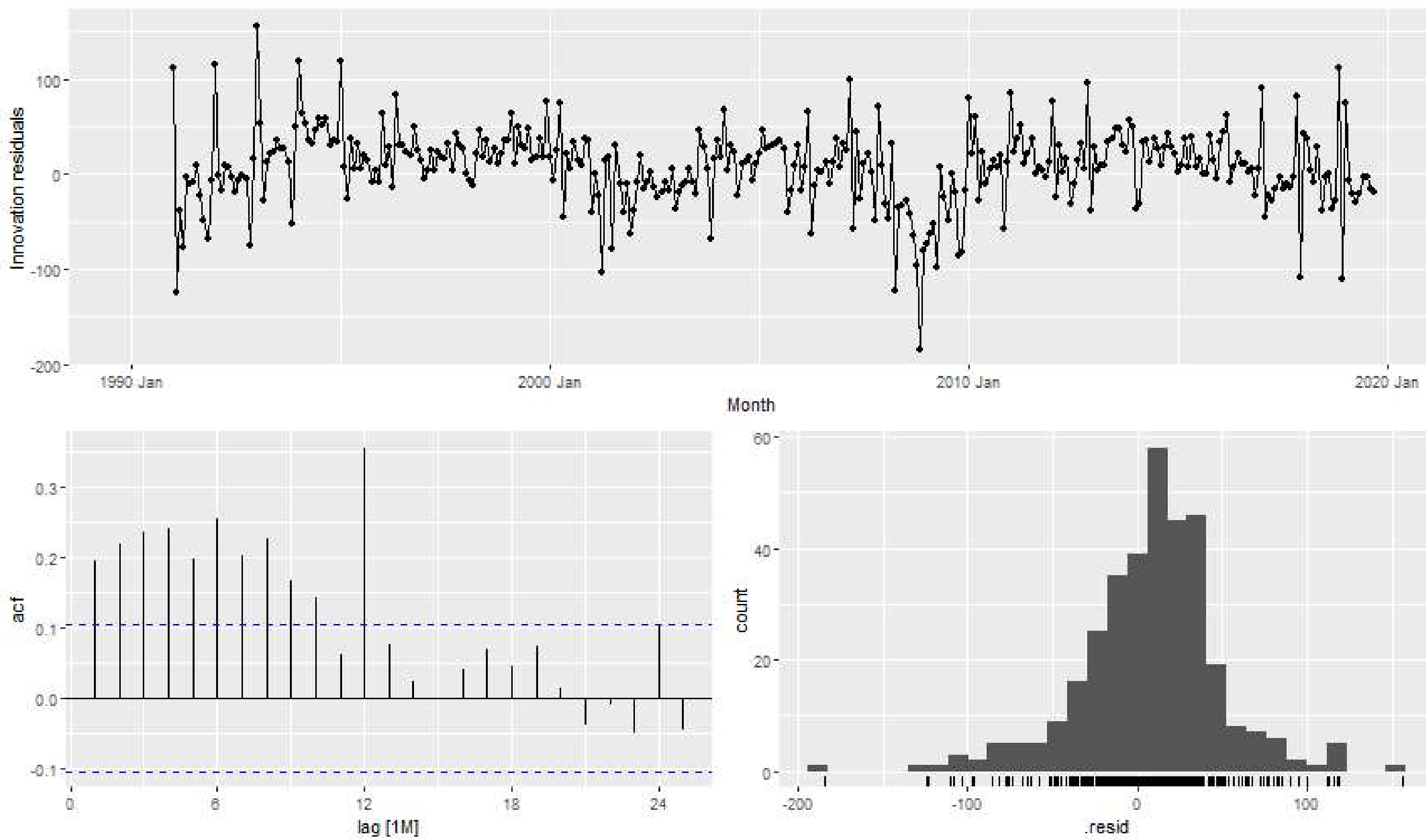


Decomposition models

`decomposition_model()` creates a decomposition model

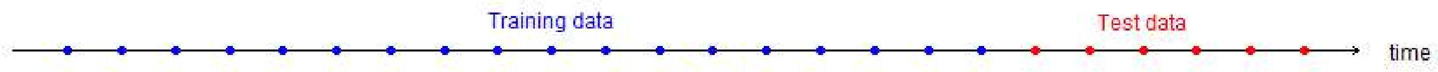
- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naive method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.

```
fit_dcmp |> gg_tsresiduals()
```



Evaluating forecast accuracy

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

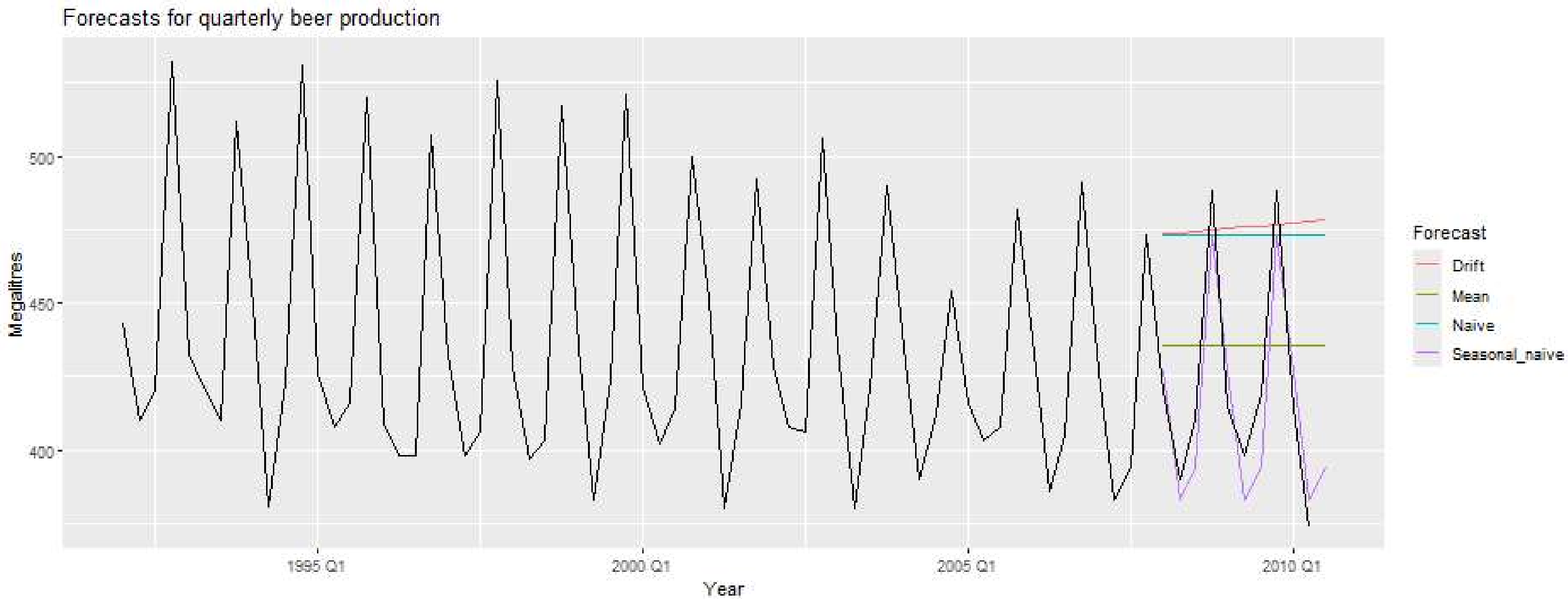
Forecast "error": the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy



Measures of forecast accuracy

$$y_{T+h} = (T+h)\text{th observation}, h = 1, \dots, H$$

$$\hat{y}_{T+h|T} = \text{ its forecast based on data up to time } T.$$

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e^2_{T+h})$$

$$\text{RMSE} = \sqrt{\text{mean}(e^2_{T+h})}$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = \frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|e_{T+h}|/Q)$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

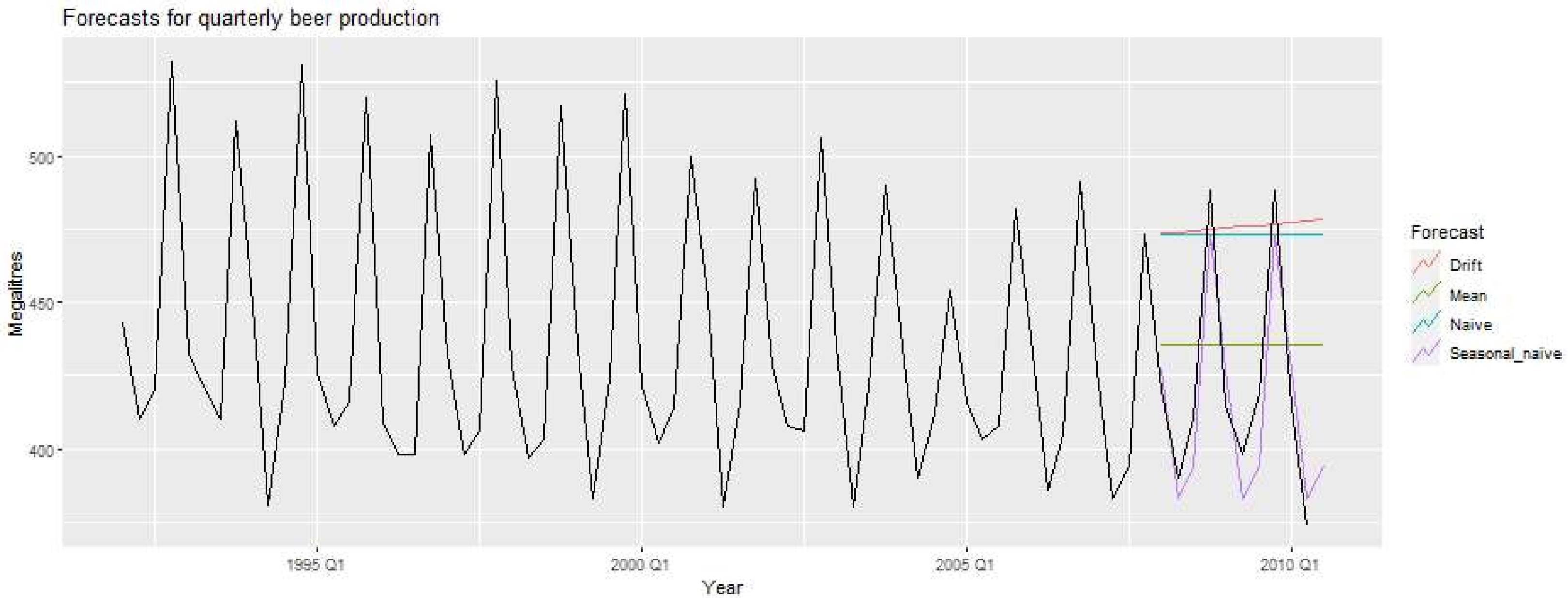
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = \frac{1}{T-m} \sum_{t=m+1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

```
recent_production <- aus_production |>
  filter(year(Quarter) >= 1992)
train <- recent_production |>
  filter(year(Quarter) <= 2007)
beer_fit <- train |>
  model(
    Mean = MEAN(Beer),
    Naive = NAIVE(Beer),
    Seasonal_naive = SNAIVE(Beer),
    Drift = RW(Beer ~ drift())
  )
beer_fc <- beer_fit |>
  forecast(h = 10)
```

Measures of forecast accuracy

```
accuracy(beer_fit)
```

```
## # A tibble: 4 × 6
##   .model         .type     RMSE   MAE  MAPE  MASE
##   <chr>          <chr>    <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Training  65.3  54.8 12.2   3.83
## 2 Mean           Training  43.6  35.2  7.89  2.46
## 3 Naive          Training  65.3  54.7 12.2   3.83
## 4 Seasonal_naive Training  16.8  14.3  3.31  1
```

```
accuracy(beer_fc, recent_production)
```

```
## # A tibble: 4 × 6
##   .model         .type  RMSE   MAE  MAPE  MASE
##   <chr>          <chr> <dbl> <dbl> <dbl> <dbl>
## 1 Drift          Test   64.9  58.9 14.6  4.12
## 2 Mean           Test   38.4  34.8  8.28 2.44
## 3 Naive          Test   62.7  57.4 14.2  4.01
## 4 Seasonal_naive Test   14.3  13.4  3.17 0.937
```

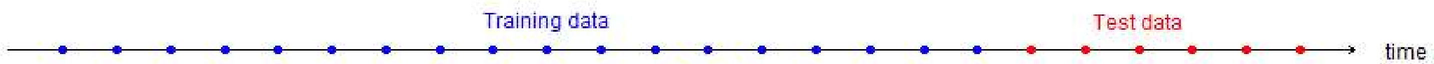
Poll: true or false?

1. Good forecast methods should have normally distributed residuals.
2. A model with small residuals will give good forecasts.
3. The best measure of forecast accuracy is MAPE.
4. If your model doesn't forecast well, you should make it more complicated.
5. Always choose the model with the best forecast accuracy as measured on the test set.

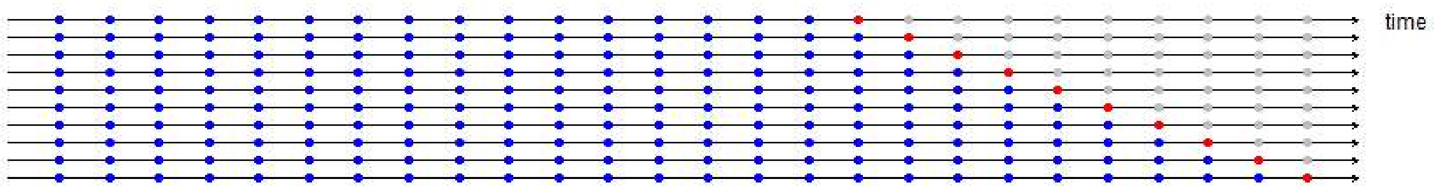
Time series cross-validation

Time series cross-validation

Traditional evaluation



Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"

Creating the rolling training sets

There are three main rolling types which can be used.

- Stretch: extends a growing length window with new data.
- Slide: shifts a fixed length window through the data.
- Tile: moves a fixed length window without overlap.

Three functions to roll a tsibble: `stretch_tsibble()`, `slide_tsibble()`, and `tile_tsibble()`.

For time series cross-validation, stretching windows are most commonly used.

A good way to choose the best forecasting model is to find the model with the smallest RMSE computed using time series cross-validation.

Time series cross-validation

Stretch with a minimum length of 3, growing by 1 each step.

```
fb_stretch <- fb_stock |>
  stretch_tsibble(.init = 3, .step = 1) |>
  filter(.id != max(.id))
```

```
## # A tsibble: 790,650 x 4 [1]
## # Key:       .id [1,255]
##    Date       Close trading_day   .id
##    <date>     <dbl>       <int> <int>
##  1 2014-01-02  54.7           1     1
##  2 2014-01-03  54.6           2     1
##  3 2014-01-06  57.2           3     1
##  4 2014-01-02  54.7           1     2
##  5 2014-01-03  54.6           2     2
##  6 2014-01-06  57.2           3     2
##  7 2014-01-07  57.9           4     2
##  8 2014-01-02  54.7           1     3
##  9 2014-01-03  54.6           2     3
## 10 2014-01-06  57.2           3     3
## 11 2014-01-07  57.9           4     3
## 12 2014-01-08  58.2           5     3
## 13 2014-01-02  54.7           1     4
```

Time series cross-validation

Estimate RW w/ drift models for each window.

```
fit_cv <- fb_stretch |>
  model(RW(Close ~ drift()))
```

```
## # A mable: 1,255 x 3
## # Key:     .id, Symbol [1,255]
##      .id Symbol `RW(Close ~ drift())`
##    <int> <chr>               <model>
## 1      1 FB            <RW w/ drift>
## 2      2 FB            <RW w/ drift>
## 3      3 FB            <RW w/ drift>
## 4      4 FB            <RW w/ drift>
## # … with 1,251 more rows
```

Time series cross-validation

Produce one step ahead forecasts from all models.

```
fc_cv <- fit_cv |>
  forecast(h=1)
```

```
## # A fable: 1,255 x 5 [1]
## # Key:     .id, Symbol [1,255]
##     .id Symbol trading_day      Close .mean
##   <int> <chr>        <dbl>     <dist> <dbl>
## 1     1 FB               4 N(58, 5.8)  58.4
## 2     2 FB               5 N(59, 2.7)  59.0
## 3     3 FB               6 N(59, 1.9)  59.1
## 4     4 FB               7 N(58, 2.2)  57.7
## # ℹ 1,251 more rows
```

Time series cross-validation

```
# Time series cross-validation accuracy
fc_cv |> accuracy(fb_stock)
# Training set (Residual accuracy)
fb_stock |> model(RW(Close ~ drift())) |> accuracy()
```

	RMSE	**MAE**	**MAPE**
Cross=validation	2.418	1.469	1.266
Training	2.414	1.465	1.261

References

- Hyndman, R. J., & Athanasopoulos, G. (2018). Forecasting: principles and practice. OTexts.