MA 5124 Financial Time Series Analysis and Forecasting

Chapter 1: Introduction to time series and forecasting
Lesson 1

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08-02-2026

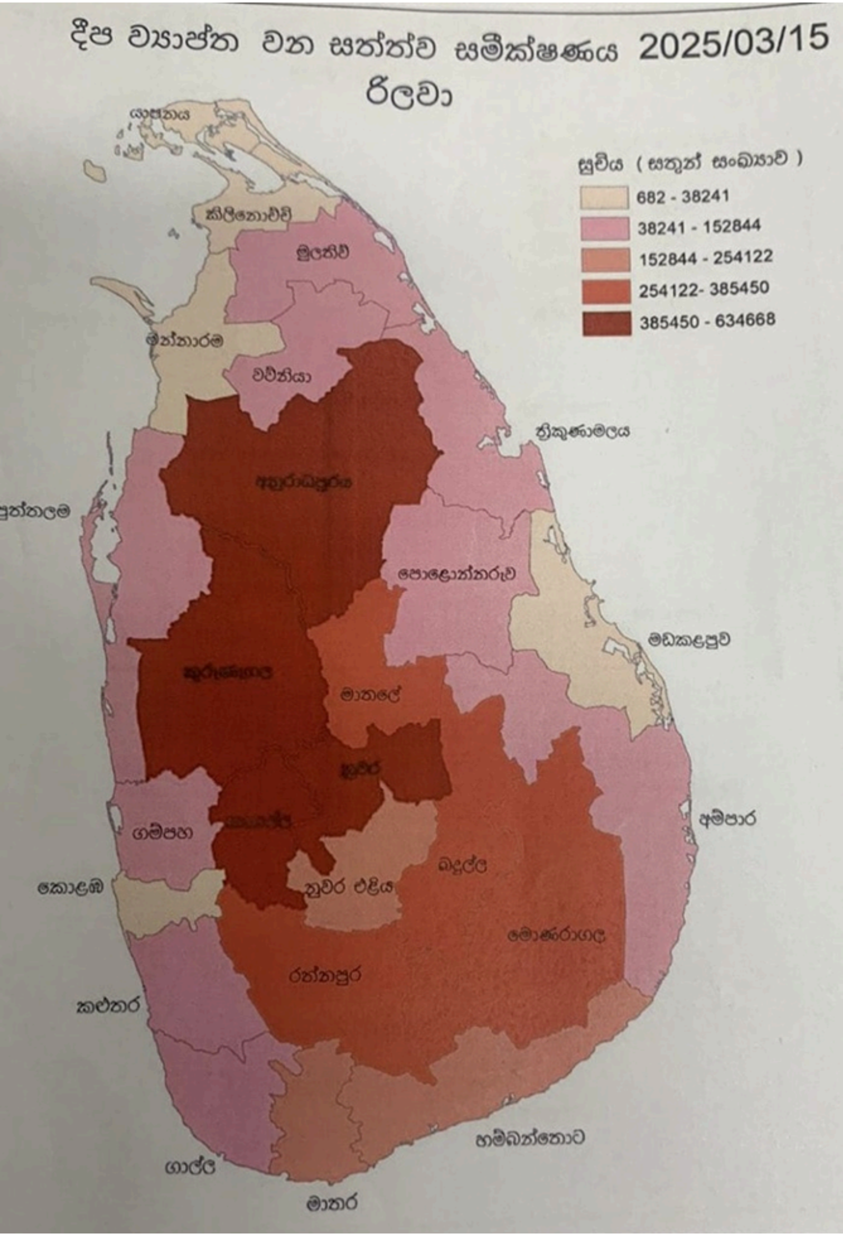
Types of Data

- Cross-sectional data
- Time series data
- Pooled data
- Panel data

1. Cross-sectional data

- A cross-sectional data set consists of a sample of individuals, households, countries or any other type of unit at **a specific point in time.**
- Sometimes, data across all units do not correspond to exactly the same time point.
- Example: A survey that collects data from questionnaire surveys of different units within a month.
- In this case, we can ignore the minor time differences in collection.

ID	Monthly Income (in LKR)
1	83000
2	150000
3	40000
4	65000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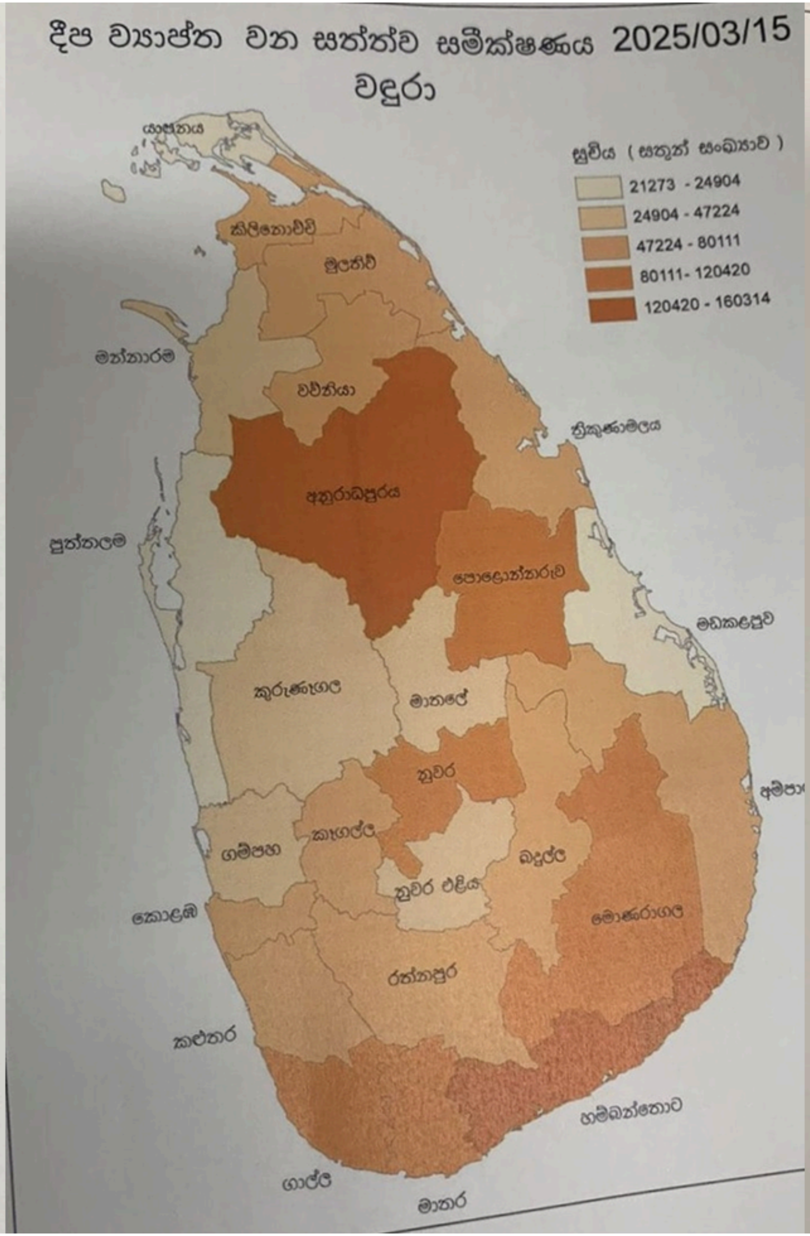
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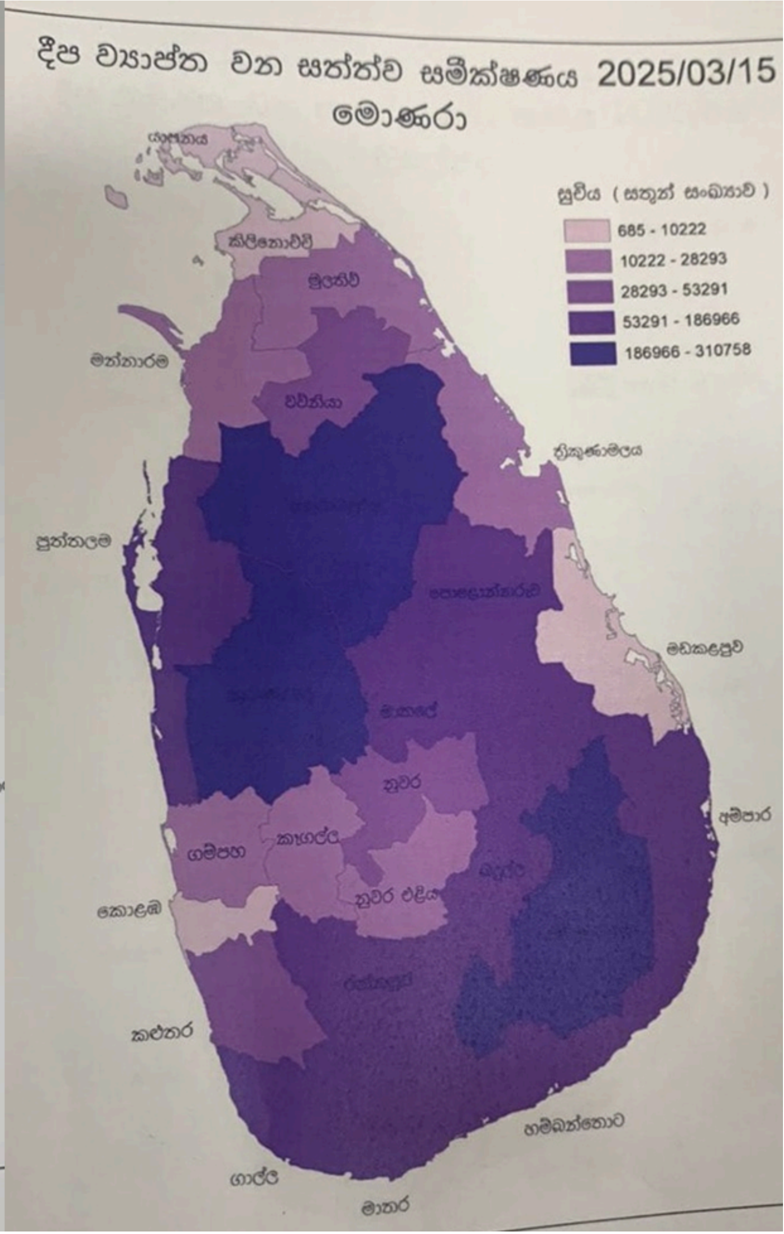
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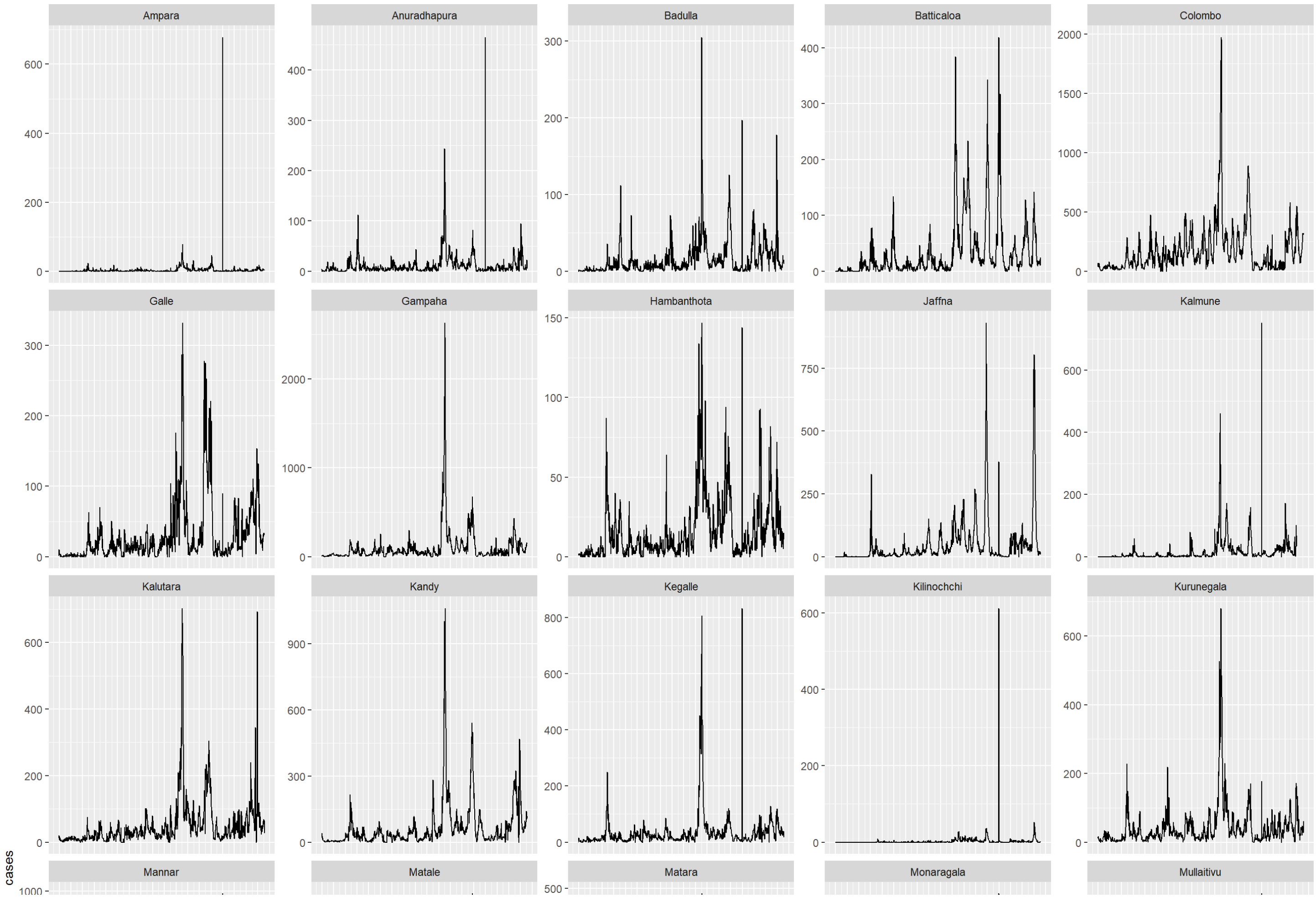
2. Time series data

- A time series is a sequence of observations taken **sequentially in time**.
- Time series data are arranged in chronological order and can have different time frequencies (eg: biannual, annual, quarterly, monthly, weekly, daily, hourly, etc.)
- Examples of time series data
  - Annual Google profits
  - Monthly rainfall
  - Weekly retail sales
  - Daily confirmed dengue cases and deaths
  - Hourly electricity demand

District-wise weekly Dengue cases from 2006 to 2025



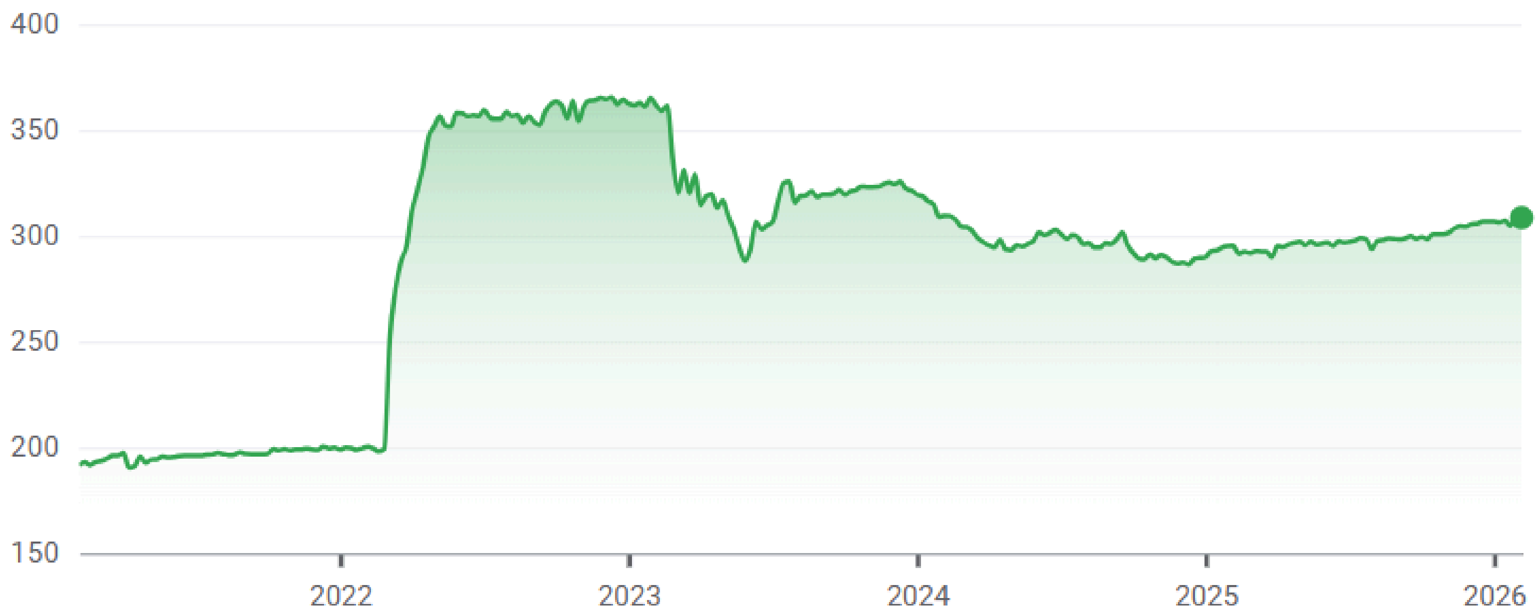
HOME > USD / LKR • CURRENCY

United States Dollar to Sri Lankan Rupee

308.4498 ↑61.16% +117.0615 5Y

Feb 6, 10:24:20 PM UTC · Disclaimer

1D 5D 1M 6M YTD 1Y 5Y MAX



Compare to

EUR / LKR	JPY / LKR	GBP / LKR	AUD / LKR
364.76	1.96	419.82	216.37
EUR ↑57.45%	JPY ↑8.13%	GBP ↑59.10%	AUD ↑47.30%

3. Pooled data

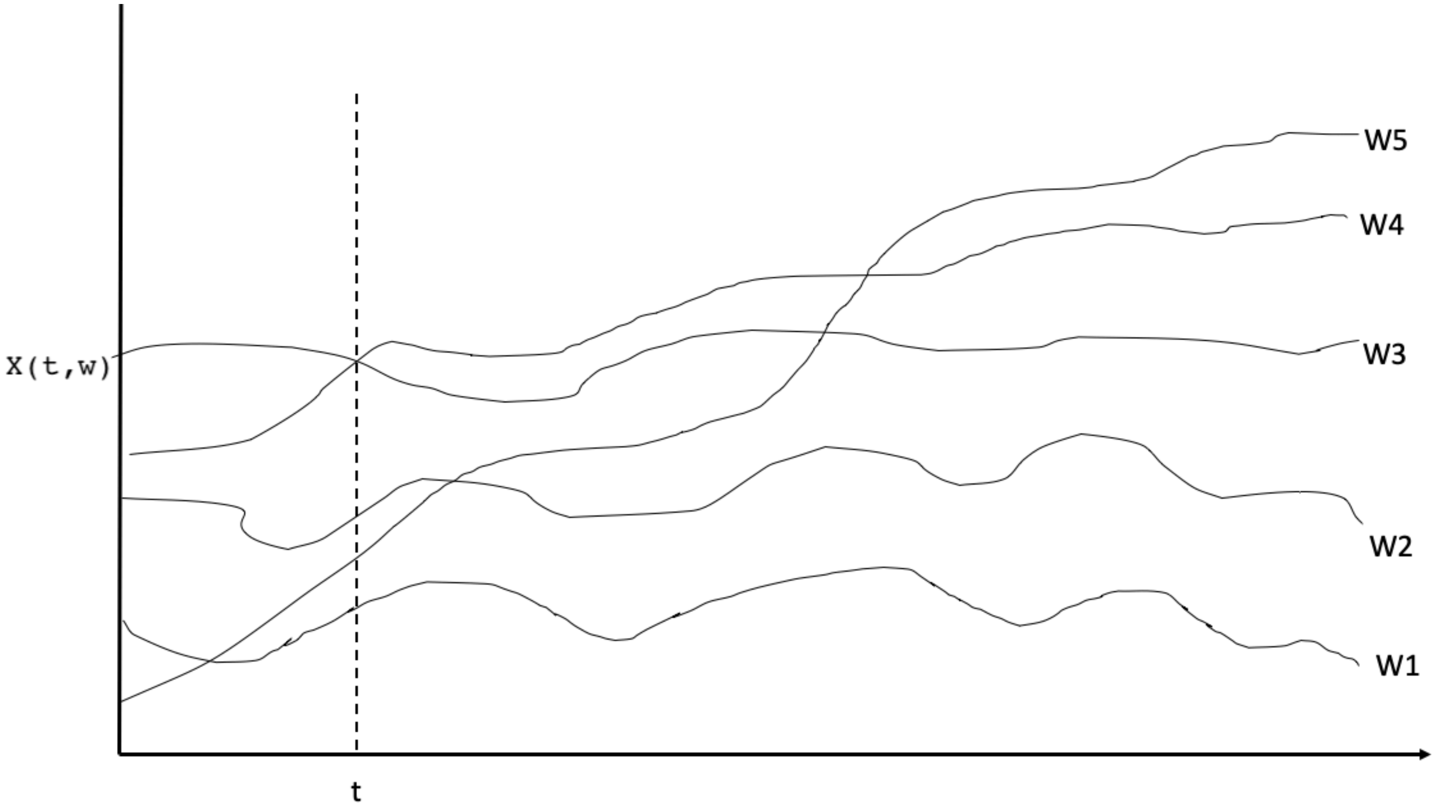
- Pooled data occur when we have a “time series of cross sections,” but the observations in each cross section do not necessarily refer to the same unit.

4. Panel data

- This is a **special type** of pooled data in which the samples of the same cross-sectional units observed over time.

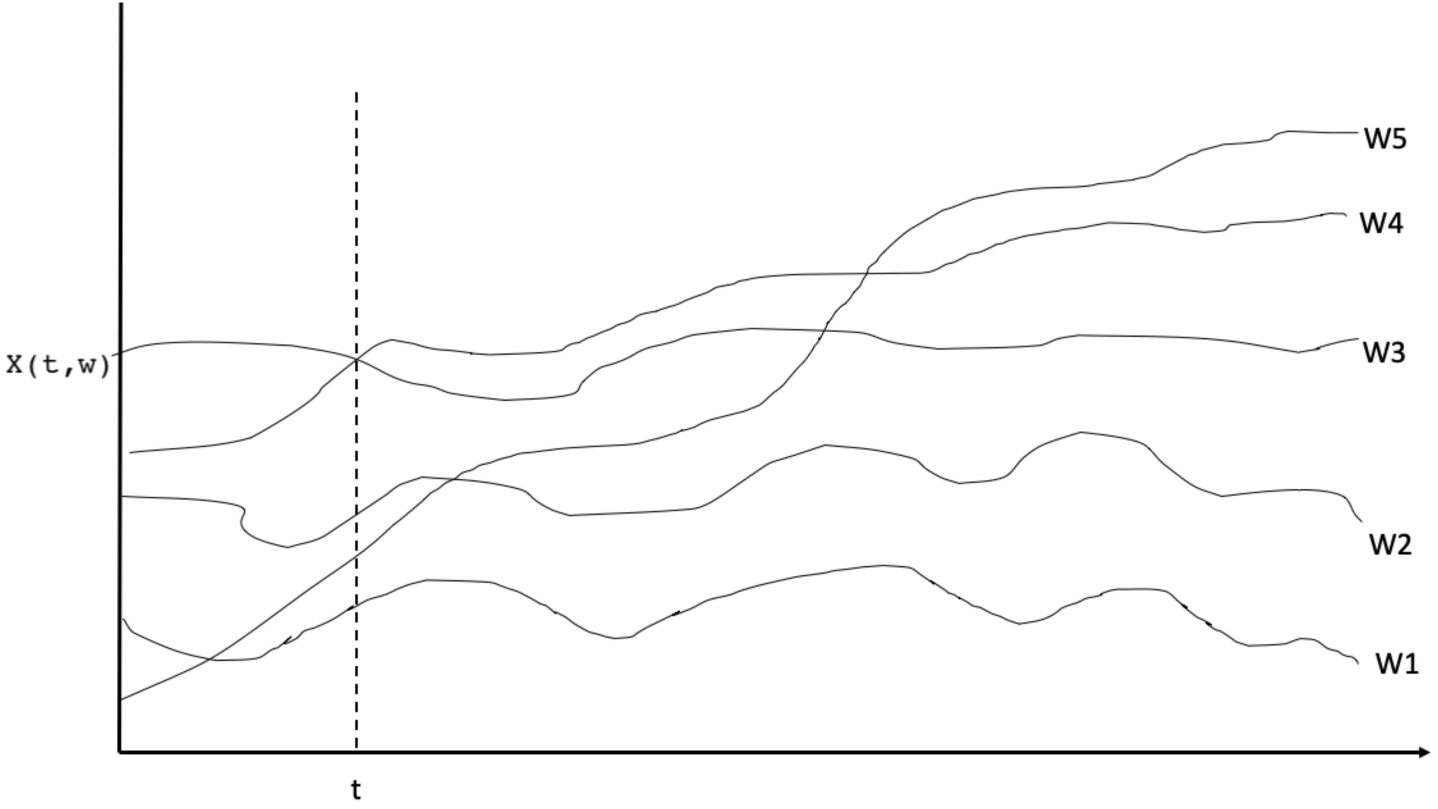
Stochastic Processes

A stochastic process is a family of indexed random variables $\{X(t, \omega); t \in T; \omega \in \Omega\}$ defined on a probability space $(\Omega, \beta, \mathbf{P})$ where T is an arbitrary set.



There are many ways of visualizing a stochastic process.

(i) For each choice of $t \in T$, $X(t, \omega)$ is a random variable.
(ii) For each choice of $\omega \in \Omega$, $X(t, \omega)$ is a function of t.
(iii) For each choice of ω and t, $X(t, \omega)$ is a number.
(iv) In general it is an ensemble (family) of functions $X(t, \omega)$ where t and w can take different possible values.



Time series data

- **The observed time series or time series to be analyzed is a particular realization of a stochastic process.**
- Anything that is observed sequentially over time is a time series.
- In this course, we will only consider time series that are observed at **regular intervals of time.**
- **Irregularly spaced time series** can also possible, but are beyond the scope of this course

Forecasting

- Forecasting is about **predicting the future** as accurately as possible, given all of the information available, including historical data and knowledge of any future events that might impact the forecasts.
- Forecasting is estimating how the sequence of observations will continue into the future.

Factors affecting forecastability

Something is easier to forecast if:

- we have a good understanding of the factors that contribute to it
- there is lots of data available;
- the forecasts cannot affect the thing we are trying to forecast.
- there is relatively low natural/unexplainable random variation.
- the future is somewhat similar to the past

Types of Methods

- **Qualitative** forecasts
  - Judgmental forecasting is the only option if no historical data (for new product, new market conditions), or if the data available are not relevant to the forecasts.
  - See fpp3 Chapter 6: https://otexts.com/fpp3/judgmental.html.
- **Quantitative** forecasts: can be applied
  - if numerical information about the past is available
  - if it is reasonable to assume that some aspects of the past patterns will continue into the future

Quantitative forecasts

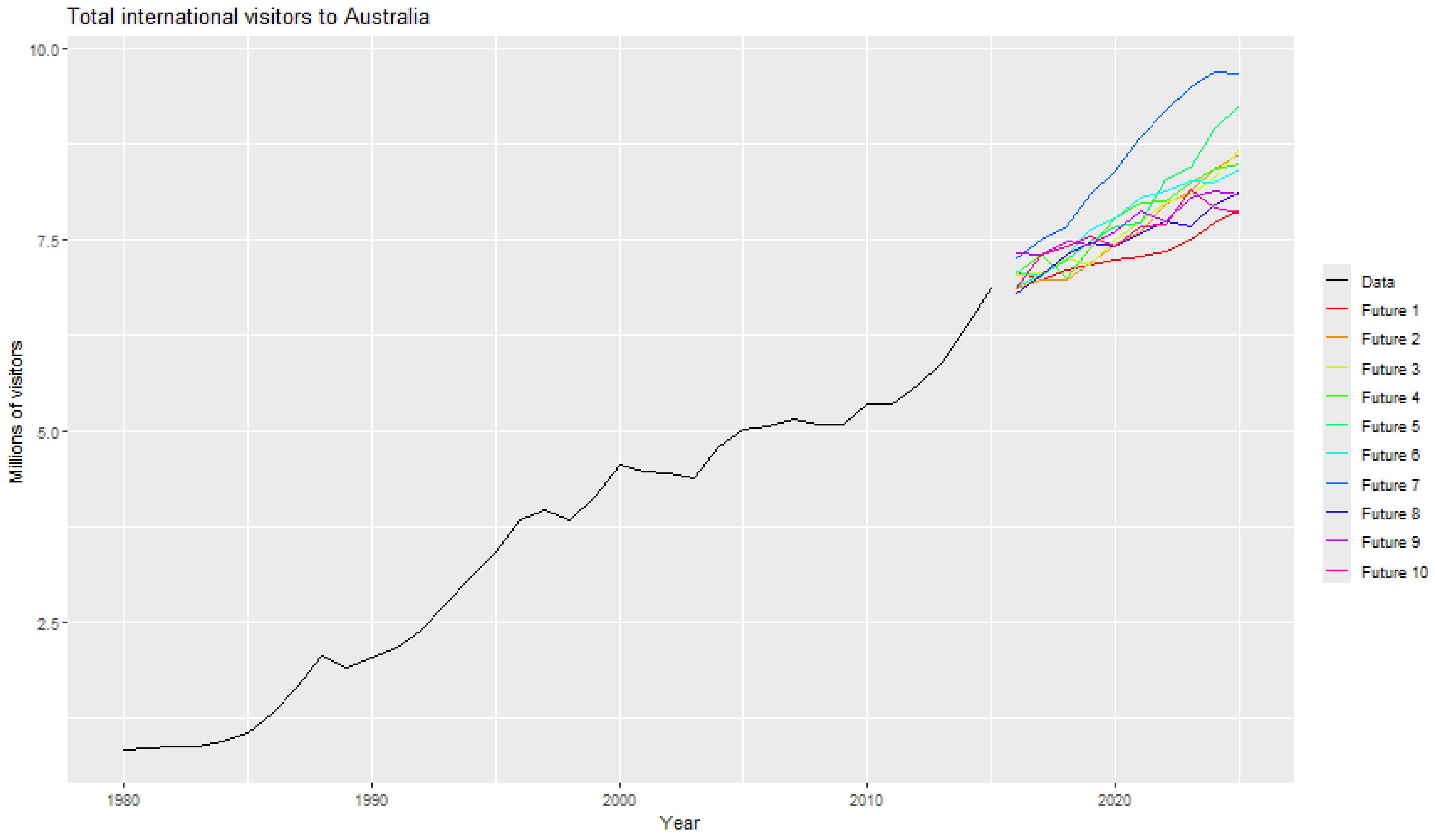
- Most quantitative forecasting problems use either
  - Time series data (collected at regular intervals over time).
  - Cross-sectional data (collected at a single point in time).

Basic steps in a forecasting task

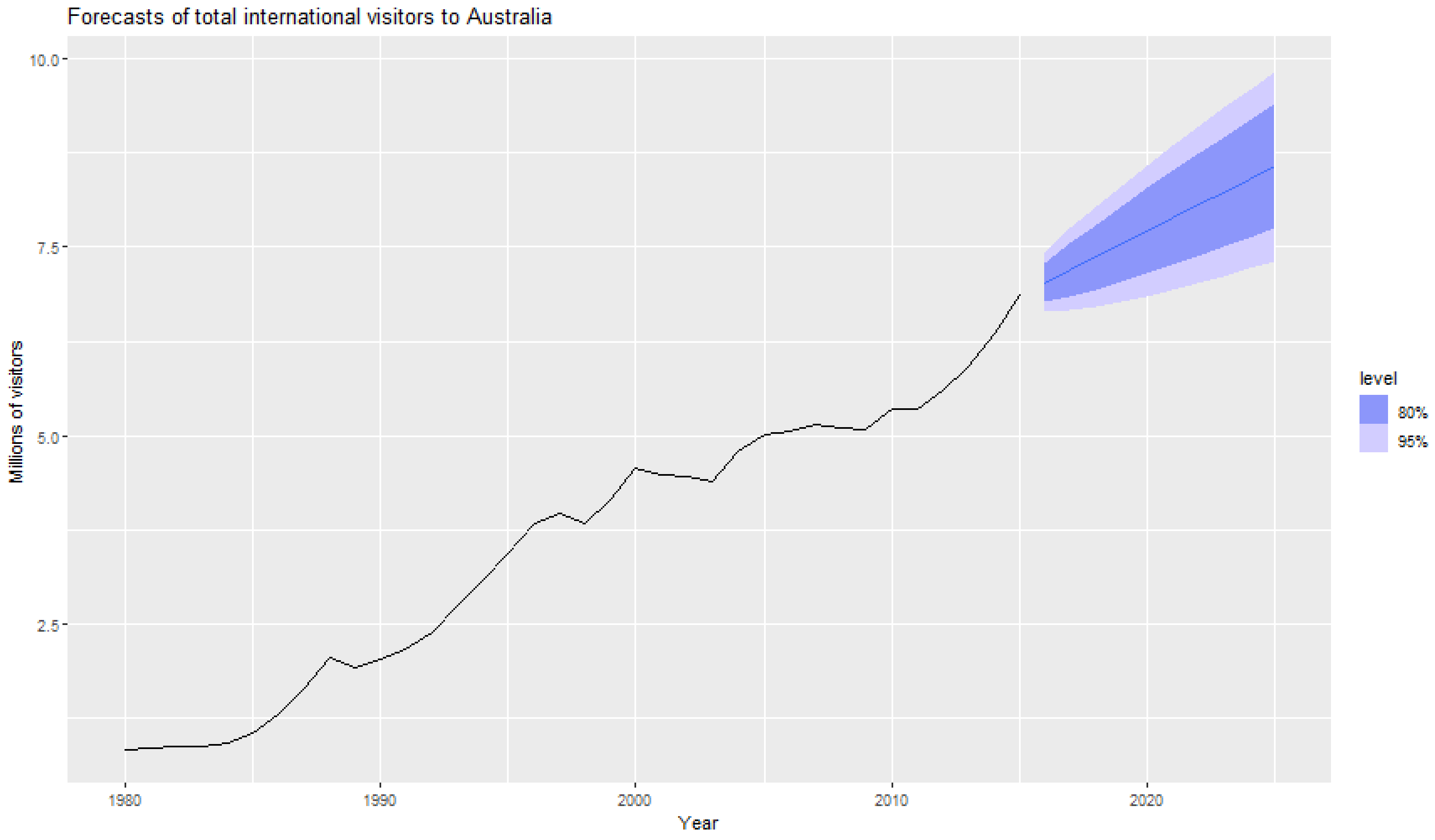
- Problem definition
- Collect data
- Preliminary (exploratory) analysis (data Visualization)
- Modelling
- Evaluate the fitted model

The statistical forecasting perspective

Sample futures



Forecast intervals



Statistical forecasting

- Thing to be forecast: a random variable, y_t.
- Forecast distribution: If $\mathcal{I}$ is all observations, then $y_t|\mathcal{I}$ means "the random variable y_t given what we know in $\mathcal{I}$".
- The **point forecast** is the mean (or median) of $y_t|\mathcal{I}$
- The **forecast variance** is $\text{var}[y_t|\mathcal{I}]$
- A prediction interval or **interval forecast** is a range of values of y_t with high probability.
- With time series, $y_{t|t-1} = y_t|\{y_1, y_2, \dots, y_{t-1}\}$.
- $\hat{y}_{T+h|T} = \text{E}[y_{T+h}|y_1, \dots, y_T]$ (an h-step forecast taking account of all observations up to time T).

Frequency of a time series: Seasonal periods

- **Frequency**: number of observation per natural time interval of measurement

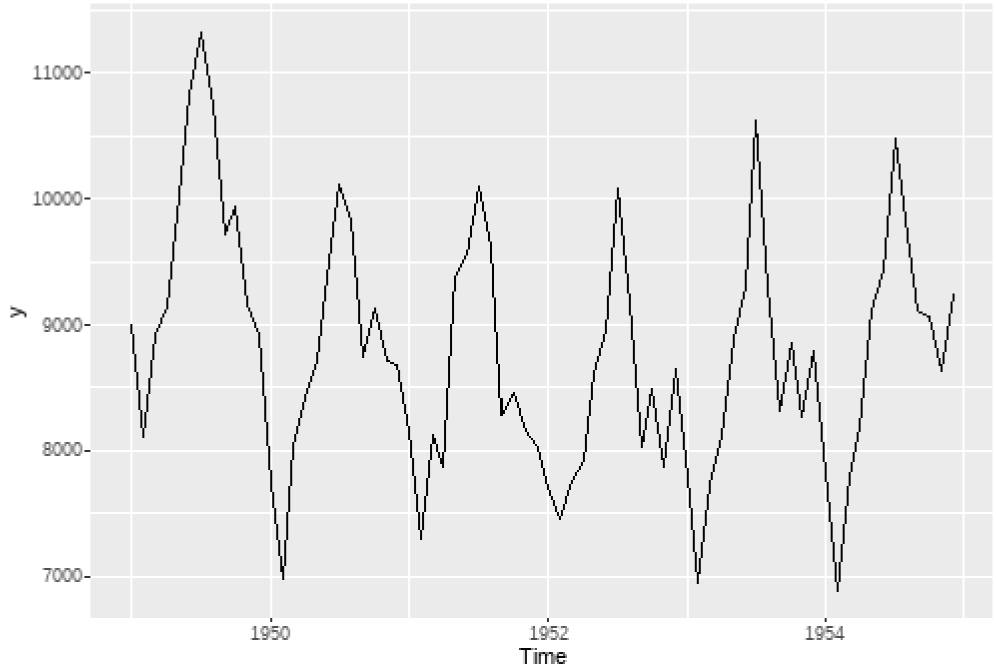
Data	Frequency
Annual	1
Quarterly	4
Monthly	12
Weekly	52 or 52.18

Frequency of a time series: Seasonal periods

- Multiple frequency setting

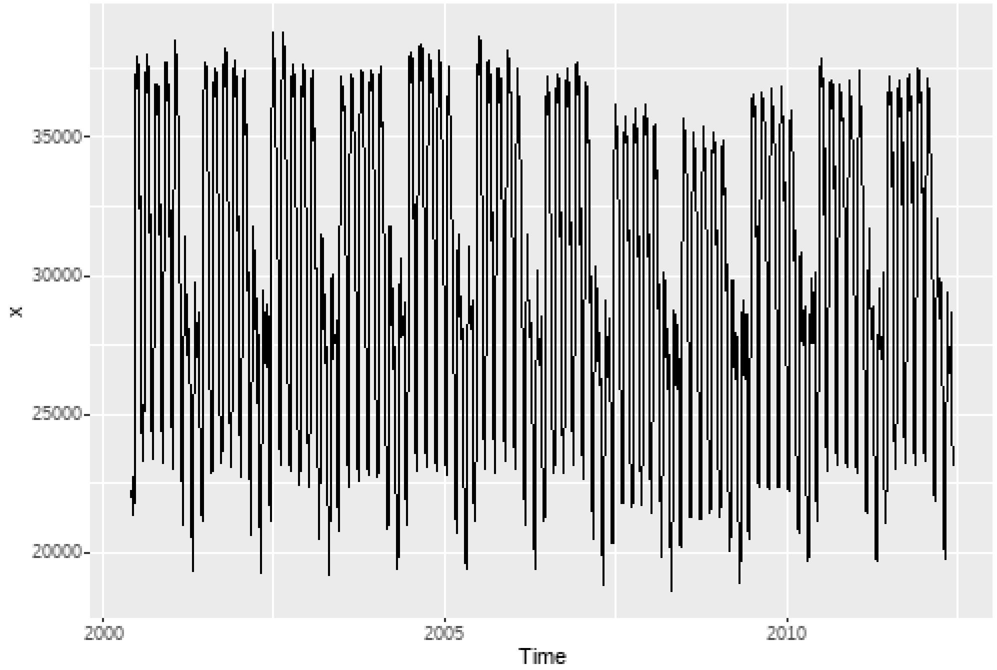
Data	Minute	Hour	Day	Week	Year
Daily				7	365.25
Hourly			24	168	8766
Half-Hourly			48	336	17532
Minutes		60	1440	10080	525960
Seconds	60	3600	86400	604800	31557600

Monthly time series



- Length of the series: 72
- Monthly seasonality

Half-hourly Time Series



- Length of the series: 4032
- Daily seasonality and weekly seasonality

Time series patterns

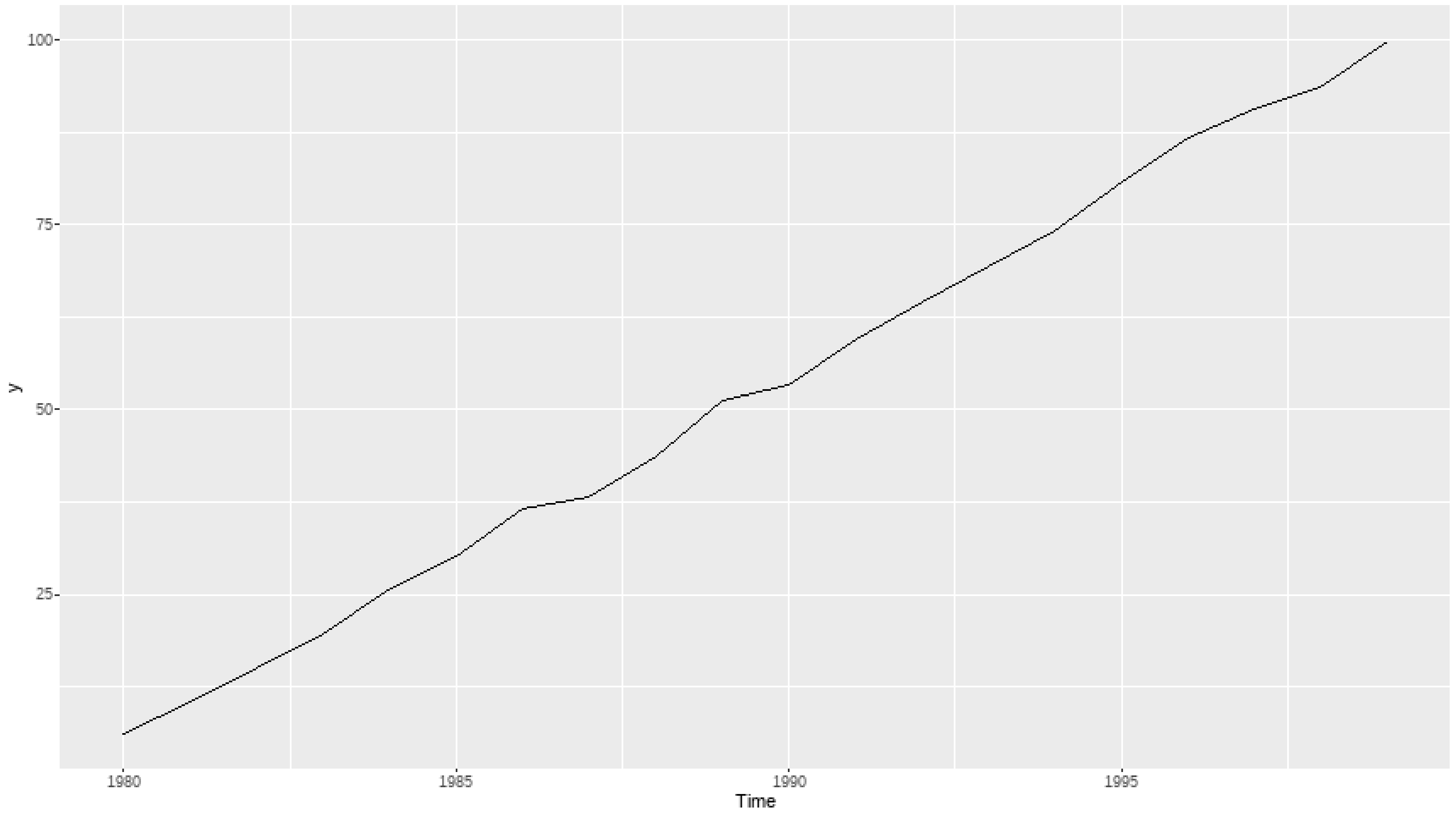
- **Trend** pattern exists when there is a long-term increase or decrease in the data.
- **Seasonal** pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).
- **Cyclic** pattern exists when data exhibit rises and falls that are not of fixed frequency (duration usually of at least 2 years).

Seasonal or cyclic?

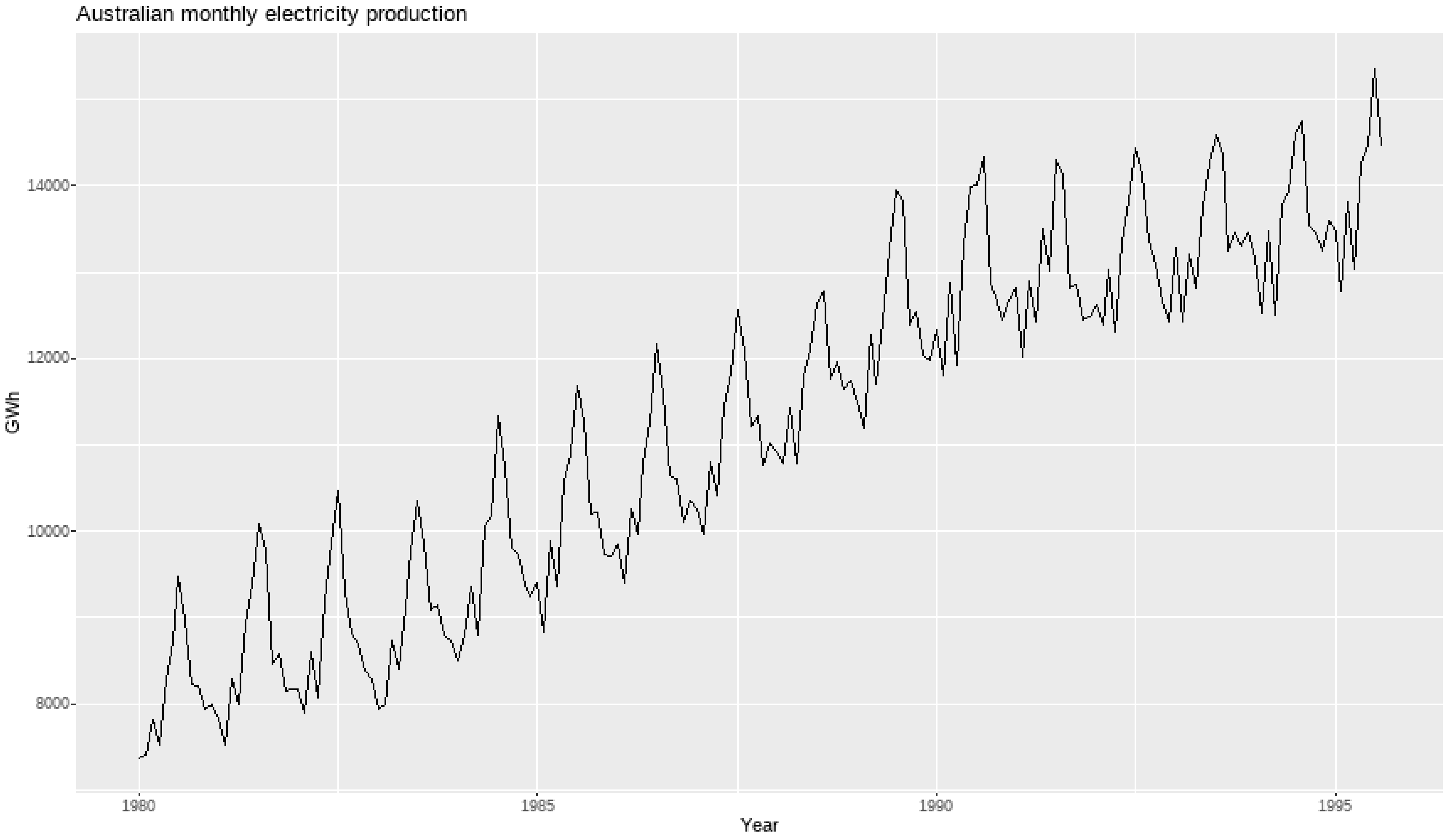
Differences between seasonal and cyclic patterns:

- seasonal pattern constant length; cyclic pattern variable length
- average length of cycle longer than length of seasonal pattern
- magnitude of cycle more variable than magnitude of seasonal pattern
- **The timing of peaks and troughs is predictable with seasonal data, but unpredictable in the long term with cyclic data.**

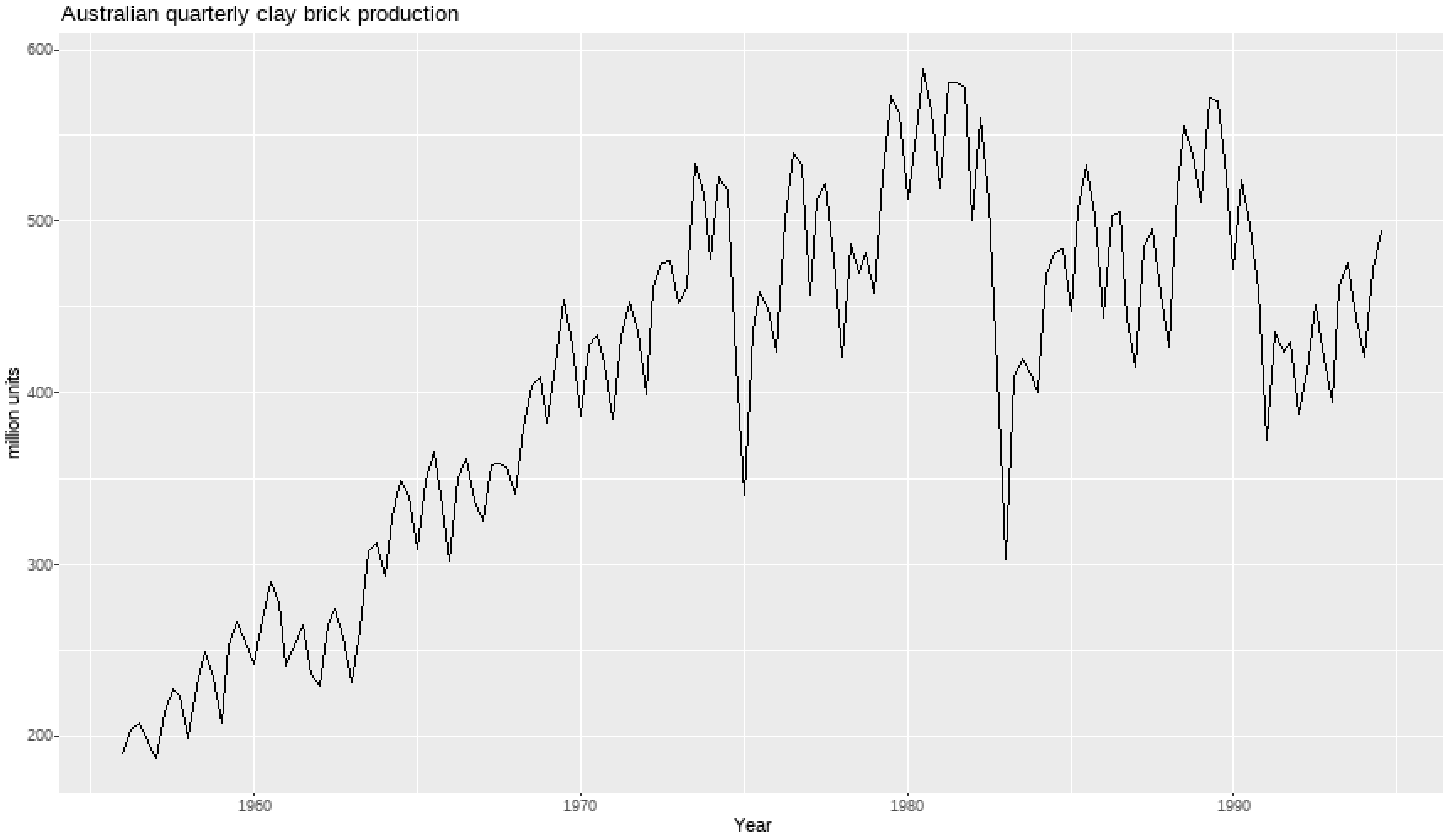
Time series patterns



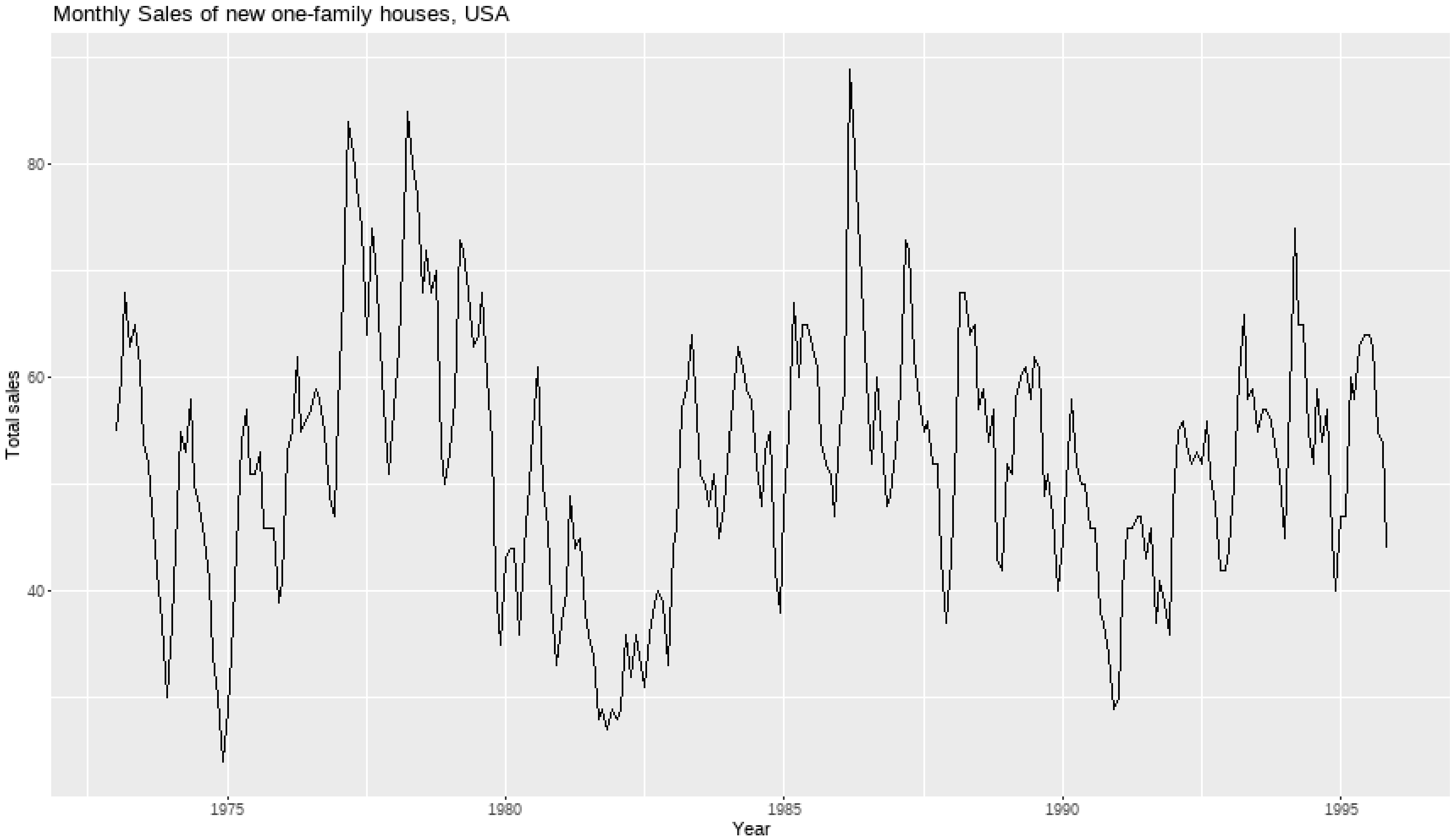
Time series patterns



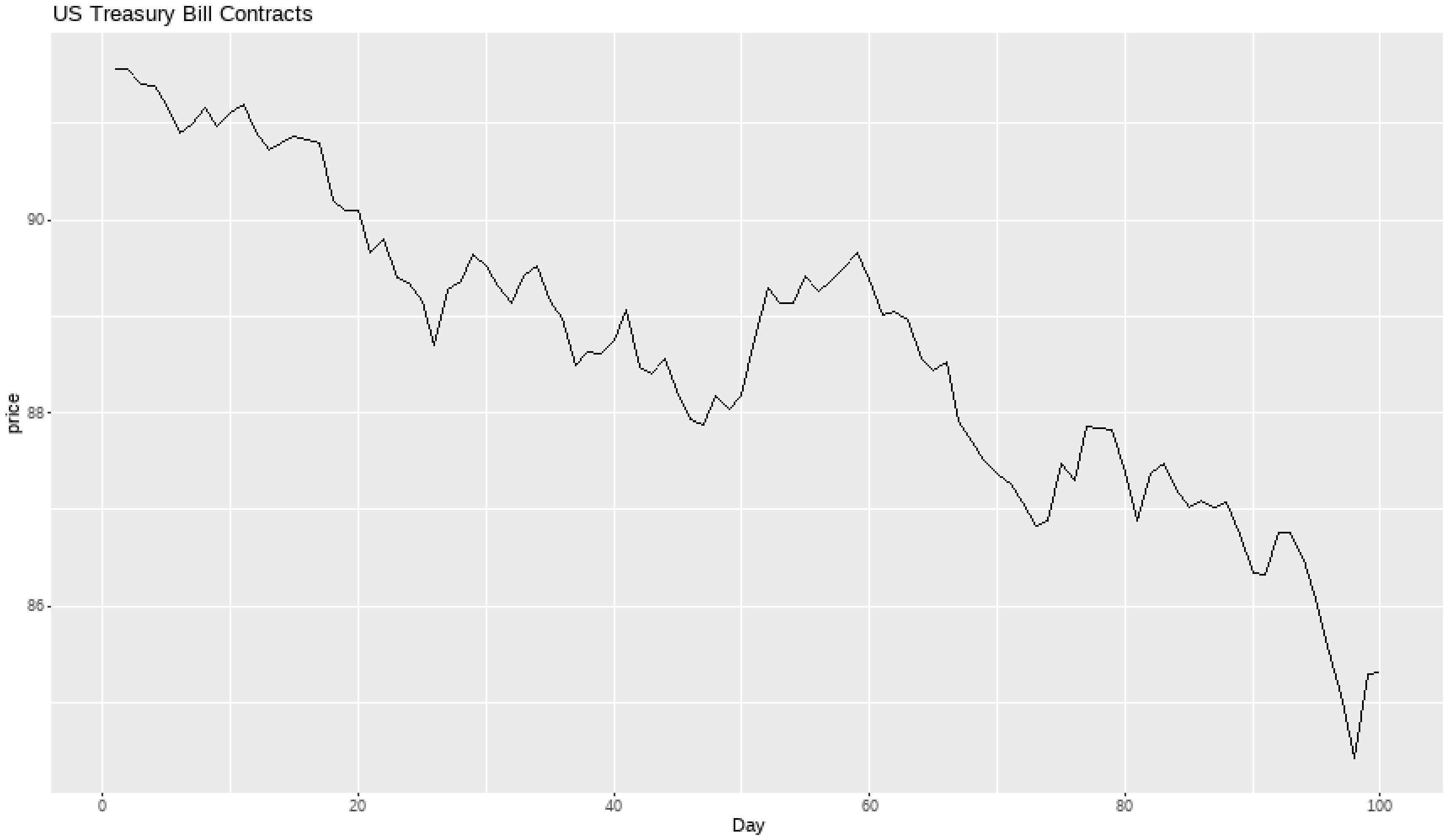
Time series patterns



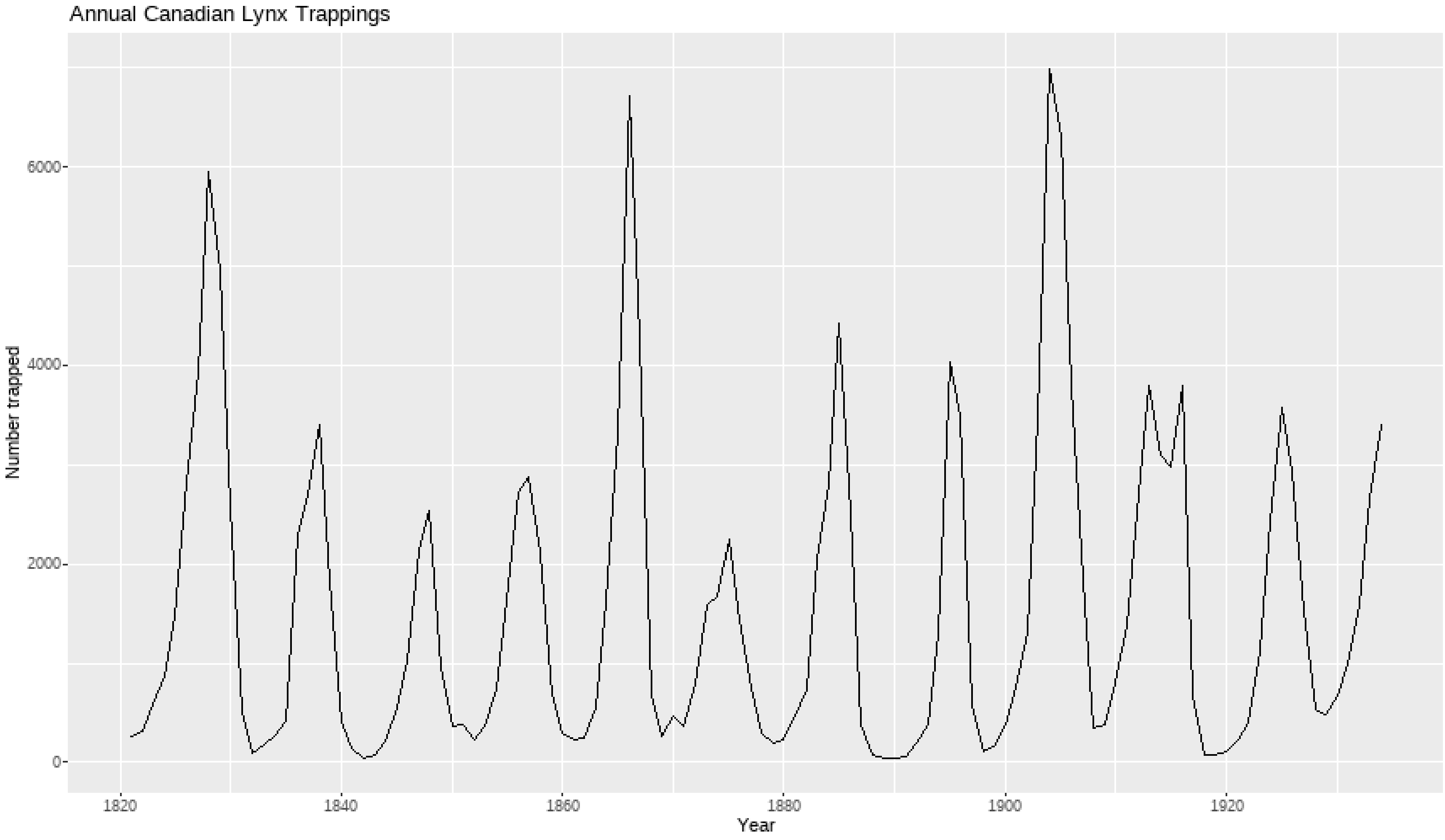
Time series patterns



Time series patterns



Time series patterns

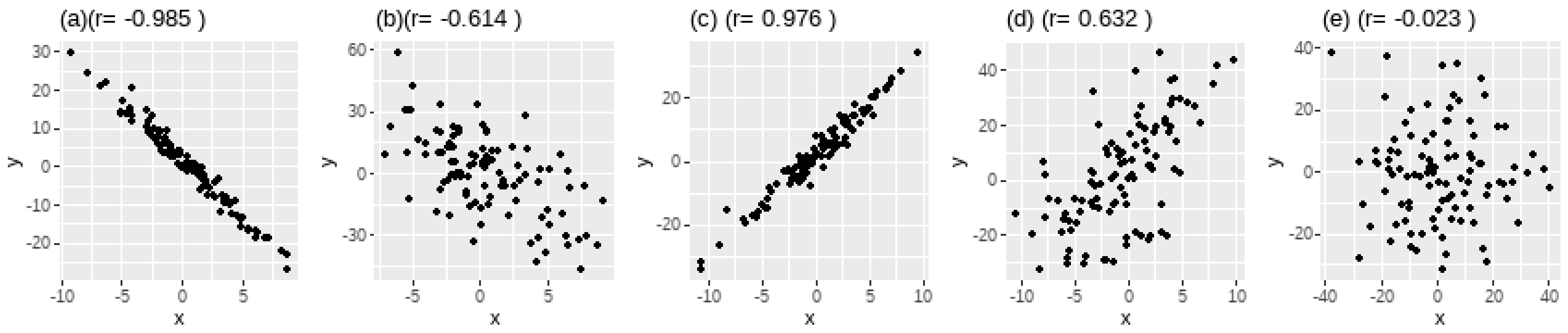


Numerical data summaries

- **Covariance** and **correlation**: measure extent of linear relationship between two variables (x and y).

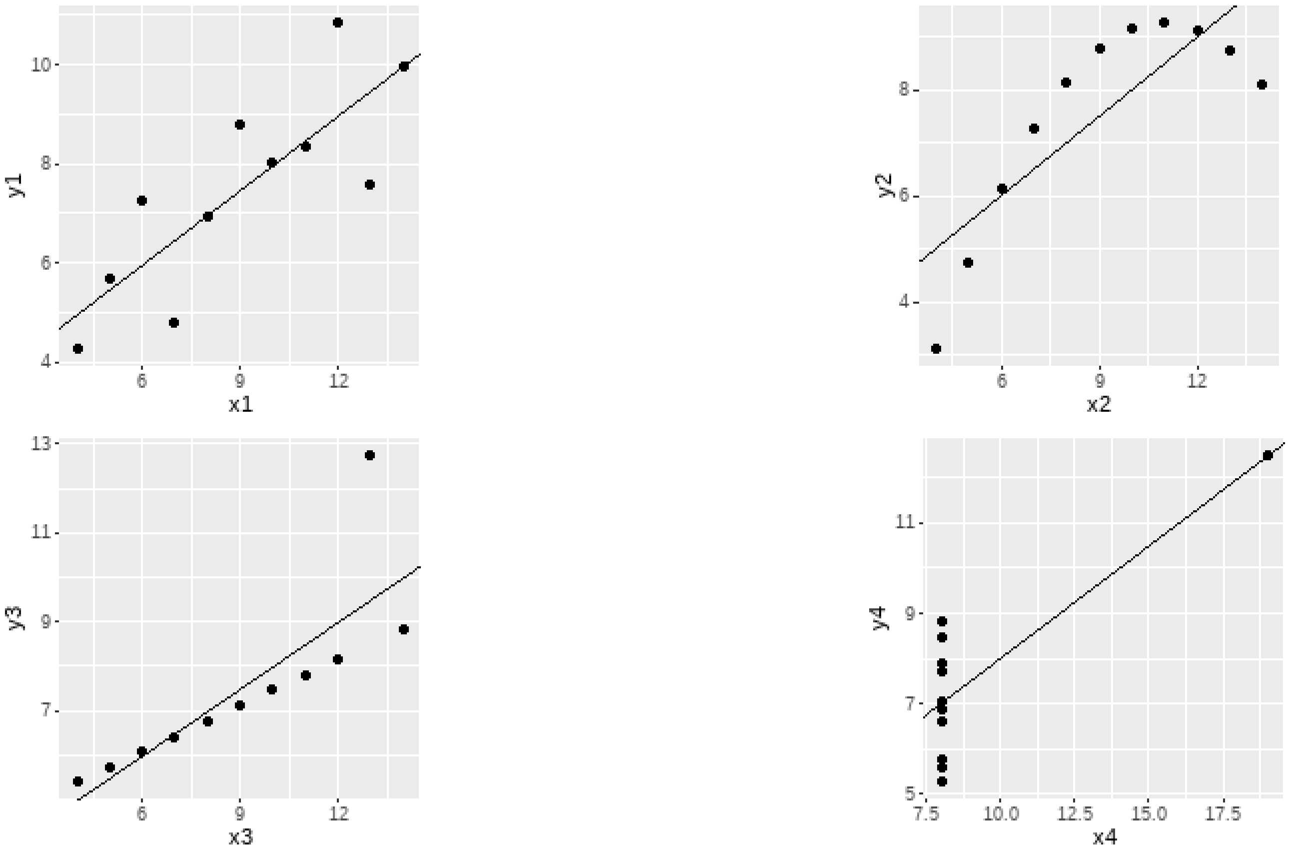
$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

- Lies between -1 and +1



Correlation coefficient

Which one has the highest correlation?



- All these have $r = 0.82$.

Autocorrelation

Autocovariance (c_k) and **auto**correlation (r_k): measure linear relationship between lagged values of a time series y.

- We measure the relationship between:

y_t and y_{t-1}

y_t and y_{t-2}

y_t and y_{t-3}

$\ldots$

y_t and y_{t-k}

- We denote the sample autocovariance at lag k by c_k and the sample autocorrelation at lag k by r_k.

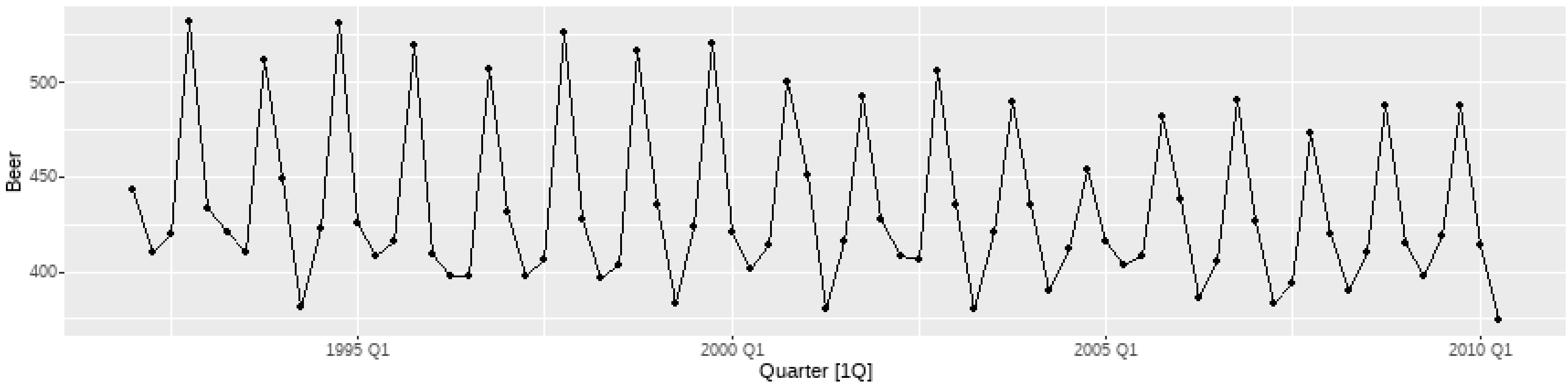
Then define

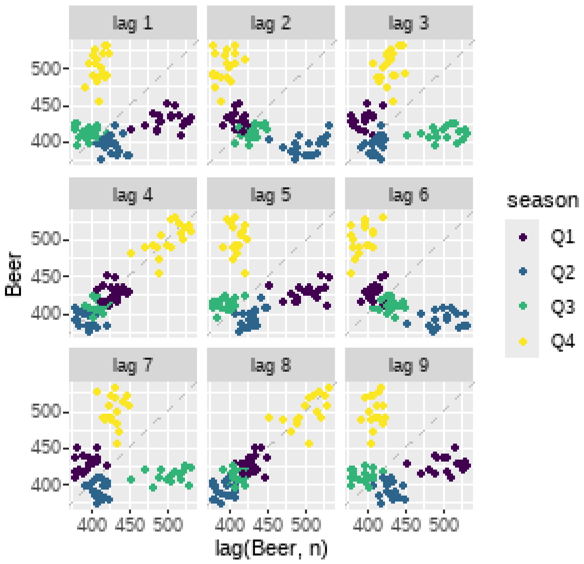
$$r_k = \frac{c_k}{c_0} = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

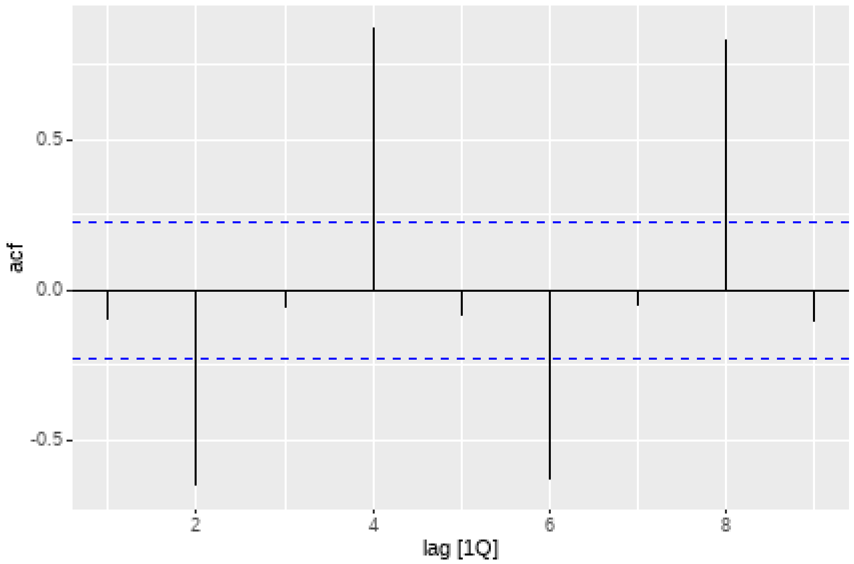
Autocorrelation

- r_1 indicates how successive values of y relate to each other
- r_2 indicates how y values two periods apart relate to each other
- r_k is almost the same as the sample correlation between y_t and y_{t-k}.

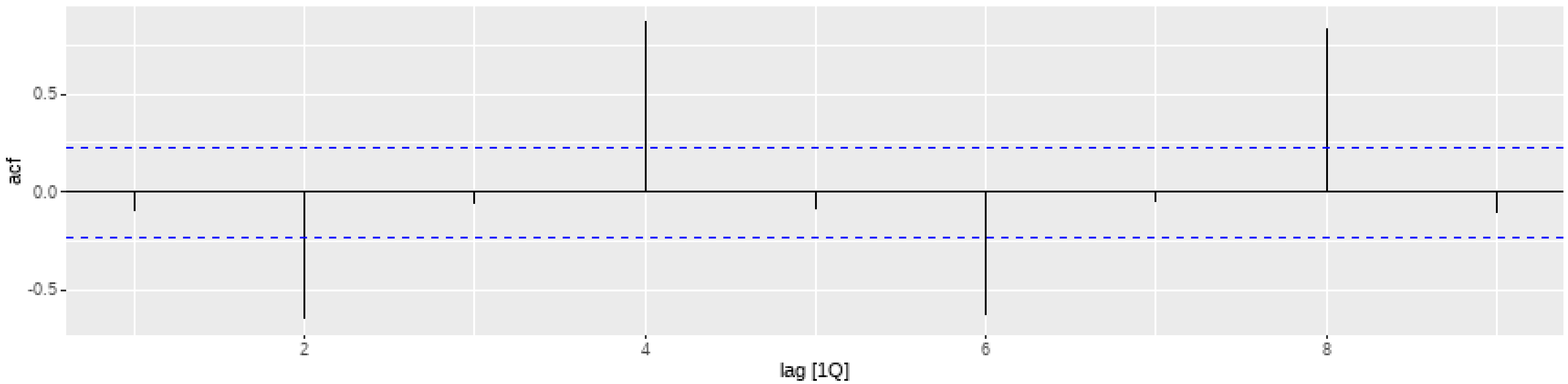
Autocorrelation: Results for first 9 lags for beer data:





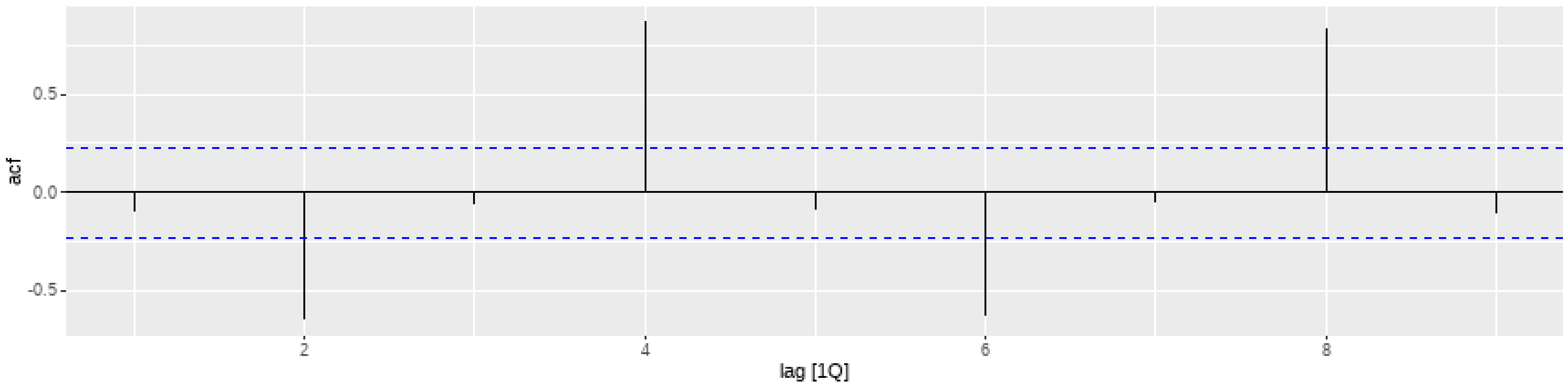


Autocorrelation: Results for first 9 lags for beer data:



- r_4 is **positive and higher** than for the other lags. This is due to the **seasonal pattern** in the data.
  - the peaks (troughs) tend to be 4 quarters apart.
  - the spikes every 4 lags after this $(r_8, r_{12}, \ldots)$ decrease in size as the lag number increases.

Autocorrelation: Results for first 9 lags for beer data:

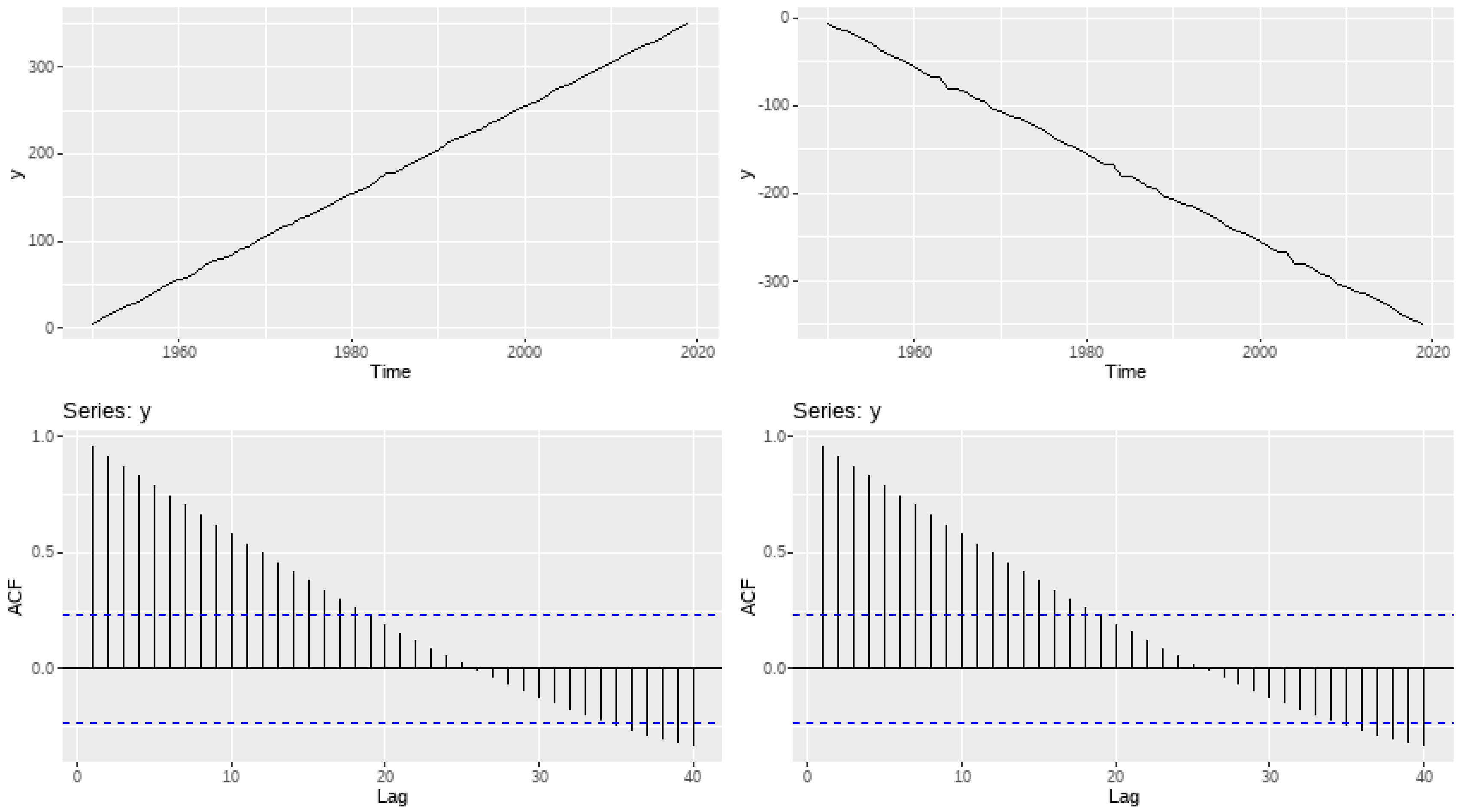


- r_2 is **more negative** than for the other lags because troughs and peaks tend to be 2 quarters apart.
  - The highest and the lowest productions are 2 quarters apart.
  - The spikes every 4 lags after this $r_6, r_{10}, \ldots$ decrease in size as the lag number increases.
- Together, the autocorrelations at lags $1, 2, \ldots,$ make up the **autocorrelation** or **ACF**.
- The plot is known as a **correlogram**.
- The dashed blue lines indicate whether the correlations are significantly different from zero.

Trend and seasonality in ACF plots

- When data have a trend, the autocorrelations for small lags tend to be large and positive.
- When data are seasonal, the autocorrelations will be larger at the seasonal lags (i.e., at multiples of the seasonal frequency)
- When data are trended and seasonal, you see a combination of these effects

Trend



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100
0
y
1960
1980
2000
2020
Time
0
-100
-200
-300
y
1960
1980
2000
2020
Time
Series: y
1.0
0.5
0.0
ACF
0
10
20
30
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Lag
Series: y
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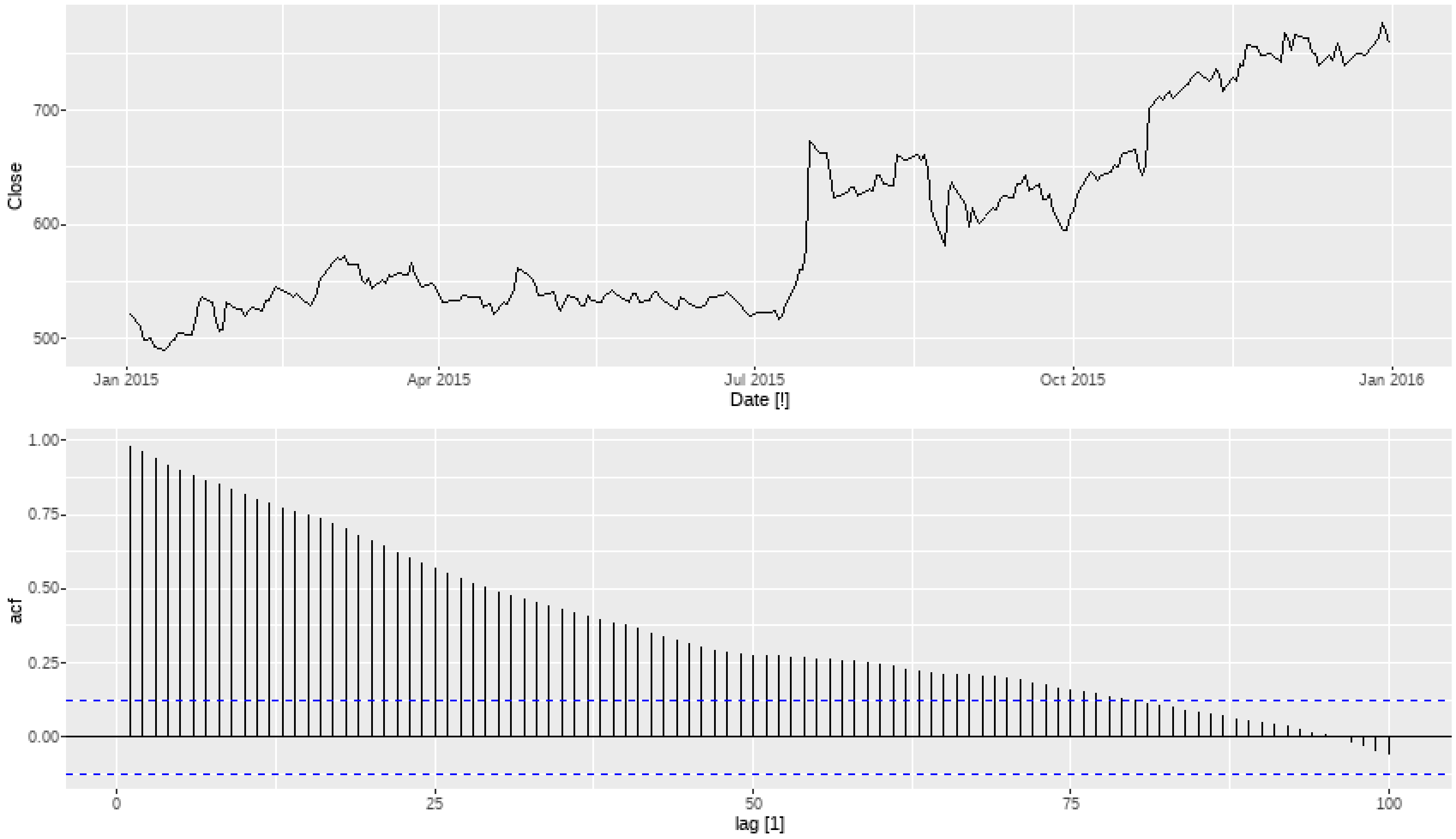
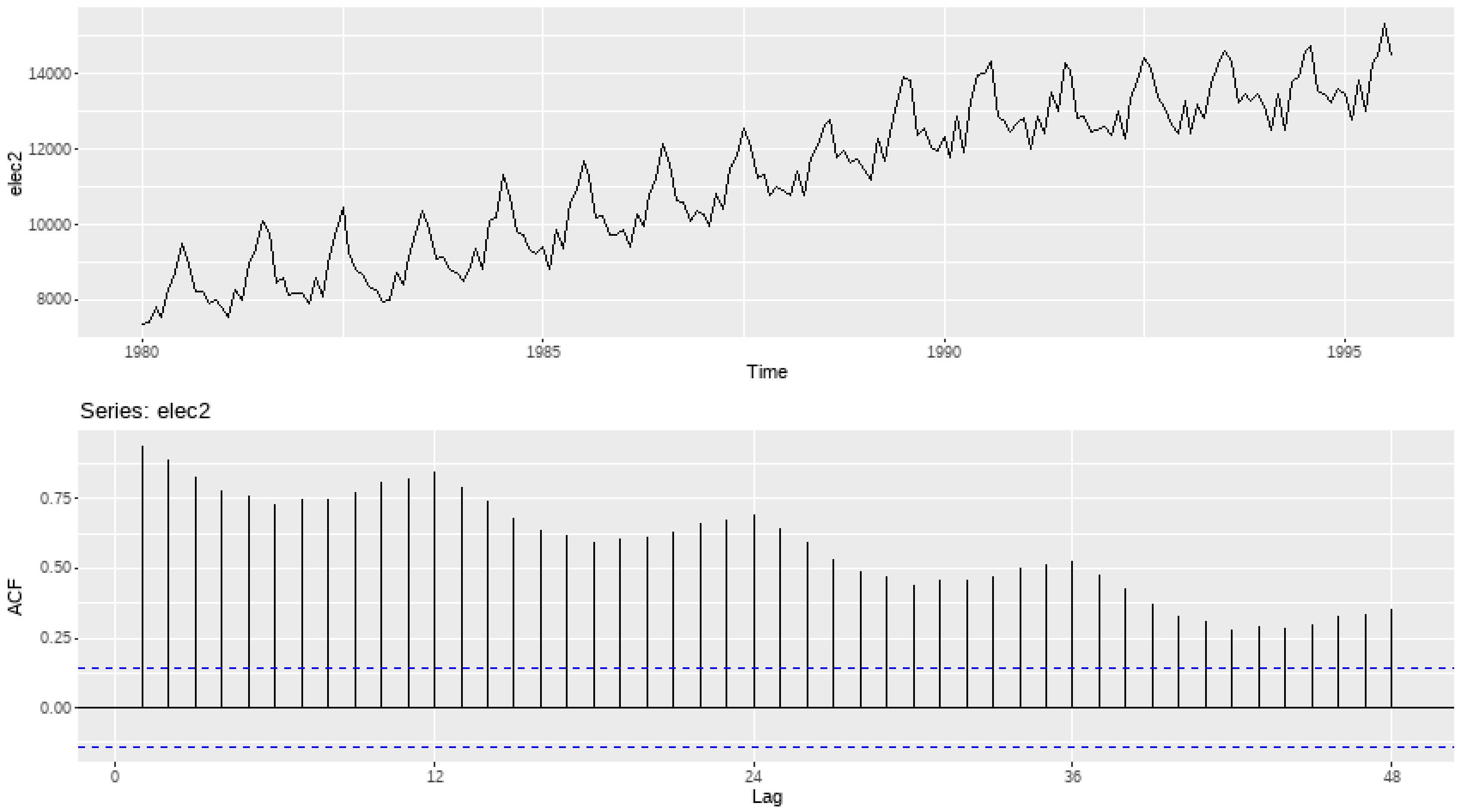
0.5
0.0
ACF
0
10
20
30
40
Lag

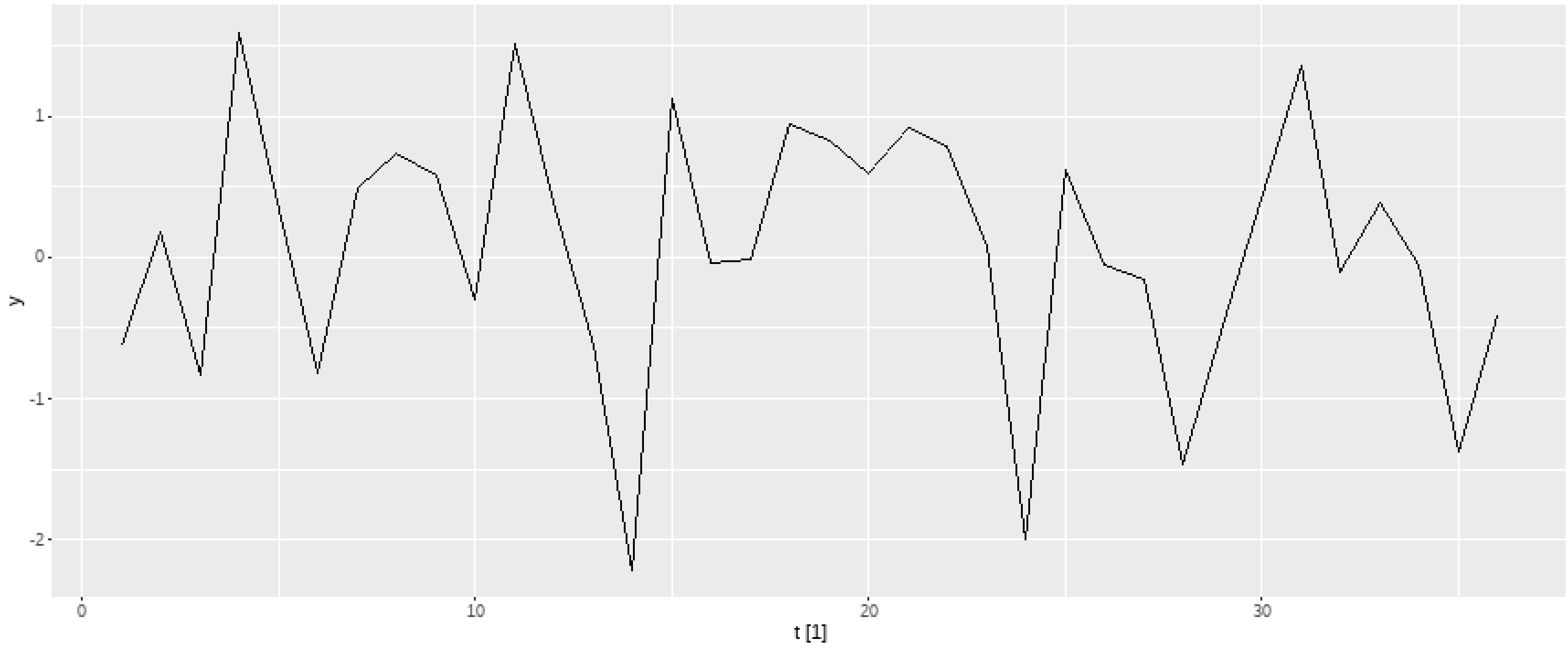
Google stock price



Australian monthly electricity production

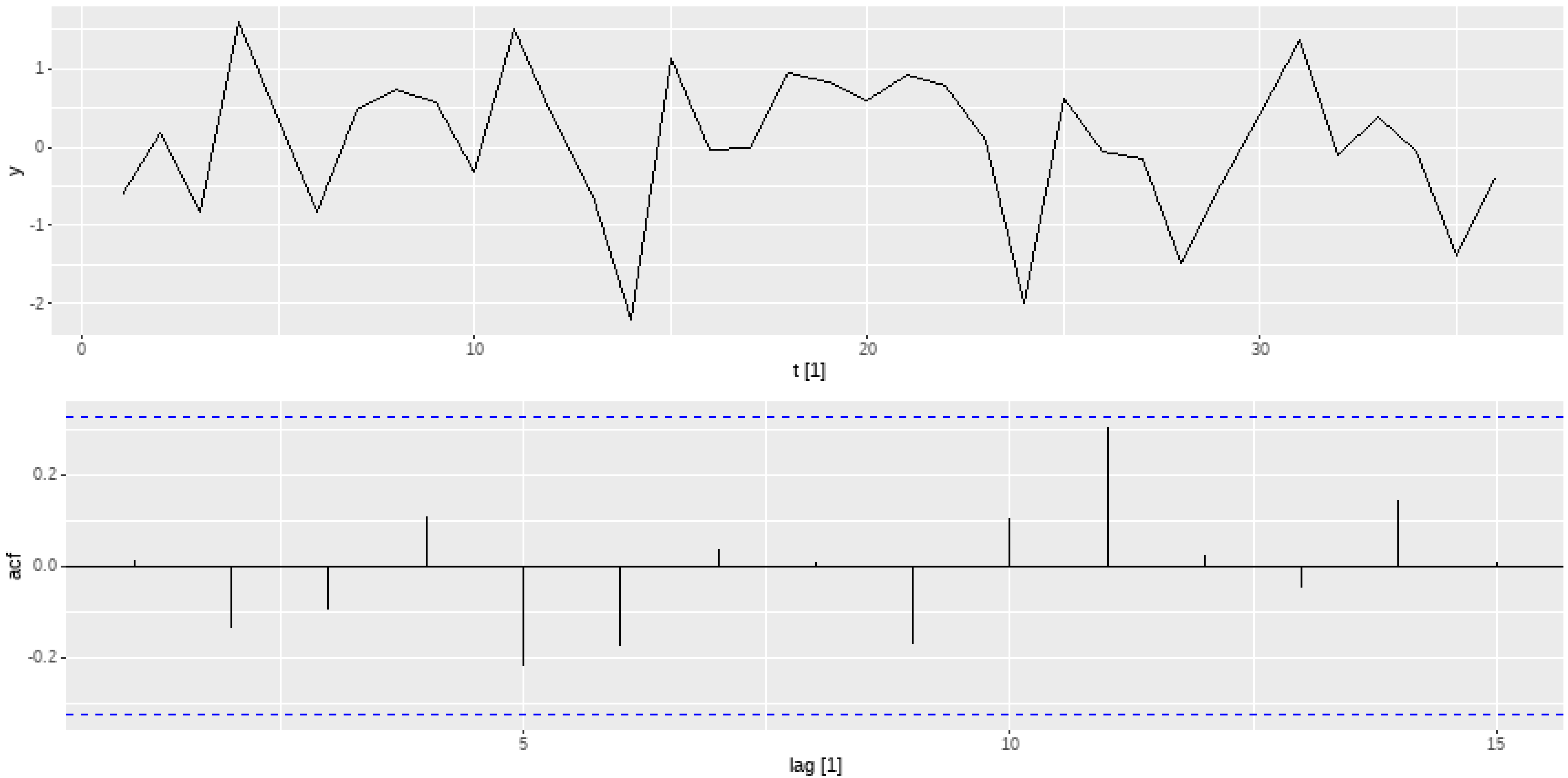


White noise



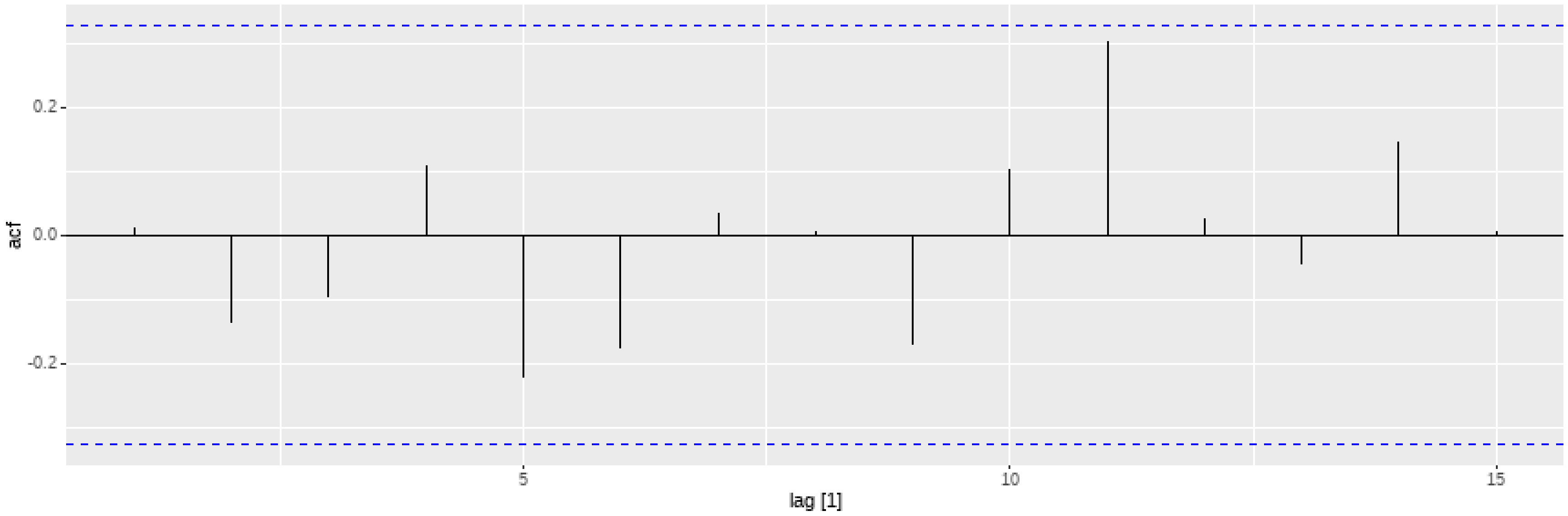
- **White noise data** is uncorrelated across time with zero mean and constant variance.
- Technically, we require independence as well.

Sample autocorrelations for white noise series



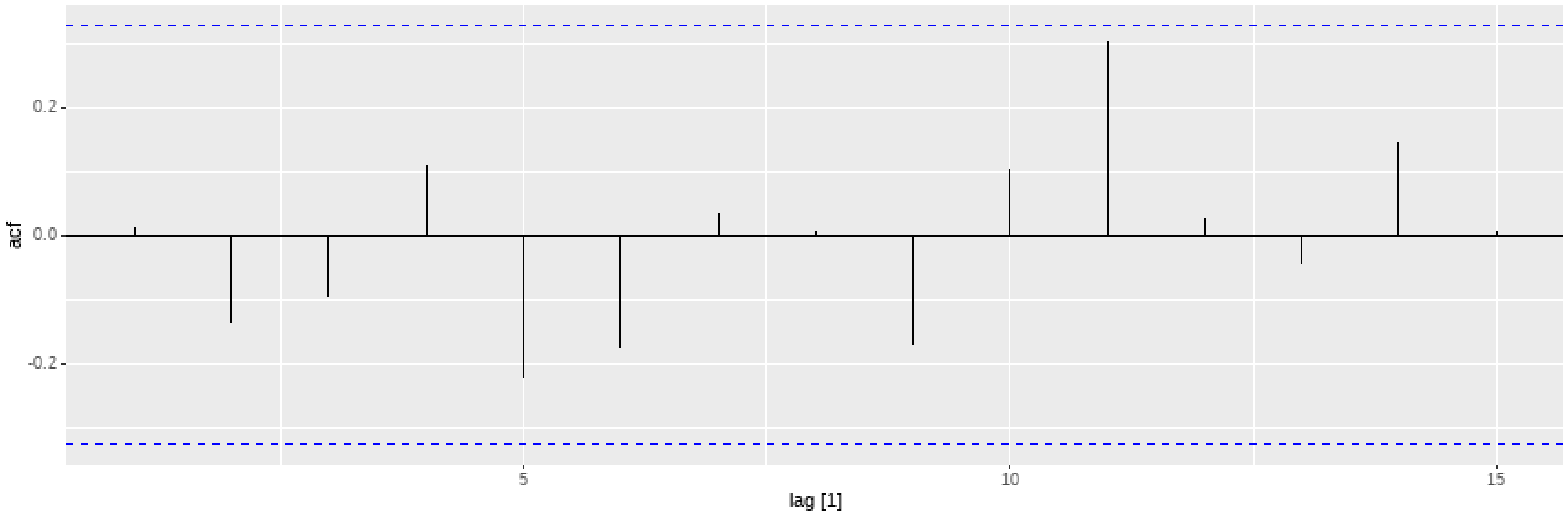
- For uncorrelated data, we would expect each one to be close to zero.
- Blue lines show 95% **critical values**.

Sampling distribution of autocorrelations



- Sampling distribution of r_k for white noise data is asymptotically $N(0, 1/T)$.
- 95% of all r_k for white noise must lie within $\pm 1.96/\sqrt{T}$.
- If this is not the case, the series is probably not WN.
- Common to plot lines at $\pm 1.96/\sqrt{T}$ when plotting ACF. These are the **critical values**.

Sampling distribution of autocorrelations

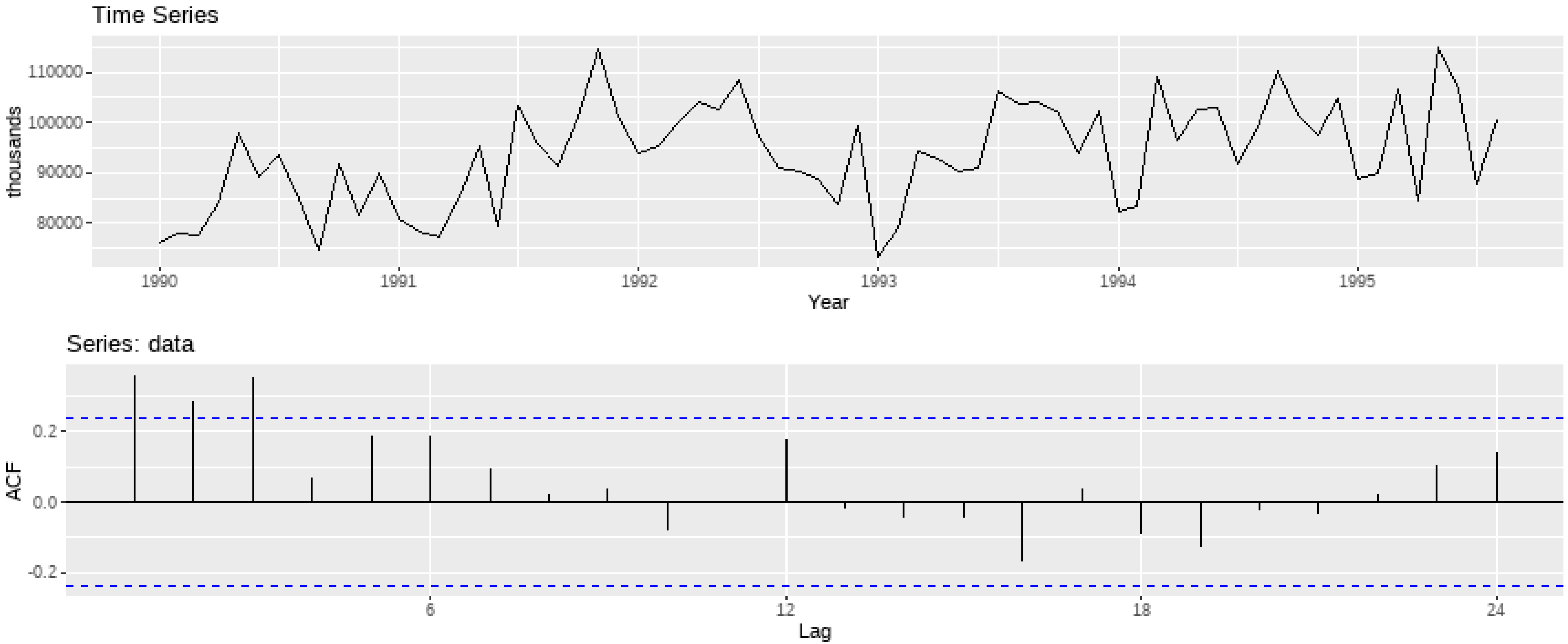


Example:

- $T = 36$ and so critical values at $\pm 1.96/\sqrt{36} = \pm 0.327$.
- All autocorrelation coefficients lie within these limits, confirming that the data are white noise.

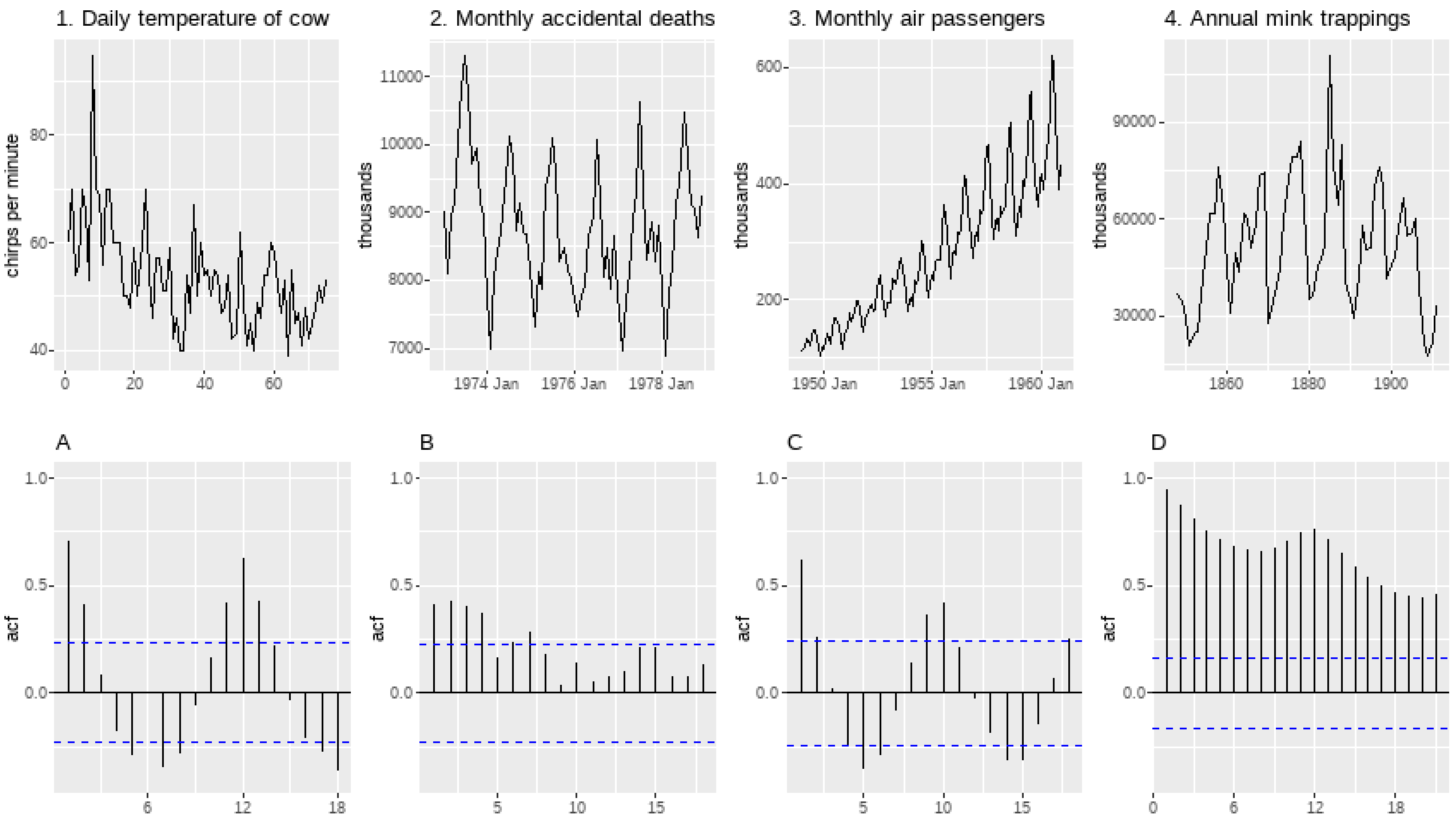
(More precisely, the data cannot be distinguished from white noise.

Example



- Difficult to detect pattern in time plot.
- ACF shows some significant autocorrelation at lags 1, 2, and 3.
- r_{12} relatively large although not significant.This may indicate some slight seasonality.
- These show the series is **not a white noise series**.

Which is which?



References

- Hyndman, R. J., & Athanasopoulos, G. (2018). Forecasting: principles and practice. OTexts.
- Mathai, A. M., & Haubold, H. J. (2008). Applications to Stochastic Process and Time Series. Special Functions for Applied Scientists, 247-295.