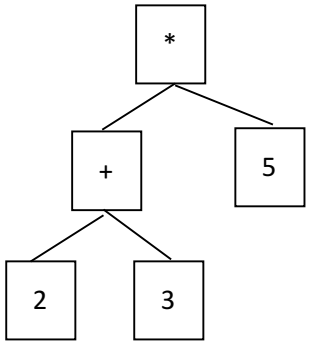
CSCI 3353 Object Oriented Design
Homework Assignment 9
Due Friday, April 20

An arithmetic expression is naturally represented as a tree. The leaves of the tree are numbers, and the internal nodes are operators. For example, the arithmetic expression

```
(2 + 3) * 5
```

corresponds to the tree



A polynomial is a generalization of an arithmetic expression, in which a leaf node can also be a variable. For example, the polynomial

```
(x + 3) * y
```

has the same representation as the above tree, except that the nodes for 2 and 5 are nodes for x and y.

In this assignment, I want you to write code to construct and manipulate polynomials. Here are the basic requirements:

- You only need to support integer values.
- You need to support three operators: PLUS, MINUS, and PRODUCT. These are binary operators – that is, their nodes in an expression tree will always have exactly two children.
- An operator's children can be any polynomial. In particular, a child could be an integer, a variable, or another operator.
- Values and variables are implemented in their own class, as are each kind of operator. These classes should be named *Number*, *Variable*, *PlusOp*, *ProductOp*, and *MinusOp*. Each class should implement the interface *Polynomial*.

The interface *Polynomial* should look like this:

```
public interface Polynomial {
  String toString();
  boolean equals(Object obj);
  boolean hasNoVariables();
  int evaluate(Map<String,Integer> m);
  Polynomial reduce();
  Iterator<Polynomial> childIterator();
}
```

Your job will be to implement these methods for the five classes that implement *Polynomial*, so here is a brief explanation of what they do.

toString

The method *toString* returns an infix representation of the polynomial. Recall that an infix expression may require parentheses. I don't care if the string returned by the method has extra, redundant parentheses.

equals

The *equals* method tests to see if the polynomial is equal to another one. This method overrides the *equals* method inherited from the class *Object* (as described in Chapter 3). Thus its argument is of type *Object*. You will need to use **instanceof** to ensure that the argument is actually a polynomial.

Two polynomials are "equal" if they belong to the same class and have the same structure, taking into consideration the fact that addition and multiplication are commutative. For example, the following polynomials are all equal:

```
(x + 3) * y
(3 + x) * y
y * (x + 3)
y * (3 + x)
```

You should not use arithmetic to determine equality. For example, the following polynomials are not equal to the above ones:

```
(x + 3) * (y + 0)
(x + (2 + 1)) * y
```

hasNoVariables

The *hasNoVariables* method returns true if none of its nodes is a variable.

evaluate

The method *evaluate* evaluates the polynomial and returns an integer. The specified map should have an entry for each variable; the variable's name is the entry's key, and its value is the entry's value. For example, consider the polynomial (x + 3) * y. If I evaluate it using the mapping [x=6, y=7], the answer is 63; if I evaluate it using the mapping [y=8, z=1, x=2] the answer is 40. It is ok for the map to have values for unmentioned variables. But if the map doesn't have a value for a mentioned variable, then the evaluation method has no choice but to throw an exception. Note that if the polynomial has no variables, then the *evaluate* method will return the same value regardless of what map is provided. A null value can denote an empty map.

reduce

The method *reduce* does "partial evaluation" of the polynomial. That is, it evaluates the polynomial as much as it can without using a map, and returns the resulting polynomial. For example, the following polynomials all reduce to (x + 3) * y:

```
(x + ((2 * 2) - 1)) * y
(x + 3) * (y + 0)
((x * 1) + 3) * ((y + 0) + (z * 0))
((x + 3) * y) + ((z + x) – (x + z))
```

The first example demonstrates that reduction evaluates all arithmetic expressions within the polynomial. The second and third examples demonstrate that reduction can make use of the special properties of 0 and 1. The fourth example demonstrates that if p and q are polynomials such that p.equals(q) is true, then reduction should transform the polynomial (p – q) to 0.

If a polynomial *p* has no variables, then *p.reduce()* returns a polynomial having a single node of type *Number*, whose value is the same as the result of calling *p.evaluate*.

Note that calling *reduce* should not change the existing polynomial. Instead, the returned polynomial should consist entirely of newly created nodes.

HINT: Use a postorder traversal of the tree.

childIterator

The method *childIterator* returns an iterator of the node's children. If the node is a leaf node (i.e. an integer or a variable), then the method should return an empty iterator, and NOT the value *null*.

WHAT YOU SHOULD DO

1. Draw a class diagram corresponding to your design for these classes. Use the composite pattern.

2. Write the Java code that implements the interface *Polynomial* and its five classes. Your code is only allowed use the **instanceof** operator to implement the *equals* method. The constructor of each operator class should have two arguments, denoting the children of the operator node. The operator classes don't need to have an *add* method.

3. Write a default method for the *Polynomial* interface called *forEach*, whose header looks like this:

```
default void forEach(Consumer<Polynomial> c) { ... }
```

The *forEach* method should implement the visitor pattern; that is, it applies the specified consumer object to each node of the polynomial.

4. Write a program, called *HW9Test.java*, that uses your code. Your program should have the following components:

- Write a static method *getVarsExternal*, having the following signature:

  ```
  public static Set<Variable> getVarsExternal(Polynomial p)
  ```

  This method should recursively call the polynomial's *childIterator* method to traverse the tree and return a set containing the variables it finds. (The reason for using a set is that it removes duplicates automatically.)

- Write a static method *getVarsInternal*, having the same signature and returning the same output. The difference is that it should perform internal iteration, using the *forEach* method that you wrote for question 3.

- Construct objects corresponding to the two polynomials:

  ```
  ((x * 1) + 3) * ((y + 0) + (z * 0))
  ((x + 3) * y) + ((z + x) – (x + z))
  ```

  For an example of how I want you to construct polynomials, download the file *SimpleHW9Test.java* from Canvas. That code creates an object corresponding to the polynomial (x+3)*y and prints it.

- For each of these polynomials:
  - Call *toString* to print it.
  - Call *evaluate* to evaluate it using the map [x=3, y=4, z=5], and print the result.
  - Call *reduce* to reduce it, and print the reduced polynomial.
  - Call *getVarsExternal* to get its set of variables, and then print those variables.
  - Call *getVarsInternal* to get its set of variables, and then print those variables.

For comparison, my *HW9Test* class prints the following:

```
The expression is (((x * 1) + 3) * ((y + 0) + (z * 0)))
It evaluates to 24
It reduces to ((x + 3) * y)
Using external iteration, its variables are x y z
Using internal iteration, its variables are x y z

The expression is (((x + 3) * y) + ((z + x) - (x + z)))
It evaluates to 24
It reduces to ((x + 3) * y)
Using external iteration, its variables are x y z
Using internal iteration, its variables are x y z
```

WHAT TO SUBMIT: You will need to create the following files:

- A document *HW9diagram.pdf* containing your class diagram from problem 1.
- The six files *Polynomial.java*, *Number.java*, *Variable.java*, *PlusOp.java*, *ProductOp.java*, and *MinusOp.java*, as described in problems 2 and 3.
- The file *HW9Test.java*, as described in problem 4.

The Java files must be in the package "hw9". Create a zip file containing these files and submit it to Canvas.