PHYS 6260 Project Progress Report

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1 Introduction

For this project, I am attempting to model stellar structure for a variety of stars to see if I can reproduce the trends seen along the zero-age main sequence (ZAMS) from stellar data.

The current setup of my model is:

1. Specify total mass and chemical composition (assumed to be constant throughout the star).
2. Calculate/approximate boundary conditions at the surface and the center based on semi-empirical scaling relations and astrophysical equations for radius r, pressure P, luminosity L, and temperature T.
3. Integrate the equations of stellar structure from the center outwards and from the surface inwards as a function of mass until the solutions meet.
4. Calculate residuals $\vec{\mathcal{R}}$ at this mass midpoint for each of the four variables above.
5. Determine how these residuals are affected by small perturbations to the approximated boundary conditions (total radius R, central pressure P_c, total luminosity L_s, central temperature T_c).
6. Use these partial results to calculate approximate perturbations to these boundary conditions that should reduce the residuals to zero.
7. Repeat until the residuals are small compared to the perturbed boundary conditions.

2 Project Status

The code for my project is located in a GitHub repository. Currently, all of my code is located in a single Jupyter Notebook to give me easier control over debugging; once the code can produce reasonable results, I will separate the code into python files so it is more modular.

2.1 Difficulties and Changes

I have come across a few major difficulties that led to noteworthy plan changes, each of which I will discuss in further detail below:

1. Correcting the ZAMS central boundary conditions
2. Properly implementing the perturbations calculations
3. Handling runtime errors during the RK4 integration

The primary issue was correcting the central boundary conditions used for ZAMS stars. Previously, I wanted to use $m = r = L = 0$, but this led to issues with calculating the derivatives $\frac{\partial \vec{z}}{\partial m}$. I tried using small values for m and $r(m)$ (atomic/microscopic scales), but this led to non-convergence issues because the derivatives and the integration range were so large. Instead, I'm assuming the core of mass $M_0 = 10^{-10}M$ is small enough to approximate constant conditions for T_c, P_c, etc., but large enough for the integration to converge by reducing the mass range. For a Sun-like star, this yields $M_0 \approx 2 \cdot 10^{20}$ kg and $r_c \approx 66$ km; though these are large on human scales, a core of this mass/radius is significantly smaller than the Moon, so I believe this approximation is reasonable. Here are my updated central boundary conditions:

- $M_0 = \lim_{m \to 0} m \implies \frac{M_0}{M} \approx 0$
- $\rho_c = \lim_{m \to M_0} \rho(m) = 115 \, \frac{3}{4\pi} \frac{M}{R^3}$
- $r_c = \lim_{m \to M_0} r(m) = \left(\frac{3}{4\pi} \frac{M_0}{\rho_c} \right)^{1/3}$
- $P_c = \lim_{m \to M_0} P(m) = 7.701 \frac{GM^2}{R^4}$
- $T_c = \lim_{m \to M_0} T(m) = \frac{2}{3} \frac{G}{k_B} \frac{M m_u}{R}$

- $L_c = \lim\limits_{m \to M_0} L(m) = M_0 \, \epsilon\Big(\vec{X}, \rho_c, T_c\Big)$

For the perturbations, I was applying them incorrectly. Previously, I had assumed they were multiplicative rather than additive. This has been resolved, and perturbations should now be calculated and applied correctly.

For the RK4 integration, I was getting runtime errors involving NaNs. Since RK4 integration involves calculations at the midpoints, there was a moment where the temperature gradient at the midpoint flipped from radiative to convective (or vice versa), which ended up yielding a negative temperature value in one of the $\vec{k}$'s This created NaNs in the calculations for ϵ. To address this, I am catching value errors for NaNs and inf during the RK4 integration; if these arise, then h is reduced and RK4 is rerun until the stepsize is small enough for the integration to yield sensible results. Since I expect this situation to arise infrequently, this seemed like a better approach than reducing the stepsize over the whole domain.

2.2 Outlook

Currently, most of the code for my project has been written and is functioning properly. I have:

1. Functions for the equation of state (P, ρ, T)
2. Functions for the remaining intermediary variables (ϵ, κ, etc.)
3. A function to calculate the stellar derivatives $\frac{\partial \vec{z}}{\partial m}$
4. A function to calculate/estimate the ZAMS boundary conditions
5. A function that performs one pass of integrating the equations of stellar structure
6. A function that calculates perturbations to the boundary conditions using the residuals $\vec{\mathcal{R}}$
7. A function that performs the full stellar integration (multiple integration passes and updating the boundary conditions)

There are a few minor errors I need to address, including how I am calculating the density from the equation of state. There may be additional runtime errors (likely involving the perturbations/residuals calculations, if they exist) I would need to address once this is fixed. I would also like to add H^- as an opacity source, as this is an important source of opacity at the surface for cooler stars that I should consider for the surface boundary conditions. I would like to have these done by the end of next week.

If I can get the ZAMS integration running properly in a reasonable amount of time, I would like to try expanding to giant stars and white dwarfs. This would require a few modifications:

- Giants and white dwarfs would need different sets of boundary conditions.
- For giant stars, I can no longer assume the chemical composition $\vec{X}$ is constant throughout the star; rather, I would need to have a set of chemical compositions $\overleftrightarrow{X} = [\vec{X}_1, \vec{X}_2, \dots]$ that change based on which one produces the most energy given the conditions at that location.
- For white dwarfs, they do not produce energy via nuclear fusion, so ϵ is irrelevant. Rather, I think I would need to calculate the luminosity as $L(m) = 4\pi r^2(m)\sigma_{SB}T^4(m)$.