Calculating scalar curvature associated with the information metric

coordinates for the statistical manifold $\{\alpha = \frac{h}{kT}, \beta = \frac{J}{kT}\}$

In[51]:=
```
coord = {α, β}
```

Out[51]= $\{\alpha, \beta\}$

$\ln Z = W = \mathrm{Log}[2] - 1/2\,\mathrm{Log}\left[1 - \left(\frac{\alpha}{1-q\,\beta}\right)^2\right] - \frac{q\beta}{2}\left(\frac{\alpha}{1-q\,\beta}\right)^2 \approx \mathrm{Log}[2] + \frac{\alpha^2}{2\,(1-q\beta)} + \frac{\alpha^4}{4\,(1-q\beta)^4}$

[upto second order approximation of lnZ]

In[80]:=
```
W = Log[2] + α^2/(2 (1 - q * β)) + α^4/(4 (1 - q * β)^4)
```

Out[80]= $$\frac{\alpha^4}{4\,(1-q\,\beta)^4} + \frac{\alpha^2}{2\,(1-q\,\beta)} + \mathrm{Log}[2]$$

the Information metric ≡ *Entropy Derivative Metric* $g_{ab} = \partial_{a,b}[\ln Z]$

In[81]:=
```
metric = {{D[D[W, coord[[1]]], coord[[1]]], D[D[W, coord[[1]]], coord[[2]]]},
   {D[D[W, coord[[2]]], coord[[1]]], D[D[W, coord[[2]]], coord[[2]]]}} // Simplify
```

Out[81]= $$\left\{\left\{\frac{1}{1-q\,\beta} + \frac{3\,\alpha^2}{(-1+q\,\beta)^4}, \frac{q\,\alpha\,(-4\,\alpha^2 + (-1+q\,\beta)^3)}{(-1+q\,\beta)^5}\right\}, \left\{\frac{q\,\alpha\,(-4\,\alpha^2 + (-1+q\,\beta)^3)}{(-1+q\,\beta)^5}, -\frac{q^2\,\alpha^2\,(-5\,\alpha^2 + (-1+q\,\beta)^3)}{(-1+q\,\beta)^6}\right\}\right\}$$

In[82]:=
```
detI = Det[{{D[D[W, coord[[1]]], coord[[1]]], D[D[W, coord[[1]]], coord[[2]]]},
   {D[D[W, coord[[2]]], coord[[1]]], D[D[W, coord[[2]]], coord[[2]]]}}] // Simplify
```

Out[82]= $$-\frac{q^2\,\alpha^6}{(-1+q\,\beta)^{10}}$$

determinant of information matric : Det[g] = - $\frac{q^2\,\alpha^6}{(1-q\beta)^{10}}$

In[83]:=
```
curv = {{{D[D[W, coord[[1]]], coord[[1]]]},
    {D[D[W, coord[[1]]], coord[[2]]]}, {D[D[W, coord[[2]]], coord[[2]]]}},
   {{D[D[D[W, coord[[1]]], coord[[1]]], coord[[1]]]}, {D[D[D[W, coord[[1]]], coord[[1]]], coord[[2]]]},
    {D[D[D[W, coord[[1]]], coord[[2]]], coord[[2]]]}},
   {{D[D[D[W, coord[[1]]], coord[[1]]], coord[[2]]]}, {D[D[D[W, coord[[1]]], coord[[2]]], coord[[2]]]},
    {D[D[D[W, coord[[2]]], coord[[2]]], coord[[2]]]}}} // FullSimplify
```

Out[83]=
$$\left\{\left\{\left\{\frac{1}{1-q\,\beta}+\frac{3\,\alpha^2}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\,\alpha\,\left(-4\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{q^2\,\alpha^2\,\left(-5\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},\right.$$
$$\left\{\left\{\frac{6\,\alpha}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},$$
$$\left.\left\{\left\{\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\},\left\{\frac{3\,q^3\,\alpha^2\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right\}\right\}\right\}$$

In[87]:=
```
curv // MatrixForm
```

Out[87]//MatrixForm=
$$\begin{pmatrix} \left(\frac{1}{1-q\,\beta}+\frac{3\,\alpha^2}{(-1+q\,\beta)^4}\right) & \left(\frac{q\,\alpha\,\left(-4\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right) & \left(-\frac{q^2\,\alpha^2\,\left(-5\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right) \\ \left(\frac{6\,\alpha}{(-1+q\,\beta)^4}\right) & \left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right) & \left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right) \\ \left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right) & \left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right) & \left(\frac{3\,q^3\,\alpha^2\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right) \end{pmatrix}$$

In[89]:= **DelCurv = FullSimplify[**

$$\left(\frac{1}{1-q\,\beta}+\frac{3\,\alpha^2}{(-1+q\,\beta)^4}\right)\left(\left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right)\left(\frac{3\,q^3\,\alpha^2\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right)-\left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right)\right.$$
$$\left.\left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right)\right)-\left(\frac{q\,\alpha\,\left(-4\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right)\left(\left(\frac{6\,\alpha}{(-1+q\,\beta)^4}\right)\left(\frac{3\,q^3\,\alpha^2\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right)-\right.$$
$$\left.\left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right)\left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right)\right)+\left(-\frac{q^2\,\alpha^2\,\left(-5\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right)$$
$$\left(\left(\frac{6\,\alpha}{(-1+q\,\beta)^4}\right)\left(-\frac{2\,q^2\,\alpha\,\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right)-\left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right)\left(\frac{q\,\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right)\right)\Big]$$

Out[89]=
$$\left\{\frac{2\,q^4\,\alpha^6}{(-1+q\,\beta)^{13}}\right\}$$

In[88]:=
$$\text{Det}\left[\left\{\left\{\left\{\frac{1}{1-q\,\beta}+\frac{3\,\alpha^2}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\,\alpha\left(-4\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{q^2\,\alpha^2\left(-5\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},\left\{\left\{\frac{6\,\alpha}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},\left\{\left\{\frac{q\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\},\left\{\frac{3\,q^3\,\alpha^2\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right\}\right\}\right\}\right]$$

Out[88]=
$$\text{Det}\left[\left\{\left\{\left\{\frac{1}{1-q\,\beta}+\frac{3\,\alpha^2}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\,\alpha\left(-4\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{q^2\,\alpha^2\left(-5\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},\left\{\left\{\frac{6\,\alpha}{(-1+q\,\beta)^4}\right\},\left\{\frac{q\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\}\right\},\left\{\left\{\frac{q\left(-12\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^5}\right\},\left\{-\frac{2\,q^2\,\alpha\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^6}\right\},\left\{\frac{3\,q^3\,\alpha^2\left(-10\,\alpha^2+(-1+q\,\beta)^3\right)}{(-1+q\,\beta)^7}\right\}\right\}\right\}\right]$$

In[91]:=
$$\text{Curv} = \text{FullSimplify}\left[\frac{-1}{2\,(\text{detI})}\,\text{DelCurv}\right]$$

Out[91]=
$$\left\{\frac{q^2}{(-1+q\,\beta)^3}\right\}$$

The scalar Curvature: $R = \frac{q^2}{(q\beta-1)^3}$