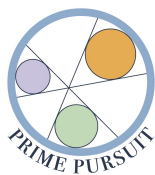


# Monthly Problems 1 [*February*]



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## Problems

1. Find all triplets of positive integers  $(a, b, c)$  such that

$$\frac{\ln(ab + bc + ac)}{\ln(\text{lcm}(a, b, c))}$$

is an integer. (*Culver Kwan*)

2.  $x, y, z, t$  are positive reals summing to 4. Prove that

$$\frac{x}{1+y^2} + \frac{y}{1+z^2} + \frac{z}{1+t^2} + \frac{t}{1+x^2} \geq 2.$$

(*Culver Kwan*)

3. Let  $P$  be a point on the circumcircle of  $\triangle ABC$  with circumcenter  $O$ . Let  $G_1, G_2, G_3$  be the centroid of  $PBC, PAC, PAB$  respectively. Let  $K$  be the first intersection of the circumcircle of  $G_1G_2G_3$  and the median of  $BC$  with respect to  $A$ . Let  $H', O'$  be the orthocenter and the circumcenter of  $\triangle G_1G_2G_3$  respectively. Prove that  $O'H' = KO$ . (*George Zhu*)
4. Let  $n$  be a positive integer. Culver and George are playing a game using  $n$  piles of stones, initially with  $1, 2, \dots, n$  stones in each of the piles respectively. Culver and George take turns to play, with Culver starting first. In a turn, a player chooses a pile with a positive number of stones remaining and discards  $4^k$  stones from the pile where  $k$  is a non-negative integer and  $4^k$  does not exceed the number of stones in the pile before the move. Whoever discards the last stone wins. If both of the players play optimally, for which  $n$  will George win? (*Culver Kwan*)