FinSearch Midterm Report

Option Pricing Models and Their Accuracy

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1 Introduction

Options are widely used financial derivatives that derive their value from underlying assets. Pricing options accurately is essential for risk management, trading strategies, and hedging. This report compares two cornerstone models of option pricing — the Binomial Option Pricing Model (BOPM) and the Black-Scholes Model (BSM). Through detailed mathematical exposition and practical application on a real-world-inspired Infosys (INFY) stock option, we assess their respective strengths, assumptions, and use-cases.

2 Overview of Option Pricing Models

2.1 Black-Scholes Model

Developed by Black and Scholes in 1973, this model provides a closed-form analytical formula for pricing European options. It assumes that the underlying asset follows a geometric Brownian motion and uses stochastic calculus for its derivation.

Assumptions

- The asset price follows a geometric Brownian motion: $dS = \mu S dt + \sigma S dW$
- Constant risk-free rate r and volatility σ
- No dividends during the option life
- European option (exercised only at expiry)
- No arbitrage, frictionless markets
- Log-normal distribution of returns

Formula (European Call)

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

2.2 Binomial Model

Proposed by Cox, Ross, and Rubinstein (1979), the Binomial model constructs a recombining price tree of the underlying asset. It is a discrete-time model that uses backward induction to compute the option value at each node.

Assumptions

- Asset price can go up (u) or down (d) at each step
- No arbitrage condition holds
- Uses risk-neutral probabilities
- Suitable for American and European options

Formulas

$$u = e^{\sigma\sqrt{\Delta t}}, \quad d = \frac{1}{u}, \quad p = \frac{e^{r\Delta t} - d}{u - d}$$

$$C = e^{-r\Delta t}(pC_u + (1-p)C_d)$$

3 Numerical Application: Infosys Call Option

3.1 Problem Parameters

Table 1: Input Parameters for the Option

Underlying Stock	Infosys (INFY)
Spot Price S_0	Rs. 1,333.30
Strike Price K	Rs. 1,340
Risk-Free Rate r	6% annually
Volatility σ	30% annually
Time to Expiry T	14 calendar days = 0.03836 years
Price at Expiry S_T	Rs. 1,349.85

We assume a European call option, written (sold) at time $t = 0$, with a 14-day maturity.

3.2 Black-Scholes Price Calculation

$$d_1 = \frac{\ln(1333.30/1340) + (0.06 + 0.5 \times 0.3^2) \times 0.03836}{0.3\sqrt{0.03836}} \approx -0.0843$$
$$d_2 = d_1 - 0.3\sqrt{0.03836} \approx -0.1733$$
$$C = 1333.30 \cdot N(d_1) - 1340 \cdot e^{-0.06 \cdot 0.03836} \cdot N(d_2)$$

Using cumulative distribution values:

$$N(d_1) \approx 0.4664, \quad N(d_2) \approx 0.4312$$

$$C \approx 1333.30 \cdot 0.4664 - 1340 \cdot 0.9977 \cdot 0.4312 \approx Rs.42.56$$

3.3 Binomial Model Price (One-Step Model)

We now apply the one-period binomial model to the given option data.

$$\Delta t = T = 0.03836 \text{ years}$$
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.3 \cdot \sqrt{0.03836}} \approx 1.0595$$
$$d = \frac{1}{u} \approx 0.9438$$
$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.06 \cdot 0.03836} - 0.9438}{1.0595 - 0.9438} \approx \frac{1.00231 - 0.9438}{0.1157} \approx 0.5045$$

Stock prices at maturity:

$$S_u = S_0 \cdot u = 1333.30 \cdot 1.0595 \approx Rs.1413.07$$
$$S_d = S_0 \cdot d = 1333.30 \cdot 0.9438 \approx Rs.1258.63$$

Option payoffs at maturity:

$$C_u = \max(S_u - K, 0) = \max(1413.07 - 1340, 0) = Rs.73.07$$
$$C_d = \max(S_d - K, 0) = \max(1258.63 - 1340, 0) = Rs.0$$

Present value of expected payoff:

$$\begin{aligned} C &= e^{-r\Delta t} \left(pC_u + (1-p)C_d\right) \\ &= e^{-0.06 \cdot 0.03836} \left(0.5045 \cdot 73.07 + 0.4955 \cdot 0\right) \\ &= 0.9977 \cdot 36.87 \approx Rs.36.79 \end{aligned}$$

Tree Diagram

$$\begin{tikzcd}[column sep=small] & 1333.30 \arrow[dl, "\text{up}"'] \arrow[dr, "\text{down}"] & \\ 1413.07\ (C_u = 73.07) & & 1258.63\ (C_d = 0) \end{tikzcd}$$

This simple one-step tree captures the core logic of the binomial model: compute future stock outcomes, evaluate option payoffs, and discount the expected value under risk-neutral probability.

3.4 Delta Comparison and Interpretation

Black-Scholes Delta

Using the Black-Scholes model:

$$\Delta_{\text{BS}} = N(d_1) \approx 0.4664$$

This implies that for a Rs. 1 increase in the stock price, the option price increases by approximately Rs. 0.47.

Binomial Model Delta (One-Step)

$$\Delta_{\text{binomial}} = \frac{C_u - C_d}{S_u - S_d} = \frac{73.07 - 0}{1413.07 - 1258.63} = \frac{73.07}{154.44} \approx 0.4732$$

So, in the one-step binomial model, a Rs. 1 increase in the stock price increases the option price by approximately Rs. 0.4732. This is very close to the Black-Scholes estimate, validating the short-term accuracy of the binomial approach.

3.5 Hedging Effectiveness

Payoff Realization

At maturity, the stock price was Rs. 1,349.85. Hence, the realized payoff for the call option is:

$$\text{Payoff} = \max(S_T - K, 0) = \max(1349.85 - 1340, 0) = Rs.9.85$$

Model Evaluation

- The Black-Scholes model priced the option at Rs. 42.56, and the binomial model priced it at Rs. 36.79.
- Both models suggest a Delta around 0.47, implying that holding approximately 0.47 shares of Infosys per option contract would form a delta-neutral hedge.

- The binomial model offers flexibility for dynamic hedging as it allows recalculating delta at each time step.

In conclusion, both models offer reasonable hedging guidance, but the binomial model is slightly better suited for discrete-time rebalancing in real trading environments.

4 Model Comparison Summary

Table 2: Comparison of Binomial and Black-Scholes Models

Aspect	**Black-Scholes**	**Binomial**
Type	Continuous-time, analytic	Discrete-time, numeric
Option Type	European only	European and American
Volatility	Constant	Can be varied stepwise
Complexity	Low (closed-form)	Medium-High (trees)
Delta Estimation	Analytical	Stepwise numerical
Hedging Accuracy	High (theoretical)	Flexible, practical
Use Cases	Standard options	Exotic, American, dividends

5 Conclusion

Both the Binomial and Black-Scholes models price the Infosys call option similarly, with small differences due to modeling frameworks. The Delta values from both models were closely aligned, making both effective for short-term hedging. However, the Binomial model's adaptability makes it more suitable for options with complex features.