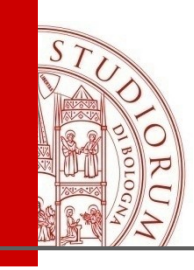
Simplex for network problem

Francesco Palmisano

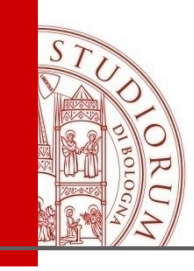
Primiano Arminio Cristino



Simplex for network problem

We have used simplex algorithm to resolve the following network problem:

- Min cost flow
- Max flow
- Max flow as min cost



Simplex: min cost conversion

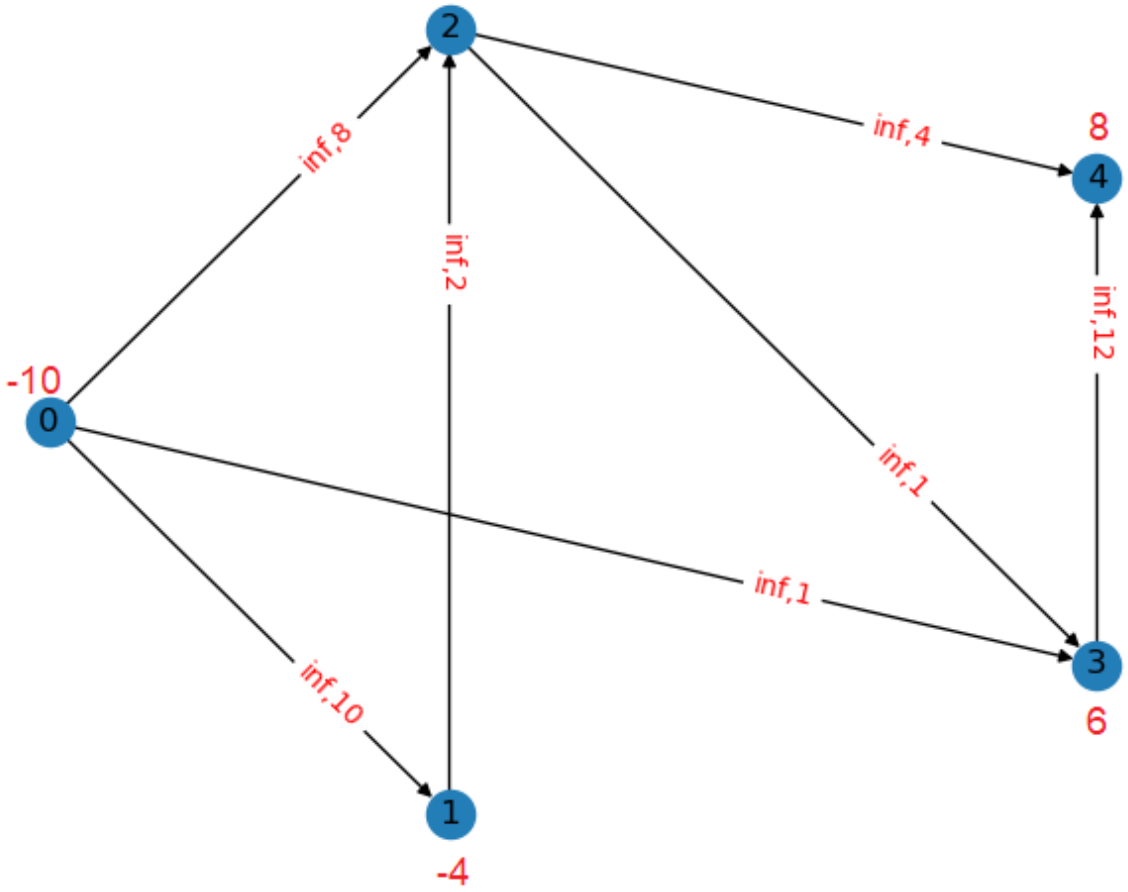
Variables of the simplex are edges of the graph. We simply build the objective function as the weighted sum of edges flows with their cost:

$$\boldsymbol{min} \sum (\boldsymbol{c_i * edge_i})$$

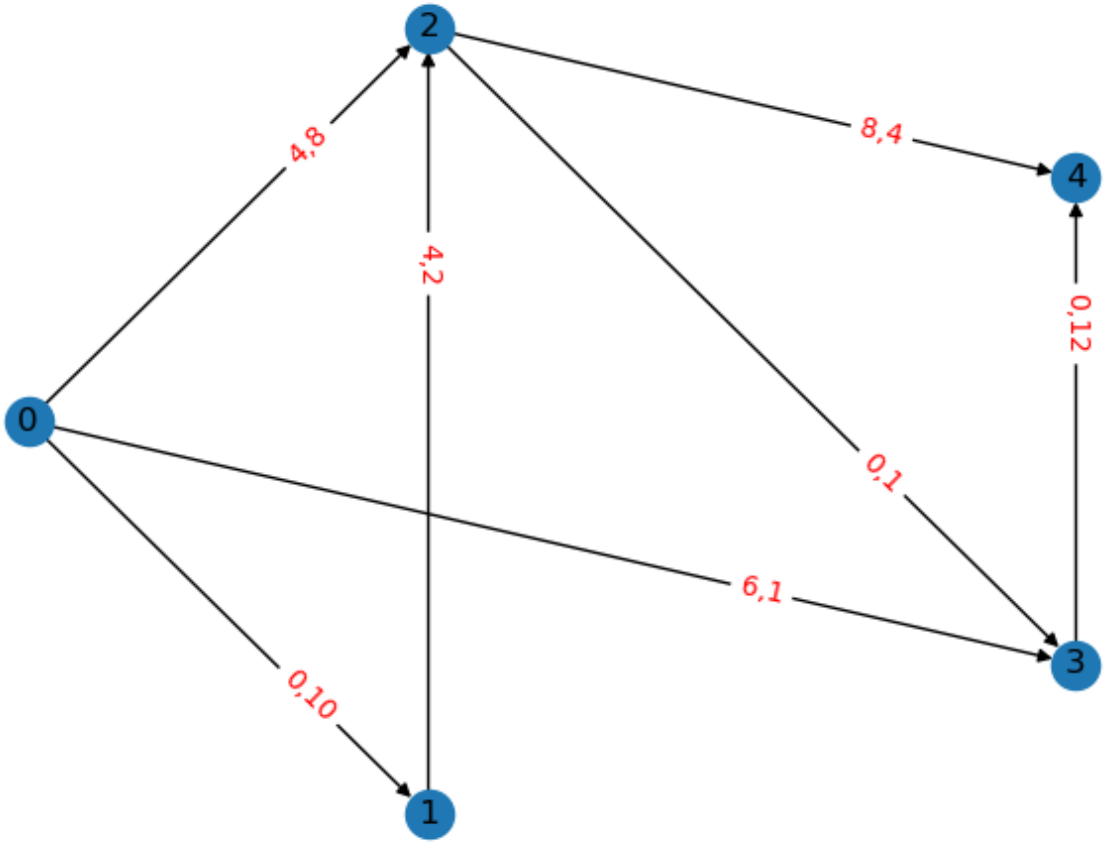
Then we add constraints as follows:

- “=” constraint to balance the flow for each node
- “≤” costraint to give an upper bound to edge flow, when capacity is defined.

Min cost flow: problem



Min cost flow: solution



Simplex: max flow convertion

Variables are the same as before. We build the objective function as the sum of edges flows from the source node:

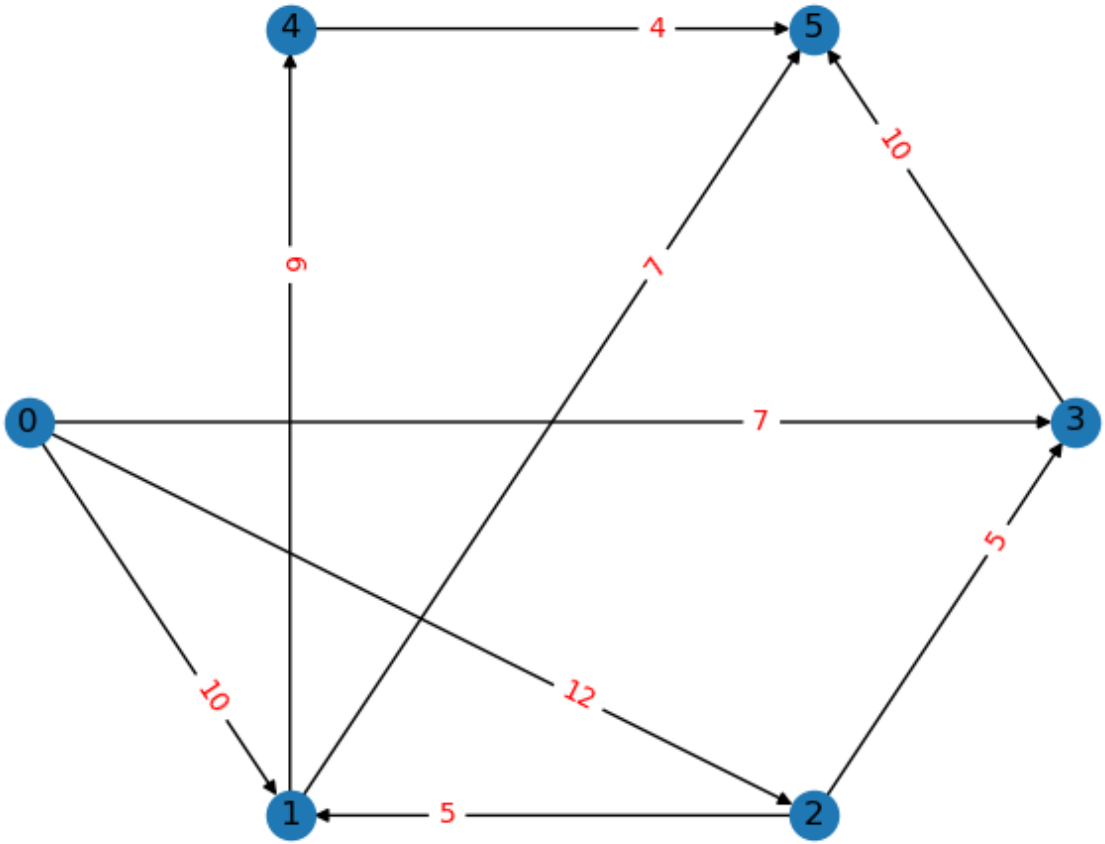
$$\boldsymbol{max} \sum \boldsymbol{edge_i} \qquad \boldsymbol{i \in FS(source)}$$

FS(node)=set of outgoing edges

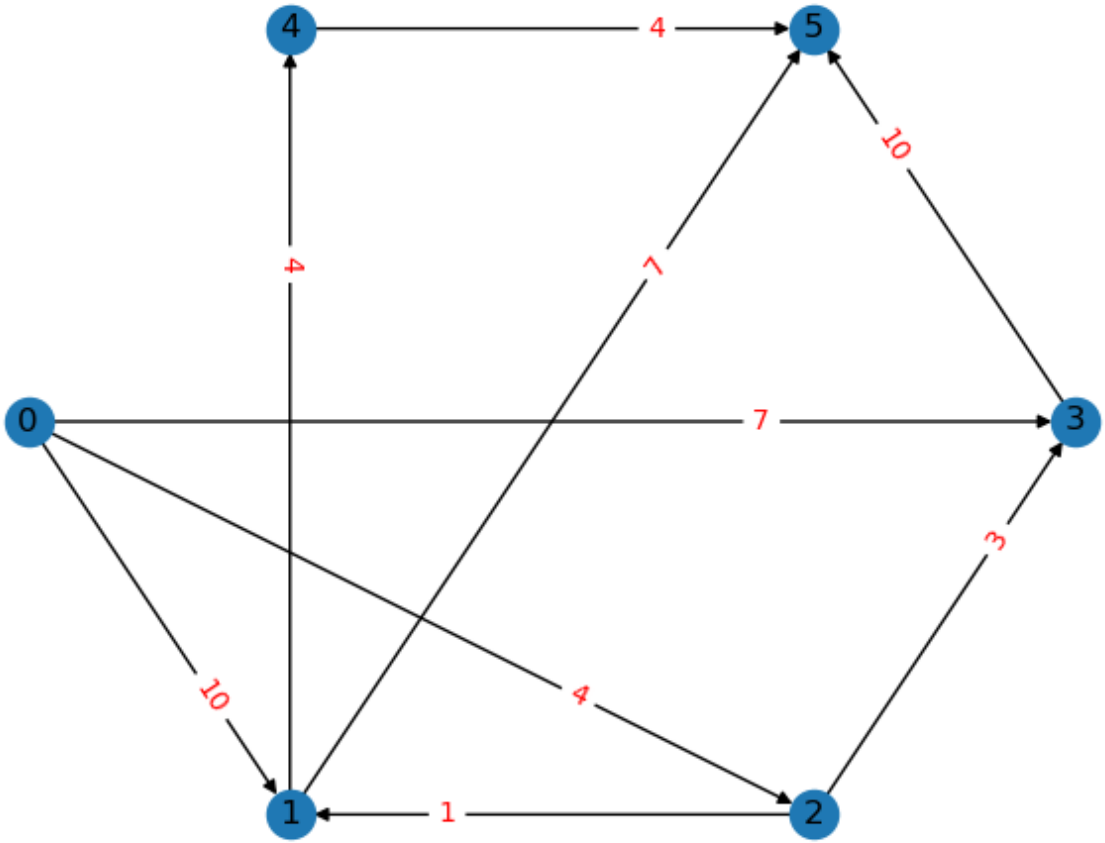
Then we add constraints as follows:

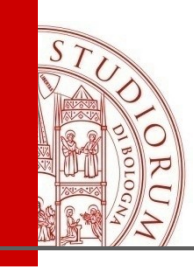
- “=” constraint to balance the flow for each intermediate node.
- “≤” costraint to give an upper bound to edge flow.

Max flow: prolem



Max flow: solution

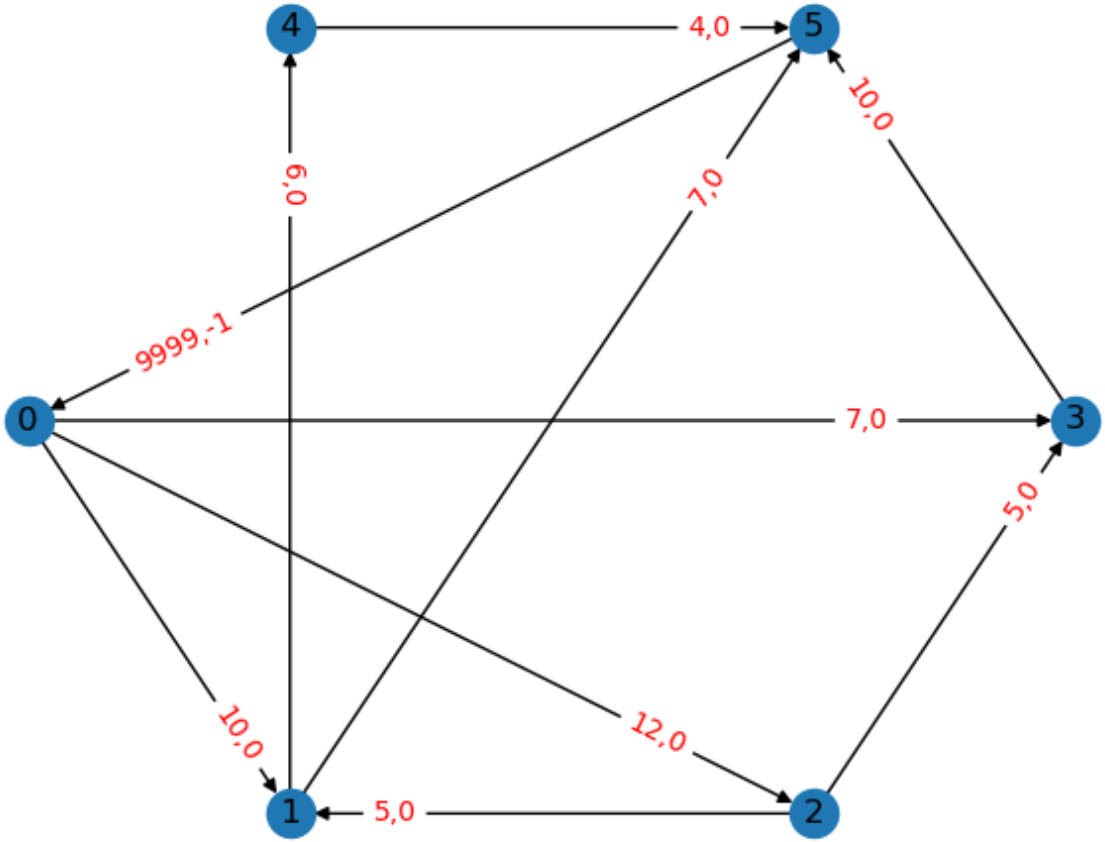




Simplex: max flow as min cost convertion

We convert the problem as a min cost one. We then add an edges from the sink to the source with -1 cost and high capacity. All the other costs are 0s.

Max flow as min cost: problem



Max flow as min cost: solution