# Practice Counting

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# Number of Hands

**Question**

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## Answer

$$\binom{52}{5} = \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2\,598\,960$$

# Two Hearts and Three Spades

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What is the number of 5-card hands with two hearts and three spades?

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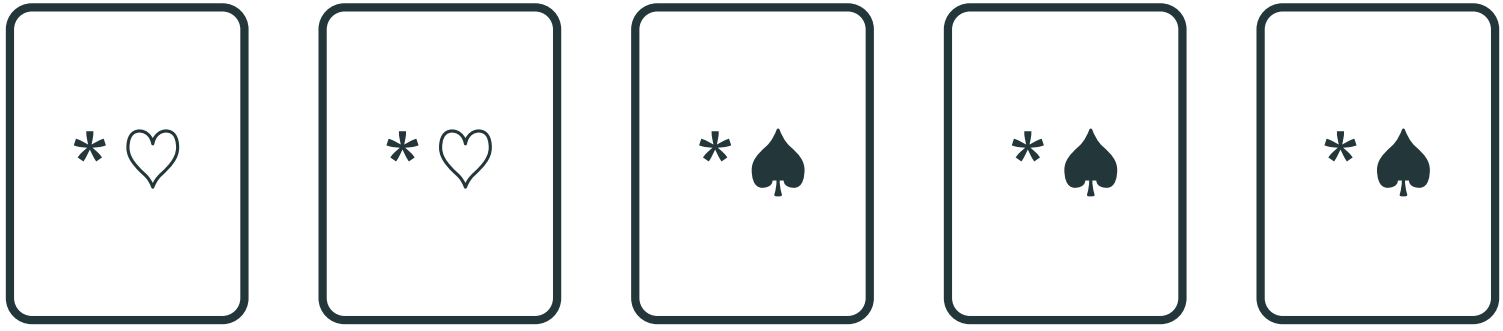


## Answer

$$\binom{13}{2}\binom{13}{3} = 22\,308$$

# 4-Digit Numbers Containing 7

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## Answer

$$10^4 - 9^4 = 3\,439$$

# Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if 7 in d:
        count += 1

print(count)
print(10**4 - 9**4)
```

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from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if 7 in d:
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print(count)
print(10**4 - 9**4)
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```
3439
3439
```

# 4-Digit Numbers with Increasing Digits

| Question |
| --- |
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What is the number of non-negative integers with at most four digits whose digits are increasing?

## Answer

$$\binom{10}{4} = 210$$

# Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
        count += 1
        print(d)

print(count)
```

# Code

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from itertools import product

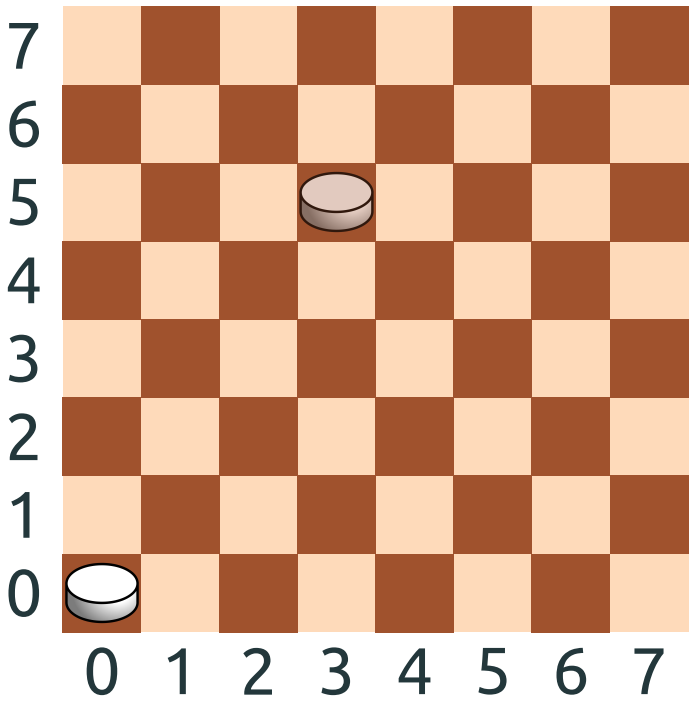
count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
        count += 1
        print(d)

print(count)
```

```
(0, 1, 2, 3)
(0, 1, 2, 4)
(0, 1, 2, 5)
(0, 1, 2, 6)
(0, 1, 2, 7)
(0, 1, 2, 8)
(0, 1, 2, 9)
(0, 1, 3, 4)
(0, 1, 3, 5)
(0, 1, 3, 6)
```

```
(4, 6, 7, 8)
(4, 6, 7, 9)
(4, 6, 8, 9)
(4, 7, 8, 9)
(5, 6, 7, 8)
(5, 6, 7, 9)
(5, 6, 8, 9)
(5, 7, 8, 9)
(6, 7, 8, 9)
210
```

# Piece on a Chessboard




A piece can move one step up or one step to the right. What is the number of ways of getting from the cell $[0, 0]$ (bottom left corner) to the cell $[5, 3]$?

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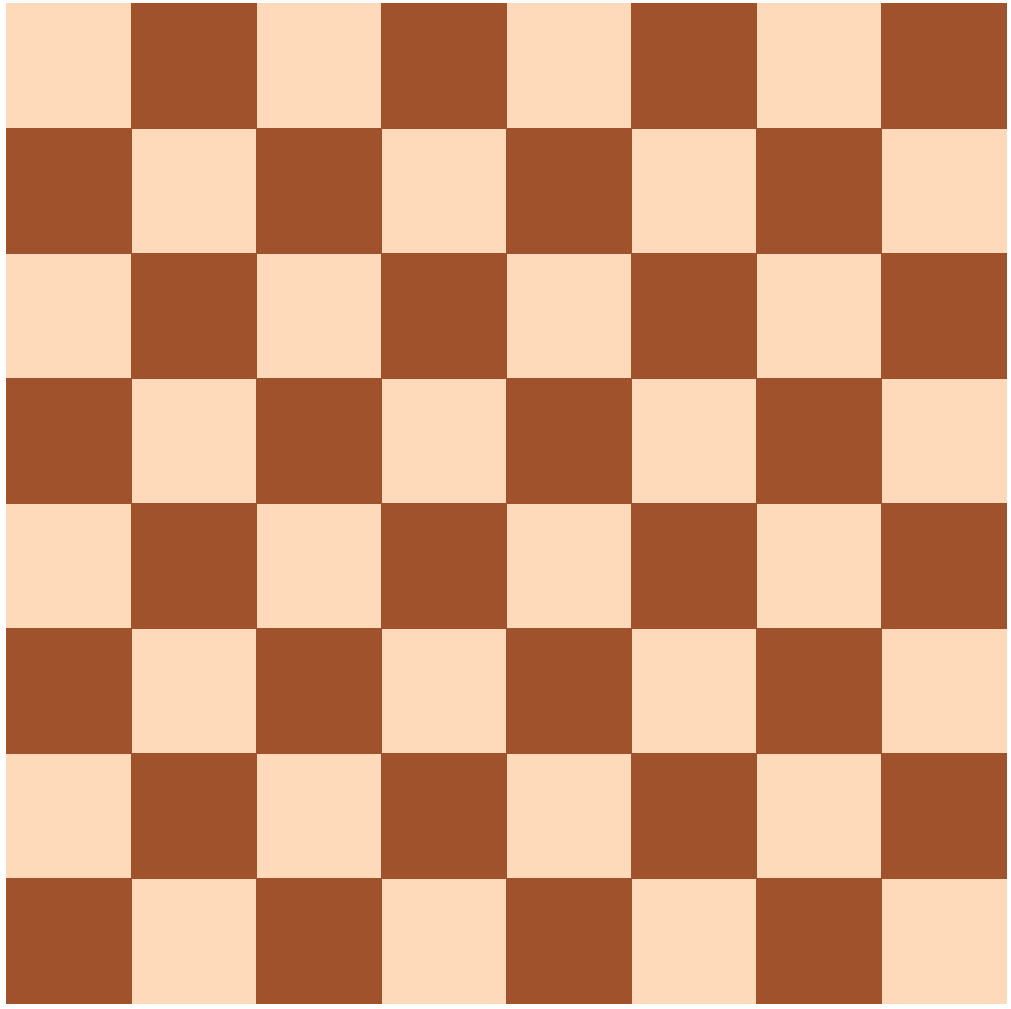
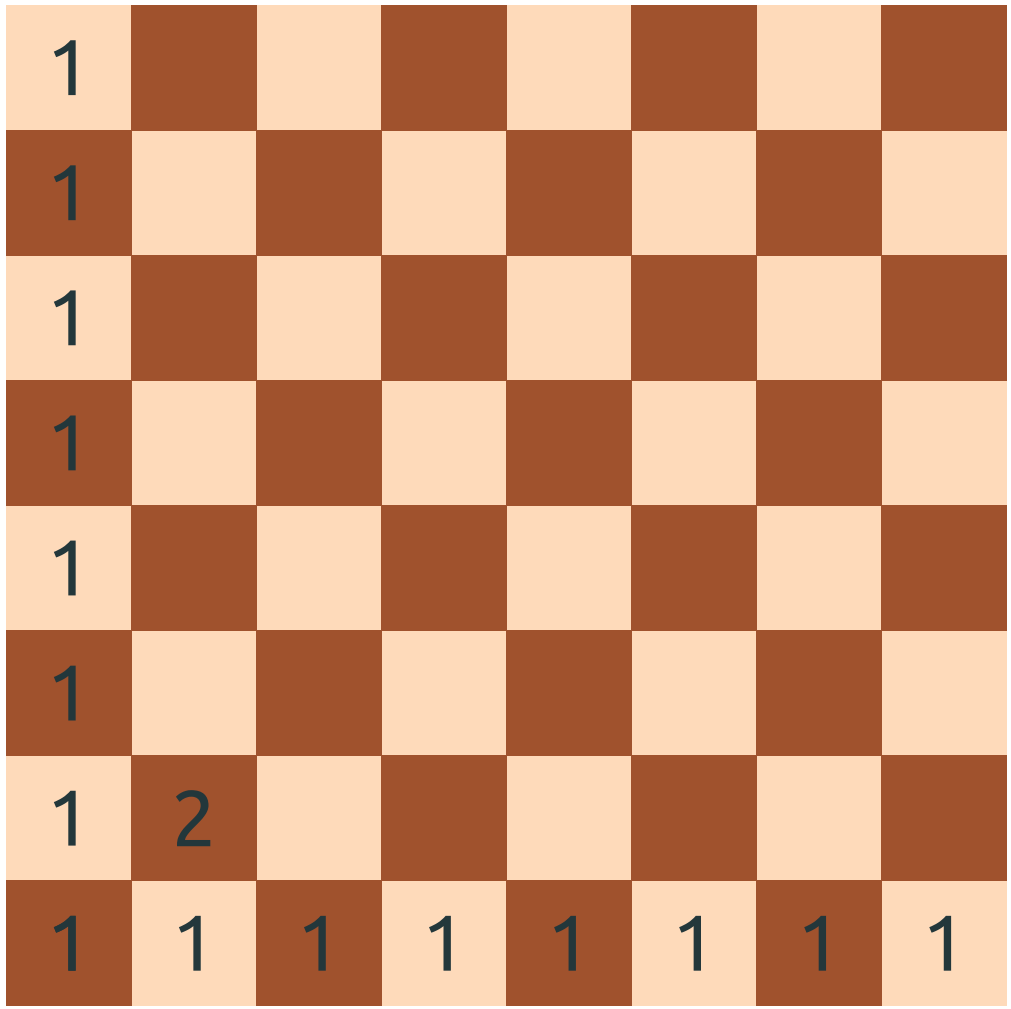
- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell $[5, 3]$
- Hence, the answer is $\binom{8}{3} = 56$

# Solution Using Pascal's Triangle



# Solution Using Pascal's Triangle



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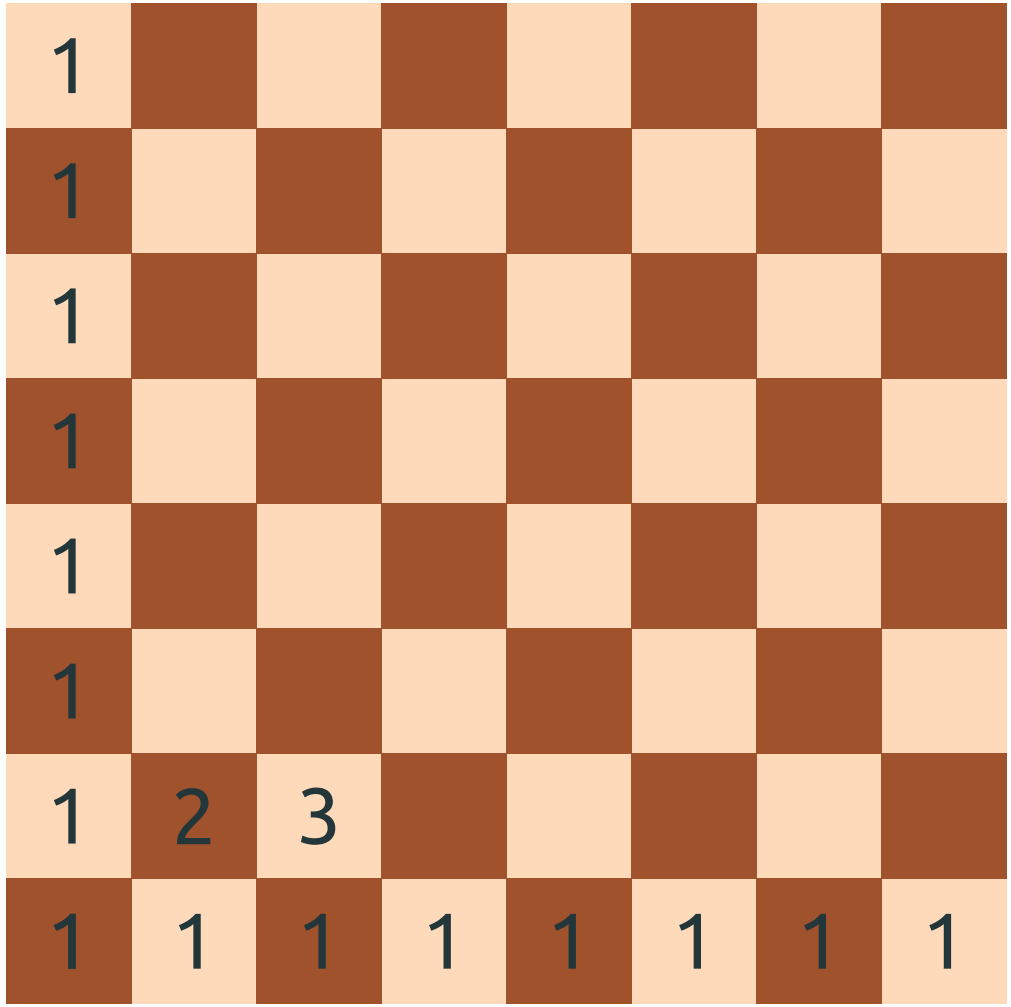

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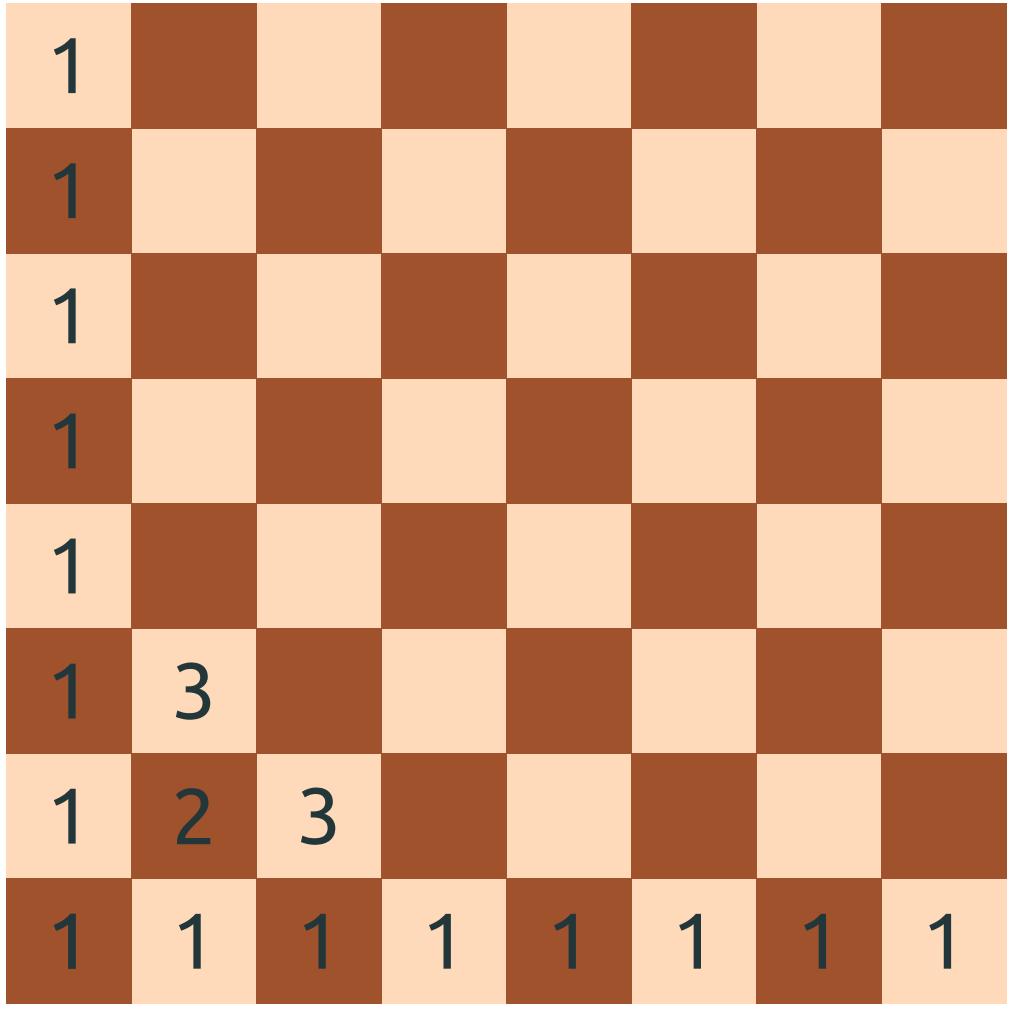

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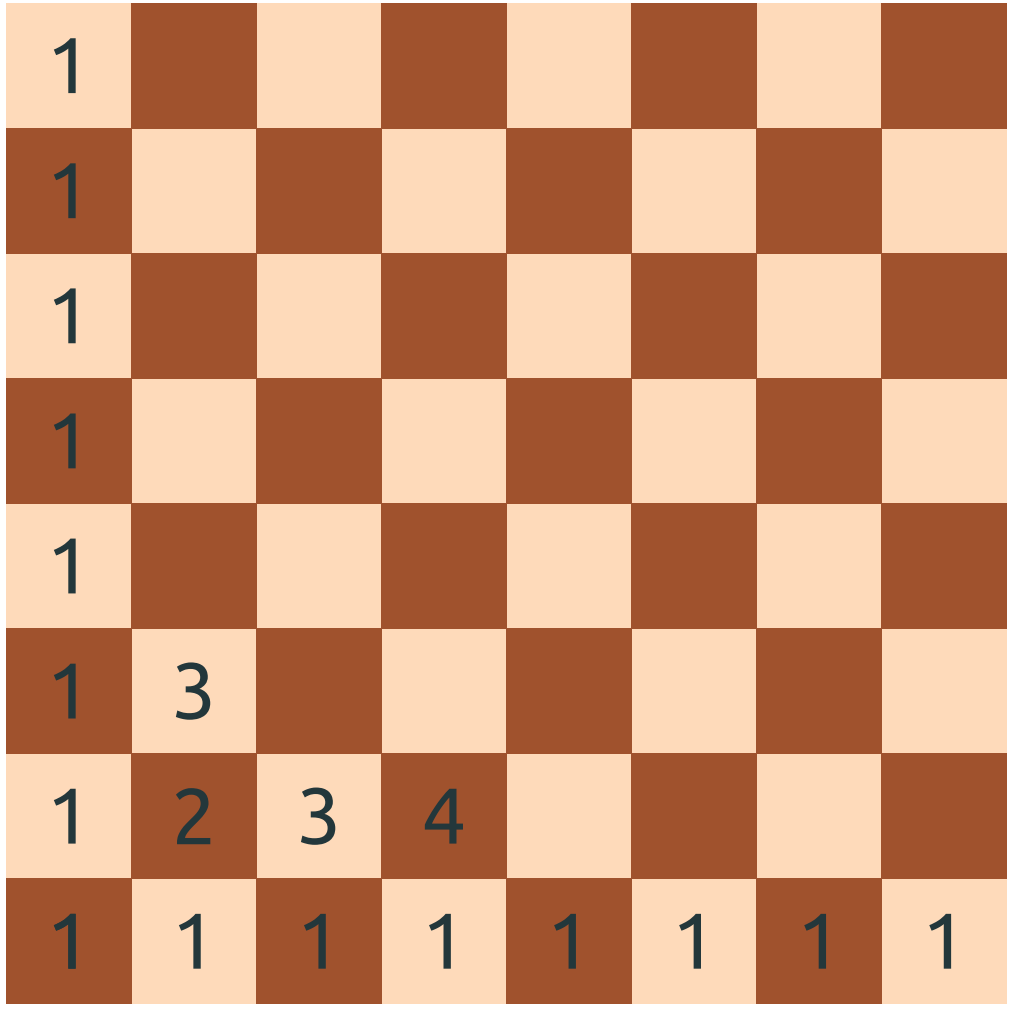

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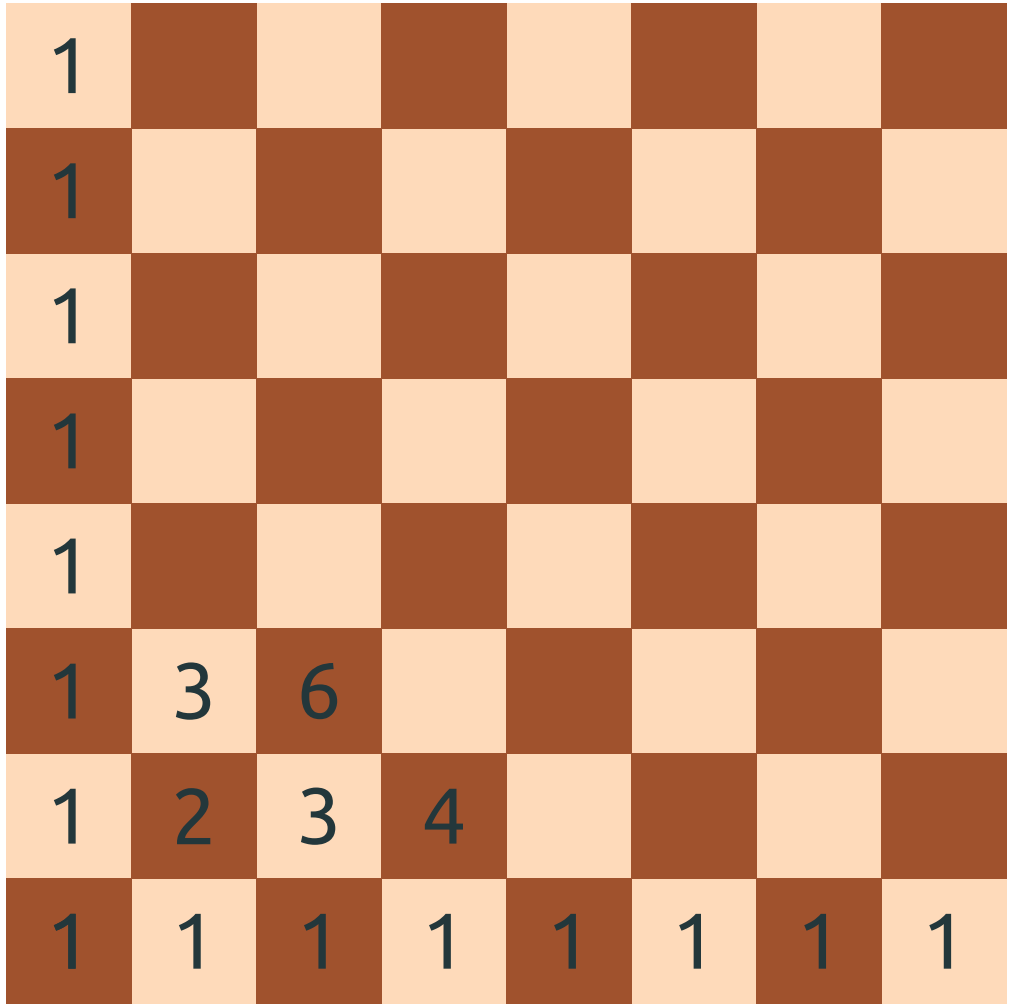

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