Problems in Combinatorics

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Outline

Distributing Assignments Among People

Distributing Candies Among Kids

Numbers with Fixed Sum of Digits

Numbers with Non-increasing Digits

Splitting into Working Groups

Distributing Assignments Among People

Problem

Suppose there are 4 people and 9 different assignments. Each person should receive one assignment. Assignments for different people should be different. How many ways are there to do it?

- We have count selections of assignments for 4 people

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- In this problem people are different, so the selection is ordered

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- No assignment can be given to two persons simultaneously, so no repetitions

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- We have count selections of assignments for 4 people
- In this problem people are different, so the selection is ordered
- No assignment can be given to two persons simultaneously, so no repetitions
- So we are dealing with k-permutations

Distributing Assignments Among People

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Suppose there are 4 people and 9 different assignments. Each person should receive one assignment. Assignments for different people should be different. How many ways are there to do it?

Persons	1	2	3	4
Number of options				

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Number of options	9			

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Persons	1	2	3	4
Number of options	9	8		

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Number of options	9	8	7	

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Persons	1	2	3	4
Number of options	9	8	7	6

Distributing Assignments Among People

Problem

Suppose there are 4 people and 9 different assignments. Each person should receive one assignment. Assignments for different people should be different. How many ways are there to do it?

- The answer is $9 \times 8 \times 7 \times 6 = 3024$

Distributing Assignments Among People

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Suppose there are 4 people and 9 different assignments. Each person should receive one assignment. Assignments for different people should be different. How many ways are there to do it?

- The answer is $9 \times 8 \times 7 \times 6 = 3024$
- Needed to count permutations

Distributing Assignments Among People

Problem

There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

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There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

- Each person receives several assignments

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- Each person receives several assignments
- Can try to look at persons one by one

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- Can try to look at persons one by one
- The first person is assigned arbitrary subset; we know how to count the number of subsets

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- The first person is assigned arbitrary subset; we know how to count the number of subsets
- The second person: the number of options left depends on what we choose for the first person

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- Seems tricky
- Idea: look from the other point of view!

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There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

- Seems tricky
- Idea: look from the other point of view!
- Don't give assignments to people, assign people to assignments instead

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There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

Assignment	1	2	3	4	5	6	7	8	9
Number of options									

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Assignment	1	2	3	4	5	6	7	8	9
Number of options	4	4	4	4	4	4	4	4	4

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There are 4 people and 9 different assignments. We need to distribute all assignments among people. No assignment should be assigned to two people. Every person can be given arbitrary number of assignments from 0 to 9. How many ways are there to do it?

- So the answer is $4^9 = 262\,144$

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- Just needed to count tuples

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- Just needed to count tuples
- But needed also to look from the other side

Outline

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Distributing Candies Among Kids

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Numbers with Non-increasing Digits

Splitting into Working Groups

Distributing Candies Among Kids

Problem

There are 15 identical candies. How many ways are there to distribute them among 7 kids?

- Giving each candy we choose one of 7 kids

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- Giving each candy we choose one of 7 kids
- Repetitions are allowed, the same kid can receive a candy several times
- Candies are identical, so the order does not matter
- We are dealing with combinations with repetitions!

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- Number of candies is the size of a combination
- Number of kids is the number of options
- So, the answer is $\binom{15+(7-1)}{(7-1)} = \binom{21}{6} = 54\,264$

More Fair Distribution

Problem

There are 15 identical candies. How many ways are there to distribute them among 7 kids in such a way that each kid receives at least 1 candy?

- The previous solution does not work, we have additional restriction

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- But we can reduce the problem to the previous one
- We have to give each kid at least one candy

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- But we can reduce the problem to the previous one
- We have to give each kid at least one candy
- So let's just do it!

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- Now we are left with $15 - 7 = 8$ candies

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- Combinations with repetitions, the size of the combination is 8, the number of options is 7

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- Now we are left with $15-7=8$ candies
- And we can distribute them as in the previous problem!
- Combinations with repetitions, the size of the combination is 8, the number of options is 7
- So, the answer is $\binom{8+(7-1)}{(7-1)} = \binom{14}{6} = 3\,003$

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- Compare to the answer to the previous problem: $54\,264$

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- So, the answer is $\binom{8+(7-1)}{(7-1)} = \binom{14}{6} = 3\,003$
- Compare to the answer to the previous problem: $54\,264$
- Vast majority of ways to share candies will leave some kids without any

Outline

- Distributing Assignments Among People
- Distributing Candies Among Kids
- Numbers with Fixed Sum of Digits
- Numbers with Non-increasing Digits
- Splitting into Working Groups

Numbers with Fixed Sum of Digits

Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

- Consider numbers as sequences of 4 digits

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- Nine options for the first one; but already unclear for the second one

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- Consider numbers as sequences of 4 digits
- We can look at the problem from the side of numbers
- Nine options for the first one; but already unclear for the second one
- Idea: look from the other side

Numbers with Fixed Sum of Digits

$$* \quad * \quad * \quad *$$

- There are four positions

Numbers with Fixed Sum of Digits

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- There are four positions
- We split the weight 9 among them

Numbers with Fixed Sum of Digits

$$0 \quad 0 \quad 0 \quad 0$$

- There are four positions
- We split the weight 9 among them
- Assume they are 0 in the beginning and add 1 nine times

Numbers with Fixed Sum of Digits

0 0 0 0

- There are four positions
- We split the weight 9 among them
- Assume they are 0 in the beginning and add 1 nine times
- Each time we pick one of the digits to increase

Numbers with Fixed Sum of Digits

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- There are four positions
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- There are four positions
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- We have combinations of size 9 among 4 options

Numbers with Fixed Sum of Digits

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- There are four positions
- We split the weight 9 among them
- Assume they are 0 in the beginning and add 1 nine times
- Each time we pick one of the digits to increase
- Order does not matter; there are repetitions
- We have combinations of size 9 among 4 options
- The answer is $\binom{9+(4-1)}{(4-1)} = \binom{12}{3} = 220$

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- Looks very similar to the previous one

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Numbers with Fixed Sum of Digits

Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 10?

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- Distribute ten ones between four positions
- Combinations of size 10 among 4 options

Numbers with Fixed Sum of Digits

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How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 10?

- Looks very similar to the previous one
- Distribute ten ones between four positions
- Combinations of size 10 among 4 options
- The answer is $\binom{10+(4-1)}{(4-1)} = \binom{13}{3} = 286$

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- Is everything right?

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How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 10?

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- Distribute ten ones between four positions
- Combinations of size 10 among 4 options
- The answer is $\binom{10+(4-1)}{(4-1)} = \binom{13}{3} = 286$
- Is everything right? Let's check!

Numbers with Fixed Sum of Digits

The following code searches through all combinations:

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import itertools as it
count = 0
for d in it.product(range(10), repeat = 4):
    if sum(d) == 10:
        count += 1
print(count)
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Here is the output

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282
```

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The answer is off by 4!

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Here is the output

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```

The answer is off by 4!

What went wrong?

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- With our approach we can assign all ten ones to the same position

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- What should we do now?

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Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 10?

- With our approach we can assign all ten ones to the same position
- But digits should be at most 9
- What should we do now?
- Just subtract the number of things that we should not have counted!

Numbers with Fixed Sum of Digits

$* \quad * \quad * \quad *$

- What we should not have counted

Numbers with Fixed Sum of Digits

$$* \quad * \quad * \quad *$$

- What we should not have counted
- Assignments of all ten ones to the same digit

Numbers with Fixed Sum of Digits

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- What we should not have counted
- Assignments of all ten ones to the same digit
- But there are just 4 of them!

Numbers with Fixed Sum of Digits

$$10 \quad 0 \quad 0 \quad 0$$

- What we should not have counted
- Assignments of all ten ones to the same digit
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Numbers with Fixed Sum of Digits

0 10 0 0

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Numbers with Fixed Sum of Digits

$$0 \quad 0 \quad 10 \quad 0$$

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Numbers with Fixed Sum of Digits

$$0 \quad 0 \quad 0 \quad 10$$

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- Assignments of all ten ones to the same digit
- But there are just 4 of them!
- So our previous answer was of by just 4

Numbers with Fixed Sum of Digits

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- What we should not have counted
- Assignments of all ten ones to the same digit
- But there are just 4 of them!
- So our previous answer was of by just 4
- The correct answer is $286 - 4 = 282$

Outline

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Distributing Candies Among Kids

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Numbers with Non-increasing Digits

Splitting into Working Groups

Numbers with Non-increasing Digits

Problem

How many four-digit numbers are there such that their digits are not increasing? Three-digit numbers are also four-digit, they just start with 0

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How many four-digit numbers are there such that their digits are not increasing? Three-digit numbers are also four-digit, they just start with 0

- If we try to count options for each position and apply the product rule, there are problems

Numbers with Non-increasing Digits

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- If we try to count options for each position and apply the product rule, there are problems
- 10 options for the first position, but for the second the number of options depends on the first number

Numbers with Non-increasing Digits

Problem

How many four-digit numbers are there such that their digits are not increasing? Three-digit numbers are also four-digit, they just start with 0

- If we try to count options for each position and apply the product rule, there are problems
- 10 options for the first position, but for the second the number of options depends on the first number
- Idea: look from the other side

Numbers with Non-increasing Digits

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- We pick digits from 0 to 9 to be in our number

Numbers with Non-increasing Digits

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- We pick digits from 0 to 9 to be in our number
- Once we picked four digits, our number is uniquely determined

Numbers with Non-increasing Digits

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Picked 3, 4 ,3, 7

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Numbers with Non-increasing Digits

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Picked 3, 4 ,3, 7

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- We pick digits from 0 to 9 to be in our number
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- We pick digits from 0 to 9 to be in our number
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- We have combinations of size 4 from 10 options

Numbers with Non-increasing Digits

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- We pick digits from 0 to 9 to be in our number
- Once we picked four digits, our number is uniquely determined
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- We have combinations of size 4 from 10 options
- The answer is $\binom{4+(10-1)}{(10-1)} = \binom{13}{9} = 715$

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Splitting into Working Groups

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There are 12 students in the class. How many ways are there to split them into working groups of size 2 to work on the same assignment?

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- This problems is more tricky
- There are several ways to solve it
- But we need to combine several ideas

Splitting into Working Groups

- One solution goes by looking from the position of working groups

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- We have to pick 2 people out of 12 in a working group

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- For the second group we have 10 people left, there are $\binom{10}{2}$ options

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- We have to pick 2 people out of 12 in a working group
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- And so on
- Overall we have $\binom{12}{2} \times \binom{10}{2} \times \binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{2}$ options

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- Are we done?

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- Overall we have $\binom{12}{2} \times \binom{10}{2} \times \binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{2}$ options
- Are we done? No!

Splitting into Working Groups

- Enumerate people by numbers from 1 to 12

Splitting into Working Groups

$$\{3,7\},\{1,5\},\{6,9\},\{11,2\},\{8,12\},\{4,10\}$$

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- For example, we have counted this splitting in the groups

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- So, what to do now?
- Apply the old idea! We have counted each splitting $6!$ times, for each permutation of 6 groups

Splitting into Working Groups

- In our first attempt the answer was $\binom{12}{2} \times \binom{10}{2} \times \binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{2}$

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- So the right answer is

$$\binom{12}{2} \times \binom{10}{2} \times \binom{8}{2} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{2} \times \frac{1}{6!} =$$

$$\frac{12 \times 11}{2} \times \frac{10 \times 9}{2} \times \frac{8 \times 7}{2} \times \frac{6 \times 5}{2} \times \frac{4 \times 3}{2} \times \frac{2 \times 1}{2} \times \frac{1}{6!} =$$

$$\frac{12!}{6! \times 2^6} = 10\,395$$

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- Next week we will proceed to Probability