

# Practice Counting

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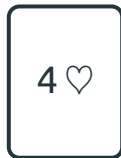
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# Number of Hands

## Question

What is the number of 5-card hands dealt off of a standard 52-card deck?



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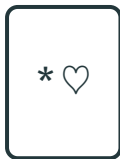
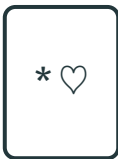
## Answer

$$\binom{52}{5} = \frac{52!}{5!47!} = \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2\,598\,960$$

# Two Hearts and Three Spades

## Question

What is the number of 5-card hands with two hearts and three spades?



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## Answer

$$\binom{13}{2} \binom{13}{3} = 22\,308$$

# 4-Digit Numbers Containing 7

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What is the number of non-negative integers with at most four digits at least one of which is equal to 7?

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## Answer

$$10^4 - 9^4 = 3\,439$$

# Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if 7 in d:
        count += 1

print(count)
print(10**4 - 9**4)
```



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3439

3439

# 4-Digit Numbers with Increasing Digits

## Question

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What is the number of non-negative integers with at most four digits whose digits are increasing?

## Answer

$$\binom{10}{4} = 210$$

# Code

```
from itertools import product

count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
        count += 1
        print(d)

print(count)
```

# Code

```
from itertools import product

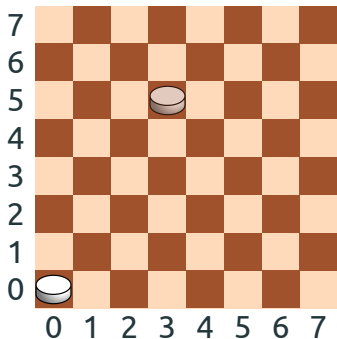
count = 0
for d in product(range(10), repeat = 4):
    if d[0] < d[1] and d[1] < d[2] and d[2] < d[3]:
        count += 1
        print(d)

print(count)
```

```
(0, 1, 2, 3)
(0, 1, 2, 4)
(0, 1, 2, 5)
(0, 1, 2, 6)
(0, 1, 2, 7)
(0, 1, 2, 8)
(0, 1, 2, 9)
(0, 1, 3, 4)
(0, 1, 3, 5)
(0, 1, 3, 6)
```

```
(4, 6, 7, 8)
(4, 6, 7, 9)
(4, 6, 8, 9)
(4, 7, 8, 9)
(5, 6, 7, 8)
(5, 6, 7, 9)
(5, 6, 8, 9)
(5, 7, 8, 9)
(6, 7, 8, 9)
210
```

# Piece on a Chessboard



A piece can move one step up or one step to the right. What is the number of ways of getting from the cell  $[0, 0]$  (bottom left corner) to the cell  $[5, 3]$ ?

# Solution

- There are exactly eight moves

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- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up



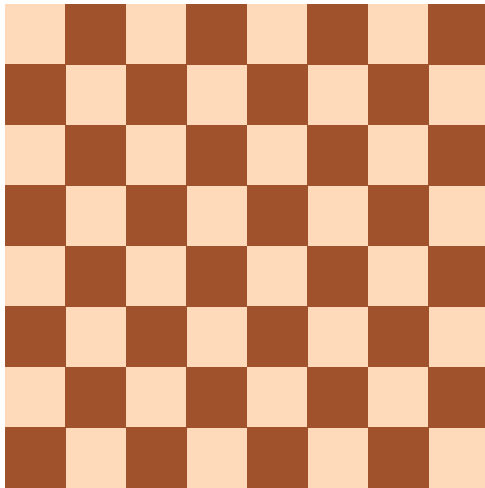
# Solution

- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell  $[5, 3]$

# Solution

- There are exactly eight moves
- Three of them should be to the right, while the remaining five should go up
- Moreover, any such combination of three moves to the right and five moves up is a valid way of getting to the cell  $[5, 3]$
- Hence, the answer is  $\binom{8}{3} = 56$

# Solution Using Pascal's Triangle



# Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1							
1	2						
1	1	1	1	1	1	1	1

# Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1							
1	2	3					
1	1	1	1	1	1	1	1

# Solution Using Pascal's Triangle

A grid of 8 rows and 8 columns. The cells are colored in a checkerboard pattern, alternating between light orange and dark brown. The numbers are placed in the light orange cells, forming a Pascal's Triangle shape. The first row has one '1' in the first column. The second row has one '1' in the first column. The third row has one '1' in the first column. The fourth row has one '1' in the first column. The fifth row has one '1' in the first column. The sixth row has '1' in the first column and '3' in the second column. The seventh row has '1' in the first column, '2' in the second column, and '3' in the third column. The eighth row has '1' in the first column, '1' in the second column, '1' in the third column, '1' in the fourth column, '1' in the fifth column, '1' in the sixth column, '1' in the seventh column, and '1' in the eighth column.

1							
1							
1							
1							
1							
1	3						
1	2	3					
1	1	1	1	1	1	1	1

# Solution Using Pascal's Triangle

A grid of 8 rows and 8 columns with a checkerboard pattern of light orange and dark brown squares. Numbers are placed in the light orange squares, representing values from Pascal's Triangle. The numbers are: Row 1: 1; Row 2: 1; Row 3: 1; Row 4: 1; Row 5: 1; Row 6: 1, 3; Row 7: 1, 2, 3, 4; Row 8: 1, 1, 1, 1, 1, 1, 1, 1.

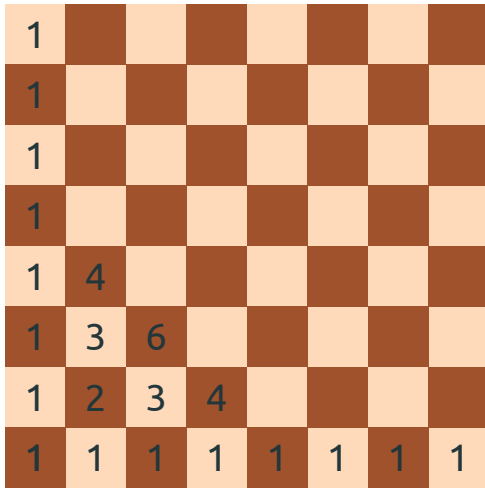
1							
1							
1							
1							
1							
1	3						
1	2	3	4				
1	1	1	1	1	1	1	1

# Solution Using Pascal's Triangle

1							
1							
1							
1							
1							
1	3	6					
1	2	3	4				
1	1	1	1	1	1	1	1



# Solution Using Pascal's Triangle



The image shows an 8x8 grid with a checkerboard pattern of light orange and dark brown squares. The values in the grid are as follows:

1							
1							
1							
1							
1	4						
1	3	6					
1	2	3	4				
1	1	1	1	1	1	1	1