Double Counting

Vladimir Podolskii

Computer Science Department, Higher School of Economics

Outline

Double counting

"Homework" Assignment Problem

Double counting

- It is useful to look at the problem from two different angles

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Double counting

- It is useful to look at the problem from two different angles
- We can get information from two sources

wikimedia.org

Double counting

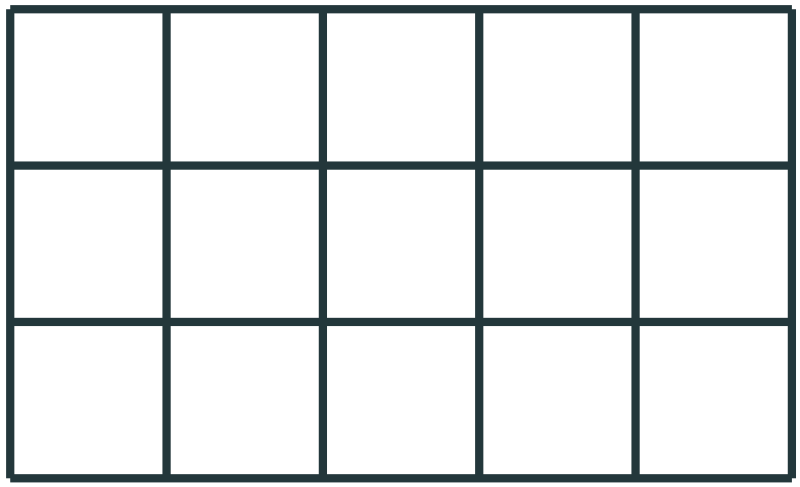
- It is useful to look at the problem from two different angles
- We can get information from two sources
- A standard special case in math: look at the number from two points of view

wikimedia.org

Example

Puzzle

Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?



Example

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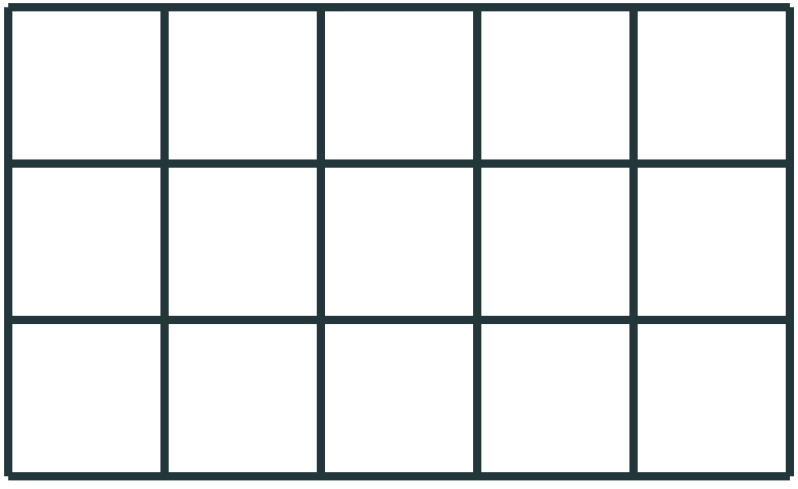
Let's try



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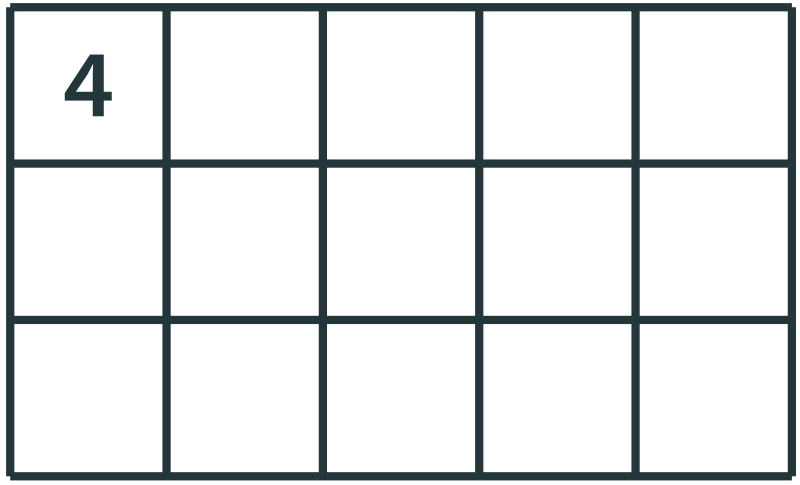

Example

Puzzle

Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	**3**			

Example

Puzzle

Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	**3**	**5**		

Example

Puzzle

Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	**3**	**5**	**6**	

Example

Puzzle

Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	**3**	**5**	**6**	**2**	**20**

Example

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

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4	**3**	**5**	**6**	**2**	**20**
3					

Example

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

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4	3	5	6	2	20
3	4				

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	3	5	6	2	20
3	4	1			

Example

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Example

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's try

4	3	5	6	2	20
3	4	1	4	8	20

Example

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3	3	4	0		
10	10	10	10		

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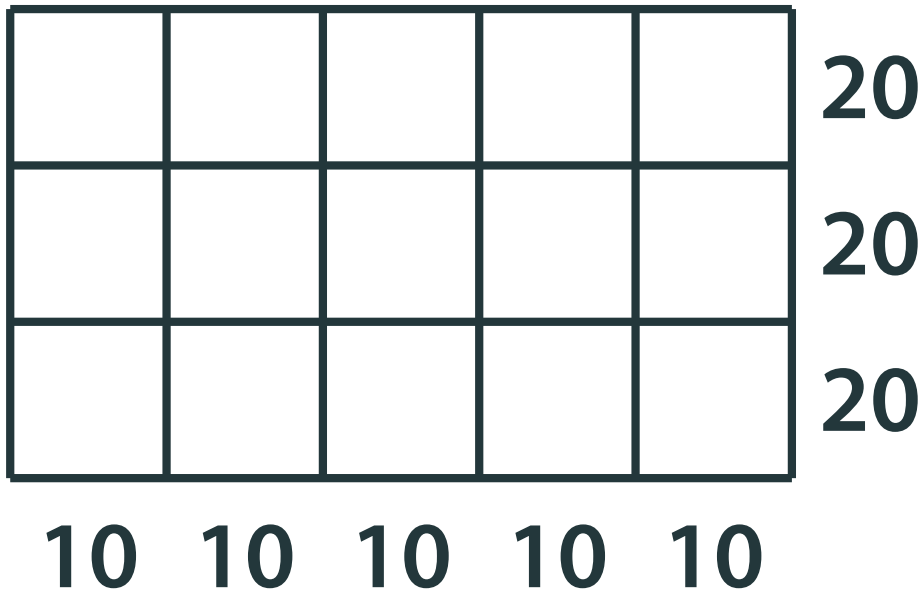

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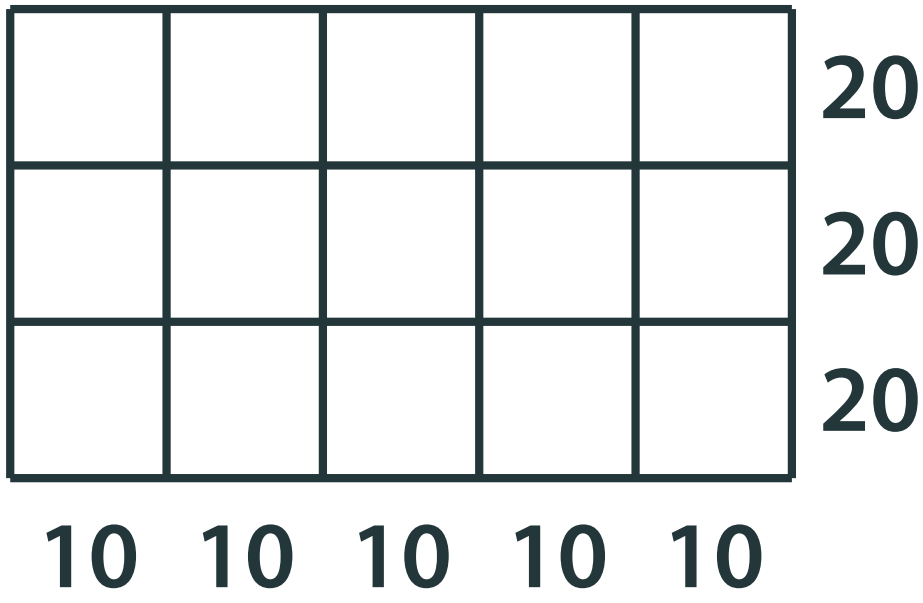

- Assume we have filled the table (reductio ad absurdum!)

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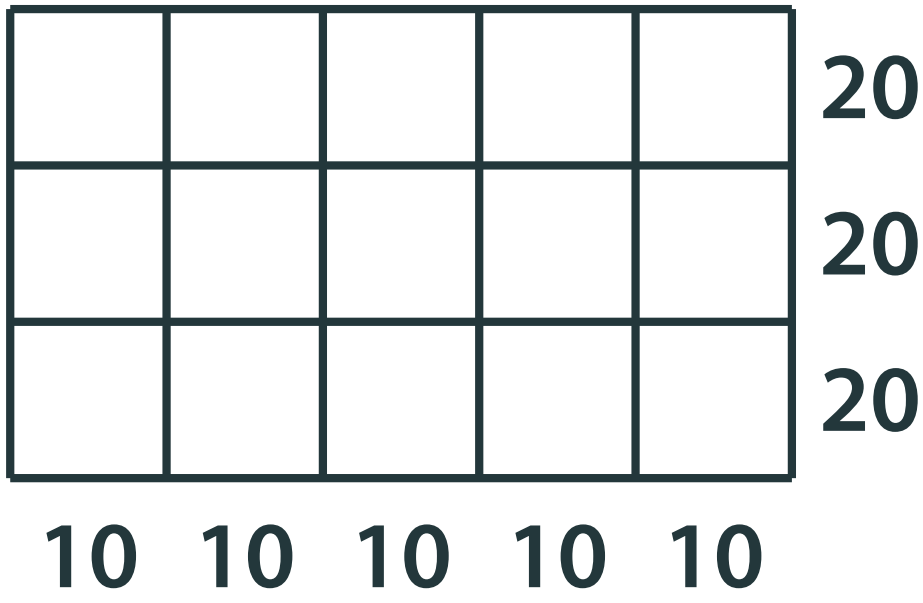

- Assume we have filled the table (reductio ad absurdum!)
- Let's look at the sum of all numbers in the table!

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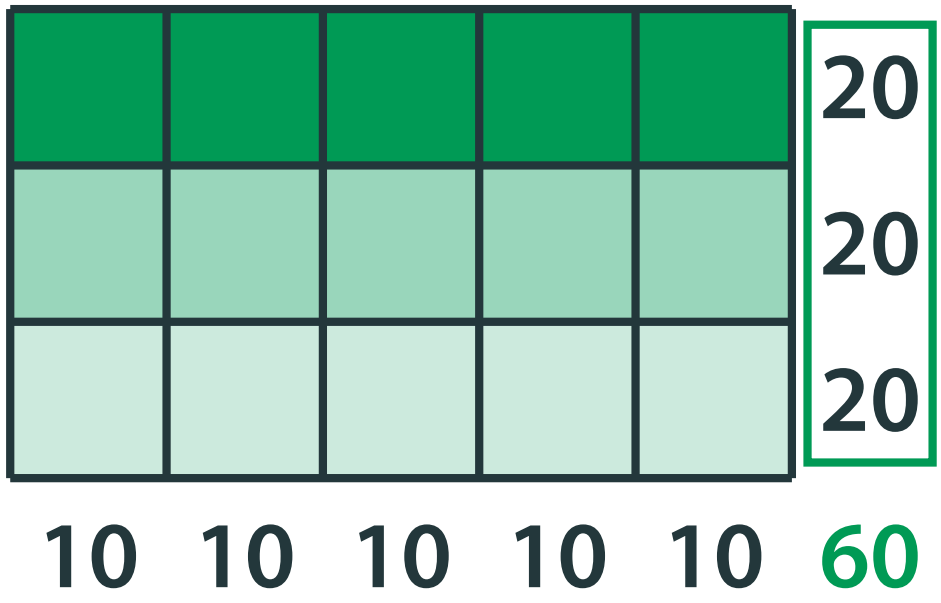
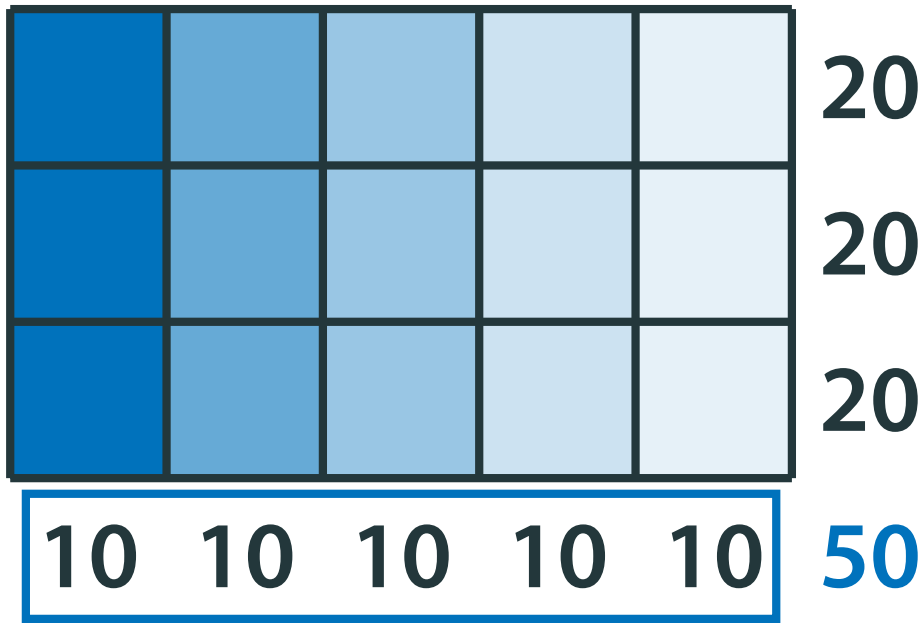


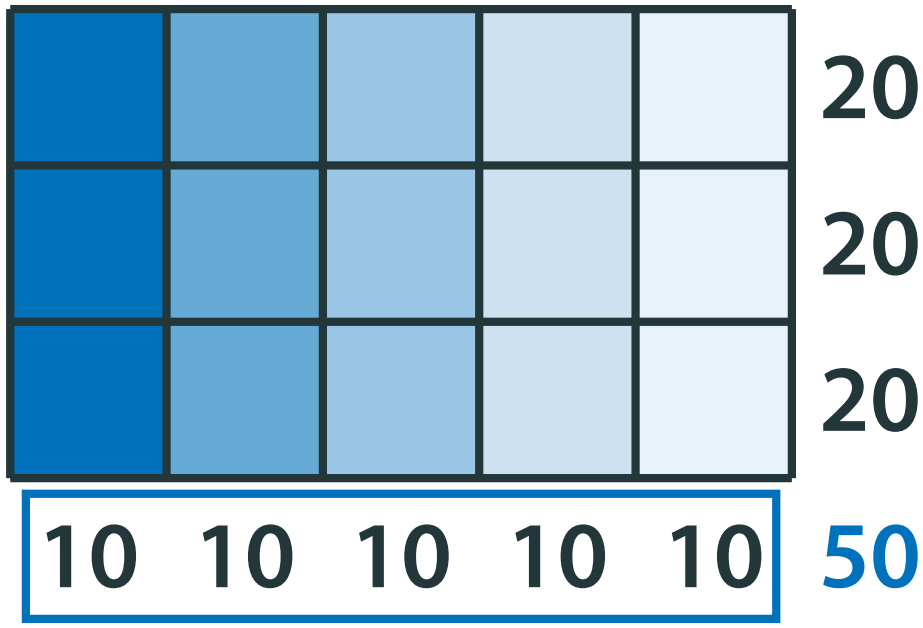
- Assume we have filled the table (reductio ad absurdum!)
- Let's look at the sum of all numbers in the table!
- The sum in each row is 20 so in total we have 60

Example



- Assume we have filled the table (reductio ad absurdum!)
- Let's look at the sum of all numbers in the table!
- The sum in each row is 20 so in total we have 60
- But the sum in each column is 10 so in total we have 50

Example



- Assume we have filled the table (reductio ad absurdum!)
- Let's look at the sum of all numbers in the table!
- The sum in each row is 20 so in total we have 60
- But the sum in each column is 10 so in total we have 50
- This is a contradiction!

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

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Is it possible to fill a 3×5 table by integers so that the sum of each row is equal to 20 and the sum of each column is equal to 10?

Let's summarize:

- Shown that it is impossible by computing some value in two ways
- Key to success: find the right value to compute

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“Homework” Assignment Problem

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Problem

Each of 20 students in a group have solved three problems from the homework assignment, and each problem was solved by two students. How many problems were in the assignment?

- Again, we have to look at some value from two sides

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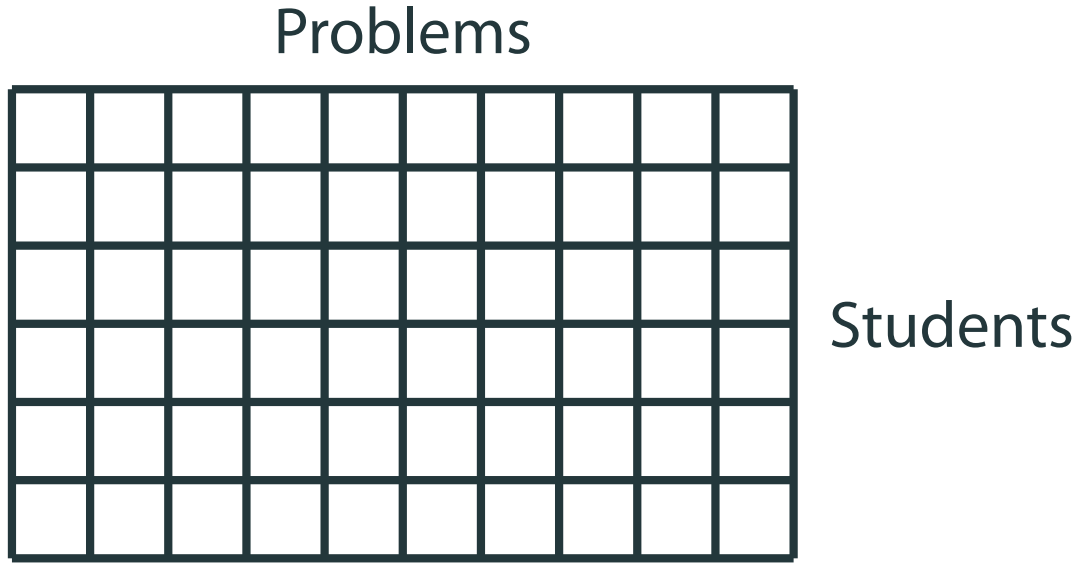
- Again, we have to look at some value from two sides
- This time we look at the total number of problem solutions by all students
- Since 20 students solved 3 problems each, in total they have 60 solutions
- Since each problem received 2 solutions, there were 60/2 = 30 problems

“Homework Assignment” Problem

The problem is similar to the previous one



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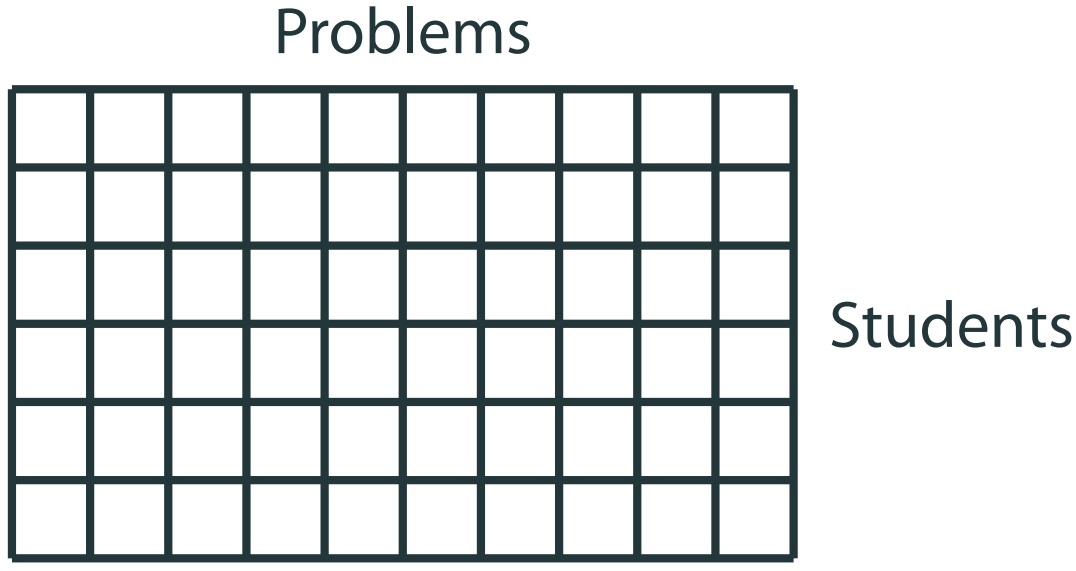

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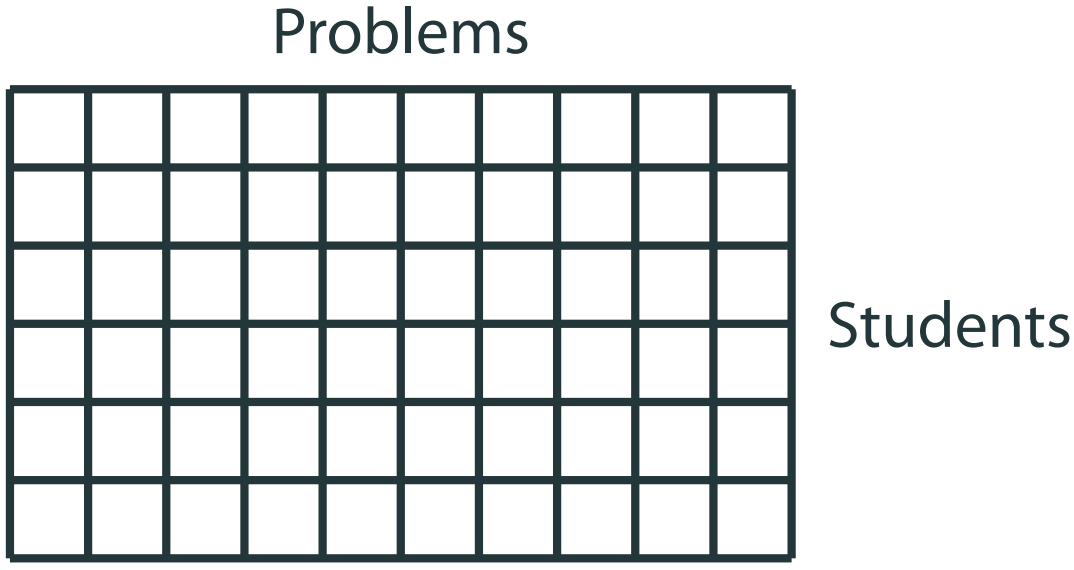

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- There are 60 ones in the table

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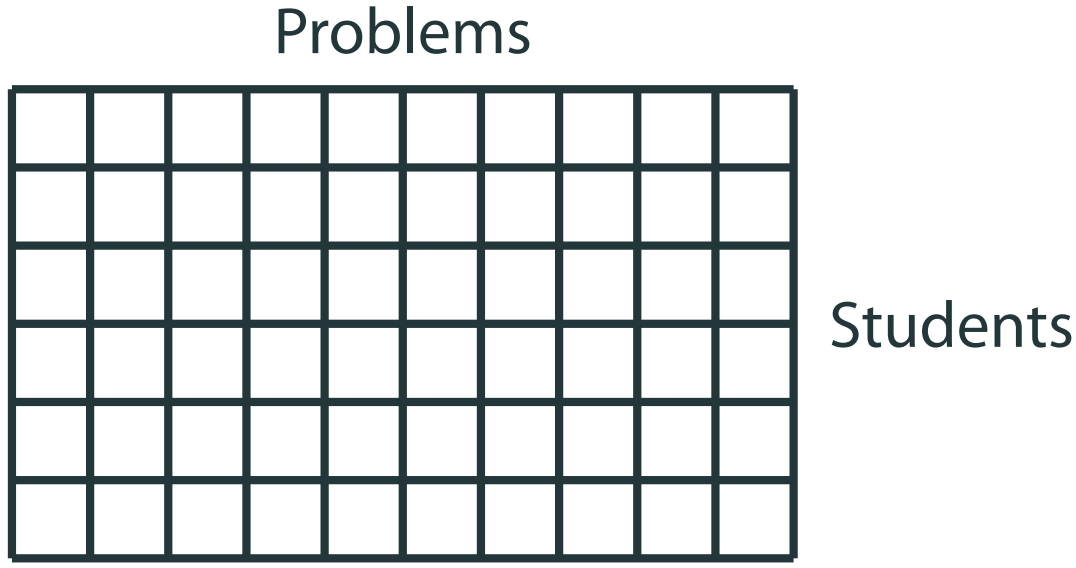

- Put 1 in the cell if the student solved the problem and 0 otherwise
- There are 60 ones in the table
- There are 30 columns