Termination

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Outline

Termination

Football Fans

Arthur's Books

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- In a more wide sense invariant is a property that changes in a right way
- Another standard use of invariants is showing termination of processes

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There are two football teams in a town. Each of the citizens is supporting one of the teams. If among someone's friends there are more fans of another team, than of his own, this person tend to switch to supporting the other team. Each day one of such persons switch. Is it possible that this switching process goes forever (assume that friendship is always mutual and that the population of the city and friendship do not change)?

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- But how can we prove it?
- We need to look at the right value

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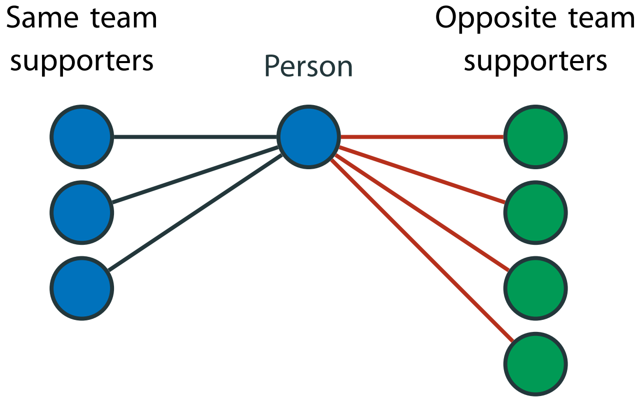

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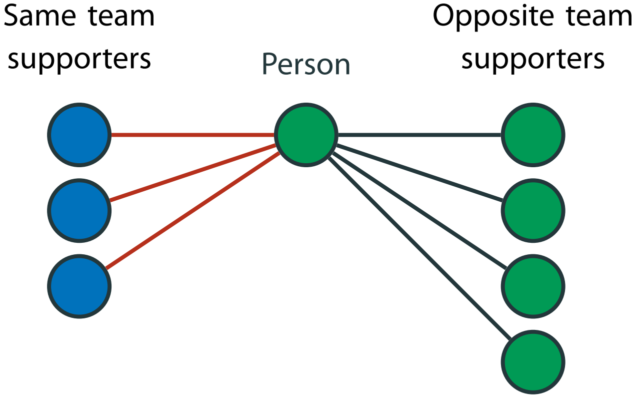

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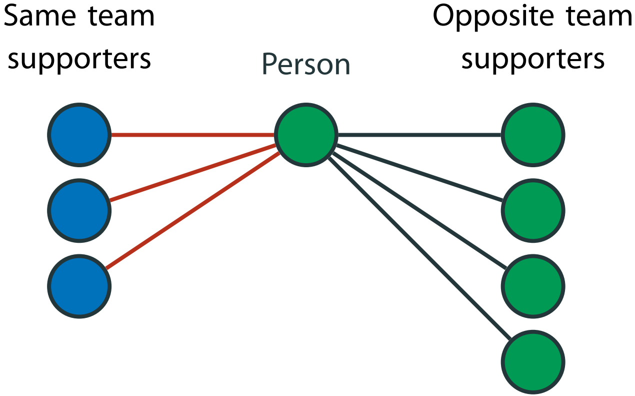

This value always decreases! The process stops

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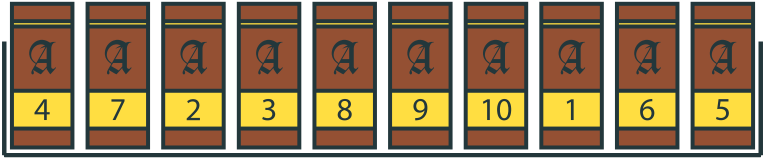
Arthur's Books

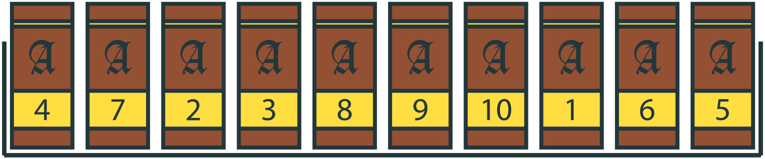
Problem

King Arthur has a shelf of his works consisting of 10 volumes, numbered $1, 2, 3, \ldots, 10$. Over the years of use the volumes got disordered. Arthur hires Merlin to sort the collection, but he does not want more than two volumes leave the shelf at once. The volumes are heavy, so it is possible only to switch two volumes on the shelf in a day. In how many days Merlin can guarantee to sort the volumes?



Arthur's Books

We can always place books in the right order in at most 9 days



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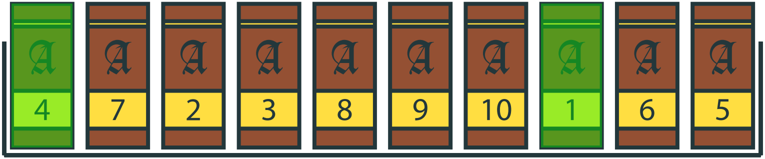

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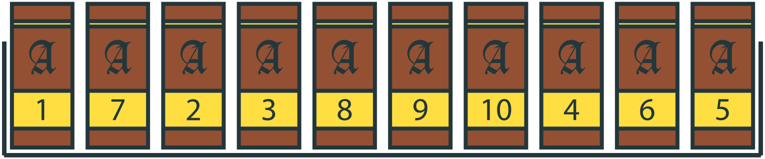

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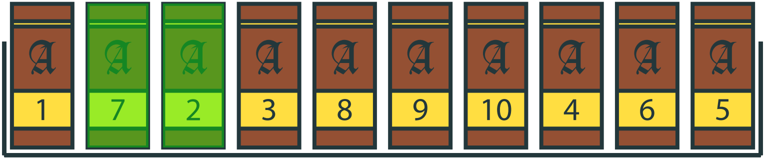

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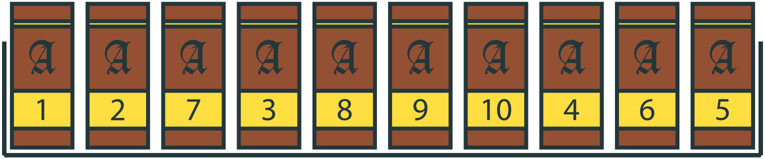

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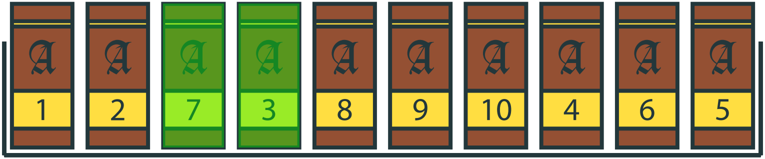

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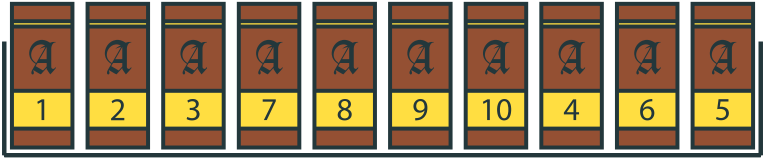

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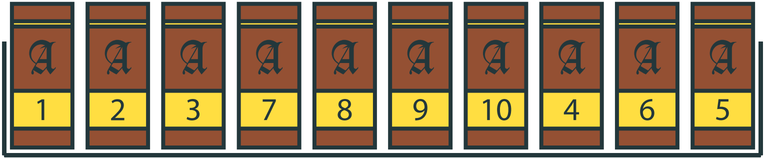

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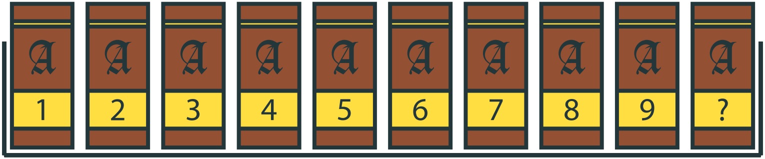

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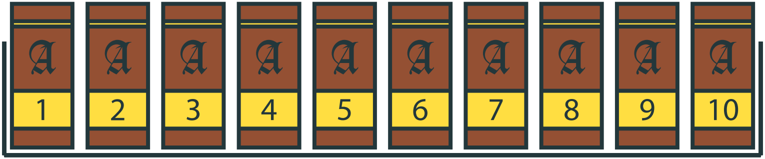

On day 1 we place volume 1 on its place
On day 2 we place volume 2 on its place
On day 3 we place volume 3 on its place
And so on ...
After 9 days we placed first 9 volumes on their places
Volume 10 must also be on its place, since it is the last one left

Arthur's Books

- Are 9 days optimal? What is the hardest permutation of books?

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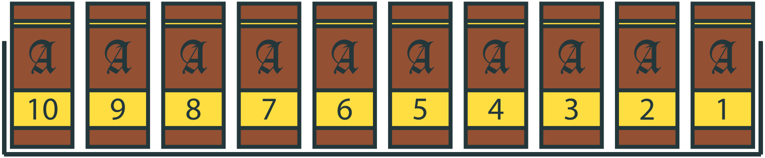

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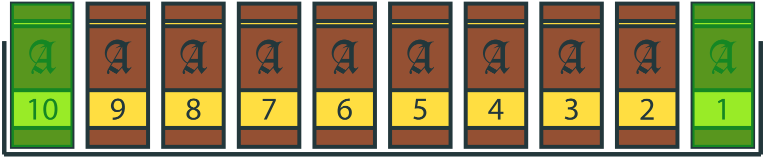

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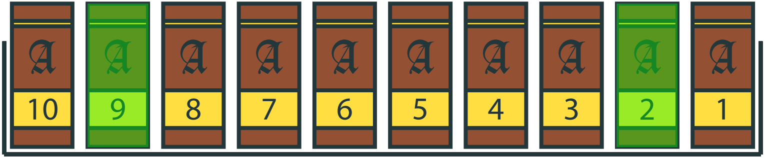

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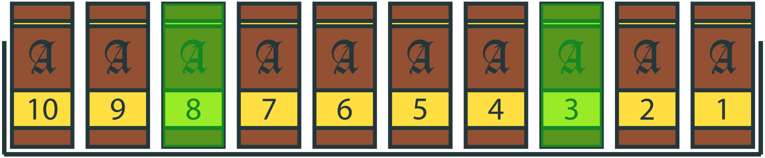

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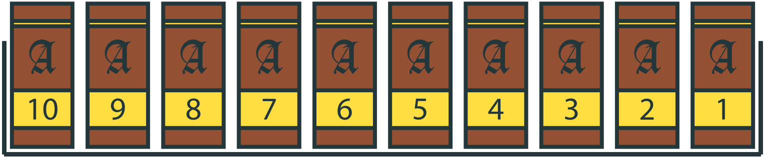

We can switch books in 5 days here

Arthur's Books

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- We need to find some invariant that:
  1. does not change fast
  2. should change substantially while ordering the book

Arthur’s Books

Recall the problem:

Puzzle

There is a sequence of 10 cells, the leftmost contains number 1 and the rightmost contains 30. Is it possible to fill other cells with numbers in such a way that consecutive numbers differ by at most 3?

1									30

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The invariant: The number of books staying to the right of their intended place

- Small in the end: equals 0

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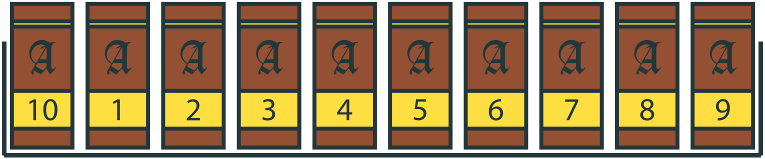

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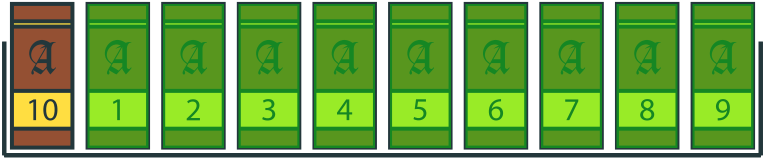

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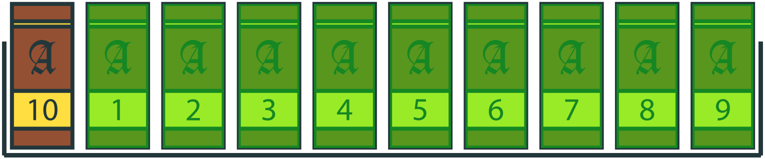

The invariant is 9 in the beginning, we need at least 9 days