Unit 12 - Week 11

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-15, 23:59 IST.

1) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with mean μ and variance σ^2 and as $n \to \infty$, $\frac{X_1^2+X_2^2+\cdots+X_n^2}{n} \xrightarrow{p} c$, for some constant c. Find an expression for c *1 point*

- $c = \frac{\sigma^2}{n}$
- $c = \mu^2 + \sigma^2$
- $c = \frac{\mu^2+\sigma^2}{n}$
- $c = \mu^2$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$c = \mu^2 + \sigma^2$

2) Let $X_1, X_2, \ldots$ be a sequence of random variables such that each $X_n \sim Bin(n, p_n)$, where $p_n = \frac{1}{4n}$. Which of the following statements is/are TRUE? *1 point*

- $X_n \xrightarrow{L} Poisson(4)$
- $X_n \xrightarrow{L} Poisson\left(\frac{1}{2}\right)$
- $X_n \xrightarrow{L} Poisson(2)$
- $X_n \xrightarrow{L} Poisson\left(\frac{1}{4}\right)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X_n \xrightarrow{L} Poisson\left(\frac{1}{4}\right)$

3) Let $X_1, X_2, \ldots . X_n$ be a sequence of random variable such that $P\left(X_n = \pm n^{\lambda}\right) = \frac{1}{2}$ for all $n = 1,2, \ldots$. Then the Strong Law of Large Number holds for which of the following? *1 point*

- $\lambda \in \mathbb{R}$
- $\lambda > 1$
- $\lambda < \frac{1}{2}$
- $\lambda > \frac{1}{2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\lambda < \frac{1}{2}$

4) Suppose 100 real numbers are chosen independently from $[0,20]$ with uniform probability. Using Chebyshev's inequality find a lower bound l for the probability that their average lies between 8 and 12. *1 point*

- $l = \frac{1}{12}$
- $l = \frac{7}{8}$
- $l = \frac{1}{8}$
- $l = \frac{11}{12}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$l = \frac{11}{12}$

5) Let $X_1, X_2, \ldots$ be independent random variables such that each X_i takes the values $-2^i, 0, 2^i$. Given that, $P\left(X_i = \pm 2^i\right) = \frac{1}{2^{2i+1}}$ and $P(X_i = 0) = 1 - \frac{1}{2^{2i}}$ *1 point*

- $Var(X_i) = 1$
- $Var(X_i) = \frac{1}{2}$
- Weak Law of Large Numbers holds
- Weak Law of Large Numbers does not hold

No, the answer is incorrect.
Score: 0
Accepted Answers:
$Var(X_i) = 1$
Weak Law of Large Numbers holds

6) Let $X_1, X_2, \ldots$ be jointly Normal with $E(X_i) = 0$ and $E(X_i^2) = 1$ for all i and $Cov(X_i, X_j) = \begin{cases} \lambda & \text{if} |j-i| = 1 \\ 0 & \text{otherwise} \end{cases}$. Which of the following is/are True? *1 point*

- WLLN holds only if $1/4 < \lambda < 1/2$
- WLLN holds if $\lambda < 1/2$
- WLLN holds if $\lambda > 0$
- WLLN does not hold for any $\lambda \in \mathbb{R}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
WLLN holds if $\lambda > 0$

7) There are n birds that sit in a row on a wire. Each bird looks left or right with equal probability. Let X be the number of birds not seen by any bird adjacent to it. Determine the constant c so that $\frac{1}{n} X \xrightarrow{p} c$. *1 point*

- $c = 1/2$
- $c = 1$
- $c = \frac{1}{4}$
- $c = 2$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$c = \frac{1}{4}$

8) Suppose it is known that the number of tickets purchased by a person standing in line at the ticket window for the cricket match of India against Australia follows a distribution that has mean μ = 3.3 and standard deviation σ = 3.0. Suppose that few hours before the start of one of these matches there are 100 eager cricket fans standing in line to purchase tickets. If only 350 tickets remain, what is the probability that all 100 people will be able to purchase the tickets they desire? [Here $\Phi(z) = P(Z \leq z)$ and $Z \sim N(0,1)$] *1 point*

- $\Phi\left(\frac{2}{3}\right)$
- $1 - \Phi\left(\frac{1}{3}\right)$
- $1 - \Phi\left(\frac{2}{3}\right)$
- $\Phi\left(\frac{1}{3}\right)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\Phi\left(\frac{2}{3}\right)$

9) Using the Weak Law of Large Numbers, determine the limit (l) of the integral $\int_0^1 \ldots \int_0^1 \frac{x_1^2+\cdots x_n^2}{x_1+\cdots x_n}\, dx_1 \ldots dx_n$ as $n \to \infty$ *1 point*

- $l = 1$
- $l = \frac{1}{3}$
- $l = \frac{1}{2}$
- $l = \frac{2}{3}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$l = \frac{2}{3}$

10) Let $X_1, X_2, \ldots$ be independent random variables with $|E(X_n)| < \infty$ for $n \geq 1$. Consider the statements *1 point*

I. The random variables are identically distributed

II. $\forall\, n, Var(X_n) < \infty$ and $\Sigma_{n=1}^{\infty} \frac{Var(X_n)}{n^2} < \infty$

Which of the following is/are TRUE?

- Strong Law of Large Numbers holds only if I holds
- Strong Law of Large Numbers holds only if II holds
- Strong Law of Large Numbers holds only if both I and II holds
- Strong Law of Large Numbers holds if I or II holds

No, the answer is incorrect.
Score: 0
Accepted Answers:
Strong Law of Large Numbers holds if I or II holds