Quadratic Integers: Quadratic integers are algebraic integers of degree two, i.e, solutions of equations of the form

$$x^2 + Bx + C = 0, \quad B, C \in \mathbb{Z}$$

e.g $\sqrt{2}$, i are quadratic integers.

Cyclotomic Integer: A number of the form

$$a_0 + a_1 \zeta + a_2 \zeta^2 + \dots + a_{n-1} \zeta^{p-1},$$

$$\zeta = e^{2\pi i/p}$$

is called cyclotomic integer, p is a prime.

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Integral Domain: A commutative ring R with unity is called integral domain if it is free from zero divisors. (An element $a \in R$, $a \neq 0$ is called zero divisor if $ab = 0 \Rightarrow b = 0$)

Euclidean Domain: An integral domain R is called Euclidean domain if $\forall\ a, b \in R$, $b \neq 0$

$\exists\ q, r$ such that

$a = bq + r$, either $r = 0$

or $d(r) < d(b)$.

Prime Ideal: An ideal P of a ring R is called prime ideal if $ab \in P \Rightarrow$ either $a \in P$ or $b \in P$.

Maximal Ideal: An ideal M of a ring R is called maximal ideal if $M \neq R$ and $\nexists$ ideal $I \neq R$ such that

$$M \subset I \subset R$$

Noetherian Ring: A ring R is noetherian if

1. Every ideal of R is finitely generated.

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2. Every ascending chain of ideals $I_0 \subset I_1 \subset \ldots$ is stationary i.e $\exists$ $r > 0$ s.t

$$I_i = I_j \quad \forall\ i, j \geq r.$$

Def: Dedekind Domain: An integral domain R is called Dedekind domain if

1. R is Noetherian
2. R is integrally closed in its fields of fractions (Integral extension)
3. Every nonzero prime ideal is a maximal ideal.

Theorem: Integral closure of a Dedekind domain is a Dedekind domain.

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Field of Fraction: Let A be an integral domain. A field K s.t $A \subset K$ is called the field of fractions of A with the property that every $c \in K$ can be written in the form

$$c = ab^{-1} \quad \text{with } a, b \in A, \; b \neq 0.$$

e.g: $\mathbb{Q}$ is the field of fractions of $\mathbb{Z}$.