Unit 11 - Week 10

Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2020-04-08, 23:59 IST.

1) Let $X_1, X_2, \ldots$ be a sequence of random variables such that *1 point*

$$F_{X_n}(x) = \begin{cases} 1 - \left(1 - \frac{1}{n}\right)^{nx} & 0 < x \le \infty \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } n > 0$$

Which of the following is correct?

- X_n does not converges in distribution
- X_n converges in distribution to $Exp(\lambda)$ for some $\lambda > 1$
- X_n converges in distribution to $Exp(1)$
- X_n does not converges in distribution to $Exp(\lambda)$ for any $\lambda > 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
X_n converges in distribution to $Exp(1)$

2) Let $X_1, X_2, \ldots$ be a sequence of random variables with cdf denoted by $F_{X_n}(x)$. For which of the following options X_n does NOT converge in distribution? *1 point*

- $F_{X_n}(x) = \left(\frac{x}{1+x}\right)^n$ for $0 < x < \infty$ and 0 otherwise
- $F_{X_n}(x) = \left(1 - \frac{1}{1+nx}\right)^n$ for $0 < x < \infty$ and 0 otherwise
- $F_{X_n}(x) = 1 - \left(1 - \frac{x}{n}\right)^n$ for $0 < x \le n$ and 0 otherwise
- $F_{X_n}(x) = 1 - (1-x)^n$ for $0 \le x \le 1$ and 0 otherwise

No, the answer is incorrect.
Score: 0
Accepted Answers:
$F_{X_n}(x) = \left(\frac{x}{1+x}\right)^n$ for $0 < x < \infty$ and 0 otherwise

3) Let $X_1, X_2, \ldots$ be a sequence of random variables. Which of the following statements is/are FALSE? *1 point*

- If $X_n \xrightarrow{p} X$ then $kX_n \xrightarrow{p} kX$ for all $k \in \mathbb{R}$
- If $X_n \xrightarrow{p} k$ then $X_n^2 \xrightarrow{p} k^2$ for all $k \in \mathbb{R}$
- If $X_n \xrightarrow{p} X$ and g is a positive valued real function i.e. $g: \mathbb{R} \to [0, \infty)$, then $g(X_n) \xrightarrow{p} g(X)$
- If $X_n \xrightarrow{p} 1$ then $X_n^{-1} \xrightarrow{p} 1$

No, the answer is incorrect.
Score: 0
Accepted Answers:
If $X_n \xrightarrow{p} X$ and g is a positive valued real function i.e. $g: \mathbb{R} \to [0, \infty)$, then $g(X_n) \xrightarrow{p} g(X)$

4) Let $X_1, X_2, \ldots$ be positive valued (i.e. $P(X_i > 0) = 1 \ \forall i$) i.i.d. random variables. Suppose the pdf of each X_i is denoted by f and $\lim_{x \to 0} f(x) = \frac{1}{4}$. Define $Y_n = n \min\{X_1, \ldots . X_n\}$. Which of the following is/are TRUE? *1 point*

- $Y_n \xrightarrow{d} Exp(2)$
- $Y_n \xrightarrow{d} Exp\left(\frac{1}{4}\right)$
- $Y_n \xrightarrow{d} Exp\left(\frac{1}{2}\right)$
- $Y_n \xrightarrow{d} Exp(4)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$Y_n \xrightarrow{d} Exp\left(\frac{1}{4}\right)$

5) Let $X_1, X_2, \ldots$ and $Y_1, Y_2, \ldots$ be two sequences of random variables such that $X_n \xrightarrow{p} 4$ and $Y_n \xrightarrow{p} 3$. Which of the following is/are TRUE? *1 point*

- $X_n Y_n \xrightarrow{p} 4$
- $\frac{Y_n}{X_n} \xrightarrow{p} \frac{3}{4}$
- $X_n^2 Y_n^2 \xrightarrow{p} 144$
- $\frac{X_n}{Y_{n-1}}, n \ge 2$ does not converge in probability

No, the answer is incorrect.
Score: 0
Accepted Answers:
$\frac{Y_n}{X_n} \xrightarrow{p} \frac{3}{4}$
$X_n^2 Y_n^2 \xrightarrow{p} 144$

6) Let $X \sim Exp(1)$. For any positive integer n, let $X_n = \frac{X}{n}$. Which of the following is TRUE? *1 point*

- X_n , $n = 1,2, \ldots.$ are dependent random variables
- X_n , $n = 1,2, \ldots.$ are independent random variables
- $\{X_n\}$ converges to 0 in probability
- $\{X_n\}$ converges to 1 in probability

No, the answer is incorrect.
Score: 0
Accepted Answers:
X_n , $n = 1,2, \ldots.$ are dependent random variables
$\{X_n\}$ converges to 0 in probability

7) Let $X_1, X_2, \ldots$ be a sequence of random variables. Which of the following is/are TRUE? *1 point*

- $X_n \xrightarrow{a.s.} X \Leftrightarrow P\{\omega: X_n(\omega) \to X(\omega) \text{ as } n \to \infty\} = 0$
- $X_n \xrightarrow{a.s.} X \Leftrightarrow P\{\omega: X_n(\omega) \to X(\omega) \text{ as } n \to \infty\} = 1$
- $X_n \xrightarrow{a.s.} X \Leftrightarrow \lim_{n\to\infty} P\{|X_n - X| > \epsilon\} = 0$ for every $\epsilon > 0$
- $X_n \xrightarrow{a.s.} X \Leftrightarrow \lim_{n\to\infty} P\{|X_n - X| < \epsilon\} = 1$ for every $\epsilon > 0$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X_n \xrightarrow{a.s.} X \Leftrightarrow P\{\omega: X_n(\omega) \to X(\omega) \text{ as } n \to \infty\} = 1$

8) Let $Y_n = \frac{k}{n}$ for $k = 0,1,2, \ldots n$ with $P\left(Y_n = \frac{k}{n}\right) = {}^nC_k \left(\frac{1}{2}\right)^n$. Let $X_n = Y_n - \frac{1}{2}$ then, which of the following is/are TRUE? *1 point*

- $X_n \xrightarrow{d} 0$
- $X_n \xrightarrow{d} \frac{1}{2}$
- $X_n \xrightarrow{p} 0$
- $X_n \xrightarrow{p} -\frac{1}{2}$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X_n \xrightarrow{d} 0$
$X_n \xrightarrow{p} 0$

9) Let $X_1, X_2, \ldots$ be a sequence of random variables. Which of the following is/are TRUE? *1 point*

- $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{p} X$
- $X_n \xrightarrow{L} k \Leftrightarrow X_n \xrightarrow{p} k$ for all $k \in \mathbb{R}$
- $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{a.s.} X$
- $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{p} X$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$X_n \xrightarrow{L} k \Leftrightarrow X_n \xrightarrow{p} k$ for all $k \in \mathbb{R}$
$X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{p} X$

10) Let $X_1, X_2, \ldots$ be i.i.d. random variables and $X_{(n)} = \max\{X_1, X_2, \ldots X_n\}$. Suppose $X_i \sim Exp(1)$ for all i and $Y_n = X_{(n)} - a_n$ for all $n > 0$, where a_n is a sequence of real numbers. For which of the following a_n, the sequence $\{Y_n\}$ converges in distribution? *1 point*

- $a_n = \frac{1}{\log(n)}$
- $a_n = e^{-n}$
- $a_n = 1/n$
- $a_n = \log(n)$

No, the answer is incorrect.
Score: 0
Accepted Answers:
$a_n = \log(n)$