## Algebraic Extension: A Field (145)

extension K of L 18 called algebraic if every element of K is algebraic over L, i.e, if every element of k is a root of some non-zero polynomial with coefficients in L.

- > Field extensions that are not algebraic are called transcendental.
- 7. R(a) is transcendental
  - 2. r(R) is a algeboraic
  - 3. c(Q) is not algebraic

Integral Extension: Let R be element definition a substing of the sting S.

XES is integral over R it

& is a scoot of a monic polynomial with coefficients in R, 1.e,

] f(x) = R[x] s.t f(x) = 0.

Def: It every element XES is integral over R, then S be integral over is said to R 092 S is integral extension of R.

-> Integral Extension is also called Integral closure.

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$$\pm(\infty) = x^2 - d$$

$$N(x) = a^2 db^2 = x x$$

Also 
$$V_L: R \rightarrow C$$
,  $i = 1,2$ 

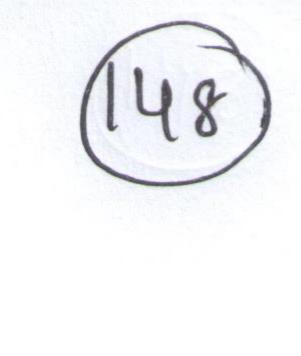
$$T_1(x) = 1.x = a+bJd$$

$$[M_{x}] = \begin{pmatrix} a & db \\ b & a \end{pmatrix}$$

$$T_{\lambda}(x) = T_{\lambda}([m_{\lambda}]) = 2a$$

$$N(x) = det([m_x])$$

$$= a^2 - db^2$$



Disouminant: 
$$\Delta$$
 $B = SI, Jd3 = S x_1, x_2^3$ 
 $x_1 = I$ 
 $x_2 = Jd$ 

$$\Delta = \det \left( \operatorname{Ta} \left( \operatorname{did} \right) \right)$$

$$= \left( \operatorname{Ta} \left( \operatorname{Jd} \right) \right)$$

$$= \left( \operatorname{Ta} \left( \operatorname{Jd} \right) \right)$$

$$= \left( \operatorname{2} \left( \operatorname{0} \right) \right) = 4 d$$

In general if

 $B = \begin{cases} X_1, X_2, \dots, X_n \end{cases}$ 

 $\Delta = \left( Tr(x_1 x_1) Tr(x_1 x_2) ... Tr(x_1 x_n) \right)$   $Tr(x_1 x_1) Tr(x_2 x_2) ... Tr(x_1 x_n)$ 

Tr (dndr)... Tr (dndn)

K = Q 3Jd

 $B_2$   $S_1$ ,  $3Jd_1$   $3Jd^2$