STUDY OF DIFFERENT CONCEPTS OF RENEWAL THEORY, POINT PATTERN ANALYSIS AND COMPARISON OF THE DISTRIBUTIONS OF 2 POINT DATASETS

Submitted by

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INTRODUCTION TO RENEWAL THEORY

STOCHASTIC PROCESS

- Collection of Random variables indexed by a variable t, which represents time.
- It can also be defined as a function of two parameters from the sample space and parameter space i.e. $X(w,t)$, where $w \in \Omega$ (sample space) & $t \in T$(parameter time).
- 4 types of stochastic processes

1. Continuous Time Continuous Space
2. Continuous Time Discrete Space
3. Discrete-Time Continuous Space
4. Discrete Time Discrete Space

MARKOV PROPERTY

- Markov property refers to the memoryless property of a stochastic process i.e. the conditional probability distribution of the present state depends only on the previous state.
- For any i, j $\in \Omega$ and n $\geq$ 0,

$$P(X_{n+1} = j \mid X_n = i, \ldots\ldots, X_0) = P(X_{n+1} = j \mid X_n = i)$$

MARKOV PROCESS

- A stochastic process satisfying Markov property is called a Markov process.

RENEWAL PROCESS

A counting process, $\{N(t), t \geq 0\}$, is a renewal process if $\{X1, X2, X3, \ldots\ldots..\}$, the sequence of non-negative random variables of interarrival time are independent and identically distributed.

Ex – Poisson Process

RENEWAL FUNCTION

Renewal Function = M(t) = E(N(t))

$$= \sum_{n=1}^{\infty} P(N(t) \geq n)$$

$$= \sum_{n=1}^{\infty} P(Tn \leq n)$$

$$= \sum_{n=1}^{\infty} F_n(t) \text{ , for } t > 0$$

RENEWAL EQUATION

$$M(t) = F(t) + \int_0^t M(t - x)\, dF(x)$$

(Volterra Integral Equation of 2nd Kind)

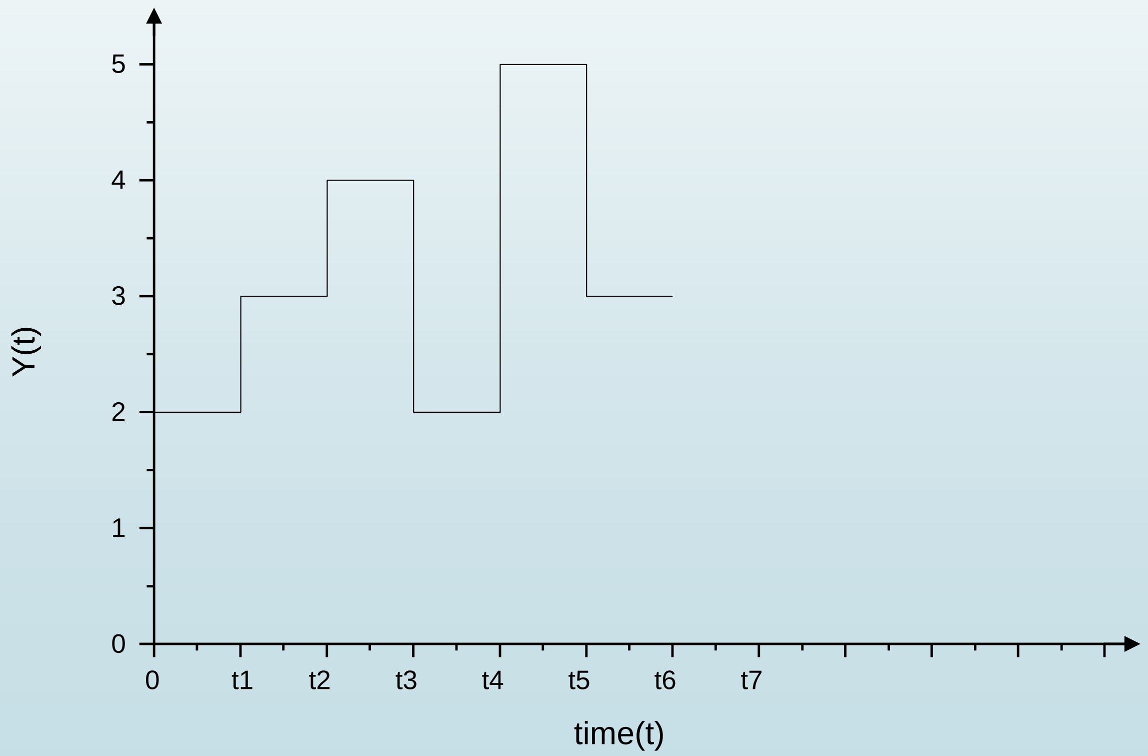
After taking Laplace Transform and solving it, we got

$$f^*(s) = \frac{s\ .\ M*(s)}{1 + s\ .\ M*(s)}$$

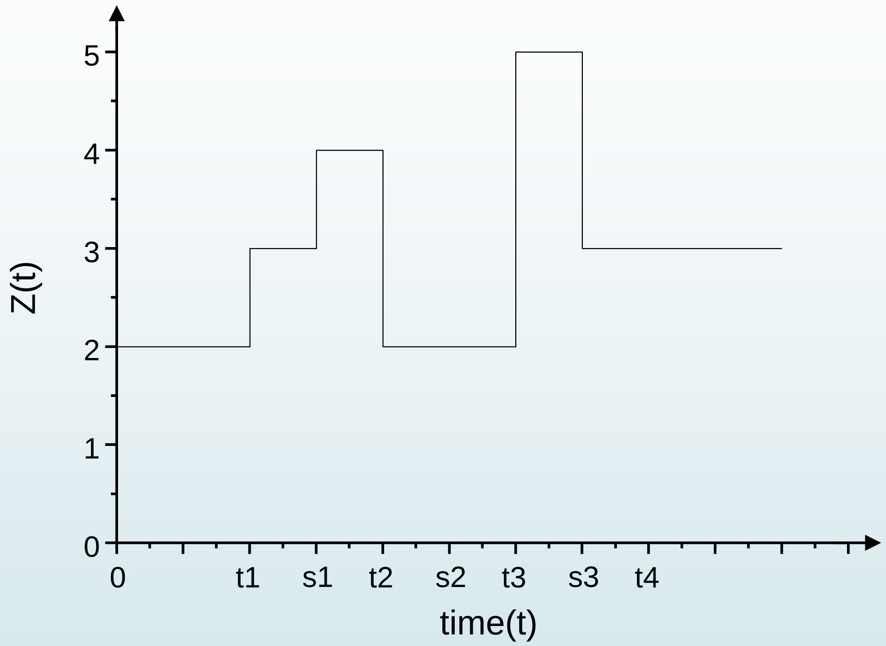
{By applying $F^*(s) = \frac{f*(s)}{s}$}

MARKOV RENEWAL PROCESS

- **Also known as Semi Markov Process.**
- **Time instants t_{n}, {n = 0,1,2,…} satisfy the Markov Property.**
- **If the Markov Property is satisfied everywhere, it is called Markov Process.**



Markov Regenerative Process(MRGP)



- t1,t2,t3,….. are the regeneration points.
- s1,s2,s3,… are the service completion times.

INTRODUCTION TO POINT PATTERN ANALYSIS

- Study of spatial arrangements in geographical space.
- 3 types: Deterministic – Outcomes are same every time.

  Stochastic – Outcomes are different.

  Independent Random Process – Independent of location.

COMPLETE SPATIAL RANDOMNESS (CSR)

- Point events occur within a given study area in a completely random fashion.
- Frequency distribution of CSR satisfies the Poisson distribution.
- Standard case or Null Hypothesis.

NEAREST NEIGHBOUR METHOD

- Finds the distance between each point and its closest neighbour, then averages these distance.
- Observed Mean NN distance = $d_{obs}(mean) = \frac{\sum_{i=1}^{n} d_{min}(si)}{n}$
- Expected Mean NN distance = $d_{exp}(mean) = \frac{1}{2}\sqrt{\frac{Area}{n}}$
- Nearest neighbour index 'd' as a difference = $d_{obs}(mean) - d_{exp}(mean)$
- Nearest neighbour index 'r' as a ratio = $\frac{d_{obs}(mean)}{d_{exp}(mean)}$

d	r	Pattern
d < 0	r < 1	Clustered
d > 0	r > 1	Random
d = 0	r = 1	Dispersed

G-FUNCTION

$$g(d) = \frac{\textbf{Number of events having minimum distance} \leq d}{\textbf{Total number of } events}$$

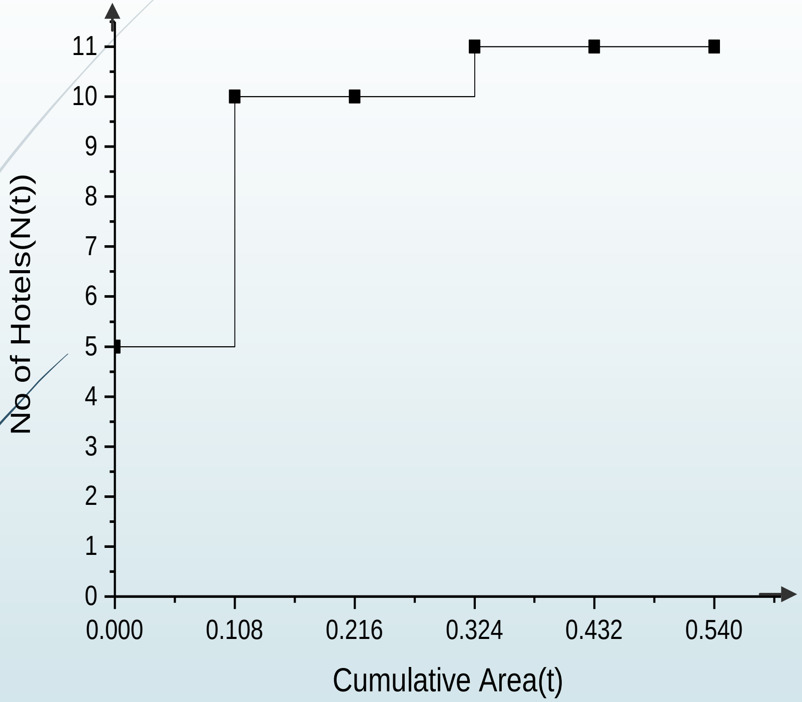
- For the clustered distribution g-function increases rapidly at short distances.
- For evenly spaced distribution it slowly increases up to event spacing.

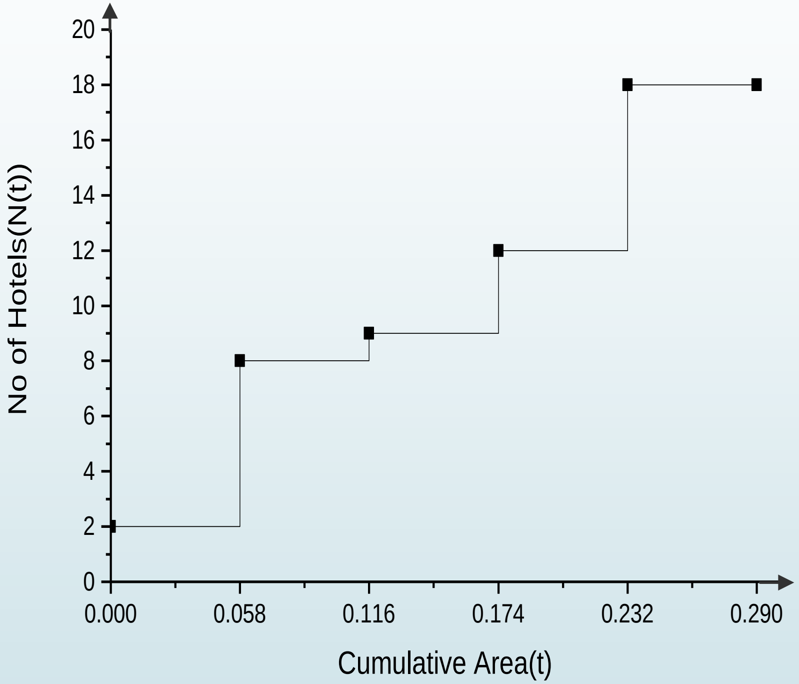
USED FORMULAE

- Mean = $\mu = x \,.\, P(X = x)$
- Variance = $Var(X) = (x - \mu)^2 \,.P(X = x)$
- Standard Deviation = $SD(X) = \sqrt{Var(X)}$
- Coefficient of Variation = $\frac{\text{Standard Deviation}}{\text{Mean}}$

$$= \frac{SD(X)}{\mu}$$

COMPARISON OF POINT DATASETS





(SAMPLE PATHS FOR BOTH THE LOCATIONS)

- Cumulative Areas are used as the regeneration points.

LOCATION – 1

Mean = 1.512

Variance = 5.816

Standard Deviation = 2.412

Coefficient of Variation = 1.595

Standard Deviation > Mean

NN Index as Difference = -0.0226 (clustered)

LOCATION – 2

Mean = 0.467

Variance = 0.184

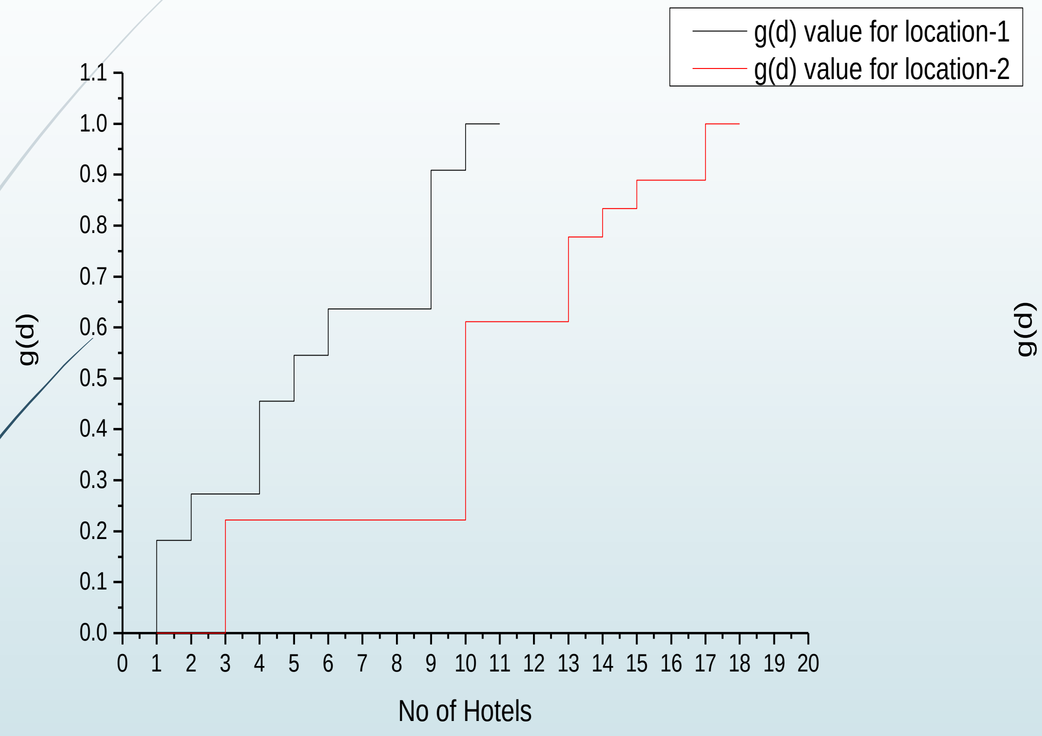
Standard Deviation = 0.429

Coefficient of Variation = 0.919

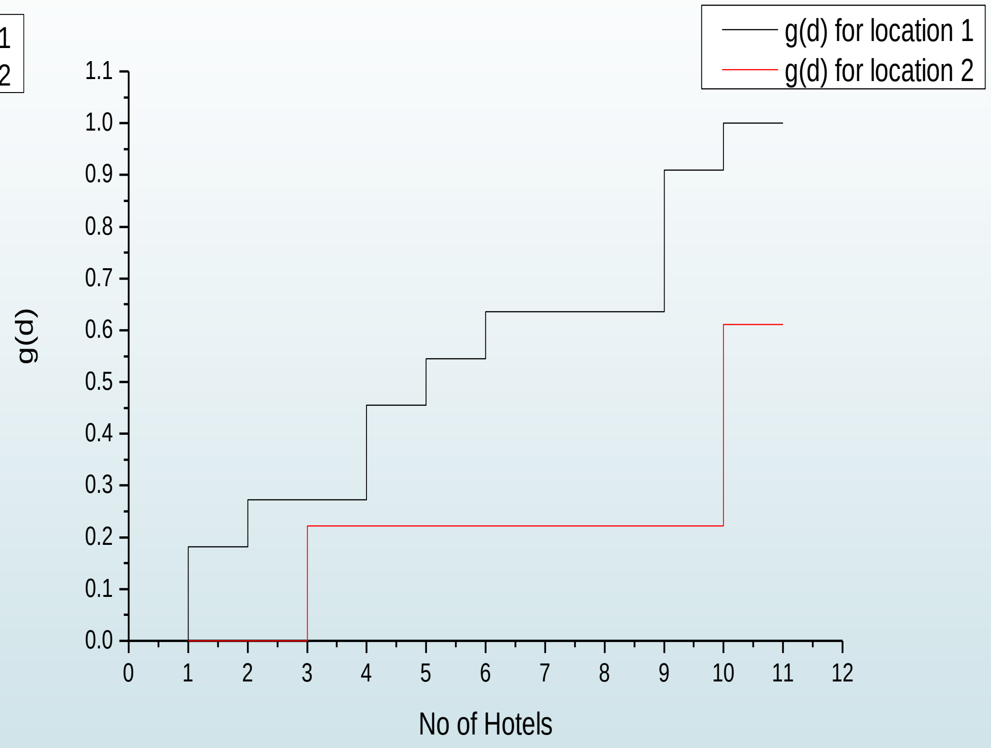
Standard Deviation < Mean

NN Index as Difference = -0.0052 (clustered)

g – Function Comparison



Comparison of g(d) values



Comparison of g(d) values of first 11 points

CONCLUSION

- Location – 2 has a better distribution of hotels.
- Not concentrated in only one place which helps people find them easily.
- In real life these methods can be applicable in analysing the locations of shopping malls and other buildings, distribution of natural resources in a particular area, to identify individual stands of specific types of trees, the location of diseased trees in forests etc.
- We are currently working on a paper showing the application of Renewal Theory on Point Pattern Analysis. We are just mentioning it as have not got all the results.

Thank You …