Question 1.

$$l = -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - u)^2$$

$$\frac{\partial l}{\partial u} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - u) = \sum_{i=1}^{n}\left(\frac{x_i}{\sigma^2} - \frac{u}{\sigma^2}\right)$$

$$\frac{\partial^2 l}{\partial u \partial u} = \sum_{i=1}^{n}\frac{-1}{\sigma^2} = \frac{-n}{\sigma^2}$$

$$\frac{\partial l}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}(x_i - u)^2$$

$$\frac{\partial l}{\partial \sigma^2 \partial \sigma^2} = \frac{n}{2\sigma^4} + (-2)\frac{1}{2\sigma^6}(x_i - u)^2$$

$$= \frac{n}{2\sigma^4} + \left(-\frac{1}{\sigma^6}(x_i - u)^2\right)$$

$$= \frac{n}{2\sigma^4} - \frac{1}{\sigma^6}(x_i - u)^2.$$

$$\frac{\partial^2 l}{\partial u \partial \sigma^2} = \frac{\partial^2 l}{\partial \sigma^2 \partial u} = -\frac{1}{\sigma^4}\sum_{i=1}^{n}(x_i - u)$$

Hessian Matrix =

$$\begin{pmatrix} \dfrac{-n}{\sigma^2} & -\dfrac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4} \\ -\dfrac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4} & -\dfrac{n}{2\sigma^4} + \dfrac{\sum_{i=1}^{n}(x_i-u)^2}{\sigma^6} \end{pmatrix}$$

So the standard error of mean $\hat{u}$ is

$$\sqrt{\frac{-n}{\sigma^2}} = -\frac{\sqrt{n}}{\sigma}$$

the standard error of standard deviation $\hat{\sigma}$ is.

$$\sqrt{-\frac{n}{2\sigma^4} + \frac{\sum_{i=1}^{n}(x_i-u)^2}{\sigma^6}}.$$

The estimated covariance is:

$$-\frac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4}.$$