In [438]:
```
import os
os.getcwd()
```

Out[438]: 'C:\\Users\\dzfal'

In [439]:
```
import pandas as pd
import numpy as np
```

In [440]:
```
data = pd.read_excel(r'OneDrive\Desktop\CPI_IJC_2012_2022.xlsx')
data
```

Out[440]:

	Year	Month	CPI	IJC
0	2012	JAN	227.842	372000
1	2012	FEB	228.329	365000
2	2012	MAR	228.807	358000
3	2012	APR	229.187	372000
4	2012	MAY	228.713	381000
...	...	...	...	...
124	2022	MAY	291.474	202000
125	2022	JUN	295.328	231000
126	2022	JUL	295.271	248000
127	2022	AUG	295.620	228000
128	2022	SEP	296.761	190000

129 rows × 4 columns

In [441]:
```
data.loc[data['Year'] < 2020, 'Period'] = 1
data.loc[data['Year'] == 2020, 'Period'] = 2
data.loc[data['Year'] > 2020, 'Period'] = 3
data['Period'] = data['Period'].astype(int)
```

In [442]:
```
data
```

Out[442]:

	Year	Month	CPI	IJC	Period
0	2012	JAN	227.842	372000	1
1	2012	FEB	228.329	365000	1
2	2012	MAR	228.807	358000	1
3	2012	APR	229.187	372000	1
4	2012	MAY	228.713	381000	1
...	...	...	...	...	...
124	2022	MAY	291.474	202000	3
125	2022	JUN	295.328	231000	3
126	2022	JUL	295.271	248000	3
127	2022	AUG	295.620	228000	3
128	2022	SEP	296.761	190000	3

129 rows × 5 columns

Question 1

In [443]:
```
col = ["CPI", "IJC"]
data_cp = data[col]
```

In [444]:
```
data_cp
```

Out[444]:

	CPI	IJC
0	227.842	372000
1	228.329	365000
2	228.807	358000
3	229.187	372000
4	228.713	381000
...	...	...
124	291.474	202000
125	295.328	231000
126	295.271	248000
127	295.620	228000
128	296.761	190000

129 rows × 2 columns

In [445]:
```
data_cp= np.log(data_cp)
data_cp["Period"] = data["Period"]
data_cp
```

Out[445]:

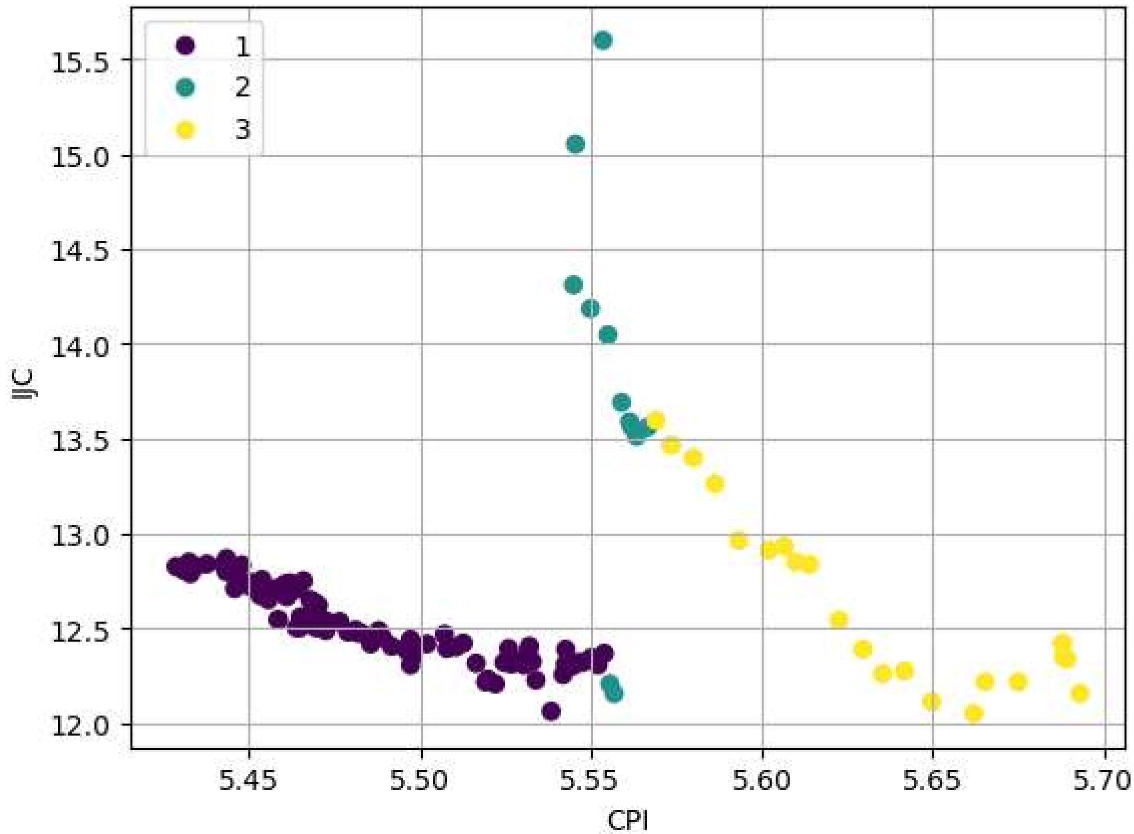
	CPI	IJC	Period
0	5.428652	12.826649	1
1	5.430788	12.807653	1
2	5.432879	12.788288	1
3	5.434538	12.826649	1
4	5.432468	12.850555	1
...	...	...	...
124	5.674951	12.216023	3
125	5.688087	12.350173	3
126	5.687894	12.421184	3
127	5.689075	12.337101	3
128	5.692927	12.154779	3

129 rows × 3 columns

In [446]:
```
import matplotlib.pyplot as plt

u, inv = np.unique(data_cp.Period.values, return_inverse=True)

scatter = plt.scatter(data_cp["CPI"], data_cp["IJC"], c=inv)
plt.legend(scatter.legend_elements()[0], u, loc=2)
plt.xlabel("CPI")
plt.ylabel("IJC")
plt.grid()
plt.show()
```



Question 2

In [447]:
```
def SWEEPOperator (pDim, inputM, origDiag, sweepCol = None, tol = 1e-7):
    ''' Implement the SWEEP operator
    Parameter
    ---------
    pDim: dimension of matrix inputM, integer greater than one
    inputM: a square and symmetric matrix, np array
    origDiag: the original diagonal elements before any SWEEPing
    sweepCol: a list of columns numbers to SWEEP
    tol: singularity tolerance, positive real
    Return
    ------
    A: negative of a generalized inverse of input matrix
    aliasParam: a list of aliased rows/columns in input matrix
    nonAliasParam: a list of non-aliased rows/columns in input matrix
    '''
    if (sweepCol is None):
        sweepCol = range(pDim)
    aliasParam = []
    nonAliasParam = []
    A = np.copy(inputM)
    ANext = np.zeros((pDim,pDim))
    for k in sweepCol:
        Akk = A[k,k]
        pivot = tol * abs(origDiag[k])
        if (not np.isinf(Akk) and abs(Akk) >= pivot and pivot > 0.0):
            nonAliasParam.append(k)
            ANext = A - np.outer(A[:, k], A[k, :]) / Akk
            ANext[:, k] = A[:, k] / abs(Akk)
            ANext[k, :] = ANext[:, k]
            ANext[k, k] = -1.0 / Akk
        else:
            aliasParam.append(k)
            ANext[:,k] = np.zeros(pDim)
            ANext[k, :] = np.zeros(pDim)
        A = ANext
    return (A, aliasParam, nonAliasParam)

def LinearRegression (X, y):
    ''' Train a linear regression model
    Parameter
    ---------
    X: A pd DataFrame, rows are observations, columns are regressors
    y: A pd Series, rows are observations of the response variable
    Return
    ------
    A list of the following entities:
    1. b: an array of regression coefficient
    2. residual_SS: residual sum of squares
    3. XtX_Ginv: a generalized inverse of the XtX matrix
    4. aliasParam: a list of aliased rows/columns in input matrix
    5. nonAliasParam: a list of non-aliased rows/columns in input matrix
    '''
    # X: A pd DataFrame, rows are observations, columns are regressors
    # y: A pd Series, rows are observations of the response variable
    Z = X.join(y)
    n_sample = Z.shape[0]
```

```
    n_param = Z.shape[1] - 1
    ZtZ = Z.transpose().dot(Z)
    diag_ZtZ = np.diagonal(ZtZ)
    eps_double = np.finfo(np.float64).eps
    tol = np.sqrt(eps_double)
    ZtZ_transf, aliasParam, nonAliasParam = SWEEPOperator ((n_param+1), ZtZ, 
    b = ZtZ_transf[0:n_param, n_param]
    b[aliasParam] = 0.0
    XtX_Ginv = - ZtZ_transf[0:n_param, 0:n_param]
    XtX_Ginv[:, aliasParam] = 0.0
    XtX_Ginv[aliasParam, :] = 0.0
    residual_SS = ZtZ_transf[n_param, n_param]
    return ([b, residual_SS, XtX_Ginv, aliasParam, nonAliasParam])
```

In [448]:
```
X = pd.get_dummies(data_cp[['Period']].astype('category'))
X = X.iloc[:,[2,1,0]]
X = X.join(data_cp[['CPI']])
X.insert(0, 'Intercept', 1)
y = data_cp['IJC']
result_list = LinearRegression(X, y)
# Label the output properly
parameter_name = X.columns
estimate = pd.Series(result_list[0], index = parameter_name)
residual_SS = result_list[1]
cov_matrix = pd.DataFrame(result_list[2], index = parameter_name, columns = p
```

In [449]:
```
residual_SS
```

Out[449]:
```
12.81199636652066
```

In [450]:
```
estimate
```

Out[450]:
```
Intercept    47.376556
Period_3      1.055882
Period_2      1.717105
Period_1      0.000000
CPI          -6.354338
dtype: float64
```

In [451]:
```
cov_matrix
```

Out[451]:

	Intercept	Period_3	Period_2	Period_1	CPI
Intercept	198.012847	5.325266	2.586715	0.0	-36.103852
Period_3	5.325266	0.201819	0.080403	0.0	-0.972911
Period_2	2.586715	0.080403	0.127816	0.0	-0.473562
Period_1	0.000000	0.000000	0.000000	0.0	0.000000
CPI	-36.103852	-0.972911	-0.473562	0.0	6.583193

Standard error should be the squareroot of the variance of the coefficients, which locate in the diagonal of the covariance matrix

In [452]:
```
import math
for i in cov_matrix.index:
    standardError = math.sqrt(cov_matrix[i][i])
    print("The standard error of the coefficient of ", i, " is: ", standardEr
```

```
The standard error of the coefficient of  Intercept  is:  14.071703783412302
The standard error of the coefficient of  Period_3  is:  0.4492431158419582
The standard error of the coefficient of  Period_2  is:  0.3575132227254053
The standard error of the coefficient of  Period_1  is:  0.0
The standard error of the coefficient of  CPI  is:  2.565773295093052
```

Question 3

Use the definition

In [453]:
```
def tss(a):
    m = a.mean()
    n = 0
    for i in a:
        n += ((i-m)**2)
    return (n)
```

In [454]:
```
1 - (residual_SS/tss(y))
```

Out[454]:
```
0.6430150381336952
```

Use the sklearn package

In [455]:
```
from sklearn.linear_model import LinearRegression
#initiate linear regression model
model = LinearRegression()
#fit regression model
model.fit(X, y)
#calculate R-squared of regression model
r_squared = model.score(X, y)
r_squared
```

Out[455]:
```
0.6430150381274364
```

The answer of these two methods agrees with each other

Question 4

In [456]:
```
from scipy.stats import f
```

In [457]:
```
summ_stat = data_cp.groupby(by = 'Period')['IJC'].describe()
summ_stat
```

Out[457]:

Period	count	mean	std	min	25%	50%	75%	max
1	96.0	12.527799	0.191160	12.061047	12.385250	12.502467	12.707604	12.868761
2	12.0	13.787805	0.989758	12.154779	13.546112	13.637455	14.215633	15.598229
3	21.0	12.644592	0.484669	12.049419	12.259613	12.421184	12.933621	13.596110

We can get the ANOVA table

In [458]:
```
# Calculate the Within Group Sum of Squares
n_sample = data_cp.shape[0]
dfW = 0.0
SSW = 0.0
for Period in [1, 2, 3]:
   group_stat = summ_stat.loc[Period]
   group_count = group_stat['count']
   group_mean = group_stat['mean']
   group_std = group_stat['std']
   dfW = dfW + (group_count - 1)
   SSW = SSW + group_std * group_std * (group_count - 1)

MSW = SSW / dfW
# Calculate the Corrected Total Sum of Squares
overall_std = np.std(data_cp['IJC'], ddof = 1)
dfT = n_sample - 1
SST = overall_std * overall_std * dfT
SSB = SST - SSW
dfB = dfT - dfW
MSB = SSB / dfB
F_stat = MSB / MSW
F_sig = f.sf(F_stat, dfB, dfW)
print('\nSource: Between Group')
print('DF = ', dfB)
print('SS = ', SSB)
print('MS = ', MSB)
print('\nSource: Within Group')
print('DF = ', dfW)
print('SS = ', SSW)
print('MS = ', MSW)
print('\nSource: Total')
print('DF = ', dfT)
print('SS = ', SST)
print('F Statistic = ', F_stat)
print('     F Sig. = ', F_sig)
print('R-Square = ', (SSB/SST))
```

```

Source: Between Group
DF =  2.0
SS =  16.94402133029339
MS =  8.472010665146694

Source: Within Group
DF =  126.0
SS =  18.945435467942588
MS =  0.1503605989519253

Source: Total
DF =  128
SS =  35.889456798235976
F Statistic =  56.34461903051773
     F Sig. =  3.310362399319542e-18
R-Square =  0.472116962526059
```

Since the F Significance value is very small, we will reject the null hypothesis. We can

conclude that the population means of IJC are different among the three periods and thus r_i is not equal to 0 for some r_i

Question 5

In this question, the residual of original data has been given in question 2, and I ran the regression again with the input variable dropped log(CPI). Then I calculated the residual of my second regression and ran the F-test to test if the residuals of these two regression are equal, because if log(CPI) does not affect the log(IJC), the residuals which can be regarded as variance should be the same.

In [461]:
```
X_b = X.drop(columns=["CPI"])
X_b
```

Out[461]:

	Intercept	Period_3	Period_2	Period_1
0	1	0	0	1
1	1	0	0	1
2	1	0	0	1
3	1	0	0	1
4	1	0	0	1
...	...	...	...	...
124	1	1	0	0
125	1	1	0	0
126	1	1	0	0
127	1	1	0	0
128	1	1	0	0

129 rows × 4 columns

In [464]:
```
y
```

Out[464]:
```
0      12.826649
1      12.807653
2      12.788288
3      12.826649
4      12.850555
         ...
124    12.216023
125    12.350173
126    12.421184
127    12.337101
128    12.154779
Name: IJC, Length: 129, dtype: float64
```

In [470]:
```
model = LinearRegression()

# fitting the data
model.fit(X_b, y)

# predicting values
y_pred = model.predict(X_b)
df = pd.DataFrame({'Actual': y, 'Predicted': y_pred})

residual = np.sum(np.square(df['Predicted'] - df['Actual']))

print(' residual sum of squares is : '+ str(np.sum(np.square(df['Predicted']
```

```
 residual sum of squares is : 18.945435467942588
```

In [473]:
```
import scipy.stats
scipy.stats.f.ppf(q=1-0.05, dfn=len(y_pred)-1, dfd=len(y_pred)-1)
```

Out[473]:
```
1.337380547457587
```

In [475]:
```
F_stats = residual/residual_SS
F_stats
```

Out[475]:
```
1.4787262598239075
```

In [477]:
```
p = 1-scipy.stats.f.cdf(F_stats, len(y_pred)-1, len(y_pred)-1)
p
```

Out[477]:
```
0.01384786482912781
```

In [478]:
```
print("The F-test statistic is: ", F_stats, "with degrees of freedom ", "(",
```

```
The F-test statistic is:  1.4787262598239075 with degrees of freedom  ( 128
,  128 ) , and the significance level is:  0.01384786482912781
```

We can reject the null hypothesis and conclude that CPI does affect the IJC

Question 6

In [498]:
```
X.loc[len(X.index)] = [1, 1, 0, 0, math.log(298.062)]
X
```

Out[498]:

	Intercept	Period_3	Period_2	Period_1	CPI
0	1.0	0.0	0.0	1.0	5.428652
1	1.0	0.0	0.0	1.0	5.430788
2	1.0	0.0	0.0	1.0	5.432879
3	1.0	0.0	0.0	1.0	5.434538
4	1.0	0.0	0.0	1.0	5.432468
...	...	...	...	...	...
125	1.0	1.0	0.0	0.0	5.688087
126	1.0	1.0	0.0	0.0	5.687894
127	1.0	1.0	0.0	0.0	5.689075
128	1.0	1.0	0.0	0.0	5.692927
129	1.0	1.0	0.0	0.0	5.697302

130 rows × 5 columns

In [499]:
```
df_yhat = X.dot(estimate)
df_yhat
```

Out[499]:
```
0      12.881064
1      12.867496
2      12.854208
3      12.843663
4      12.856819
         ...
125    12.288414
126    12.289640
127    12.282134
128    12.257655
129    12.229859
Length: 130, dtype: float64
```

In [574]:
```
yhat_new = df_yhat[len(df_yhat) - 1].astype(float)
yhat_new
```

Out[574]:
```
12.229858891340308
```

In [575]:
```
pred_origin = pow(math.e, yhat_new)
pred_origin
print("The predicted IJC of Oct 2022 is:", pred_origin)
```

```
The predicted IJC of Oct 2022 is: 204814.2789884431
```

N_n **(Xb,σ^2 X(X^t X)^(−1) X^t)**

In [576]:
```
aa = X[-1:]
aa
```

Out[576]:

	Intercept	Period_3	Period_2	Period_1	CPI
129	1.0	1.0	0.0	0.0	5.697302

In [577]:
```
a = np.linalg.inv(X.T.dot(X))
b = aa.dot(a)
proj = b.dot(aa.T.to_numpy()).values
proj = proj[0][0].astype(float)
proj
```

Out[577]: 0.06207747940416741

We can use the residual_SS in question 2 as σ^2

In [590]:
```
from scipy.stats import t
dof = len(y) - 5
t_critical = t.ppf(0.975, df = dof)
t_critical
```

Out[590]: 1.9792801165796825

In [594]:
```
rss = residual_SS.astype(float)
c = t_critical * math.sqrt(rss/dof * proj)
ci_lower = yhat_new - c
ci_upper = yhat_new + c
ori_ci_lower = pow(math.e, ci_lower)
ori_ci_upper = pow(math.e, ci_upper)
print("The confidence interval of original mean prediction is: (", ori_ci_low
```

```
The confidence interval of original mean prediction is: ( 174790.49175115686
,  239995.25636255418 )
```

Question 7

In [365]:
```
y_IJC
```

Out[365]:
```
array([ 372000,  365000,  358000,  372000,  381000,  372000,  372000,
        377000,  376000,  364000,  388000,  362000,  366000,  342000,
        375000,  331000,  353000,  340000,  334000,  325000,  319000,
        347000,  312000,  332000,  340000,  341000,  330000,  345000,
        312000,  308000,  303000,  303000,  290000,  291000,  291000,
        285000,  281000,  317000,  269000,  269000,  275000,  275000,
        269000,  279000,  272000,  275000,  265000,  276000,  282000,
        269000,  271000,  278000,  263000,  262000,  266000,  261000,
        247000,  265000,  255000,  244000,  241000,  221000,  236000,
        232000,  253000,  247000,  245000,  247000,  260000,  241000,
        243000,  248000,  223000,  202000,  205000,  200000,  224000,
        241000,  222000,  225000,  221000,  225000,  244000,  237000,
        227000,  204000,  173000,  210000,  222000,  240000,  220000,
        224000,  224000,  230000,  221000,  235000,  200000,  190000,
       5946000, 3446000, 1639000, 1446000, 1260000,  881000,  795000,
        773000,  737000,  773000,  803000,  704000,  658000,  574000,
        427000,  405000,  414000,  381000,  376000,  280000,  240000,
        211000,  214000,  182000,  171000,  202000,  202000,  231000,
        248000,  228000,  190000], dtype=int64)
```

In [366]:
```
observed_bar = y_IJC.mean()
observed_bar
```

Out[366]:
```
406193.7984496124
```

In [367]:
```
X = X[:-1]
```

In [371]:
```
pred_IJC = pow(math.e, X.dot(estimate))
pred_IJC = pred_IJC.to_numpy()
pred_IJC
```

Out[371]:
```
array([ 392803.19623504,  387509.80560336,  382394.36294749,
        378383.40594285,  383394.12505474,  385413.45567116,
        384706.89704234,  370803.71470123,  359756.24764153,
        353652.01924892,  357449.29716474,  357724.43879815,
        353254.51856143,  341305.70164833,  347467.66644108,
        352113.37473585,  351188.13320874,  345922.28088634,
        341650.3933297 ,  336512.87137663,  335707.96037124,
        334568.4483002 ,  330673.59600361,  325171.26218601,
        320206.66515629,  317975.94764172,  313880.72743346,
        310187.95711201,  306463.16713688,  303902.88577257,
        301738.42611002,  302045.38493709,  301908.016597  ,
        302287.97616134,  305929.43191039,  311994.4515188 ,
        324924.88427579,  319740.08289549,  314320.49853196,
        312246.31532028,  305781.81857528,  300457.9554452 ,
        297446.92433765,  297454.86483137,  301738.42611002,
        299848.12871163,  297581.94610231,  299623.81689977,
        300498.12592414,  303049.55686608,  297081.92668572,
        289951.37508084,  285633.26613951,  280645.9102458 ,
        281545.83541774,  278259.88944715,  273666.05732779,
        269627.06833011,  267615.90336994,  263362.47201515,
        256695.87646312,  254113.18843801,  254868.88648443,
        252879.18712363,  254126.42393016,  253076.68737716,
        252550.41630477,  246460.65486224,  238611.61843794,
        237439.81155912,  233453.59511711,  230352.21763004,
        224887.94773984,  220930.29010563,  220132.7713086 ,
        216992.37508555,  213535.42262282,  211710.29701731,
        210615.36885653,  208360.94719501,  205802.28854844,
        202601.46904367,  203648.30560655,  204192.051391  ,
        204608.68798934,  201217.0435759 ,  195562.77846626,
        191268.64995344,  190498.81411452,  190328.22684889,
        187795.17760332,  186499.34374078,  184581.80281854,
        181373.65434438,  179225.07708655,  177140.7660093 ,
        976279.63147745,  968521.49021742,  988769.75167773,
       1040692.42345365, 1044574.10997848, 1012156.02889333,
        979619.67158281,  955016.41228625,  940878.17486724,
        937164.23907142,  928767.87838165,  909910.50283635,
        462520.68372805,  449879.07772703,  432041.87449899,
        414850.05231434,  396817.13037661,  375396.15337815,
        364754.66822319,  357116.59666491,  347932.66877386,
        329371.83144003,  315106.43999769,  303829.48528942,
        291665.77823335,  277305.737444  ,  256406.49014309,
        251063.72033059,  236069.8134471 ,  217165.1881264 ,
        217431.71378842,  215805.74437497,  210587.26749324])
```

In [372]:
```
rel_error = sum(np.square(y_IJC - pred_IJC)) / sum(np.square(y_IJC - observed
```

In [374]:
```
print("The relative error is: ", rel_error)
```

```
The relative error is:  0.6997043214964035
```