Question 1.

$$l = -\frac{n}{2}\log_e(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-u)^2$$

$$\frac{\partial l}{\partial u} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-u) = \sum_{i=1}^{n}\left(\frac{x_i}{\sigma^2} - \frac{u}{\sigma^2}\right)$$

$$\frac{\partial^2 l}{\partial u \partial u} = \sum_{i=1}^{n}\frac{-1}{\sigma^2} = \frac{-n}{\sigma^2}$$

$$\frac{\partial l}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}(x_i-u)^2$$

$$\frac{\partial l}{\partial\sigma^2\partial\sigma^2} = \frac{n}{2\sigma^4} + (-2)\frac{1}{2\sigma^6}(x_i-u)^2$$

$$= \frac{n}{\partial\sigma^4} + \left(-\frac{1}{\sigma^6}(x_i-u)^2\right)$$

$$= \frac{n}{\partial\sigma^4} - \frac{1}{\sigma^6}(x_i-u)^2.$$

$$\frac{\partial^2 l}{\partial u\partial\sigma^2} = \frac{\partial^2 l}{\partial\sigma^2\partial u} = -\frac{1}{\sigma^4}\sum_{i=1}^{n}(x_i-u)$$

Hessian Matrix =

$$\begin{pmatrix} \dfrac{-n}{\sigma^2} & -\dfrac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4} \\ -\dfrac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4} & -\dfrac{n}{2\sigma^4} + \dfrac{\sum_{i=1}^{n}(x_i-u)^2}{\sigma^6} \end{pmatrix}$$

So the standard error of mean $\hat{u}$ is

$$\sqrt{\frac{-n}{\sigma^2}} = -\frac{\sqrt{n}}{\sigma}$$

the standard error of standard deviation $\hat{\sigma}$ is.

$$\sqrt{-\frac{n}{2\sigma^4} + \frac{\sum_{i=1}^{n}(x_i-u)^2}{\sigma^6}}.$$

The estimated covariance is:

$$-\frac{\sum_{i=1}^{n}(x_i-u)}{\sigma^4}.$$

Question 2

1)

In [39]:
```
import numpy as np
import pandas as pd
import os
import math
import matplotlib.pyplot as plt
os.getcwd()
os.chdir(r'C:\Users\Primo\OneDrive\Desktop\2022Fall\Stats Analysis\A2')
```

In [4]:
```
u1357 = pd.read_csv('U1357.csv')
```

In [8]:
```
import random
def sample_wr (inData):
    n = len(inData)
    outData = np.empty((n,1))
    for i in range(n):
        j = int(random.random() * n)
        outData[i] = inData[j]
    return outData
```

In [16]:
```
u1357 = u1357.to_numpy()
print(type(u1357))
```

```
<class 'numpy.ndarray'>
```

In [33]:
```
a = []
b = []
random.seed(20221013)
for i in range(10000):
    sample = sample_wr(u1357)
    sample_a = np.mean(sample) - math.sqrt(3) * np.std(sample)
    sample_b = np.mean(sample) + math.sqrt(3) * np.std(sample)
    a.append(sample_a)
    b.append(sample_b)
```

In [34]:
```
est_a = np.asarray(a)
est_b = np.asarray(b)
mme_a = np.mean(est_a)
mme_b = np.mean(est_b)
print("The mme of a is: ", mme_a, "and the mme of b is: ", mme_b)
```

```
The mme of a is:  -2.2842488835227077 and the mme of b is:  3.6841455112603647
```

2)

In [35]:
```
a_std = np.std(est_a)
b_std = np.std(est_b)
print("The standard deviation of a_hat is: ", a_std, "and the standard deviation of b
```

```
The standard deviation of a_hat is:  0.05898983395157765 and the standard deviation o
f b is:  0.05915200193513628
```

3)

In [38]:
```
a_ci_low = mme_a - 1.96 * a_std
a_ci_high = mme_a + 1.96 * a_std
b_ci_low = mme_b - 1.96 * b_std
b_ci_high = mme_b + 1.96 * b_std
print("The 95% interval for a is: (", str(a_ci_low),",",str(a_ci_high),") and for b is
```

```
The 95% interval for a is: ( -2.3998689580678 , -2.1686288089776156 ) and for b is: (
3.5682075874674974 , 3.800083435053232 )
```

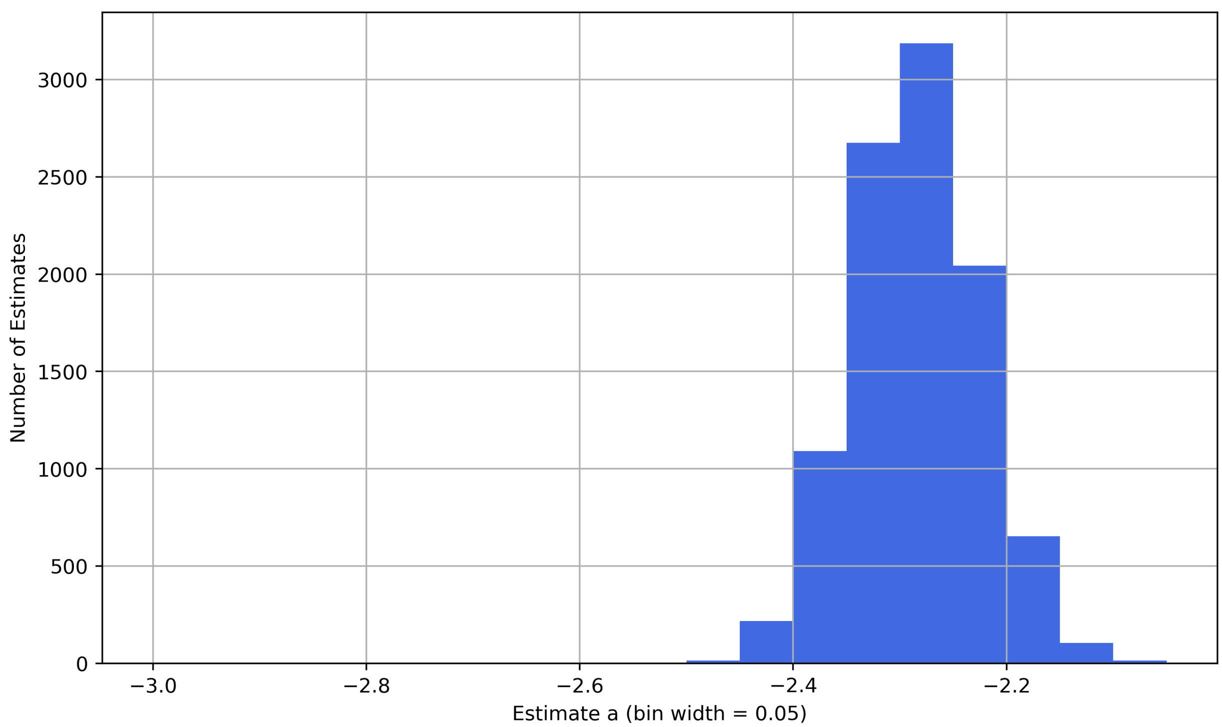
4)

In [46]:
```
est_a.min()
```

Out[46]:
```
-2.5059856420940347
```

In [54]:
```
a_bin_min = math.floor(est_a.min())
a_bin_max = math.ceil(est_a.max())
a_bins = np.arange(a_bin_min, a_bin_max, 0.05)
```

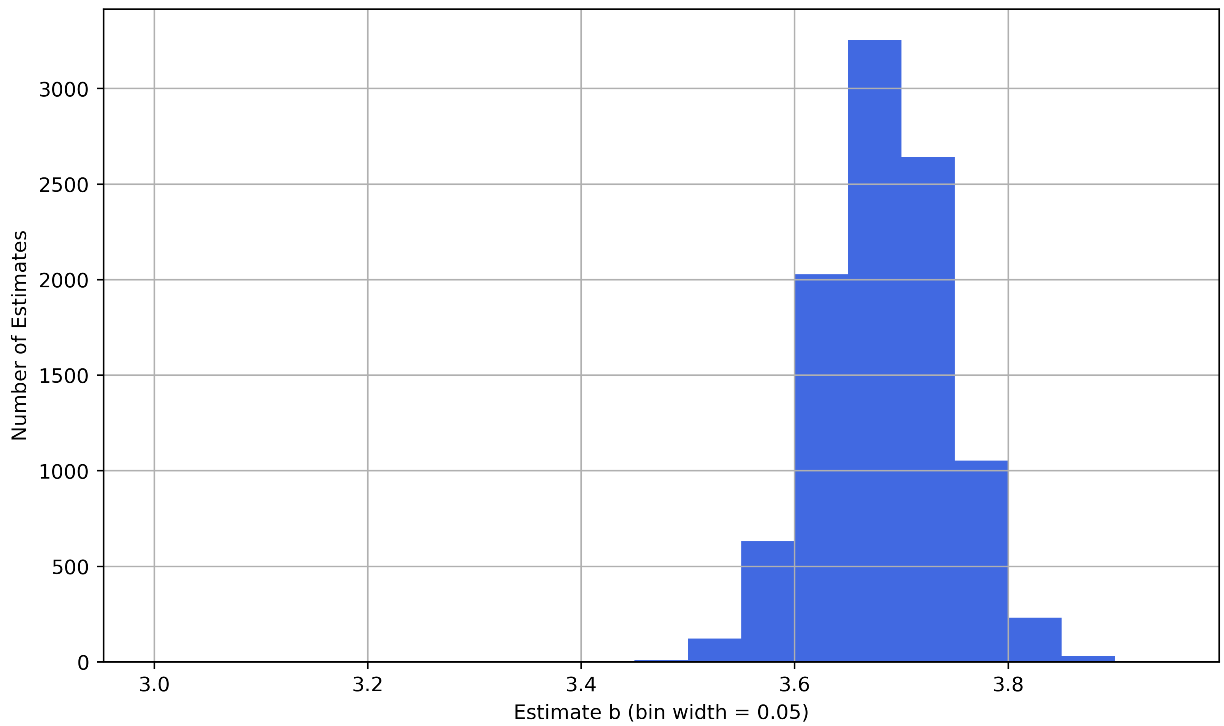
In [55]:
```
plt.figure(figsize = (10,6), dpi = 400)
plt.hist(est_a , bins = a_bins, color = 'royalblue')
plt.xlabel('Estimate a (bin width = 0.05)')
plt.ylabel('Number of Estimates')
plt.grid(axis = 'both')
plt.show()
```



In [58]:
```
b_bin_min = math.floor(est_b.min())
b_bin_max = math.ceil(est_b.max())
b_bins = np.arange(b_bin_min, b_bin_max, 0.05)
```

In [59]:
```
plt.figure(figsize = (10,6), dpi = 400)
plt.hist(est_b , bins = b_bins, color = 'royalblue')
```

```
plt.xlabel('Estimate b (bin width = 0.05)')
plt.ylabel('Number of Estimates')
plt.grid(axis = 'both')
plt.show()
```



Question 3

1)

In [61]:
```
b_pressure = pd.read_excel('BloodPressure.xlsx')
```

In [66]:
```
b_pressure.describe()
```

Out[66]:

	Day	Systolic	Diastolic
count	2140.000000	2044.000000	2044.000000
mean	1070.500000	116.214775	74.436888
std	617.909109	8.734646	5.072111
min	1.000000	92.000000	55.000000
25%	535.750000	110.000000	71.000000
50%	1070.500000	116.000000	74.000000
75%	1605.250000	121.250000	78.000000
max	2140.000000	156.000000	94.000000

In [64]:
```
b_pressure["Systolic"].isnull()
b_pressure["Diastolic"].isnull()
```

Out[64]:
```
0       False
1       False
2       False
3       False
4       False
        ...
2135     True
2136     True
2137    False
2138    False
2139    False
Name: Diastolic, Length: 2140, dtype: bool
```

In [84]:
```
b_pressure.dropna(how='any', inplace=True)
```

Since we know neither the population mean nor the population variance, we can perform t-test in this example.

In [87]:
```
d_systolic = b_pressure["Systolic"].values
n = len(d_systolic)
```

In [90]:
```
from scipy.stats import t
h0_mu = 120.0
n = len(d_systolic)
x_mean = np.mean(d_systolic)
x_std = np.std(d_systolic)
t_stat = np.sqrt(n) * (x_mean - h0_mu) / x_std
t_abs = abs(t_stat)
p_value = t.cdf(-t_abs, (n-1)) + t.sf(t_abs, (n-1))
```

In [92]:
```
print("The test statistic is: ", t_stat)
```

```
The test statistic is:  -19.59715921477689
```

In [97]:
```
t_critical = t.ppf(0.975, (n-1))
x_lower = h0_mu - t_critical * x_std / np.sqrt(n)
x_upper = h0_mu + t_critical * x_std / np.sqrt(n)
print("The x_mean is: ", x_mean)
print("The test rejection region is: (", str(x_lower), ",", str(x_upper), ")")
```

```
The x_mean is:  116.21477495107632
The test rejection region is: ( 119.62120516855424 , 120.37879483144576 )
```

In [100…
```
print("The critical value is: ", t_critical, ", so the rejection region is etiher t >
```

```
The critical value is:  1.961125831524342 , so the rejection region is etiher t > 1.9
6 or t < -1.96 for two-tail test with significance level 5%
```

In [94]:
```
print("The test significance value (aka p-value is: )", p_value)
```

```
The test significance value (aka p-value is: ) 1.6882775350454464e-78
```

In [101…
```
print("We have enough evidence to reject the null hypothesis if we set significance le
```

```
We have enough evidence to reject the null hypothesis if we set significance level at
5%
```

2)

In [102…
```
d_diastolic = b_pressure["Diastolic"].values
n = len(d_diastolic)
```

In [103…
```
from scipy.stats import t
h0_mu = 80.0
n = len(d_diastolic)
x_mean = np.mean(d_diastolic)
x_std = np.std(d_diastolic)
t_stat = np.sqrt(n) * (x_mean - h0_mu) / x_std
t_abs = abs(t_stat)
p_value = t.cdf(-t_abs, (n-1)) + t.sf(t_abs, (n-1))
```

In [104…
```
print("The test statistic is: ", t_stat)
```

```
The test statistic is:  -49.599317082088646
```

In [105…
```
t_critical = t.ppf(0.975, (n-1))
x_lower = h0_mu - t_critical * x_std / np.sqrt(n)
x_upper = h0_mu + t_critical * x_std / np.sqrt(n)
print("The x_mean is: ", x_mean)
print("The test rejection region is: (", str(x_lower), ",", str(x_upper), ")")
```

```
The x_mean is:  74.43688845401174
The test rejection region is: ( 79.78003806507189 , 80.21996193492811 )
```

In [106…
```
print("The critical value is: ", t_critical, ", so the rejection region is etiher t >
```

```
The critical value is:  1.961125831524342 , so the rejection region is etiher t > 1.9
6 or t < -1.96 for two-tail test with significance level 5%
```

In [107…
```
print("The test significance value (aka p-value is: )", p_value)
```

```
The test significance value (aka p-value is: ) 0.0
```

In [108…
```
print("We have enough evidence to reject the null hypothesis if we set significance le
```

```
We have enough evidence to reject the null hypothesis if we set significance level at
5%
```

3)

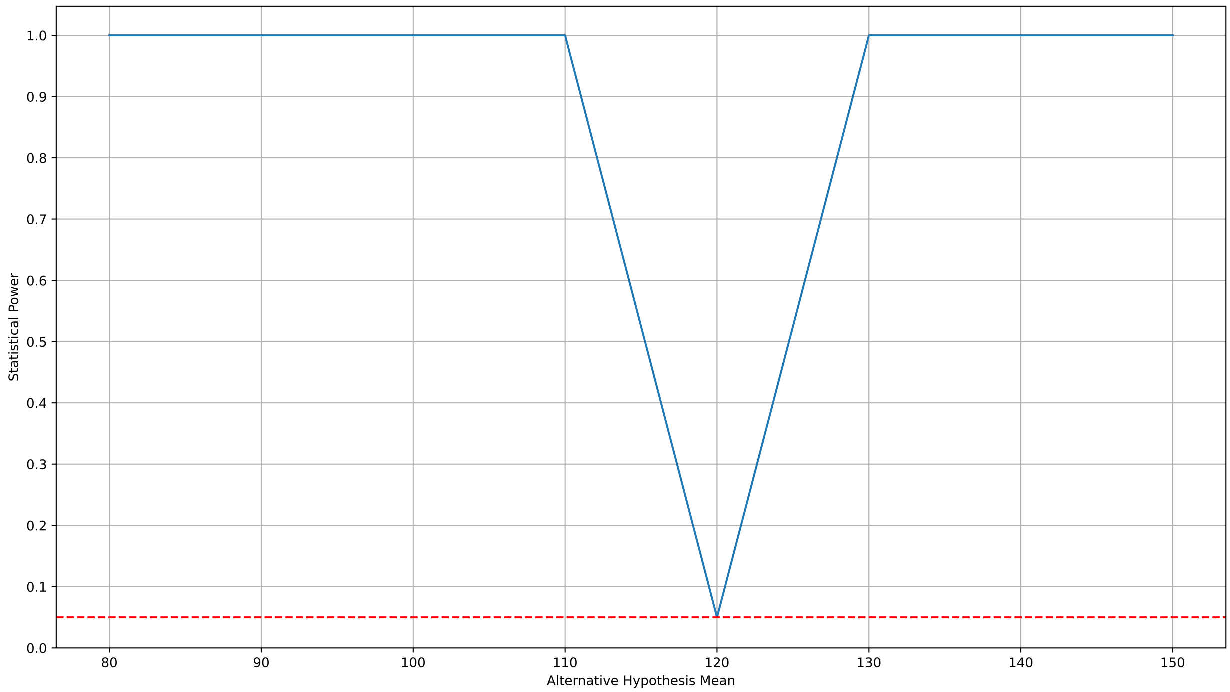
In [140…
```
from scipy.stats import norm
h0_mu = 120
h0_std = 8
n = len(d_systolic)
x_mean = np.mean(d_systolic)
z_critical = norm.ppf(0.975)
x_lower = h0_mu - z_critical * h0_std / np.sqrt(n)
x_upper = h0_mu + z_critical * h0_std / np.sqrt(n)
```

In [144…
```
xbar_std = h0_std / np.sqrt(n)

mu_value = np.arange(80, 160, 10)
stat_power = []
for h1_mu in mu_value:
    prob = norm.cdf(x_lower, loc = h1_mu, scale = xbar_std) + norm.sf(x_upper, loc = h
    stat_power.append(prob)
```

In [145…
```
plt.figure(figsize = (16,9), dpi = 1600)
```

```
plt.plot(mu_value, stat_power)
plt.axhline(y = 0.05, linestyle = '--', color = 'red')
plt.xlabel('Alternative Hypothesis Mean')
plt.ylabel('Statistical Power')
plt.xticks(range(80,160,10))
plt.yticks(np.arange(0.0,1.1,0.1))
plt.grid(axis = 'both')
plt.show()
```



In [146…

```
df = pd.DataFrame(stat_power, index=mu_value, columns=['Statistical Power'])
df
```

Out[146]:

	Statistical Power
80	1.00
90	1.00
100	1.00
110	1.00
120	0.05
130	1.00
140	1.00
150	1.00

In [ ]: