# 계량경제학 HW3

2020-15709 나승환

1. $TestScore_i = \beta_0 + \beta_1 hsize + \beta_2 hsize^2 + \beta_3 female + \beta_4 black + \beta_5 (female \times black) + u_i$로 두자.

(i) female $= 0$, black $= 1$, hsize $= 3$

$$\widehat{TestScore} = 42.84 + 2.40 - 0.81 - 7.08 = 37.35$$

(ii) T-Statistic for $hsize^2 = \left|\frac{-0.09}{0.022}\right| \approx 4.1$

즉 $hsize^2$ 항은 유의수준 95%에서 유의하다 할 수 있고,

model을 hsize에 대해 정리하고 나머지 term을 하나의 값으로 고정해 합쳐 상수 C로 정리하면 model: $-0.09h^2 + 0.8h + C$.

$= -0.09\left(h - \frac{40}{9}\right)^2 + C - \frac{16}{90}$이므로

hsize $= \frac{40}{9}$에서 TestScore가 (다른 변수가 각각 특정한 값으로 given되면) 최대화된다.

(iii) nonblack female : female $= 1$, black $= 0$ - $(a_1)$

nonblack male : female $= 0$, black $= 0$ - $(a_2)$

$(a_1)$에서 $\widehat{TestScore} = 42.84 + 0.8h - 0.09h^2 - 1.88$

$(a_2)$에서 $\widehat{TestScore} = 42.84 + 0.8h - 0.09h^2$

따라서 being female은 $-1.88$점의 효과가 있다고 추정되고 (estimated difference) $\left|-\frac{1.88}{0.20}\right| = 9.4 > 1.96$ 이므로 이 계수는 유의하다 (at 유의수준 95%)

(iv) $(a_3)$ nonblack male : female $= 0$, black $= 0$

$(a_4)$ black male : female $= 0$, black $= 1$

$(a_3)$에서 $\widehat{TestScore} = 42.84 + 0.8h - 0.09h^2$

$(a_4)$에서 $\widehat{TestScore} = 42.84 + 0.8h - 0.09h^2 - 7.08$

Being black given male은 $-7.08$점의 estimated difference를 가지고 $H_0 : \beta_4 = 0$, $H_1 : \beta_4 \neq 0$으로 두면

T-Statistic for test $= \left|-\frac{7.08}{0.53}\right| \approx 13.36 > 1.96$이므로 95% 유의수준에서 $H_0$을 기각한다.

(v) $(a_5)$ black female : black $= 1$, female $= 1$

$(a_6)$ nonblack female : black $= 0$, female $= 1$

$(a_5)$에서 $\widehat{TestScore} = C - 1.88 - 7.08 + 2.60$,

$(a_6)$에서 $\widehat{TestScore} = C - 1.88$, where $C = 42.84 + 0.80h - 0.09h^2$. ($h$ = hsize)

즉 Estimated Difference는 $2.60 - 7.08 = -4.48$

Being black given female은 $-4.48$의 차이를 가질 것으로 추산되며, $H_0 : \beta_4 = 0 \wedge \beta_5 = 0$ $H_1$ : not $H_0$이므로 제약식이 2개인 F-test 등을 사용할 수 있다.

$$F \approx \frac{1}{2}\left(\frac{9.4^2 + 13.36^2 - 2\hat{\rho}_{t_4,t_5} \cdot 9.4 \cdot 13.36}{1 - \hat{\rho}^2_{t_4,t_5}}\right)$$ 이므로,

$\hat{\rho}_{t_4,t_5}$의 값을 구해야 한다.

2. $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i = P(Y_i = 1)$, $P(Y_i = 0) = 1 - \bar{Y}$.

$\hat{g}_0 = P(Y_i = 0, \hat{Y}_i < 0.5)$

$\hat{g}_1 = P(Y_i = 1, \hat{Y}_i > 0.5)$ 에서

$$\hat{P} = \frac{1}{n}\left\{\underbrace{nP(Y_i=0)}_{\#\text{ of } Y_i=0} \cdot \hat{g}_0 + \underbrace{nP(Y_i=1)}_{\#\text{ of } Y_i=1} \cdot \hat{g}_1\right\}$$

$$= P(Y_i = 0)\hat{g}_0 + P(Y_i = 1)\hat{g}_1$$

$$= \hat{g}_0(1 - \bar{Y}) + \hat{g}_1\bar{Y}$$

# Econometrics HW3 Part 3

Na SeungChan

## HW2 Q3 : Previous Data Analysis

해당 파트는 HW2의 제출 내용을 그대로 복사해 온 것으로, 데이터 전처리의 편의를 위해서만 사용되고 문제풀이의 내용이 아님.

```
dataset <- read.csv('co2-emissions-vs-gdp.csv') %>%
  filter(Year == 2018) %>%
  filter(Code != '', Code != 'OWID_WRL')

dataset1 <- read.csv('energy.csv') %>%
  filter(Year == 2018) %>%
  filter(Code != '', Code != 'OWID_WRL')

full_data <- left_join(dataset, dataset1)
```

```
## Joining, by = c("Entity", "Code", "Year")
```

```
glimpse(full_data)
```

```
## Rows: 238
## Columns: 9
## $ Entity                                          <chr> "Afghanistan", "Alb~
## $ Code                                            <chr> "AFG", "ALB", "DZA"~
## $ Year                                            <int> 2018, 2018, 2018, 2~
## $ Annual.CO..emissions..per.capita.               <dbl> 0.2948759, 1.732364~
## $ GDP.per.capita                                  <dbl> 1934.555, 11104.166~
## $ X417485.annotations                             <chr> "", "", "", "", "",~
## $ Population..historical.estimates.               <dbl> 36686788, 2877019, ~
## $ Continent                                       <chr> "", "", "", "", "",~
## $ Primary.energy.consumption.per.capita..kWh.person. <dbl> 1144.532, 14483.855~
```

Data form our world in data. Focused on countries in 2018 only. First dataset is from https://ourworldindata.org/grapher/co2-emissions-vs-gdp , Second dataset is from https://ourworldindata.org/energy

full_data are the merged data of above two.

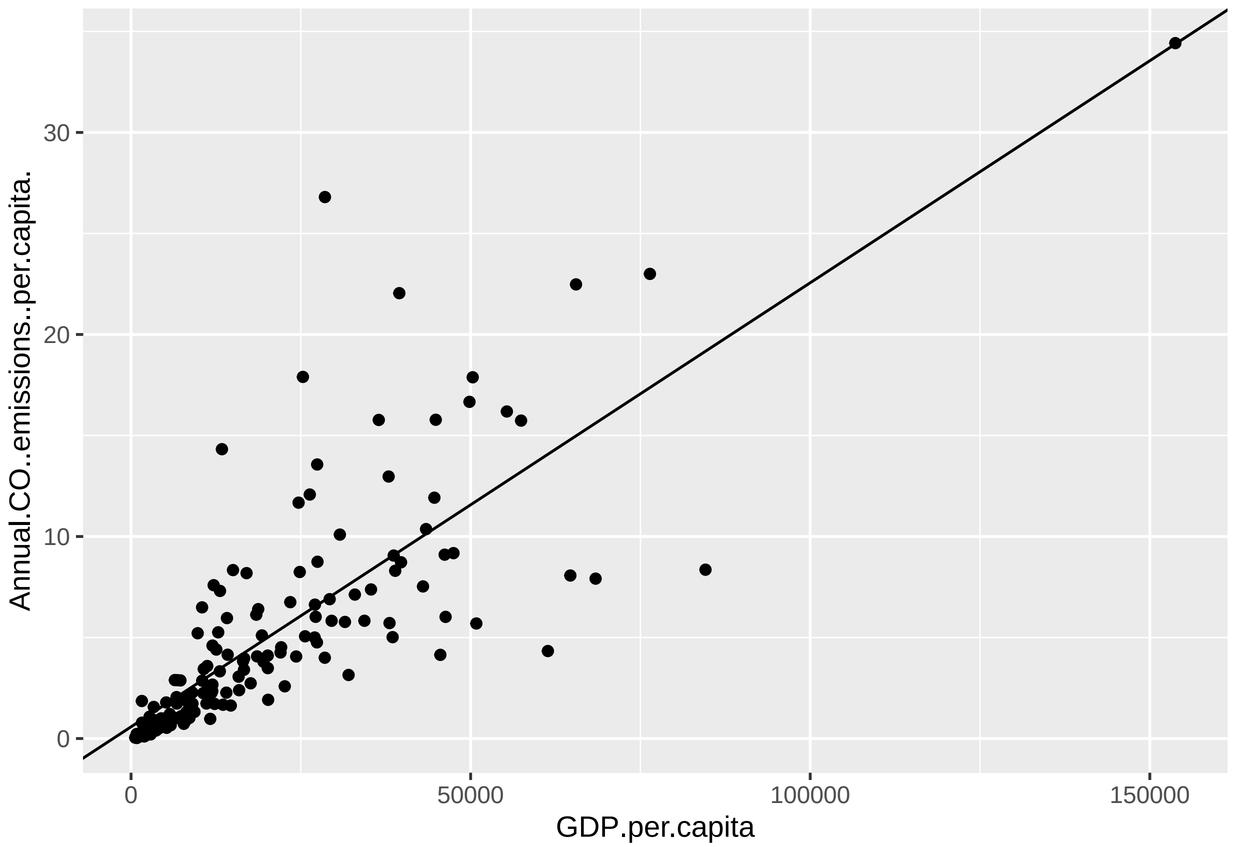
```
lm_a <- lm_robust(Annual.CO..emissions..per.capita. ~ GDP.per.capita, data = full_data)
summary(lm_a)
```

```
##
## Call:
## lm_robust(formula = Annual.CO..emissions..per.capita. ~ GDP.per.capita,
##     data = full_data)
##
## Standard error type:  HC2
##
## Coefficients:
```

```
##               Estimate Std. Error t value  Pr(>|t|) CI Lower  CI Upper  DF
## (Intercept)    0.5783892  2.564e-01   2.256 2.544e-02 0.072006 1.0847728 162
## GDP.per.capita 0.0002198  1.863e-05  11.798 1.400e-23 0.000183 0.0002565 162
##
## Multiple R-squared:  0.6284 ,    Adjusted R-squared:  0.6262
## F-statistic: 139.2 on 1 and 162 DF,  p-value: < 2.2e-16
```

```
ggplot(data = full_data, aes(x = GDP.per.capita, y = Annual.CO..emissions..per.capita.)) +
  geom_point() +
  geom_abline(intercept = 0.5783892, slope = 0.0002198)
```

```
## Warning: Removed 74 rows containing missing values (geom_point).
```



Research Question is about the causal correlation between the GDP per capita and annual CO2 emission per capita.

se(intercept) : 2.564e-01, se(slope) : 1.863e-05

p-value for slope Pr(>|t|) = <2.2e-16 ***. This means the significance of the regression model.

Add additional variables X2, ⋯, Xk that may affect Y and estimate the multiple regression model. Compare the coefficient estimate of X1 with (i). Note that the main research question is unchanged. The additional variables are supposed to eliminate possible omitted variable bias in the simple regression.

```
lm_b <- lm_robust(Annual.CO..emissions..per.capita. ~ GDP.per.capita + Primary.energy.consumption.per.capita..k
summary(lm_b)
```

```
##
## Call:
```

```
## lm_robust(formula = Annual.CO..emissions..per.capita. ~ GDP.per.capita +
##     Primary.energy.consumption.per.capita..kWh.person., data = full_data)
##
## Standard error type:  HC2
##
## Coefficients:
##                                                    Estimate Std. Error t value
## (Intercept)                                       6.754e-01  2.466e-01   2.739
## GDP.per.capita                                    6.932e-05  5.621e-05   1.233
## Primary.energy.consumption.per.capita..kWh.person. 1.017e-04  4.283e-05   2.375
##                                                    Pr(>|t|)   CI Lower
## (Intercept)                                         0.00686  1.884e-01
## GDP.per.capita                                      0.21928 -4.168e-05
## Primary.energy.consumption.per.capita..kWh.person.  0.01873  1.714e-05
##                                                    CI Upper  DF
## (Intercept)                                       1.1624382 161
## GDP.per.capita                                    0.0001803 161
## Primary.energy.consumption.per.capita..kWh.person. 0.0001863 161
##
## Multiple R-squared:  0.7447 ,    Adjusted R-squared:  0.7416
## F-statistic: 75.44 on 2 and 161 DF,  p-value: < 2.2e-16
```

Now I added X2 as ‘energy use per capita.’ This can be helpful because some of developed countries may use more non-fossil fuel energy.

p-value for ‘energy use per capita’ is smaller, and adjusted R^2 is bigger for the second model. But, ‘the usage of eco-frendily energy’ may be more helpful for this purpose, but making data like that was seriouly hard data wrangling(keys are not consistent). Making those data can improve the quality of this research.

## HW3 - Q3

You may continue on your previously collected dataset, or select another cross- sectional data. Let the variables of interest be Y and X1. You may also consider X2, ⋯, Xk that may directly affect Y . The main research question is ”Does X1 affect Y ? If so, how much?”.

(i) Conduct a nonlinear regression of Y on X1 controlling for X2, ⋯, Xk and draw the fitted curve on the scatter plot and compare it with the fitted line of the linear regression.

(ii)Provide the standard errors for the point estimates. Interpret the result.

### HW3 - Q3.(i)

```
temp_med <- median(na.omit(full_data$Primary.energy.consumption.per.capita..kWh.person.))

full_data_bined <- full_data %>%
  mutate(HiEnergy = ifelse(Primary.energy.consumption.per.capita..kWh.person. >= temp_med, 1, 0))

remove(temp_med)

head(full_data_bined)
```

```
##           Entity Code Year Annual.CO..emissions..per.capita. GDP.per.capita
## 1    Afghanistan  AFG 2018                         0.2948759       1934.555
## 2        Albania  ALB 2018                         1.7323643      11104.166
## 3        Algeria  DZA 2018                         4.1479607      14228.025
## 4 American Samoa  ASM 2018                                NA             NA
## 5      Andorra  AND 2018                   6.5922117          NA
## 6       Angola  AGO 2018                   0.7283953    7771.442
##   X417485.annotations Population..historical.estimates. Continent
## 1                                  36686788
## 2                                   2877019
## 3                                  41927008
## 4                                     48445
## 5                                     75034
## 6                                  31273538
##   Primary.energy.consumption.per.capita..kWh.person. HiEnergy
## 1                                           1144.532        0
## 2                                          14483.855        0
## 3                                          16012.625        0
## 4                                          29712.482        1
## 5                                                 NA       NA
## 6                                           3165.137        0
```

Hienergy 변수를 생성하여 통제를 진행하였다. 기준은 na.omit를 사용하여 na values인 국가들을 제외한 뒤의 중앙값이다. 단, 이와 같은 'NA 무작정 제거'가 통계적으로 문제가 될 수 있는데, 해당 데이터에서는 국가의 행정력이 잘 미치지 못하는 국가일수록 na가 발생할 가능성이 높아 NA에 체계적인 경향이 발생할 수 있기 때문이다. 차후 이와 같은 문제를 검토하기 위한 통계적 기법에 따른 연구를 수행하여야 할 것이다.

```
lm_c <- lm_robust(log(Annual.CO..emissions..per.capita.) ~ GDP.per.capita + HiEnergy + GDP.per.capita:HiEnergy ,
summary(lm_c)
```

```
##
## Call:
## lm_robust(formula = log(Annual.CO..emissions..per.capita.) ~
##     GDP.per.capita + HiEnergy + GDP.per.capita:HiEnergy, data = full_data_bined)
##
## Standard error type:  HC2
##
## Coefficients:
##                           Estimate Std. Error t value  Pr(>|t|)   CI Lower
## (Intercept)             -1.5929412  1.566e-01 -10.174 4.617e-19 -1.9021573
## GDP.per.capita           0.0002086  1.957e-05  10.658 2.223e-20  0.0001699
## HiEnergy                 2.9736159  1.910e-01  15.570 7.276e-34  2.5964458
## GDP.per.capita:HiEnergy -0.0001918  1.978e-05  -9.694 9.086e-18 -0.0002308
##                           CI Upper  DF
## (Intercept)             -1.2837251 160
## GDP.per.capita           0.0002472 160
## HiEnergy                 3.3507860 160
## GDP.per.capita:HiEnergy -0.0001527 160
##
## Multiple R-squared:  0.8265 ,    Adjusted R-squared:  0.8233
## F-statistic: 204.5 on 3 and 160 DF,  p-value: < 2.2e-16
```

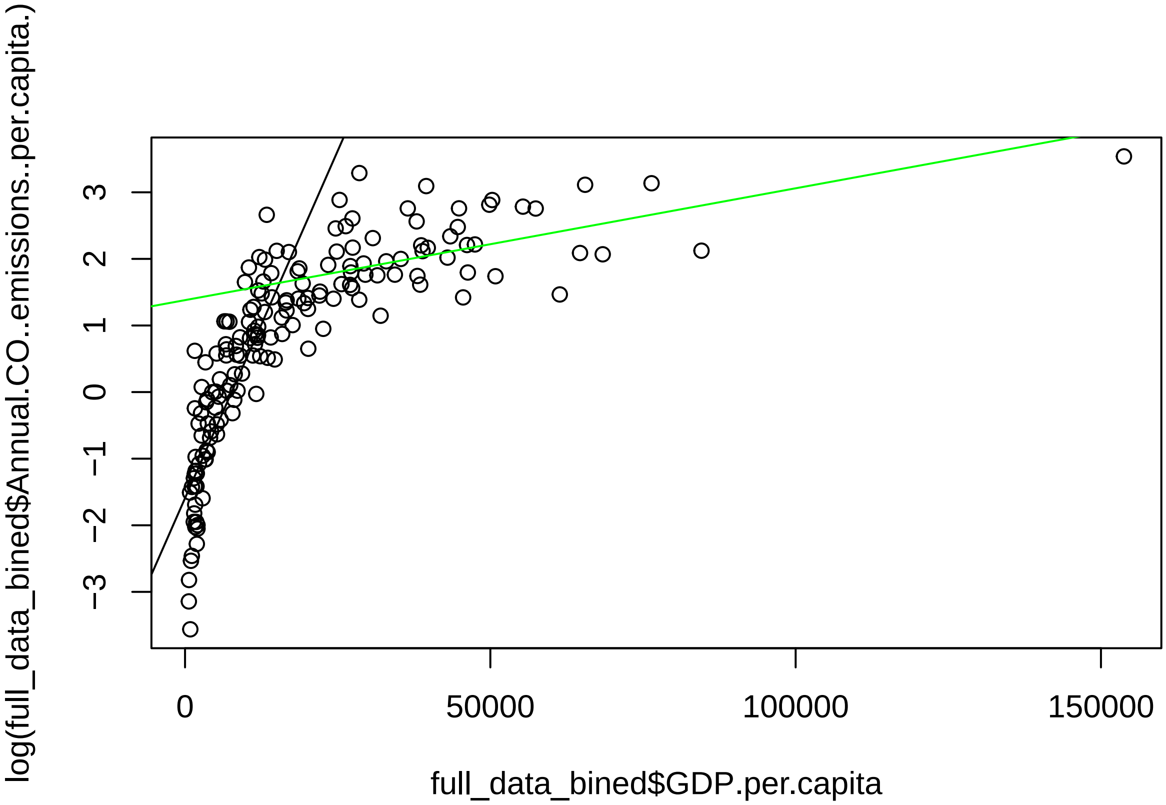
```
lm_c$coefficients
```

```
##             (Intercept)          GDP.per.capita                HiEnergy
##           -1.5929411999            0.0002085654            2.9736158920
## GDP.per.capita:HiEnergy
##           -0.0001917723
```

```
plot(full_data_bined$GDP.per.capita, log(full_data_bined$Annual.CO..emissions..per.capita.))
```

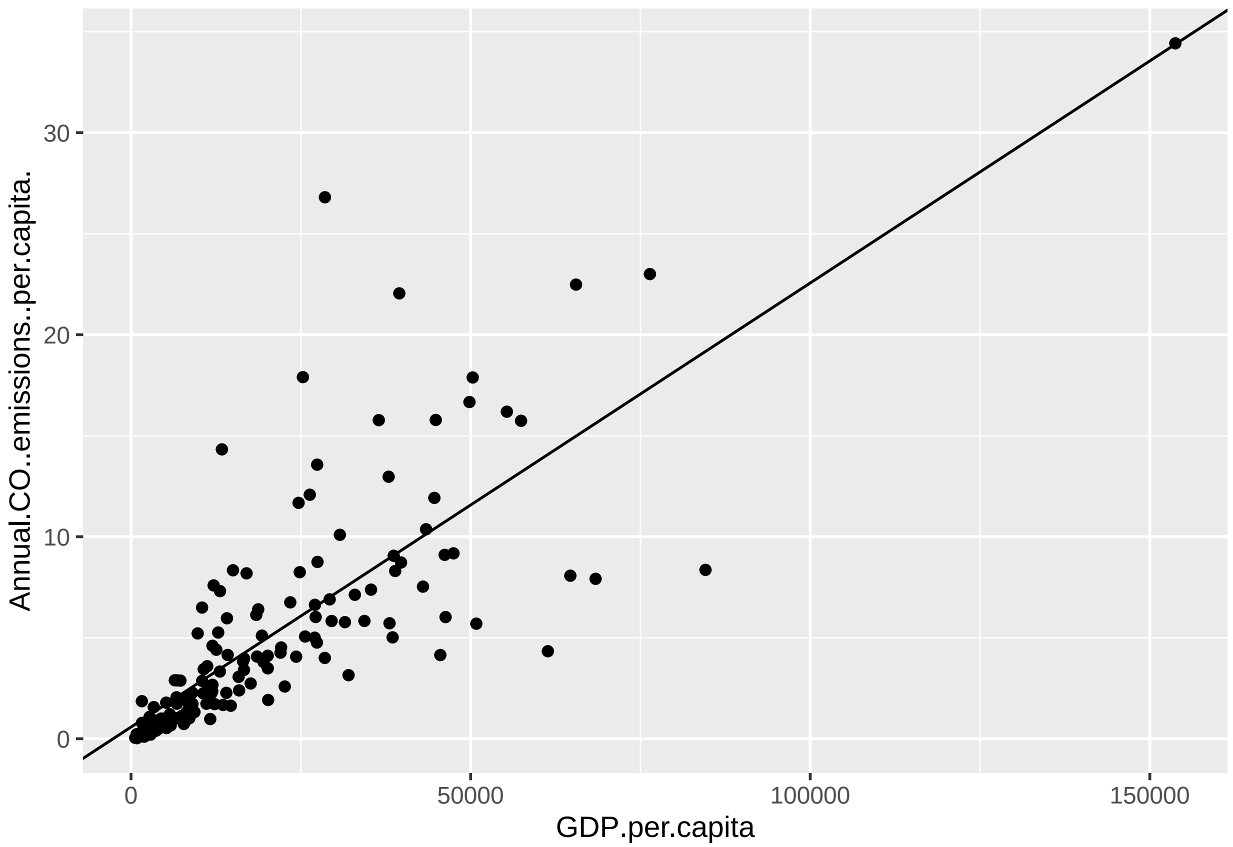
```
abline(coef = c(lm_c$coefficients[1], lm_c$coefficients[2]))

abline(coef = c(lm_c$coefficients[1] + lm_c$coefficients[3], lm_c$coefficients[2] + lm_c$coefficients[4]), col = 'green'
```



```
ggplot(data = full_data, aes(x = GDP.per.capita, y = Annual.CO..emissions..per.capita.)) +
  geom_point() +
  geom_abline(intercept = 0.5783892, slope = 0.0002198)
```

```
## Warning: Removed 74 rows containing missing values (geom_point).
```



아래쪽 그림의 linear model과 비교했을 때 교호작용 term을 포함한 model은 가파른 기울기의 한 직선(Hi = 0)과 상대적으로 완만한 직선(Hi = 1)으로 구분되며, 교호작용 term의 유의성이 있을 것이라고 추측할 수 있다.

**HW3 - Q3.(ii)**

```
summary(lm_c)
```

```
##
## Call:
## lm_robust(formula = log(Annual.CO..emissions..per.capita.) ~
##     GDP.per.capita + HiEnergy + GDP.per.capita:HiEnergy, data = full_data_bined)
##
## Standard error type:  HC2
##
## Coefficients:
##                          Estimate Std. Error t value  Pr(>|t|)   CI Lower
## (Intercept)             -1.5929412  1.566e-01 -10.174 4.617e-19 -1.9021573
## GDP.per.capita           0.0002086  1.957e-05  10.658 2.223e-20  0.0001699
## HiEnergy                 2.9736159  1.910e-01  15.570 7.276e-34  2.5964458
## GDP.per.capita:HiEnergy -0.0001918  1.978e-05  -9.694 9.086e-18 -0.0002308
##                          CI Upper  DF
## (Intercept)             -1.2837251 160
## GDP.per.capita           0.0002472 160
## HiEnergy                 3.3507860 160
## GDP.per.capita:HiEnergy -0.0001527 160
```

```
##
## Multiple R-squared:  0.8265 ,    Adjusted R-squared:  0.8233
## F-statistic: 204.5 on 3 and 160 DF,  p-value: < 2.2e-16
```

HiEnergy = 0인 경우의 회귀선은 log(y) = -1.5929 + 0.0002086x

HiEnergy = 1인 경우의 회귀선은 log(y) = lm_c$coefficients[1] + lm_c$coefficients[3] + lm_c$coefficients[2] + lm_c$coefficients[4]x

Interpret the result.

즉 x의 변화량으로 E(Y|D,X)의 변화량을 나눈 값은 lm_c$coefficients[2] + lm_c$coefficients[3]D이다.