1.3.1. $f(x)>0,\ f(x-1)>0$이므로 $\frac{f(x)}{f(x-1)}\ge 1$임을 보이자.

$$\frac{f(x)}{f(x-1)} = \frac{\frac{n!}{x!(n-x)!}}{\frac{n!}{(x-1)!(n-x+1)!}} \times \frac{p^x(1-p)^{n-x}}{p^{x-1}(1-p)^{n-x+1}}$$

$$= \frac{p}{1-p}\times\frac{n-x+1}{x} = \frac{p}{1-p}\left(\frac{n+1}{x}-1\right)$$

이제 $\frac{p}{1-p}\left(\frac{n+1}{x}-1\right)\ge 1$을 풀면 $\frac{p}{1-p}>0$이므로

$$\frac{n+1}{x}\ge\frac{1-p}{p}+1=\frac{1}{p} \quad \therefore x\le p(n+1)$$

즉 $x\le p(n+1)$이면 $f(x)>f(x-1)$이므로

f는 해당 범위에서만 증가하고 $x+1>p(n+1)$인 ~~x에서~~ x가 정수가 되도록 하면 된다.

즉 $x=\lfloor p(n+1)\rfloor$일 때 $f(x-1)\le f(x)\ge f(x+1)$

이고, 이는 $f(0)\le\cdots\le f(x-1),\ f(x+1)\ge\cdots f(n)$이다.

1.3.2. $E(X(X-1)\cdots(X-r+1)) = E\left(\frac{X!}{(X-r)!}\right)$

$$=\sum_{x=0}^{n}\frac{x!}{(x-r)!}\times\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$=\sum_{x=r}^{n}\frac{n!}{(n-r)!}\times\frac{(n-r)!}{(n-x)!(x-r)!}p^x(1-p)^{n-x}$$

$$=\frac{n!}{(n-r)!}\sum_{x-r=0}^{n}\binom{n-r}{x-r}p^r\cdot p^{x-r}(1-p)^{(n-r)-(x-r)}$$

$$=\frac{n!}{(n-r)!}p^r\{p+(1-p)\}^{n-r}$$

$$=p^r\cdot n(n-1)\cdots(n-r+1)$$

1.3.5. $P(X\le x)=\sum_{k=1}^{x}p(1-p)^{k-1}=1-(1-p)^x$

$P(X>x)=(1-p)^x$

$P(X>k+j\mid X>k)=\frac{(1-p)^{k+j}}{(1-p)^k}$

$=(1-p)^j=P(X>j)$

1.3.8. $X_1, X_2,\cdots$가 독립이므로 다음 성공까지의 시행 수 $W_1, W_2-W_1,\cdots,W_r-W_{r-1}$ 끼리는 서로 독립이고 동일한 분포를 따른다.(기하분포)

즉 $cov(W_1,W_r)=cov(W_1,W_1)+cov(W_1,W_2-W_1)+\cdots+cov(W_1,W_r-W_{r-1})$

$=Var(W_1)=\frac{1-p}{p^2}$

2. $a_{N,x}=P(X_N=x)=\binom{D}{x}\binom{N-D}{n-x}\Big/\binom{N}{n}$

$$=\frac{n!(N-n)!D!(N-D)!}{N!x!(D-x)!(n-x)!(N-n-D+x)!}\times\frac{(N-x)!}{(N-x)!}$$

$$=\binom{n}{x}\times\frac{Np(Np-1)\cdots(Np-x+1)}{N(N-1)\cdots(N-x+1)}\times\frac{(N-Np)(N-Np-1)\cdots(N-Np-n+x+1)}{(N-x)(N-x-1)\cdots(N-n+1)}$$

$$\lim_{N\to\infty}a_{N,x}=\binom{n}{x}\left(\frac{Np}{N}\right)^x\left(\frac{N-Np}{N}\right)^{n-x}=\binom{n}{x}p^x(1-p)^{n-x}$$

$$\lim_{N\to\infty}P(X_N\le x)=\lim_{N\to\infty}\sum_{k=0}^{x}a_{N,k}=\sum_{k=0}^{x}\binom{n}{k}p^k(1-p)^{n-k}$$

$=P(Z\le x)$

3. $\underbrace{\quad}_{w_1-1}\ \underset{W_1}{\circ}\ \underbrace{\quad}_{w_2-w_1-1}\ \underset{W_2}{\circ}\ \cdots\ \underset{W_{r-1}}{\circ}\ \underbrace{\quad}_{w_r-w_{r-1}-1}\ \underset{W_r}{\circ}$

(a) $P(W_1=w_1,\cdots,W_n=w_n)=(1-p)^{w_r-r}p^r$ $(0<w_1<\cdots<w_r,\ w_i\in\mathbb{Z})$

(b) $P(W_r=w_r)=\binom{w_r-1}{r-1}p^r(1-p)^{w_r-r}$

$$P(W_1,\cdots,W_{r-1}\mid W_r=t)=\frac{1}{\binom{t-1}{r-1}}$$ $(0<w_1<\cdots<w_{r-1}<t)$ $(w_i\in\mathbb{Z})$