

$$3.1. f(x) > 0, f(x-1) > 0 \Rightarrow \frac{f(x)}{f(x-1)} \geq 1 \text{ 일 때 } x \leq p(n+1)$$

$$\frac{f(x)}{f(x-1)} = \frac{\frac{n!}{x!(n-x)!}}{\frac{n!}{(x-1)!(n-x+1)!}} \times \frac{p^x(1-p)^{n-x}}{p^{x-1}(1-p)^{n-x+1}}$$

$$= \frac{p}{1-p} \times \frac{n-x+1}{x} = \frac{p}{1-p} \left(\frac{n+1}{x} - 1 \right)$$

$$\text{이제 } \frac{p}{1-p} \left(\frac{n+1}{x} - 1 \right) \geq 1 \text{ 을 풀면 } \frac{n+1}{x} \geq \frac{1-p}{p} + 1 = \frac{1}{p}$$

$$\frac{n+1}{x} \geq \frac{1-p}{p} + 1 = \frac{1}{p} \therefore x \leq p(n+1)$$

즉 $x \leq p(n+1)$ 이면 $f(x) > f(x-1)$ 이므로
 f 는 해당 범위에서만 증가하고 $x+1 > p(n+1)$ 이
 ~~x 가 정수가 되도록 하면 된다.~~

즉 $x = \lfloor p(n+1) \rfloor$ 일 때 $f(x-1) \leq f(x) \geq f(x+1)$
 이고, 이는 $f(0) \leq \dots \leq f(x-1), f(x+1) \geq \dots \geq f(n)$ 이다.

$$1.3.2. E(X(X-1) \dots (X-r+1)) = E\left(\frac{x!}{(x-r)!}\right)$$

$$= \sum_{x=0}^n \frac{x!}{(x-r)!} \times \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=r}^n \frac{n!}{(n-r)!} \times \frac{(n-r)!}{(n-x)!(x-r)!} p^x (1-p)^{n-x}$$

$$= \frac{n!}{(n-r)!} \sum_{x=r}^n \binom{n-r}{x-r} p^r p^{x-r} (1-p)^{(n-r)-(x-r)}$$

$$= \frac{n!}{(n-r)!} p^r \{p + (1-p)\}^{n-r}$$

$$= p^r \cdot n(n-1) \dots (n-r+1)$$

$$1.3.5. P(X \leq x) = \sum_{k=1}^x p(1-p)^{k-1} = 1 - (1-p)^x$$

$$P(X > x) = (1-p)^x$$

$$P(X > R+j | X > R) = \frac{(1-p)^{R+j}}{(1-p)^R}$$

$$= (1-p)^j = P(X > j)$$

1.3.8. X_1, X_2, \dots 가 독립이므로 다음 성공까지의
 시행 수 $W_1, W_2 - W_1, \dots, W_r - W_{r-1}$ 가
 서로 독립이고 동일한 분포를 따른다. (기하분포)

$$\therefore \text{COV}(W_1, W_r) = \text{COV}(W_1, W_1) +$$

$$\text{COV}(W_1, W_2 - W_1) + \dots + \text{COV}(W_1, W_r - W_{r-1})$$

$$= \text{Var}(W_1) = \frac{1-p}{p^2}$$

$$2. a_{n,x} = P(X_n = x) = \frac{\binom{n}{x} \binom{n-D}{n-x}}{\binom{n}{x}}$$

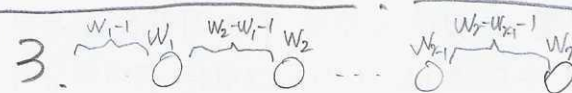
$$= \frac{n!(n-n)! D! (n-D)!}{n! x! (D-x)! (n-x)! (n-n-D+x)!} \times \frac{(n-x)!}{(n-x)!}$$

$$= \binom{n}{x} \times \frac{N(N-1) \dots (N-x+1)}{N(N-1) \dots (N-x+1)} \times \frac{(N-Np)(N-Np-1) \dots (N-Np-n+x+1)}{(N-x)(N-x-1) \dots (N-n+1)}$$

$$\lim_{N \rightarrow \infty} a_{n,x} = \binom{n}{x} \left(\frac{Np}{N} \right)^x \left(\frac{N-Np}{N} \right)^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\lim_{N \rightarrow \infty} P(X_N \leq x) = \lim_{N \rightarrow \infty} \sum_{k=0}^x a_{N,k} = \sum_{k=0}^x \binom{n}{k} p^k (1-p)^{n-k}$$

$$= P(Z \leq x)$$



$$(a) P(W_1 = W_1, \dots, W_n = W_n) = (1-p)^{W_r-r} p^r \quad (W_i \in \mathbb{Z})$$

$$(b) P(W_r = W_r) = \binom{W_r-1}{r-1} p^r (1-p)^{W_r-r}$$

$$P(W_1, \dots, W_{r-1} | W_r = t) = \frac{1}{\binom{t-1}{r-1}} \quad (0 \leq W_1 \leq \dots \leq W_{r-1} < t, W_i \in \mathbb{Z})$$