ECON 476
Fall 2023

PROBLEM SET II

1. Hirshleifer and Riley's textbook p. 133 # 5.

2. Hirshleifer and Riley's textbook p. 162 # 4.

(Note)
(1) Equation (4.3.8) in the textbook (p. 156) should change to

$$\frac{[\mu_a - (1+R_1)P_a^A]\sigma_F}{\sigma_{aF}} = \theta = \frac{\mu_F - (1+R_1)P_F^A}{\sigma_F}$$

(2) Also, in (F),

$$P_a^A = \frac{1}{1+R}(\mu_a - \frac{\overline{\alpha}}{J}\sigma_a^2)$$

should change to

$$P_a^A = \frac{1}{1+R}(\mu_a - \frac{\overline{\alpha}}{J}\omega_a\sigma_a^2),$$

where ω_a is the proportion of asset a in the market portfolio F.

3. Hirshleifer and Riley's textbook p. 163 # 5.

(Note) In (D), N should change to J.

4. Suppose there are two types of buyers. Type α has a valuation of 1 and type β has a valuation of 2.

(a) Characterize equilibrium bidding behaviors of both types of buyers in an open ascending bid auction.

(b) Suppose there are just two buyers. Each buyer is of type α with probability 1/2 and type β with probability 1/2. Confirm that the seller's equilibrium expected revenue in an open ascending bid auction is 1.25.

(c) If the object is sold in a sealed high bid auction, explain why type α buyers will, in equilibrium, bid 1 and type β buyers will adopt a purely mixed strategy $G(b)$.

(d) Confirm that $G(b) = (b-1)/(2-b)$ is type β buyers' equilibrium mixed strategy. Also, show that $\underline{b} = 1$ and $\overline{b} = 1.5$.

(e) Hence, show that the revenue equivalence theorem continues to hold.

5. There are two risk-neutral buyers, B_1 and B_2, whose valuation is one out of three possible values, V_0, V_1, V_2, where $V_0 = 0$ and $0 < V_1 < V_2$, and each buyer's exact valuation is a private value. The joint probability distribution, which is common knowledge, is given by the following (3×3) matrix.

$$\begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix},$$

where p_{ij} denotes the probability that B_1's valuation is V_i and B_2's valuation is V_j. Assume that those values are symmetric but affiliated. That is, for example,

$$\begin{aligned} \frac{p_{10}}{p_{10}+p_{11}} &\geq \frac{p_{20}}{p_{20}+p_{21}} \\ \frac{p_{10}}{p_{10}+p_{11}+p_{12}} &\geq \frac{p_{20}}{p_{20}+p_{21}+p_{22}} \\ \frac{p_{10}+p_{11}}{p_{10}+p_{11}+p_{12}} &\geq \frac{p_{20}+p_{21}}{p_{20}+p_{21}+p_{22}}. \end{aligned} \tag{1}$$

(a) Find each type's equilibrium expected payoff in the open-(ascending) bid auction.

(b) Find each type's equilibrium expected payoff in the (first) sealed-bid auction.

(c) Using (1) above, show that the seller's expected revenue in the open-(ascending) bid auction is greater than that in the (first) sealed-bid auction.

6. A worker with a marginal value product of θ can earn $w_r = k\sqrt{\theta}$ in self-employment. Marginal value products vary across the population according to the uniform distribution, i.e.,

$$\theta \sim U[0,2].$$

Firms cannot measure individual productivity, so all workers are paid a wage equal to the average of the marginal value products of those employed.

(a) Find the condition for k under which every worker will choose self-employment. Also, find the condition for k under which all the workers will choose to be employed.

(b) Assume that $k = 1$. And, each worker can take a free test which can only tell whether his marginal value product is in [0,1] or in [1,2]. Who will take the test and who will not? Also, who will be self-employed and who will be employed?

(c) Assume that $k = 1$. And, each worker can take a test at the expense of c which can reveal his marginal value product exactly. Who will take the test and who will not? Also, find the condition for c under which there exist some workers who will choose to take the test.

(d) Continuing from (c), the test is not perfect in the sense that if a worker with θ takes the test, the test reveals that the worker's marginal value product is $\theta + d$ with probability 1/2 and $\theta - d$ with probability 1/2. Will more or less workers take the test than in (c)? (Hint: After the test result is revealed, each worker can decide whether to be self-employed or employed.)

7. Consider a fire insurance market which is operated by a risk-neutral insurance company (monopolist) and many identical insureds. Each insured is risk-averse in the sense that his preference scaling function is given by $v(x) = log_{10}x$. Each insured has a house which will be sold tomorrow. The market value of the house is 100 if there is no fire but becomes 10 if it is burnt down. The probability that there is a fire on each house is p. The insurance company can do price discrimination, i.e., the insurance company can offer a set of $\{(R, y)\}$ where a specific insurance package (R, y) implies that if an insured pays R to the insurance company today, the insurance company will

pay y back to the insured provided that there is a fire on the insured's house tomorrow.

(a) By denoting x_g (each insured's income when there is no fire) on the X-axis and x_b (each insured's income when there is a fire) on the Y-axis, denote each insured's MRS (marginal rate of substitution) at (x_g, x_b) as a function of x_g and x_b. Also, show that each insured's MRS at any point on the 45 degree line must be $\frac{1-p}{p}$.

(b) Show that the insurance company's iso-profit lines on the (X_g, X_b)-space are straight lines with an absolute slope $\frac{1-p}{p}$.

(c) Using (a) and (b), show that the insurance company's optimal selling strategy is to offer (R, y) to every insured which is a full insurance and extract all the consumer surplus. (Hint: Use a graph)

(d) Now, assume that there are two types of insureds with an equal number in the market. The type F (careful type) insured's probability that there is a fire is 1/3 whereas that of the type L (careless type) insured is 1/2. Also, assume that the insurance company can identify who is who. Let (R_F, y_F) be the insurance package the insurance company offers to the type F insureds and (R_L, y_L) be the insurance package the insurance company offers to the type L insureds. Find the insurance company's optimal insurance packages (R_F^*, y_F^*) and (R_L^*, y_L^*) numerically.

(e) Continuing from (d), now assume that the insurance company cannot identify who is who. To find the insurance company's optimal selling strategy for (R_F, y_F) and (R_L, y_L) in this case, define the insurance company's optimization program.

(f) Let (R_F^o, y_F^o) and (R_L^o, y_L^o) be the optimal insurance packages in (e). Then, it is already well proven that the type F insureds strictly prefer (R_F^o, y_F^o) to (R_L^o, y_L^o). Given this, show that, at the optimum, the insurance company cannot extract all the consumer from the type L insureds whereas it can extract all the consumer surplus from the type F insureds. Also, show that, at the optimum, the type L insureds are indifferent between (R_F^o, y_F^o) and (R_L^o, y_L^o).

8. Kinko, which is a monopolist, is renting copy machines and selling copy papers. Kinko's cost of renting a copy machine is equal to 0 and its cost of producing and selling a copy paper is 5. Each consumer needs 1 copy machine and a number of copy papers to satisfy his need for copy service. There are two types of consumers in the market with the same proportion, i.e., the half of the population are type 1 consumers (denoted by B_1) and the other half of the population are type 2 consumers (denoted by B_2). B_1's demand for copy service is $p_1 = 35 - q_1$ and B_2's demand for copy service is $p_2 = 45 - q_2$. Kinko is renting a copy machine and selling copy papers to each consumer as a bundle by charging (q, R), where q is the number of papers in the bundle and R is the price for the bundle.

(a) Assume that Kinko can identify the group to which a particular consumer belongs. Define Kinko's optimization program and solve for Kinko's optimal bundling packages. Also, find Kinko's expected profit from a consumer.

(b) Now, Kinko cannot identify the group to which a particular consumer belongs but still offer the same bundling packages that you have in (a). What will happen? Also, what is Kinko's expected profit from a consumer.

(c) Define Kinko's optimization program in the case of (b) and explain each constraint in the program.

(d) Solve for the program in (c), and also find Kinko's expected profit from a consumer.

(Note: In solving (d), you do not have to prove which constraint is binding and which constraint is not binding at the optimum. Just use the results you have learned in the class.)