3.8.1 $X_t = 0.3X_{t-1}+\varepsilon_t$, $\varepsilon_t \overset{iid}{\sim}(0,1^2)$

$n=100$, $\hat\mu=\bar X=0.1$이다.

(1) 정리 3.1.1에 의해 $\sqrt{n}(\hat\mu-\mu)\xrightarrow{d} N\left(0,\frac{\sigma^2}{(1-\phi)^2}\right)$

문제의 값을 대입하면 $10(\bar X-\mu)\xrightarrow{d}N\left(0,\frac{1}{0.49}\right)$

이므로 $\mu \xrightarrow{d} N\left(0.1,\frac{1}{7^2}\right)$이다.

μ의 95% CI : $0.1\pm1.96\cdot\frac{1}{7} \Rightarrow(-0.18,0.38)$

(2) 유의수준 5% 검정을 위해 $\mu_0=0$이 μ의 95% CI에 들어가는지 여부가 활용 가능하다.

$0\in(-0.18,0.38)$이므로 $H_0:\mu=0$을 기각할 수 없다.

3.8.3. 예 3.3.3에 따라 Yule-Walker 방정식은

$$\begin{cases}\rho(1)=\phi_1+\rho(1)\phi_2\\ \rho(2)=\rho(1)\phi_1+\phi_2\end{cases}\Rightarrow\begin{cases}\hat\phi_1=\dfrac{\hat\rho(1)\{1-\hat\rho(2)\}}{1-\hat\rho(1)^2}\\[2ex] \hat\phi_2=\dfrac{\hat\rho(2)-\hat\rho(2)^2}{1-\hat\rho(1)^2}\end{cases}$$ 이다.

$\hat\rho(1)=0.6$, $\hat\rho(2)=0.3$을 대입하면

$\hat\phi_1=\frac{21}{32}$, $\hat\phi_2=-\frac{6}{91}$이다.

3.8.4. CBS 부등식에 따라 다음이 성립한다.

$$\{(X_1-\bar X)(X_{1+h}-\bar X)+\cdots+(X_{n-h}-\bar X)(X_n-\bar X)\}^2=\{n\hat\gamma(h)\}^2$$

$$\le\{(X_1-\bar X)^2+\cdots+(X_{n-h}-\bar X)^2\}\times\{(X_{1+h}-\bar X)^2+\cdots+(X_n-\bar X)^2\}$$

$$\le\{(X_1-\bar X)^2+\cdots+(X_n-\bar X)^2\}^2=\{n\hat\gamma(0)\}^2$$ ($\because$ 양수 항 추가)

즉 $\hat\gamma(h)^2\le\hat\gamma(0)^2\Rightarrow|\hat\gamma(h)|\le|\hat\gamma(0)|\Rightarrow|\hat\rho(h)|\le1$.

Timeseries_Analysis_HW3

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2025-04-25

06

```r
set.seed(1)
phi <- runif(1, -1, 1)
theta <- runif(1, -1, 1)
set.seed(1)
Xt <- arima.sim(n = 100, model = list(ar = phi, ma = theta))
Xt
```

```
## Time Series:
## Start = 1
## End = 100
## Frequency = 1
##   [1]  1.86592877 -0.87188639 -0.31204424 -1.90947290  2.58685499 -1.54582734
##   [7]  0.72026784  0.61018377  0.29366739  0.24614727  0.65164699  0.24149467
##  [13] -0.23872491 -1.89646401  2.01801558 -1.16106488  0.40307885 -1.61994434
##  [19]  0.65772393  0.23176836  1.14309473 -0.98636464  0.87654772 -0.56403860
##  [25] -1.09877447  0.45249764 -0.50036773  0.27619092  0.98566615  0.01958149
##  [31] -0.36889085 -0.03828099  0.77971430  0.01274078 -0.83709874 -0.13875957
##  [37]  0.61060124  0.38892888 -0.49130111  1.14025222 -0.36199790 -0.54407211
##  [43]  0.75280718 -1.56965873  2.45800355  0.46113986 -1.08997971 -0.43903533
##  [49]  1.04265932 -0.76975081  2.79715806 -1.96527770  1.62145627 -0.90883510
##  [55] -0.32420691  0.53093348 -2.10224128  2.91309174 -1.58775510  2.87804642
##  [61] -1.42989459 -0.16096325  0.86778569 -1.49726869 -0.31254280  0.75864289
##  [67] -0.87362026  0.52419099 -0.17177786 -0.52797306 -0.17028724  0.09012143
##  [73]  1.17039384 -2.37375957  2.09685386 -0.80234080  1.35423098 -1.21118491
##  [79]  1.01583926 -0.30394534 -0.46828603  1.56623653  0.11694997  0.34859062
##  [85]  1.24426931 -0.43089047 -1.21734618  0.32414012 -1.23001482  0.41665237
##  [91] -0.69469617  0.52657648 -1.16864812  0.93907471 -1.13541062  2.46718664
##  [97] -0.89234792  1.14537043 -0.38575259  1.76483111
```

07

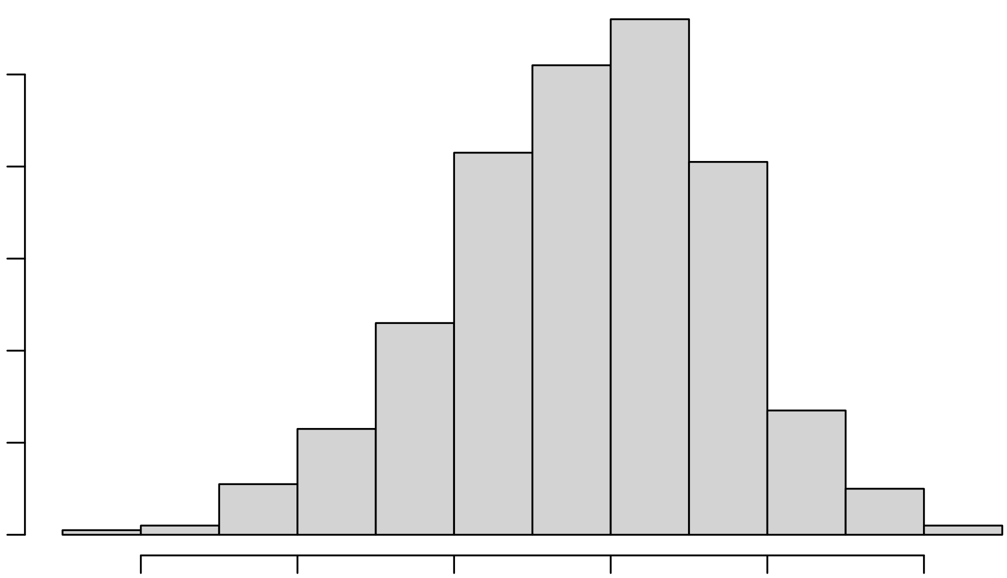
난수 생성

```r
phis <- c()

set.seed(1)
for (i in 1:500) {
  # time-series data generation
  Xt <- arima.sim(n = 100, model = list(ar = 0.5))

  # saving LSE
  phi_hat <- as.numeric(lm(formula = Xt[-length(Xt)] ~ Xt[-1] + 0)$coef)
  phis <- append(phis, phi_hat)
}

head(phis)
```

```
## [1] 0.5187736 0.4342065 0.3910718 0.5330833 0.4705040 0.4030723
```

히스토그램

```r
modified_data <- 10*(phis - 0.5)

hist(modified_data)
```



이론적 값과 비교

```r
mean(modified_data)
```

```
## [1] -0.1416625
```

```r
var(modified_data)
```

```
## [1] 0.7773941
```

```r
# 이론적 값
0
```

```
## [1] 0
```

```r
0.75
```

```
## [1] 0.75
```

$\sigma^2 = 1$이다. $E(X_1^2) = \frac{4}{3}$이다. 즉 이론적 분산은 0.75, 이론적 평균은 0이다.

```r
set.seed(1)
Xt <- arima.sim(model = list(order = c(1,0,0), ar = 0.5), n = 100)
Xt
```

```
## Time Series:
## Start = 1
## End = 100
## Frequency = 1
##   [1]  1.614242003  1.196964238 -0.022758462 -2.226079118  0.011891359
##   [6] -0.038987929 -0.035684228  0.925994097  1.284218243  1.236010443
##  [11]  1.536982593  1.550627597  0.849878782 -1.564412305 -0.162380405
##  [16] -0.137318942 -0.224454978 -1.582979873 -1.269639991 -0.216878436
##  [21]  1.250240334  0.522332440  0.648837831  0.270613875 -1.241752619
##  [26] -1.035870873 -0.912225390 -0.515426092  0.842312326  1.184331911
##  [31]  0.427642359 -0.039540500  0.677193125  0.895259761 -0.241125814
##  [36] -0.828058064 -0.049447070  0.743809390  0.259558483  1.010886968
##  [41]  0.903549364 -0.160251711  0.260993836 -0.998866178  0.933590613
##  [46]  2.447195205  0.856376126 -0.615946563  0.261746346 -0.004181431
##  [51]  2.399527045  1.160523520  1.270001122  0.663002720 -0.411771849
##  [56] -0.017093625 -1.813505441  0.558802141  0.432654409  2.388938875
##  [61]  1.669978966  0.125043052  0.673247880 -0.597473692 -1.552370246
##  [66] -0.484738888 -0.685661317 -0.341725307 -0.096521329 -0.637781611
##  [71] -0.887559538 -0.578958384  0.888607804 -1.079262898  0.054314739
##  [76]  0.360107740  1.243153708  0.317392930  0.528715275  0.531456428
##  [81] -0.276791817  1.069471898  1.695138564  1.547782932  2.360724920
##  [86]  1.738848886 -0.407167766 -0.776849297 -1.613037263 -1.279919268
##  [91] -1.260326311 -0.588047283 -1.204945290 -0.444443872 -0.876806580
##  [96]  1.328883979  1.381149466  1.600748962  1.184559839  2.274456000
```

LSE 계산

```
as.numeric(lm(formula = Xt[-length(Xt)] ~ Xt[-1] + 0)$coef)
```

```
## [1] 0.5187736
```

09

데이터 입력

```
Xt <- read.csv('ex_ch3_8.txt', sep = '\n')
Xt <- ts(as.vector(Xt)$data)
Xt
```

```
## Time Series:
## Start = 1
## End = 14
## Frequency = 1
##  [1]  72  71  59  44  74  78  81  64  86  87 104  88  84  62
```

자유아카데미 홈페이지 자료실에서 다운로드할 수 있는 '시계열 분석 이론 및 SAS 실습 2판' 교재의 데이터 파일을 내려받아 활용하였다. 파일명이 교재와 다른데, 3장에서 파일을 활용하는 문제가 하나뿐이어서 이름이 3.8 문제에 대응하는 것으로 되어 있는 파일을 활용하였다.

CSS 사용(최소제곱추정량)

```
Xt_LSE <- arima(x = Xt, order = c(1, 0, 1), method = 'CSS')
summary(Xt_LSE)
```

```
##           Length Class  Mode
## coef       3     -none- numeric
## sigma2     1     -none- numeric
## var.coef   9     -none- numeric
## mask       3     -none- logical
## loglik     1     -none- numeric
## aic        1     -none- logical
## arma       7     -none- numeric
## residuals 14     ts     numeric
## call       4     -none- call
## series     1     -none- character
## code       1     -none- numeric
## n.cond     1     -none- numeric
## nobs       1     -none- numeric
## model     10     -none- list
```

$\hat{\phi}_{LSE} = 0.4405$

$\hat{\theta}_{LSE} = 0.0347$

$\hat{\mu}_{LSE} = 74.8414$

ML 사용(최대가능도추정량)

```
Xt_MLE <- arima(x = Xt, order = c(1, 0, 1), method = 'ML')
summary(Xt_MLE)
```

```
##           Length Class  Mode     
## coef       3     -none- numeric  
## sigma2     1     -none- numeric  
## var.coef   9     -none- numeric  
## mask       3     -none- logical  
## loglik     1     -none- numeric  
## aic        1     -none- numeric  
## arma       7     -none- numeric  
## residuals 14     ts     numeric  
## call       4     -none- call     
## series     1     -none- character
## code       1     -none- numeric  
## n.cond     1     -none- numeric  
## nobs       1     -none- numeric  
## model     10     -none- list     
```

$\hat{\phi}_{MLE} = 0.4109$

$\hat{\theta}_{MLE} = 0.0342$

$\hat{\mu}_{MLE} = 74.4687$