1.

(1) $Bt^r = (t-1)^r$, $(1-B)t^r = \Delta t^r = t^r - (t-1)^r$

$\Delta^2 t^r = \{t^r - (t-1)^r\} - \{(t-1)^r - (t-2)^r\}$

$= r(r-1)t^{r-2} + \cdots$ 이고, 이를 반복하면

$\Delta^k t^r = r(r-1)\cdots(r-k+1)t^{r-k} + \cdots$ 이고, 최고차항 차수는 $r-k$이므로

$\Delta^r t^r = r(r-1)\cdots 1 = r!$

(2) 이항정리를 적용

$(1-B)^r \varepsilon_t = \sum_{j=0}^{r} \binom{r}{j} 1^{r-j}(-B)^j \varepsilon_t = \sum_{j=0}^{r} \binom{r}{j} (-1)^j \varepsilon_{t-j}$

(3) (1), (2)에 의해 $(1-B)^r X_t =$

$(1-B)^r \{a_0 + a_1 t + \cdots + a_r t^r + \varepsilon_t\}$가 되므로

정상시계열의 선형결합이어서 정상이다.

5. (식 4.2.2.) $T_n = \dfrac{\sum_{t=1}^{n} X_{t-1}\varepsilon_t / n}{\sum_{t=1}^{n} X_{t-1}^2 / n^2}$

$\frac{\sigma^2}{n} W_n\left(\frac{t-1}{n}\right)^2 = \left(\frac{1}{\sqrt{n}\sigma}\sum_{j=1}^{t-1}\varepsilon_j\right)^2 \times \frac{\sigma^2}{n} = \frac{1}{n^2}\left(\sum_{j=1}^{t-1}\varepsilon_j\right)^2$이므로,

$\sum_{t=1}^{n} X_{t-1}^2 = \left(\sum_{j=1}^{t-1}\varepsilon_j\right)^2$임을 보이자.

6. (식 4.2.3.) $W_n(s)=\frac{1}{\sqrt{n}\sigma}\sum_{j=1}^{[ns]}\varepsilon_j,\ 0\le s\le 1.$

$\sqrt{\frac{t}{s}}\sigma\left(W_n\left(\frac{t}{n}\right)-W_n\left(\frac{t-1}{n}\right)\right),\ (ns=t)$

$=\sqrt{n}\sigma\left[\frac{1}{\sqrt{n}\sigma}\left\{\sum_{j=1}^{[t]}\varepsilon_j-\sum_{j=1}^{[t]-1}\varepsilon_j\right\}\right]=\varepsilon_t$

9. $Y_t=(1-B)X_t=(1-B)\Theta(B)\varepsilon_t$

$=\varepsilon_t+\eta_1\varepsilon_{t-1}+\eta_2\varepsilon_{t-2}+\eta_3\varepsilon_{t-3}$, 이다.

$(\eta_1=\theta_1-1,\ \eta_2=\theta_2-\theta_1,\ \eta_3=-\theta_2)$

· $\gamma(0)=cov(Y_t,Y_t)=Var(Y_t)$

$=Var(\varepsilon_t+\eta_1\varepsilon_{t-1}+\eta_2\varepsilon_{t-2}+\eta_3\varepsilon_{t-3})$

$=(1+\eta_1^2+\eta_2^2+\eta_3^2)\sigma^2$

· $\gamma(1)=cov(Y_t,Y_{t+1})$

$=cov(\varepsilon_t+\eta_1\varepsilon_{t-1}+\eta_2\varepsilon_{t-2}+\eta_3\varepsilon_{t-3},\ \varepsilon_{t+1}+\eta_1\varepsilon_t+\eta_2\varepsilon_{t-1}+\eta_3\varepsilon_{t-2})$

$=(\eta_1+\eta_1\eta_2+\eta_2\eta_3)\sigma^2$

· $\gamma(2)=cov(Y_t,Y_{t+2})$

$=cov(\varepsilon_t+\eta_1\varepsilon_{t-1}+\eta_2\varepsilon_{t-2}+\eta_3\varepsilon_{t-3},\ \varepsilon_{t+2}+\eta_1\varepsilon_{t+1}+\eta_2\varepsilon_t+\eta_3\varepsilon_{t-1})$

$=(\eta_2+\eta_1\eta_3)\sigma^2$

· $\gamma(3)=cov(Y_t,Y_{t+3})=\eta_3\sigma^2$