01.03.21

Polynomial

$$f(x) = a_0 \cdot x^n + a_1 \cdot x^{n-1} + a_2 \cdot x^{n-2} + a_3 \cdot x^{n-3} + \cdots + a_r \cdot x^{n-r} + \cdots + a_n$$

where $a_0, a_1, a_2, a_3 \cdots a_n \in R$.

$a_0 \neq 0$; If $a_0 = 0$, then given polynomial is not a n^{th} degree.

Quadratic Expression

$ax^2 + bx + c = 0$ Equation

$\Leftrightarrow$ Quadratic in equation.

$a \rightarrow$ co-efficient of x^2

$b \rightarrow$ co-efficient of x

$c \rightarrow$ constant term.

co-efficient

$ax^2 + bx + c = 0$

Numerical co-efficient

04.03.21

A.P

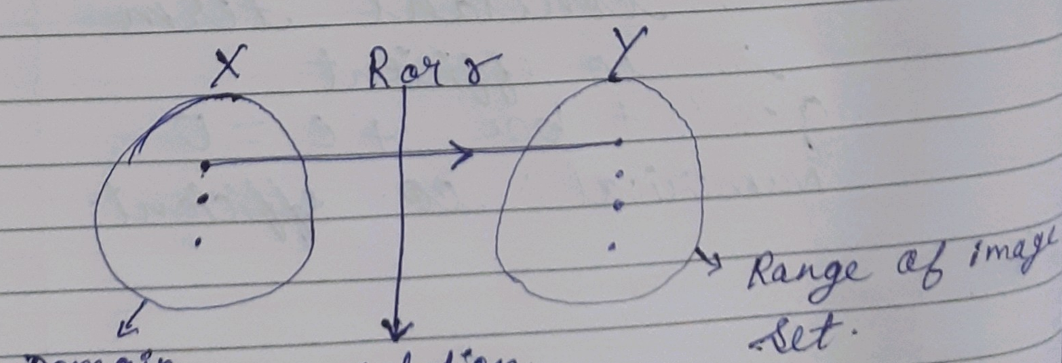
Sequence Progression :

Definition - A sequence is a function whose domain is the set of natural numbers.

Ex. $\{1, 2, 3, 4, \ldots\}$

$$a_n = \langle n \rangle$$
$$= \langle 2n \rangle \text{ where } n \in N$$
$$= \langle 2n-1 \rangle \text{ where } n \in N$$
$$= \left\langle \frac{(-1)^n}{n} \right\rangle$$

Real Sequence : A sequence whose range is a sub-set of real numbers is called real sequence.



(x, y)

xRy

Give first three terms of the sequence defined by $a_n = \frac{n}{n^2+1}$, where $n \in N$.

$n = 1, 2, 3 \ldots$

$a_1 = \frac{1}{1^2+1} = \frac{1}{2}$

$a_2 = \frac{2}{2^2+1} = \frac{2}{5}$

$a_3 = \frac{3}{3^2+1} = \frac{3}{10}$

Arithmetic Progression

A sequence is called an A.P. if the difference of a term and the previous term is always a constant. The constant term is called a common difference. It is denoted 'd'.

Let 'a' be the 1st term and d be c.d. then a; $a+d$; $a+2d$; $a+3d$ . . . . . .

Set of odd natural numbers.

$a_n = \langle 2n-1 \rangle$; $n \in N$

$\{1, 3, 5, 7, \ldots\}$

~~and~~ $3-1=2$; $5-3=2$; $7-5=2$ . . . . .

$a_1 = 1$; c.d. (d) $= 2$

$a_n = \langle 2n \rangle$

$\{2, 4, 6, 8, \ldots\}$

$a_1 = 2$; $a_2 = 4$; $a_3 = 6$, ...

$a_2 = a_1 = 4-2 = 2$. $a_3 - a_2 = 6-4 = 2$

General term of an A.P.

Let a be the 1st term and 'd' be c.d. then,

$$a_1 = a = a + (1-1)d$$
$$a_2 = a + d = a + (2-1)d$$
$$a_3 = a + 2d = a + (3-1)d \ldots$$
$$\ldots$$
$$a_n = a + (n-1)d$$

This a_n term is called general

Term of an A.P

a_n or, T_n or t_n

$$\boxed{a_n = a + (n-1)d}$$

05-03-21

Shridhra Acharya Formula

$$ax^2 + bx + c = 0 \; ; a \neq 0$$

Multiplying each term a

~~Lst~~, we get,

$$a^2x^2 + abx + ca = 0$$

$$(ax)^2 + 2ax \cdot \frac{b}{2} + \frac{b^2}{4} + ca - \frac{b^2}{4} = 0$$

Date
Page

$$\left(ax+\frac{b}{2}\right)^2-\left(\frac{\sqrt{b^2-4ac}}{2}\right)^2=0$$

$$\left(ax+\frac{b}{2}\right)^2-\left(\sqrt{b^2-4ac}\right)^2=0$$

$$ax+\frac{b}{2}-\frac{\sqrt{b^2-4ac}}{2}\left(ax+\frac{b}{2}+\frac{\sqrt{b^2-4ac}}{2}\right)=0$$

$$ax+\frac{b}{2}-\frac{\sqrt{b^2-4ac}}{2}=0$$

$$\therefore ax=\frac{-b}{2}-\frac{\sqrt{b^2-4ac}}{2}$$

$$x=\frac{\sqrt{b^2-4ac}}{2a}$$

$$ax+\frac{b}{2}+\frac{\sqrt{b^2-4ac}}{2}=0$$

$$x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\alpha \cancel{x}=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$\beta \cancel{x}=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of A.P

$S = \frac{n}{2}\{a + l\}$

where $a \rightarrow$ 1st term
$l \rightarrow$ last term

n = no. of terms in a given A.P

$S = \frac{n}{2}\{2a + (n-1)d\}$

Arithmetic mean

Let a, A & b be in A.P
then, $A - a = b - A$
$2A = a + b$
$A = \frac{a+b}{2}$

'A' is termed as A.M between two given numbers a & b.

G.P - A sequence of non zero numbers is called a geometric progression, if the ratio between two two successive terms is always a constant called common ratios. Eg:- 2, 4, 8, 16, 32 ....

$\frac{4}{2} = 2$, $\frac{8}{4} = 2$, and so on.

3, 6, 12, 24 ..... is also in G.P.

a, ar, ar^2, ar^3 ....

General term of G.P

T_n or $t_n = ar^{n-1}$

$\boxed{T_n = ar^{n-1}}$

n^{th} term of a G.P from the end.
Consider G.P contains 'M' nos. of terms

$\therefore$ n^{th} term from the end
$= (m-n+1)^{th}$ from the beginning.
$= a \cdot r^{m-n+1-1}$
$= a \cdot r^{m-n}$

Sum of finite G.P :

$$S = a \cdot \left(\frac{1 - r^n}{1 - r} \right), \text{ where } (r) < 1$$

~~∴ n^{th} term fro~~

08.03.21 Sum of infinite G.P :

$$S_\infty = \frac{a}{1 - r}$$

Harmonic Progression (H.P)

A sequence is said to be in H.P if the reciprocal of its terms are in A.P.

If the sequence $a, a_2, a_3 \dots a_n$ are in H.P then $\frac{1}{a}, \frac{1}{a_2}, \frac{1}{a_3} \dots \frac{1}{a_n}$ are in AP.

General term of H.P

$$a_n = \frac{1}{a + (n-1)d}$$

Note :(i) There is no general formula for finding the sum of finite H.P.

ii) No term of H.P can be zero

$\frac{a}{0} \Rightarrow \infty$ (undefined or infinite)

iii) For H.P whose first term is 'a' and second term is 'b' then, n^{th} term $= \frac{ab}{b+(n-1)(a-b)}$

iv) If a, b, c are in H.P then,

$b = \frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$

eg:-

2, 4, 6, 8 ---- are in AP.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ ... are in H.P

(1) Sum of finite series

Sigma notation :

$S = 1+2+3+4+ \dots + n$.

$$\sum_{n=1}^{n} n = 1+2+3+4+ \cdots + n$$

$$S = \sum n = \frac{n(n+1)}{2}$$

(2) Sum of 1st n odd natural nos.

$$S = 1+3+5+7+ \cdots + (2n-1)$$

$$\sum (2n-1) = 1+3+5+7+ \cdots + (2n-1)$$

$$= n^2$$

(3) Sum of 1st n even natural nos.

$$S = 2+4+6+8+ \cdots + 2n$$

$$\sum 2n = n(n+1)$$

(4) Sum of Squares of 1st n natural nos

$$S = 1^2+2^2+3^2+4^2+ \cdots + n^2$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

(5) Sum of the cubes of 1st n natural nos.

$$S = 1^3+2^3+3^3+ \cdots + n^3.$$

$$\sum n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

Recognisation of A.P; G.P; H.P

If a, b, c be three successive terms of a sequence.

(i) if $\frac{a-b}{b-c} = \frac{a}{a}$ then a, b, c are in A.P.

(ii) if $\frac{a-b}{b-c} = \frac{a}{b}$ then a, b, c are in G.P

(iii) if $\frac{a-b}{b-c} = \frac{a}{c}$ then, a, b, c are in H.P.

Logarithm

Laws of indices

i) $a^m \times a^n = a^{m+n}$

$2^2 \times 2^{-1} = 2^{2+(-1)} = 2^1 = 2$

(ii) $a^m \div a^n = a^{m-n}$, if $m > n$.

$2^5 \div 2^3 = 2^{5-3} = 2^2 = 4$

(iii) $(a^m)^n = a^{m \times n}$

$(3^0)^5 = 3^{0 \times 5} = 3^0 = 1$

(iv) $a^0 = 1$

(v) $a^{-n} = \frac{1}{a^n}$

(vi) $\sqrt{a} = a^{1/2}$

(vii) $\sqrt[n]{a}$ = n^{th} root of a = $a^{1/2}$

$\sqrt[3]{8} = 8^{1/3} = (2^3)^{1/3} = 2$

(vii) $(a^m)^{1/n} = \sqrt[n]{a^n} = a$

(viii) $(a \times b)^m = a^m \times b^m$

(x) $(a \pm b)^m \neq a^m \pm b^m$

(x) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$; $b \neq 0$

11·03·21

Change of Base Formula

$\log_b m - \log_a m \times \log_b a = \frac{\log_a m}{\log_a b}$

Sol: $\log_b m = x$

$= b^x = m$ —— (I)

let $\log_a^m = y$

$\Rightarrow a^y = m^a$ — (II)

and let $\log_b^a = z$

$\Rightarrow b^z = a$ — (III)

from (I), (II) & (III)

we get,

$b^x = a^y = (b^x)^y$

$x = yz$

$\log_a^m y = \frac{1}{K} \log_a^m$

$\log_a^m y = x$

$m(a^x)^x$ —— (I)

and $\log_a^m = y$

$m = a^y$ — (II)

From (i) & (ii)

$(a^k)^x = a^y$

$k \cdot x = y$

$\therefore x = \frac{1}{k} \cdot y$

$\log_{a^k}^{m} = \frac{1}{k} \cdot \log_a^m$

If $a^x = N$; $a > 0$ & $a \neq 1$
$N > 0$, also $\log_a^N = x$

$$\boxed{a^{\log_a N} = N}$$

$$\boxed{\log_b^a \times \log_a^b = 1}$$

Characteristics and mantissa of a logarithmic number.

Characteristics :

1) zero 2) Positive 3) Negative

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Date

Page

Eg:- (i) $\log_{10} 729$ $\quad$ $729.00 = 3$ digits no.

7.29×10^2

Char $= 2$

(ii) $\log 72.9$

72.9

7.29×10^1

Char $= 1$

(iii) $\log 7.29$

7.29×10^0

Char $= 0$

(iv) $\log .729$

$.729 = 7.29 \times 10^{-1}$

Char $= -1$

(v) $\log 0.0729$

$0.0729 = 7.29 \times 10^{-2}$

$\therefore$ Char $= -2$

12.03.21

Q. Find the value of x, if it is given that $\log_{10} 3 = 0.4771213$

$\log 7 = 0.8450930$ 8

$\log 11 = 1.041$

Given: $3^x + 7^{2x+1} = 11^{x+5}$

Taking log to the bas 10 on both sides, we get,

$\log 3^x + \log 7^{2x+1} = \log 11^{x+5}$

$x \cdot \log 3 + (2x+1) \cdot \log 7 = (x+5) \cdot \log 11$

$x \cdot (0.4771213) + (2x+1)(0.8450980)$
$= (x+5)(1-0413927)$

$(0.4771213) + 2 \times 0.8450980 - 1.0413927$
$= -0.8450980 + 5 \times 1.041392$

$\therefore x = \dfrac{5 \times 1.041392 - 0.8450980}{0.4771213 + 2 \times 0.8450980 - 1.0413927}$

$x = 3.8740298$

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Date ______

Page ______

Q Evaluate (i) $\log_{\sqrt{5}} 0.008$ (ii) $\log_{2\sqrt{3}} 144$

(iii) $\log_{a^2} b \div \log_{\sqrt{a}} (b)^2$

Sol (i) $\log_{\sqrt{5}} 0.008 = x$

$\Rightarrow (\sqrt{5})^x = 0.008$

$\Rightarrow 5^{x/2} = \frac{8}{1000} = \frac{1}{5^3}$

$5^{x/2} = 5^{-3}$

$\frac{x}{2} = -3$

$\therefore x = 2(-3) = -6$

(iii) $\dfrac{\log_{a^2} b}{\log_{\sqrt{a}} (b)^2} = \dfrac{\frac{1}{2} \log_a b}{2 \cdot \log_a (b)^2}$

$\Rightarrow \cancel{\frac{1}{4} \cdot \log_a b^a}$

$$= \frac{1}{4} \cdot \frac{\log_a b^a}{2 \log_a b}$$

$$= \frac{1}{8} \cdot \frac{\cancel{\log_a b}}{\cancel{\log_a b}}$$

$$= \frac{1}{8}$$

Q. i) Prove that $\log_3^{\log_2 \log_{\sqrt{3}}^{81}} = 1$

Sol. $\log_3^{\log_2 2 \cdot \log_3^{3^4}}$

$= \log_3^{\log_2 2 \cdot 4 \cdot \log_3^{3}}$

$= \log_3^{\log_2^{8}}$

$= \log_3^{\log_2^{2^3}} = \log_3^{3 \log_2^{2}}$

$= \log_3^{3} = 1$

ii) $\log_a^x \times \log_b^y = \log_b^x \times \log_a^y$

Sol L.H.S $= \log_a^x \times \log_b^y$

$= \log_b^x \times \log_a^b \times \log_a^y \times \log_b^a$

$= \log_b^x \times \log_a^y \times \log_a^b \times \log_b^a$

$= \log_b^x \times \log_a^y$

15.03.21

Q. If $a^2+b^2=7ab$, then prove that $\log\frac{(a+b)}{3} = \frac{1}{2}\{\log a + \log b\}$

2. If $a^2+b^2 = 23ab$, then prove that $\log\frac{(a+b)}{5} = \frac{1}{2}\cdot\{\log a + \log b\}$

3. Prove that $\log(1+2+3) = \log 1 + \log 2 + \log 3$.

4. Prove that $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c} = 1$

5. Prove that $x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} = 1$

6. P.T., if $\frac{\log a}{y-x} = \frac{\log b}{z-x} = \frac{\log c}{x-y}$

then, $a^x \cdot b^y \cdot c^z = 1$

7. If $\frac{\log x}{q-r} = \frac{\log y}{r-p} = \frac{\log z}{p-q}$

P.T. $x^{q+r} \cdot y^{r+p} \cdot z^{p+q} = x^{p} \cdot y^{q} \cdot z^{r}$

8. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then

P.T. $a^{b+c} \cdot b^{c+a} \cdot c^{a+b} = 1$

9. If $\frac{yz \cdot \log(yz)}{(y+z)} = \frac{zx \cdot \log(zx)}{(z+x)} = \frac{xy \cdot \log(xy)}{(x+y)}$

then, P.T. $x^{x} = y^{y} = z^{z}$

10. If $\frac{\log x}{\beta-\gamma} = \frac{\log y}{\gamma-\alpha} = \frac{\log z}{\alpha-\beta}$

P.T. $x \cdot yz = 1$

Solutions

1. $a^2+b^2 = 7ab$

Adding 2ab on both sides

$a^2+b^2+2ab = 7ab+2ab = 9ab$

$\Rightarrow (a+b)^2 = 9ab$

$\therefore a+b = \sqrt{9ab} = 3(ab)^{1/2}$

$$\frac{a+b}{3} = (ab)^{1/2}$$

$$\log\frac{(a+b)}{3} = \log(ab)^{1/2}$$
$$= \frac{1}{2}\cdot\{\log a + \log b\}$$

3. $\log(1+2+3)$

$\log 6 = \log(1\times 2\times 3)$

$= \log 1 + \log 2 + \log 3$

4. $\dfrac{\log a}{y-z} = \dfrac{\log b}{z-x} = \dfrac{\log c}{x-y} = K$

$\log a = K(y-z)$

$x\cdot\log a = K\cdot x(y-z)$

$\log a^x = k \cdot x(y-z)$

i.e. $a^x = 10^{kx(y-z)}$ — (i)

Similarly,

$\log b = k(z-x)$

$y \cdot \log b = k \cdot y(z-x)$

$\log b^y = k \cdot y(z-x) = b^y = 10^{k \cdot y(z-x)}$ (ii)

$\frac{\log c}{x-y} = k \Rightarrow \log c = k(x-y)$

$z \cdot \log c = k \cdot z(x-y)$

$\log c^z = k \cdot z(x-y)$

$c^z = 10^{k \cdot z(x-y)}$ — (iii)

$a^x \cdot b^y \cdot c^z = 10^{kx(y-z)} \times 10^{ky(z-x)} \times 10^{kz(x-y)}$

$= 10^{k\{xy - zx + yz - xy + zx - yz\}}$

$= 10^{k \cdot 0} = 10^0 = 1$

$a^x \cdot b^y \cdot c^z = 1$

5. $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c}$

$$\frac{1}{\log_b^b + \log_b^a + \log_b^c} + \frac{1}{\log_c^c + \log_c^a + \log_c^b} + \frac{1}{\log_a^a + \log_a^b + \log_a^c}$$

$$\frac{1}{\log_b^{abc}} + \frac{1}{\log_c^{abc}} + \frac{1}{\log_a^{abc}}$$

$$\log_{abc}^{b} + \log_{abc}^{c} + \log_{abc}^{a}$$

$$\log_{abc}^{b \cdot ca} = 1$$

6. $x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y}$

Taking log on both sides, we get,

$$\log^P = \log \left\{ x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} \right\}$$

$$= \log x^{\log y - \log z} + \log y^{\log z - \log x} + \log z^{\log x - \log y}$$

$$= (\log y - \log z) \cdot \log x + (\log z - \log x) \cdot \log y + (\log x - \log y) \cdot \log z.$$

$$= \log y \cdot \log x - \log z \cdot \log x + \log z \cdot \log y - \log x \cdot \log y + \log x \cdot \log z - \log y \cdot \log z = 0$$

18-03-21

Partial Fraction

Polynomial fraction: A rational expression is called polynomial, If it is integral with respect to every letter entering into that expression.

A polynomial is written as.

$$a_0 \cdot x^n + a_1 \cdot x^{n-1} + a_2 \cdot x^{n-2} + \ldots + a_{n-1} \cdot x^{n-1}.$$

is known as ordered polynomial.

if $a_0 \neq 0$ then the above polynomial is of nth degree.

Rational function

Rational function is a function of the form

$$y = f(x) = \frac{g(x)}{h(x)}$$, where $g(x)$

and $h(x)$ are polynomial.

It is defined for all real values of x excluding the $g(x)$ let $(h(x) = 0$

$$\frac{x-2}{x^2-2x-3} = \frac{g(x)}{h(x)}$$

Rational function

i) Proper Rational function
ii) Improper Rational function.

1) Proper Rational function

A rational function is said to be proper rational function i.e $g(x) \div h(x)$ where $h(x) \neq 0$.

for all real values of x,

If degree of $h(x) >$ degree of $g(x)$ the rational fraction is proper.

Ex- $\frac{x-2}{x^2+2x-3} = \frac{g(x)}{h(x)}$

Numerator of $g(x) = x-2$
degree of numerator $= 1$

Denominator of $h(x) = x^2+2x-3$
degree of denominator $= 2$

degree of $h(x) >$ degree of $g(x)$

$\therefore$ the given rational no. is a proper rational function.

ii) <u>Improper rational function</u>

A rational fract is Improper rational function, if degree of ~~h(x)~~ $h(x) \leq$ degree of $g(x)$.

Ex- $\dfrac{x^3-8}{x^2+5x-2}$ $\quad g(x) = x^3-8$

$h(x) = x^2+5x-2$

degree of $g(x) = 3$

degree of $h(x) = 2$

degree of $g(x) \geq$ degree of $h(x)$

so, it is called improper rational function.

$\Rightarrow \dfrac{x^3-8}{x^2+5x-2}$

$$
\begin{array}{r|l}
 & x-5 \\
\hline
x^2+5x-2 & x^3-8 \\
 & x^2+5x^2-2x \\
 & \;\;-\qquad + \\
\hline
 & -5x^2+2x-8 \\
 & -5x^2-25x+80 \\
 & \;\;+\qquad +\qquad - \\
\hline
 & 27x-18
\end{array}
$$

$$\frac{x^3-8}{x^2+5x-2} = (x-5) + \frac{27x-18}{x^2+5x-2}$$

Partial function

The process of decomposing a given proper or improper rational function into a no. of constituent of proper rational function.

Types of partial function

i) Distinct Linear factors of denominator.
ii) Repeated Linear factors of denominators.
iii) Repeated distinct and Quadratic factors.
iv) Repeated Quadratic factors.

i) Distinct Linear Factor

Check the degree of numerator and degree of denominator, if degree of numerator < degree of denominator then it is proper.

If degree of numerator $\geq$ degree of denominator then the fraction is improper.

Divide the numerator by denominator and express it in mixed fraction.

$$\frac{g(x)}{h(x)} = h(x) + \frac{u(x)}{h(x)}$$

19.03.21

i) Distinct linear factors of denominator

$$\frac{g(x)}{f(x)} = \frac{g(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)}$$

$$\therefore \frac{g(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)} = \frac{A}{a_1x+b_1} + \frac{B}{a_2x+b_2} + \cdots + \frac{Z}{a_nx+b_n}$$

Q. To resolve into partial fraction of the following: