

(i)

$$\frac{mx+n}{(x-a)(x-b)}$$

(ii)

$$\frac{7x-1}{1-5x+6x^2}$$

Sol i) $p(x) = mx+n$
Degree of $p(x) = 1$

$$q(x) = (x-a)(x-b) = x^2 - (a+b)x + ab$$

∴ degree of $q(x) = 2$

i.e. degree of $p(x) <$ degree of $q(x)$

∴ the function is proper rational function.

$$\frac{mx+n}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

=

$$m x + n = A(x-b) + B(x-a)$$

To determine A : Put $x-a=0$
 $m \cdot a + n = A \cdot (a-b)$

$$\therefore A = \frac{ma+n}{a-b}$$

To determine B : Put $x-b=0$

$$\therefore x = b$$

$$m \cdot b + n = B \cdot (b-a)$$

$$\therefore B = \frac{mb+n}{b-a}$$

$$\frac{mx+n}{(x-a)(x-b)} = \frac{ma+n}{a-b} \cdot \frac{1}{x-a} + \frac{xb+n}{b-a} \cdot \frac{1}{x-b}$$

93)

$$\frac{7x-1}{1-5x+6x^2}$$

$$P(x) = 7x-1$$

Degree of $P(x) = 1$

$$q(x) = 1-5x+6x^2$$

Degree of $q(x) = 2$

Degree of $P(x) <$ Degree of $q(x)$

\therefore fraction is improper rational function

$$\frac{7x-1}{1-3x-2x^2+6x^2}$$

$$= \frac{7x-1}{(1-3x)-2x(1-3x)}$$

$$= \frac{7x-1}{(1-2x)(1-3x)}$$

$$= \frac{7x-1}{(1-2x)(1-3x)} = \frac{A}{1-2x} + \frac{B}{1-3x}$$

$$= A \cdot (1-3x) + B (1-2x)$$

$$7x-1 = A(1-3x)B(1-2x)$$

To determine A : Put $x-2=0$
 $\therefore x = y_2$

$$7x\frac{1}{2}-1 = A \cdot \left(1-3x\frac{1}{2}\right)$$

$$\frac{5}{2} = \frac{1}{2}A$$

$$\therefore A = -5$$

To determine B : Put $1-3x=0$
 $x = \frac{1}{3}$

$$7x\frac{1}{3}-1 = B(1-2 \cdot y_3) = \frac{1}{3}B$$

$$\frac{4}{3} = \frac{1}{3}B \Rightarrow B = 4.$$

$$\frac{7x-1}{(1-2x)(1-3x)} = \frac{-5}{(1-2x)} + \frac{4}{(1-3x)}$$

(iii). $\frac{x^2+7x+2}{x^2+7x+10}$

$$p(x) = x^2 + 7x + 2$$

$$\text{Degree of } p(x) = 2$$

$$q(x) = x^2 + 7x + 10$$

$$\text{Degree of } q(x) = 2$$

$$\text{Degree of } p(x) = \text{degree of } q(x)$$

Fraction is improper rational function.

$$\begin{array}{r}
 & \underline{1} \\
 x^2 + 7x + 10) & x^2 + 7x + 2 \\
 & \underline{x^2 + 7x + 10} \\
 & \underline{\underline{- - -}} \\
 & -8
 \end{array}$$

$$\frac{x^2 + 7x + 2}{x^2 + 7x + 10} = 1 + \frac{-8}{x^2 + 7x + 10}$$

$$\frac{8}{x^2 + 7x + 10} = \frac{8}{(x+2)(x+5)}$$

$$\frac{8}{(x+2)(x+5)} = \frac{A}{(x+2)} + \frac{B}{x+5}$$

$$= \frac{A(x+5) + B(x+2)}{(x+2)(x+5)}$$

$$8 = A(x+5) + B(x+2)$$

To determine A : Put $x+2=0$
 $x = -2$

$$8 = A(-2+5) = 3A$$

$$A = 8/3$$

To determine B : Put $x+5=0$
 $x = -5$

$$8 = B \cdot (-5+2) = -3B$$

$$\therefore B = -8/3$$

$$\frac{8}{(x+2)(x+5)} = \frac{8}{3(x+2)} + \frac{-8}{3(x+5)}$$

$$\frac{x^2 + 7x + 2}{x^2 + 7x + 10} = 1 - \left\{ \frac{8}{3(x+2)} - \frac{8}{3(x+5)} \right\}$$
$$= 1 - \frac{8}{3} \left[\frac{1}{x+2} - \frac{1}{x+5} \right]$$