$$\frac{g(x)}{f(x)} = \frac{g(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)}$$

$$\therefore \frac{g(x)}{(a_1x+b_1)(a_2x+b_2)\cdots(a_nx+b_n)} = \frac{A}{a_1x+b_1}$$

$$\frac{A}{a_1x_1+b_1} + \frac{B}{a_2x+b_2} + \cdots + \frac{B\ x}{a_nx+b_n}.$$

19/03/2021.

Q To resolve into partial ~~fact~~ fraction of the following:

(i) $\frac{mx+n}{(x-a)(x-b)}$ (ii) $\frac{7x-1}{1-5x+6x^2}$

(i) $p(x) = mx+n$

Degree of $p(x) = 1$

$q(x) = (x-a)(x-b) = x^2-(a+b)x+ab.$

$\therefore$ degree of $q(x) = 2.$

i.e. degree of $p(x) <$ degree of $q(x)$

$\therefore$ the given function is proper rational function

$$\frac{mx+n}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$mx + n = A(x-b) + B(x-a)$

To determine A : Put $x - a = 0$

i.e $x = a$

$m \cdot a + n = A \cdot (a-b)$

$\therefore A = \dfrac{ma+n}{a-b}$

To determined B : Put $x - b = 0$

$\therefore x = b$

$m \cdot b + a = B(b-a)$

$\therefore B = \dfrac{mb+n}{b-a}$

$$\frac{mx+n}{(x-a)(x-b)} = \frac{ma+n}{a-b} \cdot \frac{1}{x-a} + \frac{mb+n}{b-a} \cdot \frac{1}{x-b}$$

(ii) $\dfrac{7x-1}{1-5x+6x^2}$

$= \quad p(x) = 7x - 1$

Degree of $p(x) = 1$

$g(x) = 1 - 5x + 6x^2$

degree of given $g(x) = 2$.

Degree of $p(x) <$ Degree of $g(x)$

= fraction is proper natural function

$\dfrac{7x-1}{1-3x-2x+6x^2}$

$\dfrac{7x-1}{(1-3x)-2x(1-3x)} = \dfrac{7x-1}{(1-2x)(1-3x)}$

$\dfrac{7x-1}{(1-3x)\cdot(1-2x)} = \dfrac{A}{1-2x} + \dfrac{B}{1-3x}$

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$$\frac{\cancel{7x-1}}{(1-2x)(1-3x)} = \frac{A(1-3x)+B(1-2x)}{(1-2x)(1-3x)}$$

$$7x-1 = A(1-3x) + B(1-2x)$$

To Determine A = Put $1-2x = 0$

$\therefore x = 1/2.$

$$7\cdot\frac{1}{2} - 1 = A(1-3\times\frac{1}{2})$$

$$\frac{5}{2} = \frac{1}{2}A.$$

$\therefore A = -5$

To determine B : Put $1-3x=0$

$x = 1/3.$

$$7\times\frac{1}{3} - 1 = B(1-2)\cdot 1/3) = \frac{1}{3}B.$$

$$\frac{4}{3} = \frac{1}{3}B \Rightarrow B=4.$$

$$\frac{7x-1}{(1-2x)(1-3x)} = \frac{-5}{1-2x} + \frac{4}{1-3x}.$$

(3) $\dfrac{x^2+7x+2}{x^2+7x+10}.$

$p(x) = x^2+7x+2.$

degree of $p(x) = 2$

$q(x) = x^2+7x+10$

Degree of $q(x) = 2.$

Degree of $p(x)$ = degree of $q(x)$

Faction is improper rational function

By dividend

$$x^2+7x+10 \overline{\smash{)}\, x^2+7x+2}$$ quotient 1

$$\begin{array}{r} x^2+7x+2 \\ -\;(x^2+7x+10) \\ \hline -8 \end{array}$$

$\Rightarrow 1 + \dfrac{(-8)}{x^2+7x+10}$.

$\Rightarrow \dfrac{8}{x^2+7x+10} = \dfrac{8}{(x+2)(x+5)}$

$$\frac{8}{(x+2)(x+5)} = \frac{A}{(x+2)} + \frac{B}{(x+5)}$$

$$\frac{8}{(x+2)(x+5)} = \frac{A(x+5)+B(x+2)}{\cancel{(x+2)}\cancel{(x+5)}}$$

$$8 = A(x+5) + B(x+2)$$

To determine A = Put $x+2=0$

$x=-2$.

$8 = A(-2+5) = 3A$.

$A = 8/3$

To determine B: Put $x+5=0$

$x=-5$

$8 = B(-5+2) = -3B$.

$\therefore B = -8/3$

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$$\frac{8}{(x+2)(x+5)} = \frac{8}{3(x+2)} + \frac{-8}{3(x+5)}$$

$$\frac{x^2+7x+2}{x^2+7x+10} \Rightarrow 1 - \left\{\frac{8}{3(x+2)} - \frac{8}{3(x+5)}\right\}.$$

$$\cancel{\frac{8}{3}\left(\frac{1}{x+2} - \frac{1}{x+5}\right)}$$

$$\frac{x^2+7x+2}{x^2+7x+10} \Rightarrow 1 - \frac{8}{3}\left\{\frac{1}{x+2} - \frac{1}{x+5}\right\}. \quad \text{Ans}$$