Q. Find the value of x, if it is given that
$\log_{10} 3 = 0.4771213$.

$\log 7 = 0.8450930$

$\log 11 = 1.04127$.

$3^x \times 7^{2x-1} = 11^{x+5}$

Given: $3^x \times 7^{2x+1} = 11^{x+5}$

Taking log to the base 10 on both sides we get,

$\log 3^x \times 7^{2x+1} = \log 11^{x+5}$

$\log 3^x + \log 7^{2x+1} = \log 11^{x+5}$

$x \cdot \log 3 + (2x+1) \log 7 = (x+5) \cdot \log 11$

$x \cdot (0.4771213) + (2x+1)(0.8450980)$
$= (x+5)(1.0413927)$

$(0.4771213 + 2 \times 0.8450980 - 1.0413927)$
$= 0.8450980 + 5 \times 1.041392$

$\therefore \; x = \dfrac{5 \times 1.041392 - 0.8450980}{0.4771213 + 2 \times 0.8450980 - 1.0413927}$

$x = 3.8740298$

Q Evaluate (i) $\log_{\sqrt{5}} 0.008$

(ii) $\log_{2\sqrt{3}} 144$

(iii) $\log_{a^2} b \div \log_{\sqrt{a}} (b)^2$.

Solution (i) $\log_{\sqrt{5}} 0.008 = x$

$= (\sqrt{5})^x = 0.008$

$= (5)^{x/2} = \frac{\cancel{8}}{\cancel{1000}} = \frac{1}{5^3}$ $\quad$ ~~500~~ ~~250~~ 125

$= 5^{x/2} = 5^{-3}$

$= \frac{x}{2} = -3$

$\therefore x = 2(-3) = -6$ Ans

(iii) $\frac{\log_{a^2} b}{\log_{\sqrt{a}} (b)^2} = \frac{\frac{1}{2}\log_a b}{2\cdot\log_a (b)^2}$

$\Rightarrow \frac{\frac{1}{4}\cdot\log_a b^a}{2\log_a b}$

$= \frac{1}{8}\cdot\frac{\cancel{\log_a b}}{\cancel{\log_a b}} = \frac{1}{8}$ Ans

Q Prove that $\log_3 \log_2 \log_{\sqrt{3}} 81 = 1$

$= \log_3 \log_2 \cancel{3}\,2\cdot\log_3 3^4$

$= \log_3 \log_2 2\cdot4\cdot\log_3 3$

$= \log_3 \log_2 8$

$= \log_3 \log_2 2^3$

$= \log_3 3\log_2 2$

$= \log_3 3 = 1$ hence proved.

(ii) $\log_a x \times \log_b y = \log_b x \times \log_a y$

L.H.S $= \log_a x \times \log_b y$

$= \log_b x \times \log_a b \times \log_a y \times \log_b a.$

$= \log_b x \times \log_a y \times \log_a b \times \log_b a.$

$= \log_b x \times \log_a y.$ hence proved.