

Q Find the value of  $x$ , if it is given that  
 not given  $\log_{10} 3 = 0.4771213$ .

$$\log 7 = 0.8450930$$

$$\log 11 = 1.0413927$$

$$3^x \cdot 7^{2x+1} = 11^{x+5}$$

$$\text{Given: } 3^x \cdot 7^{2x+1} = 11^{x+5}$$

Taking log to the base 10 on both sides we get,

$$\log 3^x \cdot 7^{2x+1} = \log 11^{x+5}$$

$$\log 3^x + \log 7^{2x+1} = \log 11^{x+5}$$

$$\begin{aligned} x \cdot \log 3 + (2x+1) \log 7 &= (x+5) \cdot \log 11 \\ x \cdot (0.4771213) + (2x+1) (0.8450980) &= (x+5) (1.0413927) \\ (0.4771213 + 2x \cdot 0.8450980) - 1.0413927 &= \\ &= 0.8450980 + 5x \cdot 1.0413927 \end{aligned}$$

$$\therefore x = \frac{5 \times 1.0413927 - 0.8450980}{0.4771213 + 2 \times 0.8450980 - 1.0413927}$$

$$x = 3.8740298$$

(i) Evaluate (i)  $\log_{\sqrt[3]{5}} 0.008$   $\log_{\sqrt[3]{5}}$

(ii)  $\log_{\sqrt[2+3]{144}}$

(iii)  $\log_{a^2} b + \log_{a^2} \frac{\log b}{\log a^2} \div \log(b)^2$

Solution (i)  $\log_{\sqrt{5}} 0.008 = x$

$$\begin{aligned} &= (\sqrt{5})^x = 0.008 \\ &= (5^{1/2})^x = \frac{8 \cdot 10^{-3}}{1000} = \frac{1}{5^3} \\ &= 5^{x/2} = 5^{-3} \\ &= \frac{x}{2} = -3 \\ \therefore x &= 2(-3) = -6 \quad \underline{\text{Ans.}} \end{aligned}$$

(iii)  $\log_{a^2} b = \frac{1}{2} \log_a b$

$$\log_{\sqrt{a}} (b)^2 = \frac{2 \cdot \log_a (b)^2}{2 \cdot \log_a a}$$

$$\Rightarrow \frac{1}{4} \cdot \frac{\log_a b}{2 \log_a a}$$

$$= \frac{1}{8} \cdot \frac{\log_a b}{\log_a a} = \frac{1}{8} \quad \underline{\text{Ans.}}$$

Q Prove that  $\log_3 \log_2 \log_{\sqrt{3}} 8^4 = 1$

$$\begin{aligned} &= \log_3 \log_2 2 \cdot \log_3 3^4 \\ &= \log_3 \log_2 2 \cdot 4 \cdot \log_3 3 \\ &= \log_3 \log_2 8 \end{aligned}$$

14. Find  $\log_{(ca)} z$  and  $z$

$$= 1$$

$\frac{1}{3}$  and  $\frac{1}{4}$ .

s between prove that  $xyz = 1$ .

$\log_z \frac{y}{x}$  prove that  $\log_x a, \log_x b$  and  $\log_x c$  are A.P.

$\therefore \frac{\log z}{\alpha - \beta}$  prove that  $\log_x a, \log_x b$  and  $\log_x c$  are A.P.

P. show that  $\log_x a, \log_x b$  and  $\log_x c$  are A.P. prove that  $xyz + 1 = 2yz$ .

$\log_x c = \frac{2}{\log_x x}$

$+ \log_x c = \log_{ca} 3a$ ; prove that  $xyz + 1 = 2yz$ .

$y = \log_{3a} 2a, z = \log_{4a} 3a$ ; prove that  $xyz + 1 = 2yz$ .

$$= \log_3 \log_2^{2^3}$$

$$= \log_3 3 \log_2^2$$

$$= \log_3^3 = 1 \quad \text{hence proved.}$$

$$(ii) \log_a^x \times \log_b^y = \log_b^x \times \log_a^y$$

$$\text{L.H.S} = \log_a^x \times \log_b^y$$

$$= \log_b^x \times \log_a^b \times \log_a^y \times \log_b^a$$

$$= \log_b^x \times \log_a^y \times \log_a^b \times \log_b^a$$

$$= \log_b^x \times \log_a^y. \quad \text{hence proved.}$$