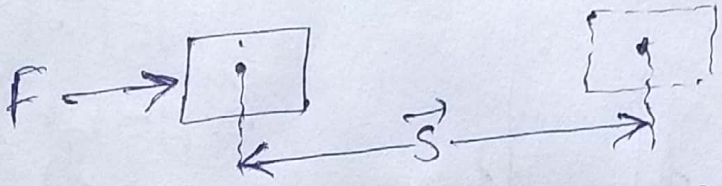
Work Energy and Power

Work:-



Work (W) = Force × displacement ×

force → vector
displacement → vector
vector × vector (cross product)
vector . vector (dot product).

Work done = Force . displacement

$$\boxed{W = \vec{F} \cdot \vec{S}}$$

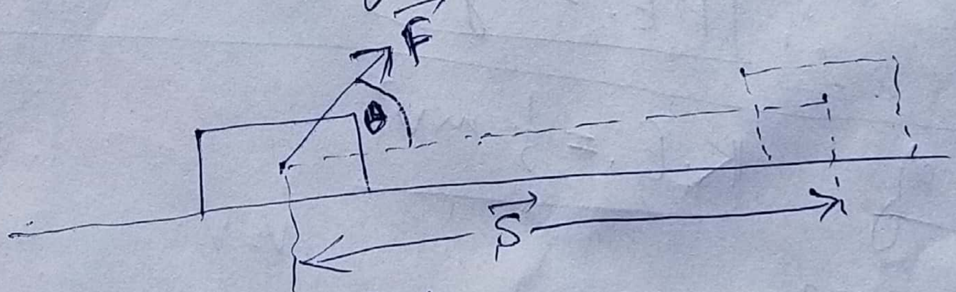
$$\Rightarrow \boxed{W = FS\cos\theta}$$

Here, $\theta = 0^\circ$

$\Rightarrow W = FS\cos 0^\circ$

$\Rightarrow \boxed{W = FS}$ as $\cos 0^\circ = 1$

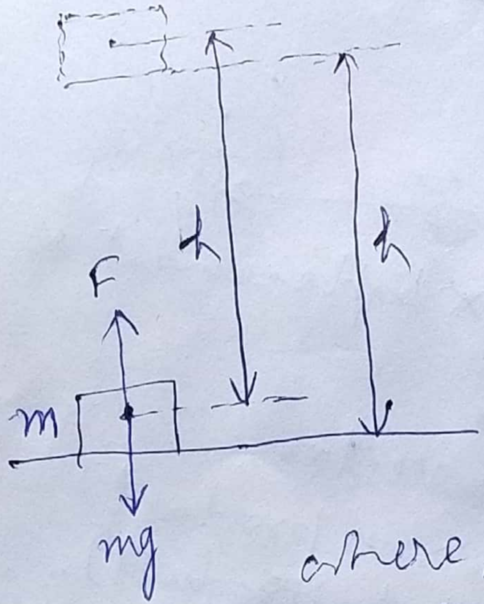
where, F = Applied force
S = Displacement produced.
θ = Angle between $\vec{F}$ and $\vec{S}$.



$W = \vec{F} \cdot \vec{S}$

$$\boxed{W = FS\cos\theta.}$$

Work done is a scalar quantity
Its S.I. Unit is joule (J)
Dimensional formula of work = $[ML^2T^{-2}]$



$W = \vec{F} \cdot \vec{h}$

$\Rightarrow W = Fh \cos 0^\circ$

$\Rightarrow W = Fh$ ——— (1)

Also, we know that

$F = mg$ ——— (2)

where,
m = Mass of the body
g = Accln due to gravity.

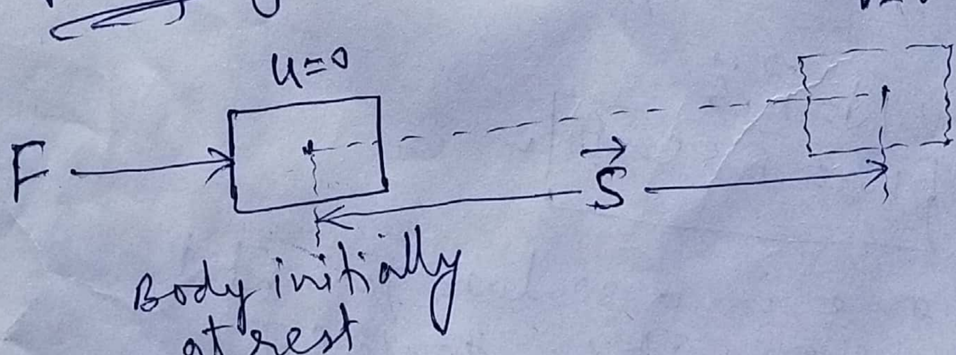
From eqns (1) & (2)

$$\boxed{W = mgh} \text{ ——— (3)}$$

From, Work energy principle
Amount of work done in lifting the body upto the height h from the surface of earth, stores as P.E. (Potential energy) into the body.

$$\therefore \boxed{P.E. = mgh}$$

Proof of $K.E. = \frac{1}{2}mv^2$



Workdone (W) in imparting velocity (P-3)
v to the body is given by

$W = \vec{F} \cdot \vec{S}$ ($\vec{F}$ & $\vec{S}$ are along same direction)

$\Rightarrow W = FS \cos 0^\circ$

$\Rightarrow W = FS$ —— (1)

Also, if a = acceln produced in the body

then, $F = ma$ —— (2)

Putting eqn (2) in eqn (1) we get,

$\Rightarrow W = maS$ —— (3)

Again we know that

$v^2 = u^2 + 2aS$

$v^2 = (0)^2 + 2aS$

$\Rightarrow v^2$

$\Rightarrow 2aS = v^2$

$\Rightarrow S = \frac{v^2}{2a}$ —— (4)

Putting eqn (4) in eqn (3), we get,

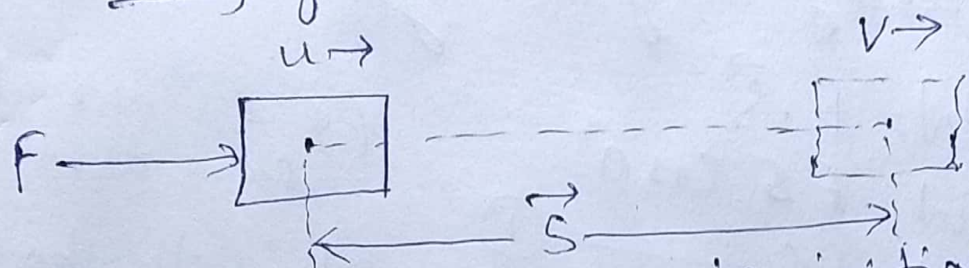
$W = ma \times \frac{v^2}{2a}$

$\Rightarrow W = \frac{1}{2} mv^2$ —— (5)

This work done in imparting velocity v to the body is it's K.E. (kinetic energy) at velocity v.

$\therefore \boxed{K.E. = \frac{1}{2} mv^2}$ Proved

Proof of Workdone = Change in K.E.



The body of mass m is initially moving with velocity u. If a force F is applied on it, it's velocity changes to v and during this time it covers a displacement S.

W = Work done by the force

$$W = \vec{F} \cdot \vec{S}$$

$$\Rightarrow W = FS\cos 0^\circ$$

$$\Rightarrow W = FS \quad \text{--- (1)}$$

Also, if a = Accln. produced in the body

$$F = ma \quad \text{--- (2)}$$

Using eqn. (2) in eqn. (1)

$$\Rightarrow W = mas \quad \text{--- (3)}$$

Again we have

$$v^2 = u^2 + 2as$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow S = \frac{v^2 - u^2}{2a} \quad \text{--- (4)}$$

Using eqn. (4) in eqn. (3) we get,

$$\Rightarrow W = ma \times \frac{v^2 - u^2}{2a}$$

$$\Rightarrow W = \frac{m(v^2 - u^2)}{2}$$

$$\Rightarrow W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$\Rightarrow$ Workdone = Final K.E. − Initial K.E.

$\Rightarrow$ Work done = Change in K.E. **Proved**