

$$\frac{x^2 + 7x + 2}{x^2 + 7x + 10} = \frac{1 - \frac{8}{3(x+2)} - \frac{8}{3(x+5)}}{}$$

$$= 1 - \frac{8}{3} \left[ \frac{1}{x+2} - \frac{1}{x+5} \right]$$

22.03.21

(ii) Repeated linear factors

$$\frac{g(x)}{(x-a)^n} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \dots$$

Ex. Resolve into partial fractions.

$$\frac{3x^2 + x - 2}{(x-2)^2(1-2x)}$$

Degree of numerator = 2

Degree of denominator = 3

Proper fraction

$$\frac{3x^2 + x - 2}{(x-2)^2(1-2x)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{1-2x}$$

$$= \frac{A(x-2)(1-2x) + B(1-2x) + Cx^2}{(x-2)^2(1-2x)}$$

$$3x^2 + x - 2 = A(x-2)(1-2x) + B(1-2x) + C(x-2)^2$$

To determine  $C$  = Put  $1-2x = 0$

$$x = \frac{1}{2}$$

$$3 \cdot (\gamma_2)^2 + \frac{1}{2} - 2 = C(\gamma_2 - 2)^2$$

$$\frac{3}{4} + \frac{1}{2} - 2 = \frac{9}{4} C$$

$$-\frac{3}{4} = \frac{9}{4} C$$

$$C = -\frac{1}{3}$$

To determine  $B$  = Put  $x-2 = 0$

$$x = 2$$

$$3 \cdot 2^2 + 2 - 2 = B(1-2 \cdot 2)$$

$$12 = -3B \Rightarrow B = -4$$

To determine A : Equating the co-efficient of  $x^2$  from both sides, we get,

$$3 = -2A + C$$

$$2A = C - 3$$

$$= -\sqrt{3} - 3 = -\frac{10}{3}$$

$$\therefore A = -5/3$$

$$\frac{3x^2 + x - 2}{(x-2)^2(1-2x)} = -\frac{5}{3} \cdot \frac{1}{x-2} + \frac{-4}{(x-2)^2} + \frac{-\sqrt{3}}{1-2x}$$

2 Resolve into partial fraction

$$\frac{g}{(x-1)(x+2)^2}$$

Degree of numerator = 0

Degree of denominator = 3

$\therefore$  Proper fraction.

$$\frac{g}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$= \frac{A(x-2)^2 + B(x-2)(x-1) + C(x-1)}{(x-1)(x+2)^2}$$

$$g = A(x+2)^2 + B(x+2)(x-1) + C(x-1)$$

To determine  $C$ : Put  $x+2=0$

$$x = -2$$

$$g = C(-2-1)$$

$$g = -3C \Rightarrow C = -3.$$

To determine  $A$ : Put  $x-1=0$

$$x = 1$$

$$g = A(1+2)^2$$

$$g = 9A \Rightarrow A = 1$$

To determine  $B$ : Equating the co-efficients of  $x^2$ ,

$$0 = A + B$$

$$A = -B = -1 \therefore B = -1$$

$$\frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} + \frac{-1}{(x+2)} + \frac{-3}{(x+2)^2}$$

Q. Dissolve into partial fractions of the following:

$$(i) \frac{3+2x-x^2}{(1+x)(1-4x)^2}$$

$$(ii) \frac{2x+5}{(x-1)^3(x-3)}$$

$$(iii) \frac{4+7x}{(2+3x)(1+x)^2}$$

$$(iv) \frac{x^2-x-1}{(x+1)(x-1)^2}$$