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A Survey on Fuzzy Logic & Fuzzy Classifiers based on Continuous Cellular Automata

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***Abstract-* Classification is one among the most studied supervised learning techniques. There are a variety of classifiers and classifying algorithms. Fuzzy Classifiers are classifiers which deals with varying degrees of membership, unlike crisp classifiers which have only a true or false value for membership of an instance to a class. Cellular Automata is one of the ways of performing computations which necessitates extremely the processing of data at a high speed. The use of cellular automata in classification is a promising area of study. Using Continuous Cellular Automata helps us to incorporate the concept of fuzziness into the Classifier using Cellular Automata. This survey studies the basic concept of fuzzy logic, cellular automata and the types of neighbourhood & diffusion rules in use, examines the concept of Continuous cellular automata and describes various defuzzification processes in brief. We also study an existing fuzzy classifier using continuous cellular automata and its working.**

***Index Terms-*Cellular Automata, Classification, Continuous Cellular Automata, Defuzzification, Fuzzy Logic.**

Introduction

1. Fuzzy logic

Fuzzy logic can represent knowledge in terms mathematical model. Because of this it can used in construction of models that can emulate the human decision making process[8].

Fuzzy logic is not fuzzy. Basically, fuzzy logic is a precise logic of imprecision and approximate reasoning [20]. More specifically, fuzzy logic can be viewed as an attempt at formalizing of two remarkable human capabilities. First, the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty, incompleteness of information, conflicting information, partiality of truth and partiality of possibility. And second, the capability to perform a wide variety of physical and mental tasks without any measurements and any computations (Zadeh 1999, 2001) Fuzzy logic has succeeded in many applications where traditional approaches have failed, because it does come to grips with the pervasive imprecision of the real world. Hence, human mind can be viewed as the role model for fuzzy logic[19]. Mendel [12] and Klir and Yuan [2] provide good introductory texts on Fuzzy Logic Systems (FLS), some examples of applications of FLSs can be found in [24], [3], and [1].

2. Pattern classifiers based on cellular automata (CA)

Pattern classifiers based on cellular automata (CA) have been explored as a result of the promising structured and spatiotemporal characteristics [31]. There is a less number of technical classifiers based on Cellular Automata reported to date. Elementary Cellular Automata (ECA) [31] consists of a group of cells arranged in one-dimension with two possible states (0 or 1). A next state for the ith cell is considered from local transition function of its nearest neighbours. [29] Now, in the case of continuous cellular automata, these group of cells can have values in the range $[0, 1]$. Also the it can be multidimensional. The transition rules for continuous cellular automata and the neighbourhood relations are similar to that in traditional cellular automata machines.

According to Wolfram [31], a continuous automaton can be described as a cellular automaton extended so the valid states a cell can take are not just discrete, but continuous, for example, the real number range $[0, 1]$. The cells remain discretely separated from each other. Such automata can be used to model certain physical reactions such as diffusion more closely[14].

3. Defuzzification

Defuzzification process is the last step in generating an output from a fuzzy inference system. The single value thus obtained after defuzzification process can be used by an expert system [21, 23]. There are a variety of defuzzification methods. Each method exhibits different performances under different situation. We cant say that a particular defuzzification method is best in all conditions. This was initially pointed out by Zadeh[16]. The most common defuzzification method is a variation of the max criterion method. These include smallest of maxima(SOM), largest of maxima(MOM), which select the smallest, the largest, or the mean output value whose membership value reaches the maximum[25].

Literature Survey

1. Fuzzy Sets and Its Properties

Zadeh [18] has introduced fuzzy set as a model to deal with imprecise, inconsistent and inexact information. Fuzzy set is a class of objects with continuous degrees of membership[26] [19]. The fuzzy set is characterized as its membership function whose values lies in the unit interval $[0, 1]$.

The base for fuzzy logic is rules. These are expressed in the form of IF-THEN statements. For example, "IF p, THEN q", where p and q are unconditional (and unqualified) fuzzy propositions[8]. An unconditional fuzzy proposition is expressed by the phrase: "X is A", where X is a variable that receives values from a universal set U, and A is a fuzzy set on U that represents a fuzzy predicate. Examples of a fuzzy predicate A are low, short, big, etc., and examples of a relevant variable X are temperature, height, and size[8].

Each fuzzy proposition is associated with a degree of truth T(p). For any value x of X, T(p) is equal to the grade of membership of x in A. Consider the fuzzy propositions given below:
P: X is A
Q: Y is B
where X receives values x from a universal set U, A is a fuzzy set on U, Y is a variable that receives values y from a universal set V, and is a fuzzy set on V.

The rule "IF X is A, THEN Y is B" combines the fuzzy propositions (p,q) into a logical implication (denoted $p \to q$)). The implication $p \to q$ signifies a fuzzy relation between p and q, and has a membership function denoted $\mu_{A\to B}(x, y)$. The function $\mu_{A\to B}(x, y)$ represents the degree of truth of the implication $p \to q$, given that X is equal to x and Y is equal to y. The IF part of an implication is called antecedent, whereas the THEN part is known as consequent. Note that a rule is also a fuzzy proposition[8].

Implication relations are used in crisp propositional logic to construct inference rules. One important type of inference rule is Modus Ponens, which has the following structure.
Premise 1: "X is A";
Premise 2: "IF X is A, THEN Y is B";
Consequence: "Y is B".
Modus Ponens is associated with the implication "A implies B" $(A \to B)$, and is the basic inference rule used in engineering applications of logic. In terms of propositions p and q, Modus Ponens is expressed as $(p \wedge (p \to q)) \to q$ [8].

Fuzzy Classification Systems are rule based fuzzy systems that perform classification by granulating the features domain by means of fuzzy partitions. Linguistic variables in the antecedent part of the rules represent features. Consequent part represents the class. A fuzzy classification rule can be expressed as
R_k : **IF** $X_1\ is\ A_{1j_1}$ **AND** $\cdots$ **AND** $X_m\ is\ A_{mj_m}$
THEN $Class\ =\ c_i$
where R_k is the identifier of k^{th} rule, X_1 to X_m are the m features of the example pattern represented by linguistic variables, A_{1j_1} to A_{mj_m} are the fuzzy sets used to represent each of the m feature values, and c_i is the fuzzy or crisp class that the example pattern belongs to.

Fuzzy set theory and fuzzy logic are a generalization of crisp set theory and traditional, dual-valued logic. In fuzzy set theory the range of the membership function characterizing a set is extended from $0, 1 to [0, 1]$ - a development that is mathematically comparable to the extension of the set of integers, **I**, to the set of real numbers, **R**. Proceeding along this path of mathematical generalizations, the next natural step is the extension of **R** to **C**, the set of complex numbers[8]. Applied to fuzzy set theory, this extension leads to the definition of the complex fuzzy set, a fuzzy set characterized by a complex-valued membership function. This novel set is the basis for complex fuzzy logic. Complex fuzzy sets were introduced in [7], where their mathematical properties are developed.

1.1. Complex Fuzzy Set

Complex fuzzy by definition allows us to specify membership by a complex number. This in effect transforms "membership in a set" into a two-dimensional concept.
Membership of any element *x*, in a complex fuzzy set

S, is given by a complex-valued grade of membership, which comprisescan amplitude term and a phase term. The amplitude term retains the traditional notion of "fuzziness", i.e., the representation of membership in a set as a value in the range $[0, 1]$. The phase term signifies the assertion of complex fuzzy set theory that, at least in some instances, a second dimension of membership is required[8].

Complex fuzzy logic is a framework which is designed in such as way as to maintain the advantages of traditional fuzzy logic, while benefiting from the properties of complex fuzzy sets. Complex fuzzy logic cannot be seen as a mere linear extension of conventional fuzzy logic. Rather, it allow a natural extension of fuzzy logic to problems that are either very difficult or impossible to address with one-dimensional grades of membership[8].

1.1.1 Definition of a Complex Fuzzy Set [7]

Definition : A Complex Fuzzy Set S, defined on a universe of discourse U, is characterized by a membership function $\mu_s(x)$ that assigns any element $x \epsilon U$ a complex-valued grade of membership in S. By definition, the values $\mu_s(x)$ may receive all lie within the unit circle in the complex plane, and are thus of the form $r_s(x).e^{jws(x)}$, where, j=/-1, $r_s(x)$ and $w_s(x)$ are both real-valued, and $r_s(x)\epsilon[0, 1]$. The complex fuzzy set S may be represented as the set of ordered pairs
$S = \{(x, \mu_s(x))|x\epsilon U\}$.

1.1.2 Interpretation of the Complex Fuzzy Set [7]

Considering the form $\mu_s(x) = r_s(x).e^{jws(x)}$, we can see that complex grades of membership are comprised of an amplitude term $r_s(x)$ and a phase term $w_s(x)$. Attempting to represent an ordinary fuzzy set in terms of a complex fuzzy set, provides initial insight into the role of the amplitude and phase terms. To represent an ordinary fuzzy set in terms of a complex fuzzy set, we need to set the amplitude term $r_s(x)$ equal to membership function, and the phase term equal to zero for all x .

This reflects the fact that the amplitude term is analogous to the membership function in traditional fuzzy set, a considering also the fact that the range of $r_s(x)$ is $[0, 1]$, like a normal real-valued grade of membership. This interpretation of the amplitude term suggests that, much like an ordinary grade of membership, the amplitude term may be regarded as representing the degree to which x is a member of the complex fuzzy set S. On the other hand, the phase term is a completely new parameter of membership, and definitely distinguishes between ordinary and complex fuzzy sets. In fact, it is this phase term, which provides the framework of complex fuzzy logic with its unique properties and sets it apart from traditional fuzzy logic[8].

1.2. Generalized Fuzzy Logic for incomplete information [26]

The fuzzy sets, fuzzy inference and fuzzy reasoning are studied to deal with incomplete information with single membership function [18]. The fuzzy set with two fold membership function will give more evidence to deal with the incomplete information rather than single membership function[26]. REN Ping [28] defined generalized fuzzy set with two fold membership function "True" and "False" and Generalized fuzzy logic is studied. Zadeh fuzzy logic is extended to generalized fuzzy logic in the following.

Generalized fuzzy set

The fuzzy set for proposition " x is A" is defined as
$A = \mu_A(x)$, where A is fuzzy set and $x\epsilon X$, $\mu_A(x)$ is fuzzy membership function. REN Ping [28] define fuzzy set with two fold membership function using True and False. Given some Universe of discourse X, the proposition " x is A" is defined as its two fold fuzzy membership function as:
$\mu_A(x) = \mu_A^{True}(x), \mu_A^{False}(x)$ or
$A = \mu_A^{True}(x), \mu_A^{False}(x)$

Where A is Generalized fuzzy set and $x\epsilon X$,
$0 <= \mu_A^{True}(x) <= 1 and, 0 <= \mu_A^{False}(x) <= 1$
$A = \mu_A^{True}(x_1)/x_1 + ... + \mu_A^{True}(x_n)/x_n$,
$\mu_A^{False}(x_1)/x_1 + ... + \mu_A^{True}(x_n)/x_n$,
$x_i\epsilon X$, where the symbol "+" represents union.

The conditions,
$\mu_A^{True}(x) + \mu_A^{False}(x) < 1$,
$\mu_A^{True}(x) + \mu_A^{False}(x) > 1$ and
$\mu_A^{True}(x) + \mu_A^{False}(x) = 1$ are interpreted as redundant, insufficient and sufficient Knowledge respectively.

2. Cellular Automata

Cellular Automata is a way to perform computations which involves processing of data at high speed. In cellular machine, a great deal of data should be processed in a short time in order to use the outcome results[4].

Self-reproduction in artificial systems was first studied by John Von Neumann in 1948. J. Von Neumman[13] and S. Ulam introduced the concept of Cellular automata to model natural physical and biological phenomena, in particular, for Von Neumanns pioneering studies of self-reproduction. [5]

Cellular Automata was designed with an aim to create Self Replication Systems that was supposed to be perfect computationally [29]. S. Ulam devised a two-dimensional imaginary finite lattice for one-cellular machine formed of components named cells [36]. These cells are in contact with each other locally. Cellular Machine is a ruptured dynamic system in time and space, which contains arrays of cells each of which can be defined in one of the limited states usually changing situation in synchronous or asynchronous paces based on the rules [15]. The next state of a cell is determined by its current state and the current states of its neighbours. The state of each cell in t time is a function of three key aspects [13]:

- The state of cell in t time
- The state of neighbouring cells in t time
- The set of governing rules on each CA cell

If we assume cellular automata to be a graph, then it can be constructed in an N-dimensional Euclidian Space. Consider the following examples. The set of points with integer coordinates, with edges connecting points that differ by at most exactly 1 in exactly one position (the Von Neumman neighbourhood) or, alternatively by at most 1 in each coordinate (the Moore neighbourhood) [5].

In a two-dimensional case the point (2,3) has neighbours (1,3), (3,3), (2,4), and (3,3) if our cellular automaton uses a Von Neumann neighbourhood, and has additional neighbours (1,2), (1,4), (3,2), (3,4) [5]. Instead, if we are using a cellular automaton with Moore neighbourhood. We can allow wrapped around or a toroidal topology by identifying nodes which differ by a fixed vector[5]. For example, in a 25 x 50 node toroidal topology with Von Neumann neighbourhood, node (1,1) has neighbours (1,2), (2,1), (25,1) and (1,50) since (25,1) is identified with node (0,1) and (1,50) is identified with (1,0).

2.1. Synchronous vs. Asynchronous Update Rules

Usually a cellular automata updates the state of all its nodes simultaneously and in discrete steps. Thus, for all discrete times t ¿= 0, if at discrete time step t each node v is in some state $q_v(t)$ then at the next discrete time step t+1 node v is in its next state $q_v(t+1)$. Thus the new state at a node is given by the local update rule as a function of the current state of that node and the finite list of all current states of all nodes in the neighbourhood of v. In this case of globally simultaneous update, we say that the cellular automaton is synchronous[5].

If the updates of the local component automata are not required to take place synchronously, but each one will be updated to its next state an unbounded number of times as (locally discrete) time goes on, then it is an asynchronous automata network. The updates are otherwise unconstrained, e.g. they may be deterministic, non-deterministic, random, sequential, etc., or even synchronous. Use of synchronous and asynchronous cellular automata to the modelling of biological systems are discussed in (Schonfisch and de Roos, 1999). Prior to this, all published models of self-reproduction in cellular automata have used only synchronous cellular automata update rules[5].

3. Fuzzy Classifier Using Continuous Cellular Automata[14]

The existing fuzzy classification using Continuous Cellular Automata [14] consists of two phases. In the first phase, data samples from the training set is used to build and set up the classification model. This model is used for performing classification in the second phase.

3.1. Phase I: Building the Classifier Model

This is also called as 'Training phase'. In this phase, first the cell array is created as the underlying data structure for implementing this method. This is an n-dimensional array where n is the dimensionality of the data set. This means that n attributes of the data samples take part in the classification. For an n-dimensional cell array, each non-boundary cell has 2n neighbours, a left and a right neighbour corresponding to each attribute. Each cell is mapped to a parallel processing resource. Therefore the total number of cells in the model is independent of data set and depends only on the availability of parallel processing units. For the basic implementation of the method, as specified in [14], each attribute is partitioned into an equal number of divisions. A cell represents one combination of one partition from each attribute. When compared with the classic fuzzy classification model, a cell represents a classification rule. Each data sample (also called as patterns) in the training set is mapped to a cell based on the attribute values of that sample. This is performed by mapping each attribute to a corresponding partition of that attribute and mapping the combination of all such partitions of all attributes to a single cell.

Once the cell that the sample belongs to has been identified, the membership values of classes in that cell are updated. The updating is done so that membership degree of a class is the percentage of data samples mapped to that cell and belonging to that class with respect to the total number of data samples mapped to that cell (belonging to any class). The values are normalized so that the sum of membership values of all classes within a cell is one.

Once all the training data has been processed, the cells, which had no patterns mapped into them, start acting like a diffusion system, in which each cell independently spreads its membership degree values into its neighbouring cells. The diffusion process stabilizes only when the new membership values of every unmapped cell, is the arithmetic mean of membership values of all its neighbours (mapped or unmapped). Once stabilization occurs the phase I is over and the classifier model is said to be built.

3.2. Phase II: Performing Classification

This is also called as 'Testing phase'. Classification is done once the classifier model has been set up in phase I. The data sample that needs to be classified is first mapped to the corresponding cell by the same cell-mapping algorithm used in phase I.

The degree of membership of this data sample in various classes is same as the membership degrees of the cell that this sample is mapped to. Now fuzzy classification is said to be completed. If the sample needs to be classified in a crisp fashion, an appropriate defuzzification method can be applied on this list of fuzzy membership degree values. The existing model is based on continuous cellular automata. This method has an accuracy as good as the existing approaches and even better in some cases.

4. Defuzzification

Defuzzification if defined mathematically, is a mapping strategy from a fuzzy set into a particular non fuzzy (crisp) space. I.e. Defuzzification is realized by a decision-making algorithm that selects the best crisp value based on a fuzzy set[32]. Detailed analysis of COG, MOM and Max Criteria(MC) strategies by Kiszka et al. [11], Kickert and Mamdani [35], Braae and Rutherford [22], Larkin[17], as well as Scharf and Mandic [9], concluded that the COA and MOM strategies usually perform better than the MC strategy, Here, neither the COA nor the MOM strategy cannot be claimed to be a better one over the other. The COA strategy yields better steady-state performance and smoother response, but is less transient than the MOM strategy[32].

In Largest of maxima method, the final output is calculated by averaging the set of those output values that have the highest possibility degrees. Two other popular techniques include the Center of Gravity (COG) or Centroid and Center of Area(COA) or Bisector methods[32].

As fuzzy concepts are more understood and accepted, the demand for developing a general and adaptable defuzzification strategy, which can generate the optimal defuzzified result also increases. Filev and Yager [6] were the first to realize that an objective defuzzification strategy cannot be generated without using a learning procedure[32], and they introduced the basic defuzzification distributions (BADD) which included a learning procedure in the defuzzification strategy. Later on, for simplicity, they also introduced the semi linear defuzzification (SLIDE) and the modified semi linear defuzzification (MSLIDE) strategies [27].

Tao Jiang and Yao Li introduced two new objective defuzzification strategies, Gaussian distribution transformation-based defuzzification (GTD) and polynomial transformation-based defuzzification (PTD), which are based on a discrete universe of discourse. The PTD strategy can be considered as a generalized defuzzification method for a wide variety of problems.

A situation specific switching of different defuzzification methods was studied by Smith [10]. This has a high implementation effort, and the switching effects may be crucial[34]. It is important to select appropriate defuzzification methods depending on the application were standard defuzzification methods fails. Thomas A Runkler proposed a fuzzy rule-based systems with a fixed defuzzification method where Selection of Appropriate Defuzzification Methods is done using Application Specific Properties[34].

There are a variety of defuzzification mechanisms [33] discusses two such defuzzification methods namely Center of Area and Centroid Approximation. A large number of defuzzification mechanisms are discussed in [30], which include True Center of Gravity, Fast Center of Gravity, Mean of Maxima (MOM), Plateau Average (PA), Weighted Plateau Average(WPA), Sparus, Capitis and Clivosus.

5. Conclusion

Fuzzy sets helps us to deal with vague information and hence to model realworld problems. They are a class of objects with continuous membership functions. Cellular automata helps us to perform computations which involves processing of data at high speeds. The use of cellular automata in Classification is a promising area of research. But yet, there is less research projects done in this is area. Classification process is aimed at finding the class label of a new instance given a set of training samples. In normal classification, we had only crisp membership values. Introducing the concept of Fuzzy logic in classification process helps us to deal with varying degrees of membership.

Fuzzy logic when applied to Cellular Automata, gives us a more flexible and accurate model named Continuous Cellular Automata. Current studies on Fuzzy Classifier using Continuous Cellular Automata tend to show better accuracy than traditional classification methods. Defuzzification procedure plays an important role in the performance of fuzzy systems because a final crisp output may be required for practical applications. The choice of defuzzification strategy, can directly affect the success of such applications.

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