$$p_4(x)=g(a)+g'(a)(x-a)+\frac{1}{2}g''(a)(x-a)^2+\frac{1}{3!}g'''(a)(x-a)^3+\frac{1}{4!}g^{(iv)}(a)(x-a)^4$$

$$g(x)=g(x)-p_4(x)+p_4(x)$$

$$\int_a^b \frac{g(x)}{(x-a)^p}dx=\int_a^b \frac{g(x)-p_4(x)}{(x-a)^p}dx+\int_a^b \frac{p_4(x)}{(x-a)^p}dx$$

where

$$\int_a^b \frac{p_4(x)}{(x-a)^p}dx=\sum_{k=0}^{4}\int_a^b \frac{g^{(k)}(a)}{k!}(x-a)^{k-p}dx=\sum_{k=0}^{4}\frac{g^{(k)}(a)}{k!(k+1-p)!}(b-a)^{k+1-p},$$

And

$$\int_a^b \frac{g(x)-p_4(x)}{(x-a)^p}dx=\int_a^b G(x)dx$$

where $G(x)=\begin{cases}\dfrac{g(x)-p_4(x)}{(x-a)^p}, & a<x\le b\\ 0, & x=a\end{cases}$

HW4 E94106185 王子豪

1. Determine the values $\int_1^2 e^x\sin(4x)dx$ with $h=0.1$ by

a. Use the composite trapezoidal rule Ans: 0.39614759

b. Use the composite Simpsons' method Ans: 0.38566360

c. Use the composite midpoint rule Ans: 0.38080480

2. Approximate $\int_1^{1.5} x^2\ln x dx$ using Gaussian Quadrature with $n=3$ and
Ans: n=3, 0.1436482150 ; Ans: n=4, 0.1436482464
$n=4$. Then compare the result to the exact value of the integral.
Ans: exact value: 0.1436482466

3. Approximate $\int_0^{\pi/4}\int_{\sin x}^{\cos x}(2y\sin x+\cos^2 x)dydx$ using

a. Simpson's rule for $n=4$ and $m=4$ Ans: 0.07739 74189

b. Gaussian Quadrature, $n=3$ and $m=3$ Ans: 0.0775173265

c. Compare these results with the exact value. Ans: 0.~~45~~

4. Use the composite Simpson's rule and $n=4$ to approximate the

improper integral a) $\int_0^1 x^{-1/4}\sin x dx$, b) $\int_1^\infty x^{-4}\sin x dx$ by use the transform

$t=x^{-1}$ (a) 0.5259288092

Ans: (b) 0.274481612 7