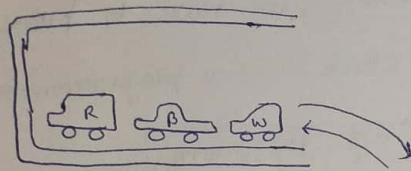


# STACK

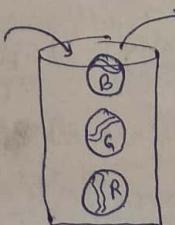
It Works on LIFO :- Last-in first out. Actually stack is a collection of elements and those elements are inserted/deleted on this discipline.

Eg :-



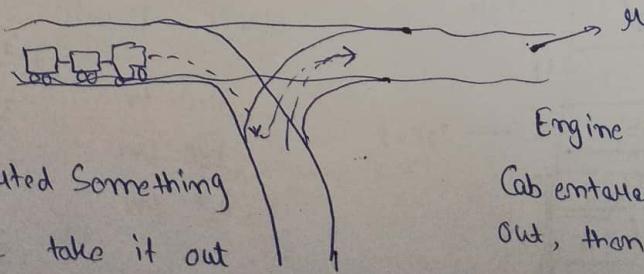
white car inserted last and white car has to come out first, this is Last in First out.

Eg :-



First ball :- Blue  $\rightarrow$  it will insert at last but will come first.

Eg :-



railway track

Note :- if we inserted something in a stack and we take it out again then it will reverse its direction by using stack.

Engine Entered first and last Cab entered last and if comes out, then Cab comes out first.

\* Now, Recursions, we know Recursions using Stack, they are working as a loop but they uses stack. and we can convert recursive function into iterative but when we convert it into iterative version, some times we have to use stack.

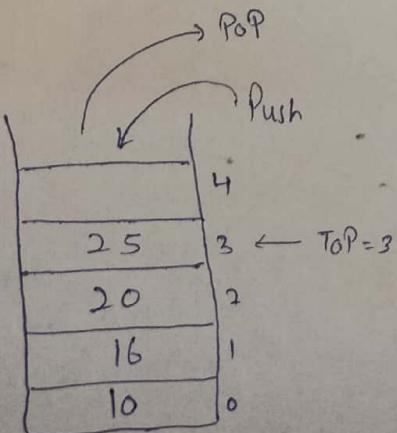
\* Recursions uses stack, we already know --- but that is a System Stack, programmer doesn't have to do anything but when we are converting it into iterative version, we have to provide a stack which is known as programmer stack. So programmer should create a stack and implement it in a program.

### \* ADT STACK :- ABSTRACT DATA TYPE STACK

Data :- 1) space for storing elements.  
2) Top Pointer.

#### Operations :-

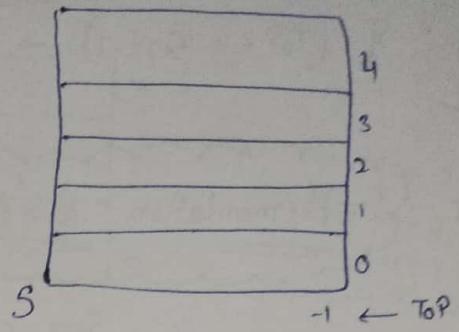
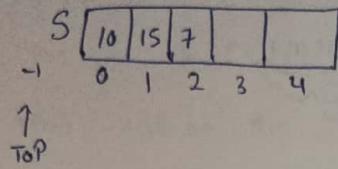
- 1) Push (x)
- 2) Pop ()
- 3) Peek (index)
- 4) Stack Top ()
- 5) is Empty ()
- 6) is Full ()



We can store values in Array & Linked List, so we use them to implement a stack.

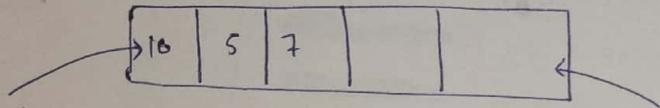
## STACK USING ARRAY :-

Size = 5



### \* Things Required for Implementing a Stack :-

- An Array
- Variable for Size
- Top Pointer



We can insert from this side also, but we will not insert from this side because if we insert from this side, we have to shift all the elements of an array which requires  $O(n)$  Time Complexity.

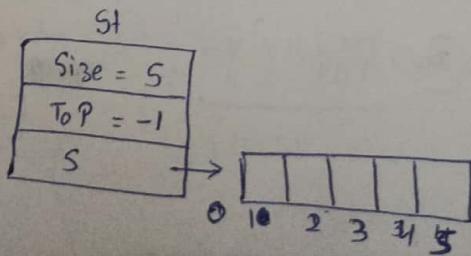
We will insert from this side because by inserting from this side, it requires  
 O(1) Time Complexity and No shifting of Elements required.

### \* Structure of Stack :-

```
struct Stack
{
    int size;
    int top;
    int *s;
};
```

### \* Initializing :-

```
int main()
{
    struct Stack st;
    printf("Enter size of stack: ");
    scanf("%d", &st.size);
    st.s = new int[st.size]; // in C++
    st.top = -1; // Pointing outside the stack if stack is empty.
}
```

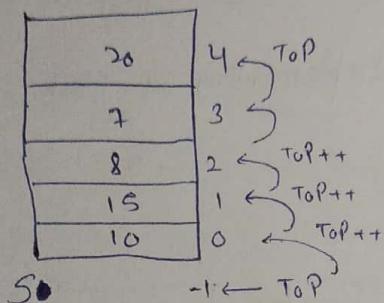


- if ( $\text{TOP} == -1$ )  $\rightarrow$  Stack Empty Condition
- if ( $\text{TOP} == \text{Size}-1$ )  $\rightarrow$  Stack full Condition

★ Implementation of Stack Using Array :-

① Push() :-

$\text{Size} = 5$



Time Complexity :-  $O(1)$

Void Push (Stack \*st, int x)

{

if ( $\text{st} \rightarrow \text{TOP} == \text{st} \rightarrow \text{Size}-1$ ) || is stack full or not

printf ("Stack overflow");

else

{

$\text{st} \rightarrow \text{TOP}++$ ;

$\text{st} \rightarrow \text{s}[\text{st} \rightarrow \text{TOP}] = x$ ;

}

② Pop() :-

int

Pop (Stack \*st, int x)

int  $x = -1$ ;

if ( $\text{st} \rightarrow \text{TOP} == -1$ )

printf ("Stack Underflow");

else

{

$x = \text{st} \rightarrow \text{s}[\text{st} \rightarrow \text{TOP}]$ ;

$\text{st} \rightarrow \text{TOP}--$ ;

}

return x;

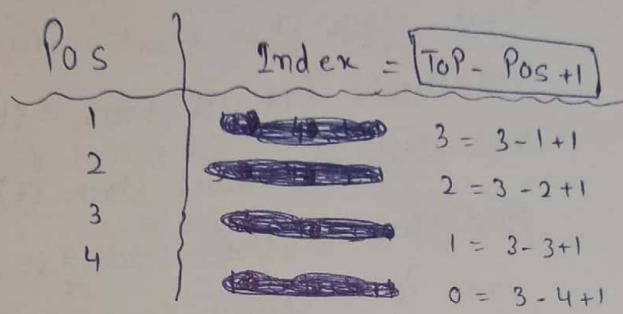


Time Complexity :-  $O(1)$

Peek() → knowing a value at given position.

it means if we want to get the 1<sup>st</sup> position, then it will be a top, we don't have to see the index, just see top position, that will be the 1<sup>st</sup> position another so on.

4	
3	$\leftarrow \text{Top} = 3$
2	$3 - 1 + 1 = 3$
1	$3 - 2 + 1 = 2$
0	$3 - 3 + 1 = 1$
	$3 - 4 + 1 = 0$



Time Complexity :-  
 $O(1)$

```
int Peek (stack st , int Pos)
{
    if (int x = -1
        (Top - Pos + 1) < 0)
        printf (" Invalid Position");
    else
        x = st. S [st. (Top - Pos + 1)];
    return x;
}
```

(4) int stack Top (stack st)  
{ if (st. Top == -1)  
 return -1;  
else  
 return st. S [st. Top];  
}

(5) int isEmpty (stack st)  
{ if (st. Top == -1)  
 return 1;  
else  
 return 0;  
}

(6) int isFull (stack st)  
{ if (st. Top == st. Size - 1)  
 return 1;  
else  
 return 0;  
}



:-

Struct Stack

{ int Size;

int top;

int \*S;

}

int main()

{

struct stack st;

void create (struct stack \*st);

void display (struct stack st);

void Push (struct stack \*st, int x);

int Pop (struct stack \*st);

int Peek (struct stack st, int index);

Create (&st);

Push (&st, 10);

Push (&st, 20);

Push (&st, 30);

Push (&st, 40);

printf ("%d", Peek (st, 3));

printf ("\n");

Display (st);

printf ("After Popping : %d", Pop (&st));

}

Void Create (struct stack \*st)

{ printf ("Enter stack size : \n");

scanf ("%d", &st->size);

st->top = -1;

st->s = (int \*) malloc (st->size \* sizeof (int));

```

Void Display (struct stack st)
{
    int i;
    for (i = st.top ; i >= 0; i--)
    {
        printf ("%d", st.s[i]);
    }
    printf ("\n");
}

Void Push (struct stack *st, int x)
{
    if (st->top == st->size - 1)
        printf ("Stack overflow\n");
    else
    {
        st->top++;
        st->s[st->top] = x;
    }
}

int PoP (struct stack *st)
{
    int x = -1;
    if (st->top == -1)
        printf ("Stack underflow\n");
    else
    {
        x = st->s[st->top--];
    }
    return x;
}

```

```
int Peek (struct stack st, int index)
{
    int x = -1;
    if (st.top - index + 1 < 0)
        printf ("Invalid index");
    else
    {
        x = st.s[st.top - index + 1];
    }
    return x;
}
```

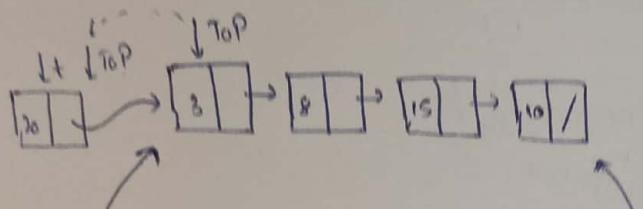
```
int isEmpty (struct stack st)
{
    if (st.top == -1)
        return 1;
    return 0;
}
```

```
int isfull (struct stack st)
{
    if (st.top == st.size - 1)
        return 1;
    return 0;
}
```

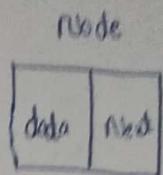
```
int stacktop (struct stack st)
{
    if (!isEmpty (st))
    {
        return st.s[st.top];
    }
    return -1;
}
```

## Stack Using Linked List :-

It also follows LIFO.



We can insert / delete from this end and it will take time which is  $O(1)$ .



```
struct Node  
{  
    int data;  
    struct Node *next;  
};
```

We can also insert / delete from this end but it will take time which is  $O(n)$  because for this side, we have to traverse through linked list.

- STACK is Empty

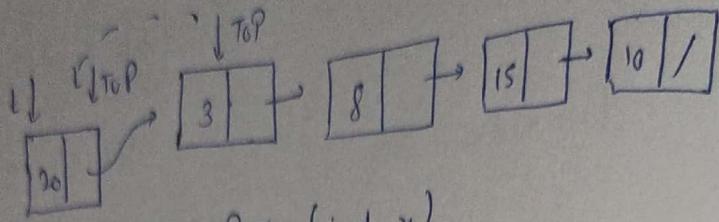
```
if (TOP == NULL)
```

- STACK is FULL

```
Node *t = new Node;
```

```
if (t == NULL)
```

# \* Implementation of Stack Using Linked List :-

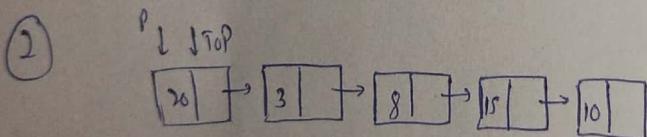


① void Push (int n)

```

{
    Node *t = new Node;
    if (t == NULL)
        cout ("Stack overflow");
    else
    {
        t->data = n;
        t->next = TOP;
        TOP = t;
    }
}
  
```

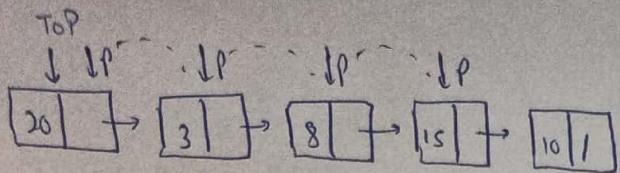
If we insert the very first value, then it will also work.



int POP ( )

```

{
    Node *P;
    int x = -1;
    if (TOP == NULL)
        cout ("Stack is Empty");
    else
    {
        P = TOP;
        TOP = TOP->next;
        x = P->data;
        free(P);
    }
    return x;
}
  
```



```

int Peek ( int Pos) {
    int i;
    Node *P = TOP;
    for (i=0; P != NULL && i < Pos-1; i++)
    {
        P = P->next;
    }
    if (P == NULL)
        return P->data;
    else
        return -1;
}

```

(4) int Stack TOP ( )

```

{
    if (TOP)
        return TOP->data;
    return -1;
}

```

(5) int isEmpty ( )

```

{
    return TOP ? 0 : 1;
}

```

(6) int isFull ( )

```

{
    Node *t = new Node;
    int x = t ? 1 : 0; → if t != NULL, then x=1 else x=0
    free(t);
    return x;
}

```

Code :-

```
Struct Node
{
    int data;
    Struct Node * next;
} * top = NULL;

int main()
{
    void Display();
    void Push(int x);
    int Pop();
    int Peek(int index);

    Push(10);
    Push(20);
    Push(30);

    Display();

    printf("%d", Pop());
    printf("%d", Peek(2));
}

void Push(int x)
{
    Struct Node * t;
    t = (Struct Node *) malloc (sizeof (Struct Node));
    if (t == NULL)
        printf("Stack is Full\n");
    else
    {
        t->data = x;
        t->next = top;
    } top = t;
}
```

+ Pop()  
Struct Node  
int x=-1;  
if (top);  
else

```

int Pop()
{
    Struct Node *t;
    int x = -1;
    if (top == NULL)
        coutf ("Stack is Empty\n");
    else
    {
        t = top;
        top = top->next;
        x = t->data;
        free(t);
    }
    return x;
}

```

```

Void Display()
{
    Struct Node *P;
    P = top;
    while (P != NULL)
    {
        coutf ("%d ", P->data);
        P = P->next;
    }
    coutf ("\n");
}

```

```

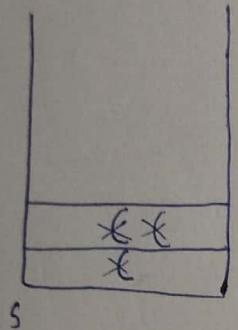
int Peek (int index)
{
    int i;
    Struct Node *P = top;
    for (i=0; P != NULL && i < index-1; i++)
        P = P->next;
    if (P != NULL)
        return P->data;
    else
        return -1;
}

```

## ★ Parenthesis Matching

:- We have check whether the parenthesis are balanced or not , that is for every opening bracket , there must be a closing bracket.

$$((a+b) * (c-d))$$



→ if stack is Empty , then parenthesis are balanced

$$(((a+b) * (c-d))) \rightarrow \text{parenthesis are not balanced}$$

### Note :-

- (i) If we are scanning through an Expression by pushing opening brackets and popping out closing brackets , In this procedure
- (ii) If at the end the stack is empty, then they are perfectly matching.
- (iii) If at the end , there will be an opening bracket remaining then it will not be matching.

If we have a closing bracket but the stack is empty , then it will not be matching.

ode :-

```
Struct Node
{
    char data;
    Struct Node * next;
}  $\Rightarrow$  top = NULL;

int main()
{
    char * exp = "((a+b)*(c+d))";
    int balanced (char * exp);
    int m = balanced (exp);
    printf ("%d", m);
}

void Push (char x)
{
    Struct Node * t;
    t = (struct node *) malloc (sizeof (Struct Node));
    if (t == NULL)
        printf ("stack is full\n");
    else
    {
        t->data = x;
        t->next = top;
        top = t;
    }
}
```

char Pop()

```
{  
    struct Node *t;  
    if (top == NULL)  
        printf ("Stack is Empty In");  
    else  
    {  
        t = top;  
        top = top->next;  
        free(t);  
    }  
}
```

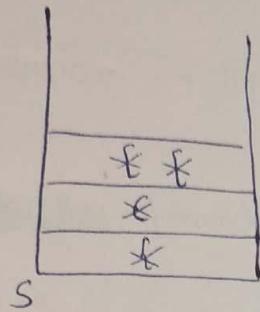
int balanced (char \*exp)

```
{  
    int i;  
    for (i=0; exp[i] != '\0'; i++)  
    {  
        if (exp[i] == '(')  
            push(exp[i]);  
        else if (exp[i] == ')')  
        {  
            if (top == NULL)  
                return 0;  
            pop();  
        }  
    }  
    if (top == NULL)  
        return 1; // True  
    else  
        return 0; // False
```

## More on Parenthesis Matching

:- Different type of brackets

$$\{ ([a+b] * [c-d]) / e \}$$



## Code

:-

```
Struct Node
{
    char data;
    Struct Node * next;
} * top = NULL;

int main()
{
    char * exp = " { ([a+b] * [c+d]) / e } ";
    int balanced (char * exp);
    int m = balanced (exp);
    printf ("%d", m);
}

Void Push (char x)
{
    Struct Node * t;
    t = (Struct Node *) malloc (size of (Struct Node));
    if (t == NULL)
        printf (" Stack is full \n");
    else
    {
        t->data = x;
        t->next = top;
        top = t;
    }
}
```

```

char top;
{
    struct Node *t;
    char x = -1;
    if (top == NULL);
        cout << "Stack is empty \n";
    else
    {
        t = top;
        top = top->next;
        x = t->data;
        free(t);
    }
    return x;
}

```

```

int Balanced (char *exp)
{
    int i;
    char x;
    for (i=0 ; exp[i] != '\0' ; i++)
    {
        if (exp[i] == '(' || exp[i] == '[' || exp[i] == '{')
            Push(exp[i]);
        else if (exp[i] == ')' || exp[i] == ']' || exp[i] == '}')
        {
            if (top == NULL)
                return 0;
            x = Pop();
            if ((exp[i] == ')' && x != '(') || (exp[i] == ']' && x != '[') || (exp[i] == '}' && x != '{'))
                return 0;
        }
        if (top == NULL)
            return 1;
    }
    return 0;
}

```

## Infix to Postfix Conversion

: -

- ① what is Postfix?
- ② why we need Postfix?
- ③ Precedence.
- ④ Manual conversion. → by Pen & paper

(A) Infix :-    ~~operator~~ operand    operator    operand

e.g. :-  $a+b$

(B) Prefix :-    operator    operand    operand

e.g. :-  $+ab$  → Add, what?  $a+b$ .

(C) Postfix :-    operand    operand    operator

e.g. :-  $ab+$  →  $a+b$ , what?, add them.

Q why we need Postfix | Prefix ?

A -  $\frac{8}{6} + \frac{3}{5} * \frac{(9-6)}{1} \frac{2^2}{3} + \frac{6}{7} / \frac{2}{4}$

we see that the order of performance of operators by jumping here and there by looking into the Expression and first which one should be performed so, to find which one should be performed, we have to scan through this Expression, so to avoid this we need Postfix | Prefix. Now, in this we will not scan this expression multiple times, we will scan only once.

$$1 \underline{8} \underline{3} \underline{9} \underline{6} \underline{-} \underline{2^2} \underline{|} \underline{*} \underline{+} \underline{6} \underline{2} \underline{|} \underline{1} \underline{+}$$

$$\textcircled{1} \quad a + b * c \quad \textcircled{2} \quad a + b + c * d$$

Now, for conversion, we need to fully

parenthesis an expression,

And Compiler need fully parenthesis function

an expression to perform and if the expression

is not parenthesis, then Compiler make it a parenthesis expression  
not physically but logically. by using Precedence.

Symbol	Precedence	Associativity
+, -	1	
*, /	2	
( )	3	



Note :- Precedence doesn't mean order of execution but it means which operation has higher Precedence, but that one in parenthesis first

$$\textcircled{1} \quad (a + (b * c))$$

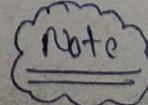
$$\begin{array}{c} \text{Prefix} \\ (a + [\times bc]) \\ / \\ + a \times bc \end{array} \qquad \begin{array}{c} \text{Postfix} \\ (a + [bc *]) \\ / \\ abc * + \end{array}$$

$$\textcircled{2}$$

$$\begin{array}{c} a + b + c * d \\ \text{Prefix} \quad \quad \quad \text{Postfix} \\ [+ ab] + [* cd] \quad a + b + cd * \\ / \quad \quad \quad | \\ + ab * cd \quad [ab+] + [cd*] \\ \quad \quad \quad | \\ \quad \quad \quad ab + cd * + \end{array}$$

$$\textcircled{3} \quad ((a+b) * (c-d))$$

$$\begin{array}{c} \text{Prefix} \quad \quad \quad \text{Postfix} \\ [+ab] * [(c-d)] \\ / \quad \quad \quad | \\ [+ab] * [-cd] \quad [ab+] * [cd-] \\ / \quad \quad \quad | \\ + ab - cd \quad ab + cd - * \end{array}$$



Note :-  
we will always  
do postfix.

## Associativity

If Precedences are equal in an expression, then we will go for Associativity.

TWO Types :- ① left - Right  
② Right - Left

① Left - Right :-

$$a + b + c - d$$

$$((a+b)+c) - d \rightarrow \text{Postfix} : - ab + c + d -$$

② Right - Left :-

$$a = b = c = s$$

$$(a = (b = (c = s))) \rightarrow \text{Postfix} : - abc s ==$$

③  $a^b^c$

$$(a^b^c) \rightarrow \text{Postfix} : - (a^b^c) = a^{[bc]} = abc^{^n}$$

④  $-a$

Prefix :-  $-a$

Postfix :-  $a-$

⑤  $(-(-a))$

Prefix :-  $--a$

Postfix :-  $a--$

⑥  $*P$

Prefix :-  $*P$

Postfix :-  $P*$

⑦  $(*(\ast P))$

Prefix :-  $\ast\ast P$

Postfix :-  $P\ast\ast$

⑧  $n!$

Postfix :-  $!n$

Prefix :-  $n$

⑨  $\log x$

Prefix :-  $\log n$

Postfix :-  $x \log$

These all are unary operations have more precedence than other operators and Right to Left Associativity.

Symbol	Precedence	Associativity
$+, -$	1	L-R
$\ast, /$	2	L-R
$\wedge$	3	R-L
$-$	4	R-L
$( )$	5	L-R
$=$		R-L

Postfix

$$\begin{aligned}
 & \# -a + b * \log n! \\
 & \downarrow \\
 & -a + b * \log [n!] \\
 & \downarrow \\
 & -a + b * [n! \log] \\
 & \downarrow \\
 & [-a] + b * [n! \log] \\
 & \downarrow \\
 & [-a] + [b n! \log] * \\
 & \downarrow \\
 & a - b n! \log + *
 \end{aligned}$$

Postfix

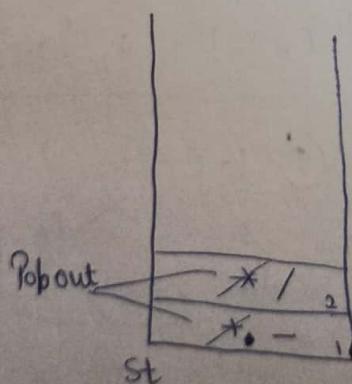
$$\begin{aligned}
 & \# -a + b * \log n! \\
 & \downarrow \\
 & -a + b * \log [!n] \\
 & \downarrow \\
 & -a + b * \cancel{\log [!n]} \\
 & \downarrow \\
 & [-a] + b * \cancel{\log [!n]} \\
 & \downarrow \\
 & [-a] + [* b \log !n] \\
 & \downarrow \\
 & [+ -a * b \log !n]
 \end{aligned}$$

\* Infix to Postfix Using Stack Method - I :-

$(a + b * c - d / e)$

"Now, We will generate a postfix Expression by Scanning above expression and taking one symbol at a time."

Symbol	Pre	Associativity
+,-	1	L-R
*,/	2	L-R



Postfix :  $a b c * + d e / -$

<u>Symbol</u>	<u>Stack</u>	<u>Postfix</u>
a	-----	a
+	+	a
b	+	ab
*	*, +	ab
c	*, +	abc
-	-	abc *+
d	-	abc *+d
/	/, -	abc *+d
e	/, -	abc *+de
-----	-----	abc *+de/-

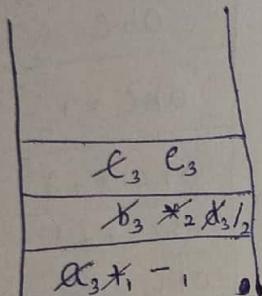
### Procedure :-

- ① Send operand to postfix.
- ② Next is operator , check if there is something in a stack or not , if it is empty then push it. and write its precedence.
- ③ For "Next" operator , look it into the stack , is there any operator or not , if there is any operator , then check its precedence and then check precedence of "Next" operator , if it is greater then push it into the stack and if it is less then pop out the symbol having higher precedence or equal precedence and push "this" operator into the stack.

## Infix to postfix using stack Method - 2

We will push operands also into the stack. Every token from infix will be pushed to stack and operands will have higher precedence than operators.

$$a+b * c - d/e$$



Symbol	Precedence	Associativity
+, -	1	L-R
*, /	2	L-R
a, b, c	3	L-R

$$\text{Postfix: } abc * + de / -$$

after end of expression, pop out all the things.

## Program :-

infix    [ a | + | b | \* | c | - | d | / | e | ]  
 0    1    2    3    4    5    6    7    8    9

char \* convert (char \* infix)

{ struct stack st; // Initialized

char \* Postfix = new char [strlen (infix)+1];

int i, j=0;

while (infix[i] != '\0')

{ if (isoperand (infix[i]))

    Postfix[j++] = infix[i++];

else

{ if (Pre (infix[i]) > Pre (stacktop (st)))

    Push (st, infix[i++]);

    Postfix[j++] = Pop (st);

}

    while (! isEmpty (st))  
 { Postfix[j++] = Pop (st);  
 Postfix[j] = '\0';

int is operand (char x)

{ if (x == '+' || x == '-' ||  
 x == '\*' || x == '/')

    return 0;

else

    return 1;

int Pre (char x)

{ if (x == '+' || x == '-')

    return 1;

else if (x == '\*' || x == '/')

    return 2;

else return 0;

de :-

Struct Node

```
{ char data;  
  Struct Node * next;  
 } * top = NULL;
```

int main()

```
{ char * infix = "a+b+c*d";  
  char * int to post (char * infix);  
  void Push (char x);  
  Push ('#'); //
```

```
char * Postfix = int to post (infix);  
printf ("%s", Postfix);
```

}

void Push (char x)

```
{ struct Node * t;  
  t = (struct Node *) malloc (size of (struct Node));
```

if (t == NULL)

printf ("stack is full in");

else

```
{ t->data = x;  
  t->next = top;  
  top = t;
```

}

}

```
char Pop ()  
{  
    struct Node *t;  
    char x = -1;  
    if (top == NULL)  
        printf ("Stack is Empty\n");  
    else  
    {  
        t = top;  
        top = top->next;  
        x = t->data;  
        free (t);  
    }  
    return x;  
}
```

```
int Pre (char x)  
{  
    if (x == '+' || x == '-')  
        return 1;  
    else if (x == '*' || x == '/')  
        return 2;  
    else  
        return 0;  
}
```

```
int isoperator (char x)  
{  
    if (x == '+' || x == '-' || x == '*' || x == '/')  
        return 1;  
    else  
        return 0;  
}
```

```

char * int to post (char * infix)
{
    int i = 0, j = 0;
    char * Postfix;
    int len = strlen (infix);
    Postfix = (char *) malloc ((len + 2) * sizeof (char));
    while (infix[i] != '\0')
    {
        if (isoperand (infix[i]))
            Postfix[j++] = infix[i++];
        else
        {
            if (Pre (infix[i]) > Pre (top->data))
                Push (infix[i++]);
            else
                Postfix[j++] = Pop();
        }
    }
    while (top != NULL)
        Postfix[j++] = Pop();
    Postfix[j] = '\0';
    return Postfix;
}

```

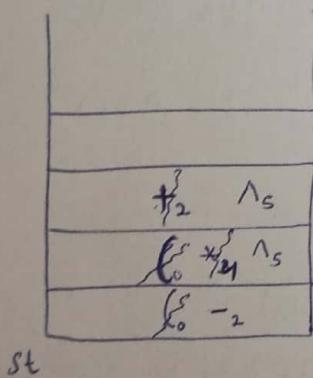
\* Student - Challenge :- Infix to Postfix conversion having  $\frac{a}{b}$  and other operators.

$$\begin{aligned}
 & ((\underline{a+b}) * c) - d^e^f \\
 & \underline{\quad / \quad} \\
 & (\underline{ab+} \underline{c *}) - d^e^f \\
 & \underline{\quad / \quad} \\
 & [\underline{ab+c} \times] - d^e^f \\
 & \underline{\quad / \quad} \\
 & [\underline{ab+c} \times] - d^e[\underline{ef}^{\wedge}] \\
 & \underline{\quad / \quad} \\
 & [\underline{ab+c} \times] - [def]^{\wedge} \\
 & \underline{\quad / \quad} \\
 & ab+c \times def^{\wedge\wedge} -
 \end{aligned}$$

Symbol	Out stack Precedence	in stack Precedence
+, -	1	2
*, /	3	4
$\wedge$	6	5
(	7	0
)	0	?

Right to Left Associativity

Outside the Stack      Inside the Stack



Postfix :- ab + c \* def ^ ^ -

Important Note :- ① When the Precedences are equal and they will be equal only in the case where we have opening bracket and closing bracket, otherwise in this Precedences, that is in stack and out stack Precedences will never be equal.

② The Purpose of giving two Precedences is that, if it is left to right then increase it, when it is going into the stack. otherwise when it is right to left, let it decrease, so if we want to add more operators, we can add, depending upon the associativity, we can increase or decrease the Precedences.

```

de) :- Struct Node
{
    int data;
    Struct Node *next;
} * top = NULL;

int main ()
{
    char * exp = "(a+b)*(c-d)^e^f";
    char * result = intopost(exp);
    printf ("%s", result);
    // printf (%s); // it will also work
}

void Push (char x)
{
    Struct Node *t;
    t = (Struct Node *) malloc (sizeof (Struct Node));
    if (t == NULL)
        printf ("Stack is full\n");
    else
    {
        t->data = x;
        t->next = top;
        top = t;
    }
}

char Pop ()
{
    Struct Node *t;
    char x = -1;
    if (top == NULL)
        printf ("Stack is Empty\n");
    else
    {
        t = top;
        top = top->next;
        x = t->data;
        free (t);
    }
    return x;
}

```

int is Empty ()

```
{  
    return ((top == NULL) ? 1 : 0);  
}
```

int is Operand (char ch)

```
{  
    if (ch == '+' || ch == '-' || ch == '*' || ch == '/' || ch == '^')  
        || ch == '<')  
    return 0;
```

else

```
    return 1;
```

}

int out Pre (char x)

```
{  
    if (x == '+' || x == '-')  
        return 2;
```

```
    if (x == '*' || x == '/')  
        return 3;
```

```
    if (x == '^')  
        return 6;
```

```
    if (x == '(')  
        return 7;
```

```
    if (x == ')')  
        return 0;
```

else

```
} return -1;
```

int in Pre (char x)

```
{  
    if (x == '+' || x == '-') return 2;
```

```
    if (x == '*' || x == '/') return 4;
```

```
    if (x == '^') return 5;
```

```
    if (x == '(') return 0;
```

```
else return 0;
```

```
} return -1;
```

```

char * intopost (char * infix)
{
    int i=0, j=0;
    char * Postfix;
    int len = strlen (infix);
    Postfix = (char *) malloc ((len+2) * (Size of (char)));
    while (infix[i] != '\0')
    {
        if (is operand (infix[i]))
            Postfix[j++] = infix[i++];
        else
        {
            if (top == NULL || outpre (infix[i]) > inpre (top->data))
                Push (infix[i]);
            else if (outpre (infix[i]) < inpre (top->data))
                Postfix[j++] = pop();
            else
            {
                pop();
                i++;
            }
        }
    }
    while (! isEmpty())
        Postfix[j++] = pop();
    Postfix[j] = '\0';
    return Postfix;
}

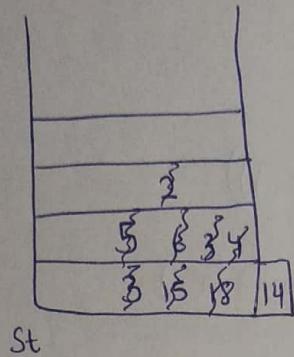
```

## Evaluation of Postfix Expression :-

Procedure :-

#  $3 * 5 + 6 / 2 - 4 \rightarrow$  infix form

Postfix form  $\rightarrow 35 * 62 / + 4 -$



$$3 * 5 = 15$$

$$6 / 2 = 3$$

$$15 + 3 = 18$$

$$18 - 4 = 14$$

Symbol	Stack	Operation
3	3	
5	5, 3	
*	15	$3 * 5 = 15$
6	6, 15	
2	2, 6, 15	<del>6 / 2 = 3</del>
/	3, 15	$6 / 2 = 3$
+	18	$15 + 3 = 18$
4	4, 18	
-	14	$18 - 4 = 14$

## Time Complexity :-

$O(n) \rightarrow$  Single Scan throughout the Postfix Expression.

## Procedure

:- ★ We will Scan through the expression by taking one symbol at a time and while Scanning do these steps as follows :-

- if it is an operand, push it into the stack , next will be an operand , then again push it into the stack.
- If we get any operator, then pop out 2-symbols from the stack and perform that operation and the first value that popped out will come on Right-hand Side and the Second value will come on Left-hand side and we will perform this operation.
- Then Push this result into the stack
- Then Proceed So on by applying above steps.

## Program

Postfix 

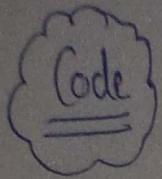
3	5	*	6	2	/	+	4	-	10
0	1	2	3	4	5	6	7	8	9

 → characters

```
int Eval (char * Postfix)
{
    struct stack st;
    int i, n1, x2, n;
    for (i=0 ; Postfix[i]!='\0' ; i++)
    {
        if (isoperand (Postfix[i]))
            Push (&st, Postfix[i] - '0'); // we are subtracting
                                         // ASCII code of '0', to convert char to integer in stack
        else
        {
            x2 = Pop (&st) ; x1 = Pop (&st)
            switch (Postfix[i])
            {
                Case '+' ; n = x1 + x2 ; Push (&st, n) ; break;
                Case '-' ; n = x1 - x2 ; Push (&st, n) ; break;
                Case '*' ; n = x1 * x2 ; Push (&st, n) ; break;
                Case '/' ; n = x1 / x2 ; Push (&st, n) ; break;
            }
        }
    }
    return Pop (&st);
}
```

St       $\leftarrow \begin{cases} = '3' - '0' \\ = 51 - 48 \end{cases}$

Conversion of character to integer



:-

```
Struct Node
{
    int data;
    Struct Node *next;
} * top = NULL;

int main()
{
    char * Postfix = "234 * + 82 / - ";
    int eval (char * Postfix);
    int P = eval (Postfix);
    printf ("Result is %d ", P);
}

Void Push (char x)
{
    Struct Node *t;
    t = (Struct Node *) malloc ( sizeof (Struct Node)
    if (t == NULL)
        printf ("Stack is full\n");
    else
    {
        t->data = x;
        t->next = top;
        top = t;
    }
}

int isoperator (char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/')
        return 0;
    else
        return 1;
}
```

```
int Pop()
```

```
struct Node *t;  
int x = -1;  
if (top == NULL)  
    die  
else  
{  
    t = top;  
    top = top->next;  
    x = t->data;  
    free(t);  
}  
return x;
```

```
int Eval (char *Postfix)
```

```
{  
    int i=0;  
    int x1, x2, m;  
    for (i=0 ; Postfix[i] != '\0' ; i++)  
    {  
        if (isoperator (Postfix[i]))  
            Push (Postfix[i] - '0');  
        else  
        {  
            x2 = Pop();  
            x1 = Pop();  
            switch (Postfix[i])  
            {  
                case '+':  
                    m = x1 + x2;  
                    break;  
                case '-':  
                    m = x1 - x2;  
                    break;  
                case '*':  
                    m = x1 * x2;  
                    break;  
                case '/':  
                    m = x1 / x2;  
                    break;  
            }  
            Push (m);  
        }  
    }  
}
```

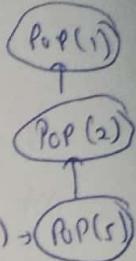
## PRACTISE SET

Ques - 1

:- Which of the following Permutations can be obtained in the output (in the same order) using a stack assuming that the input is the sequence 1, 2, 3, 4, 5 in that order?

- (a) 3, 4, 5, 1, 2
- (b) 3, 4, 5, 2, 1
- (c) 1, 5, 2, 3, 4
- (d) 5, 4, 3, 1, 2

→ Push(1)  
↓  
Push(2)  
↓  
Push(3)  
↓  
Pop(3) → Push(4) → Pop(4) → Push(5) → Pop(5)



Ques - 2

:- The Postfix Expression for the infix Expression  $A + B * (C + D) / F + D * E$  is

Answer - 2

$$\begin{aligned}
 & A + B * (C + D) / F + D * E \\
 & \quad | \\
 & A + B * [C D +] / F + D * E \\
 & \quad | \\
 & A + [B C D + * F] / + D * E \\
 & \quad | \\
 & A + [B C D + * F] + [D E *] \\
 & \quad | \\
 & [A B C D + * F] + [D E *] \\
 & \quad | \\
 & A B C D + * F / + D E * +
 \end{aligned}$$

Ques - 3 :- Which of the following is essential for converting an infix expression to the postfix form efficiently?

- Answer - 3 :-
- (i) An operator stack
  - (ii) An operand stack
  - (iii) A Parse tree.
  - (iv) An operand stack or operator stack

Ques - 4 :- Assume that the operators  $+, -, \times$  are left associative and  $\wedge$  is right associative. The order of Precedence (from highest to lowest) is  $\wedge, \times, +, -$ . The postfix Expression corresponding to the infix expression  $a+b * c-d^{\wedge}e^{\wedge}f$  is

Answer - 4 :-

$$\begin{array}{c} a+b * c-d^{\wedge}e^{\wedge}f \\ / \\ a+b * c-d^{\wedge}[ef\wedge] \\ / \\ a+b * c-[def\wedge\wedge] \\ / \\ a+[bc\wedge\wedge]-[def\wedge\wedge] \\ [abc\wedge\wedge\wedge]-[def\wedge\wedge] \\ / \\ abc\wedge\wedge\wedge+def\wedge\wedge\wedge- \end{array}$$

Ques-5 :-

The following Postfix Expression with Single digit  
is evaluated using a stack.

$8 \ 2 \ 3 \ ^ \ | \ 2 \ 3 \ * \ + \ 5 \ 1 \ * \ -$

Note that  $\wedge$  is the exponentiation operation. The top two  
Elements of the stack after the first  $*$  is evaluated are.

Answer-5 :-

Symbol	Stack	Operation
8	8	
2	2, 8	
3	3, 2, 8	
$\wedge$	8, 8	$2^3 = 8$
/	1	$8/8 = 1$
2	2, 1	
3	3, 2, 1	
*	6, 1	$2*3 = 6$

So, the required answer is  $6, 1$

Ques-6 :-

Let  $S$  be a stack of size  $n \geq 1$ . Starting with the empty stack, suppose we push the first  $n$  natural numbers in sequence, and then perform  $n$  pop operations. Assume that Push and Pop operation take  $X$  seconds each, and  $Y$  seconds elapse b/w the end of one such stack operation and the start of the next operation. For  $m \geq 1$ , define the Stack-Life of ' $m$ ' as the time elapsed from the end of  $\text{Push}(m)$  to the start of the pop operation that removes  $m$  from  $S$ . The average Stack-life of an element of this stack is :-

Ques - 6 :-

Now, Stack life time of last- Element =  $Y$  — ①

(bcz it is popped as soon as it is pushed  
on the stack)

$$\begin{aligned}\text{Now, Stack life time of } (n-1) \text{ th element} &= Y + 2x + 2y \\ &= 2x + 3y\end{aligned}$$

$$\begin{aligned}\text{Stack life time of } (n-2) \text{ th element} &= 2x + 3y + 2x + 2y \\ &= 4x + 5y\end{aligned}$$

$$\text{So, Stack life time of } 1^{\text{st}} \text{ element} = 2(n-1)x + (2n-1)y$$

$$\begin{aligned}\text{Now, Sum of all time spans of all the elements} &= 2(n-1)x + \\ &\leq (2n-1)y\end{aligned}$$

$$\begin{aligned}&= 2x(1+2+\dots+(n-1)) + y(1+3+5+\dots+(2n-1)) \\ &= (2x)\left(\frac{n(n-1)}{2}\right) + y\left(\frac{n}{2}\right)(1+2n-1) \\ &= n(n-1)x + n^2y \\ &= n[(n-1)x + ny]\end{aligned}$$

$$\text{Now, Average stack life of an element} = \frac{n[(n-1)x + ny]}{n}$$

$$\begin{aligned}&= nx - x + ny \\ &= n(x+y) - x\end{aligned}$$

Question -7 :- A single Array  $A[1 \dots \text{MAXSIZE}]$  is used to implement two stacks. The two stack grow from opposite ends of the array. Variable  $\text{top}_1$  &  $\text{top}_2$  ( $\text{top}_1 < \text{top}_2$ ) point to the address of the topmost element in each of the stacks. If the space is to be used efficiently, then condition for stack full is :-

(a)  $(\text{top}_1 = \text{MAXSIZE}/2)$  and  $(\text{top}_2 = \text{MAXSIZE}/2 + 1)$

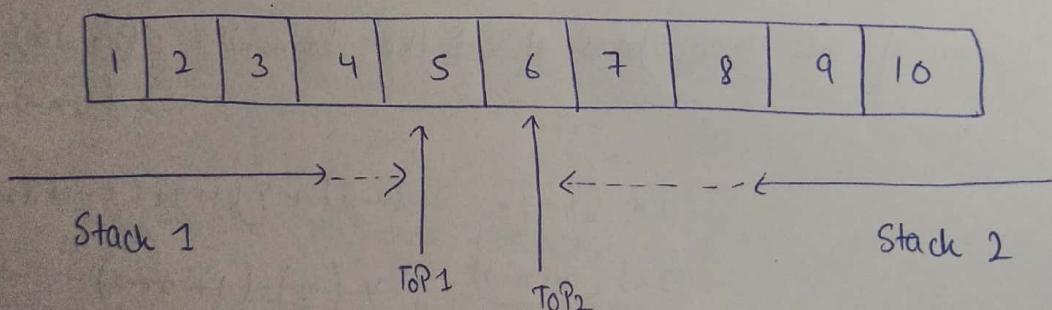
(b)  $\text{top}_1 + \text{top}_2 = \text{MAXSIZE}$

(c)  $(\text{top}_1 = \text{MAXSIZE}/2)$  or  $(\text{top}_2 = \text{MAXSIZE})$

✓ (d)  $\text{top}_1 = \text{top}_2 - 1$

Answer -7 :-

Let  $\text{MAXSIZE} = 10$



Now, if we are to use space efficiently then size of the any stack can be more than  $\text{MAXSIZE}/2$ . Both stacks will grow from both the ends and if any of the stack reached near to the other top then stacks are full. So the condition will be  $\text{top}_1 = \text{top}_2 - 1$

[ $\because$  given that  
 $\text{top}_1 < \text{top}_2$ ]