

Determining the Intruder's Location in a Given Network: Locating-Dominating Sets in a Graph

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Abstract

The exact location of an intruder (e.g. burglar, fire, etc.) in a given network or graph can be determined using the concept of locating-dominating set in a graph. In this paper the concepts of locating, strictly locating, and locating-dominating sets in a graph will be considered. Corresponding parameters will be discussed and some of their relationships will be given. It is shown that the L -domination number $\gamma_L(G)$ of a connected graph G of order $n \geq 2$ is $n - 1$ if and only if $G = K_n$ or $G = K_{1,n-1}$. If G is a connected graph and $\gamma_L(G) = 2$, then $3 \leq |V(G)| \leq 5$. The locating-dominating sets in the joins of graphs are characterized in terms of the other concepts and the associated L -domination numbers are determined subsequently.

Keywords: locating, strictly locating, locating dominating, join

Introduction

Let $G = (V(G), E(G))$ be a simple connected graph. The **neighborhood** of $v \in V(G)$ is the set $N_G(v) = N(v) = \{x \in V(G) : xv \in E(G)\}$. The **degree** of $v \in V(G)$, denoted by $\deg(v)$, is equal to the cardinality of $N_G(v)$ and the **maximum degree** of G is $\Delta(G) = \max\{\deg(x) : x \in V(G)\}$. A subset S of $V(G)$ is a **dominating** set in G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. It is a **locating set** in G if $N_G(u) \cap S \neq N_G(v) \cap S$ for all $u, v \in V(G) \setminus S$. Set S is said to be a **strictly locating set** if it is locating

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