Derived Formulas for the nth Derivative of Select Functions

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Abstract

Calculus is an important subject since it exists in most of university courses. Derivatives is one of the important concepts of calculus which is a precondition topic for most of mathematics courses and other courses in different fields of studies. The main aim of this study was to derive formulas for determining the nth derivative of some standard functions in selected forms. Also, it aimed to evaluate the Taylor Series Expansion and Maclaurin Series of select functions using the derived formulas for the nth derivative. In this study, basic research was employed. Expository method was used in developing the algorithms. Proofs through mathematical induction were presented to guarantee the generalization of the assertion. The following formulas for the nth derivative of selected functions were derived: $(ax^m)^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n},\ m > 0$; $(ax^m)^{(n)} = a\frac{(-1)^n\prod_{j=0}^{n-1}(m+j)}{x^{m+n}},\ m < 0$; $[(ax+b)^m]^{(n)} = (a)^n\left[\prod_{j=0}^{n-1}(m-j)\right](ax+b)^{m-n}, n = 1,2,3,\ldots;\ m > 0$; and $[(ax+b)^m]^{(n)} = \frac{(a)^n(-1)^n\prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}, n = 1,2,3,\ldots;\ m < 0$. The derived nth derivative formulas were applied to Taylor Series Expansion and Maclaurin Series.

Keywords: *derivatives, differential calculus, expository method, nth derivative, maclaurin series, taylor series, Philippines*

Introduction

Calculus is an important subject since it exists in most of university courses such as economy, engineering, statistics, science, and all mathematical courses like numerical analysis, statistic, differential equation, and operation research.

Calculus is one of the fundamental and underlying branches of mathematics. It was known before as infinitesimal calculus, which is focused on functions, infinite series and sequences, limits, derivatives, and anti-derivatives (Guce, 2013). For most students in mathematics, science, and engineering, calculus is the entry-point to undergraduate mathematics.

Differential calculus is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus, the other being integral calculus, the study of the area beneath a curve. The primary objects of study in differential calculus are the derivative of a function, related notions such as the differential, and their applications. The derivative of a function at a chosen input value describes the rate of change of the function near that input value. The process of finding a derivative is called differentiation.

Finding the nth derivative means to take a few derivatives and examine for a pattern. If one exists, then a formula for the nth derivative can be devised. The usual rules of differentiation to a function will be applied to find each successive derivatives to arrive at the nth. Solving for the nth derivative of some functions is a recurring task especially if successive differentiations will be performed. When solving for the derivative of function one time, the first derivative will be obtained. Differentiating the new function another time will result to the second derivative. Likewise, a third, fourth or fifth application of the rules of differentiation will lead to the third derivative, fourth derivative and fifth derivative, respectively. Having an nth derivative formula will help the teachers and the students to easily solve for the nth derivative they want to find without performing long and repetitive solutions.

The nth derivative formula can be applied to Taylor Series Expansion and Maclaurin Series since those series involve nth derivatives. An nth derivative formula can be incorporated to Taylor and Maclaurin series, and the result may be expressed in compact form or expanded form. Because of this, the nth term of an expanded Taylor or Maclaurin series can already be determined even without working on lengthy and tedious computations.

The main aim of this study was to derive formulas for determining the nth derivative of some standard functions in selected forms. It also aimed to evaluate the Taylor Series Expansion and Maclaurin Series of select functions using the derived formulas for the nth derivative. The formulas that were generalized were different from existing equations regarding nth derivatives. This study is hoped to be a contribution to the body of mathematical, scientific, and theoretical knowledge as well as to uncover new facts and learn more accurately the characteristics of discovered facts without any direct functional utility. Basic research was conducted with any expected practical use in the end. It primarily aims to generate new knowledge and understand nature and its laws. The new knowledge provides foundation from which the practical applications of it may be drawn.

Methods

This study utilized the expository method of research. Journals, books and electronic references are the sources of data for the conduct of the study. The researcher read, analyzed, and scrutinized several related studies on nth derivatives to achieve the desired objectives. Pertinent data needed in solving the nth derivatives were gathered from various learning resources which include but not limited to internet and library. Comparative analysis on the different concepts from different research has been undertaken to consolidate the ideas presented in this paper. Furthermore, consultation with the experts on the field as sought to widen and deepen the understanding and presentation of all the information.

Afterwards, proofs of conjectures were illustrated. Mathematical induction was employed to guarantee the generalization of the assertion. In the principle of mathematical induction, the conjecture satisfied two important properties which are the base case and the inductive hypothesis. Subsequently, algorithms in finding the nth derivative of select functions as well as examples were presented. Finally, the derived formulas were applied to Maclaurin Series and Taylor Series Expansion since those series also involve nth derivatives.

Results and Discussion

The following formulas for the nth derivative of selected functions were derived. The nth derivative formulas were applied to Taylor Series Expansion and Maclaurin Series.

1. Derived formulas for the nth derivative of the functions ax^m

$$(ax^m)^{(n)} = \begin{cases} \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n}, \ m > 0 \\ a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{x^{m+n}}, \ m < 0 \end{cases}$$

1.1. Power Rule on the nth derivative of ax^m if $m > 0$, $m \epsilon \mathbb{R}$

Let $m > 0$, then $(ax^m)^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n}$, where $n = 1,2,3, \dots$; $a \epsilon \mathbb{R}$; $m \epsilon \mathbb{R}$; and $m < n$.

I. Base Case

For n = 1, $(ax^m)^{(1)} = amx^{m-1} = \left[\prod_{j=0}^{1-1}(m-j)\right]ax^{m-1}$

For $n = 2$, $(ax^m)^{(2)} = a(m)(m-1)x^{m-2} = \left[\prod_{j=0}^{2-1}(m-j)\right]ax^{m-2}$

$$(ax^m)^{(2)} = \left[\prod_{j=0}^{1}(m-j)\right]ax^{m-2}$$

II. Induction Steps

Suppose $n = k$, $(ax^m)^{(k)} = \left[\prod_{j=0}^{k-1}(m-j)\right]ax^{m-k}$, then $(ax^m)^{(k+1)} = \left\{\left[\prod_{j=0}^{k-1}(m-j)\right]ax^{m-k}\right\}'$

$$= \left[\prod_{j=0}^{k-1}(m-j)\right](m-k)ax^{(m-k)-1}$$

$$= \left[\prod_{j=0}^{(k+1)-1}(m-j)\right]ax^{m-(k+1)}$$

By the Principle of Mathematical Induction,

$$(ax^m)^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n},$$

where $n = 1,2,3, \dots$; $a \in \mathbb{R}$; $m > 0$; $m \in \mathbb{R}$; and $m < n$.

1.2. Power Rule on the n^th derivative of ax^m if $m < 0$, $m \in \mathbb{R}$

Let $m < 0$, then $(ax^m)^{(n)} = a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{x^{m+n}}$, where $n = 1,2,3, \dots$; $a \in \mathbb{R}$ and $m \in \mathbb{R}$.

I. Base Case

For $n = 1$, $(ax^m)^{(1)} = -mx^{-m-1}a = a\frac{(-1)m}{x^{m+1}} = a\frac{(-1)^1 \prod_{j=0}^{1-1}(m+j)}{x^{m+1}}$

For $n = 2$, $(ax^m)^{(2)} = (-m)(-m-1)ax^{-m-2} = a\frac{(-1)^2(m)(m+1)}{x^{m+2}}$

$$= a\frac{(-1)^2 \prod_{j=0}^{2-1}(m+j)}{x^{m+1}}$$

$$= a\frac{(-1)^2 \prod_{j=0}^{1}(m+j)}{x^{m+1}}$$

II. Induction Steps

Suppose for $n = k$, $(ax^m)^{(k)} = a\frac{(-1)^k \prod_{j=0}^{k-1}(m+j)}{x^{m+k}}$, then

$$= (ax^m)^{(k+1)} = a\left[\frac{(-1)^k \prod_{j=0}^{k-1}(m+j)}{x^{m+k}}\right]'$$

$$= a\left\{(-1)^k\left[\prod_{j=0}^{k-1}(m+j)\right]x^{-(m+k)}\right\}'$$

$$= a(-1)^k\left[\prod_{j=0}^{k-1}(m+j)\right][-(m+k)]x^{-(m+k)-1}$$

$$= a(-1)^k\left[\prod_{j=0}^{k-1}(m+j)\right][-(m+k)]x^{-m-k-1}$$

$$= a(-1)^k\left[\prod_{j=0}^{k-1}(m+j)\right][-(m+k)]x^{-m-(k+1)}$$

$$= a\frac{(-1)^k(-1)\left[\prod_{j=0}^{k-1}(m+j)\right](m+k)}{x^{m+(k+1)}}$$

$$= a\frac{(-1)^{k+1}\prod_{j=0}^{(k+1)-1}(m+j)}{x^{m+(k+1)}}$$

By the Principle of Mathematical Induction,

$$(ax^m) = a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{x^{m+n}}$$

where $n = 1,2,3,\ldots$; $a \epsilon \mathbb{R}$; $m < 0$; and $m \epsilon \mathbb{R}$.

2. Derived Formulas for the n^th Derivative of the Function $(ax+b)^m$

$$[(ax+b)^m]^{(n)} = \begin{cases} (a)^n\left[\prod_{j=0}^{n-1}(m-j)\right](ax+b)^{m-n}, n = 1,2,3,\ldots;\ m > 0 \\ \\ \dfrac{(a)^n(-1)^n\prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}, n = 1,2,3,\ldots;\ m < 0 \end{cases}$$

2.1. Power Rule on the n^th derivative of $(ax+b)^m$ if $m > 0, m \in \mathbb{R}$

Let $m > 0$, then $[(ax+b)^m]^{(n)} = (a)^n\left[\prod_{j=0}^{n-1}(m-j)\right](ax+b)^{m-n}$

where $n = 1,2,3,\ldots$; $a \in \mathbb{R}$; $m \in \mathbb{R}$; and $m < n$.

I. Base Case

For $n = 1, [(ax+b)^m]^{(1)}$

$$= m(ax+b)^{m-1}(a)^1 = \left[\prod_{j=0}^{1-1}(m-j)\right](ax+b)^{m-1}(a)^1$$

$$= \left[\prod_{j=0}^{0}(m-j)\right](ax+b)^{m-1}(a)^1$$

For $n = 2, [(ax+b)^m]^{(2)}$

$$= m(m-1)(ax+b)^{m-2}(a)^2 = \left[\prod_{j=0}^{2-1}(m-j)\right](ax+b)^{m-2}(a)^2$$

$$= \left[\prod_{j=0}^{1}(m-j)\right](ax+b)^{m-2}(a)^2$$

II. Induction Steps

Suppose $n = k$,

$$[(ax+b)^m]^{(k)} = m(ax+b)^{m-k}(a)^k = \left[\prod_{j=0}^{k-1}(m - j)\right](ax+b)^{m-k}(a)^k$$

then $[(ax+b)^m]^{(k+1)} = \left\{\left[\prod_{j=0}^{k-1}(m - j)\right](ax+b)^{m-k}(a)^{k+1}\right\}'$

$$= \left[\prod_{j=0}^{k-1}(m - j)\right](m-k)(ax+b)^{m-k}(a)^{k+1}$$

$$= \left[\prod_{j=0}^{k+1-1}(m - j)\right](ax+b)^{m-k}(a)^{k+1}$$

By the Principle of Mathematical Induction,

$$[(ax+b)^m]^{(n)} = \left[\prod_{j=0}^{n-1}(m - j)\right](ax+b)^{m-n}(a)^n$$

where $n = 1,2,3, \ldots$; $a \epsilon \mathbb{R}$; $m > 0$; $m \epsilon \mathbb{R}$; and $m < n$.

2.2. Power Rule on the n^th derivative of $(ax+b)^m$ if $m < 0$, $m \epsilon \mathbb{R}$

Let $m < 0$, then $[(ax+b)^m]^{(n)} = \frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}(a)^n$ where $n = 1,2,3, \ldots$; $a \epsilon \mathbb{R}$ and $m \epsilon \mathbb{R}$.

I. Base Case

For $n = 1$, $[(ax+b)^m]^{(n)} = -m(ax+b)^{-m-1}(a)^1$

$$= \frac{(-1)m}{(ax+b)^{m+1}}(a)^1$$

$$= \frac{(-1)^1 \prod_{j=0}^{1-1}(m+j)}{(ax+b)^{m+1}}(a)^1$$

$$= \frac{(-1)^1 \prod_{j=0}^{0}(m+j)}{(ax+b)^{m+1}}(a)^1$$

For $n = 2$, $[(ax+b)^m]^{(n)} = -m(-m-1)(ax+b)^{-m-2}(a)^2$

$$= \frac{(-1)^2(m)(m+1)}{(ax+b)^{m+2}}(a)^2$$

$$= \frac{(-1)^2 \prod_{j=0}^{2-1}(m+j)}{(ax+b)^{m+2}}(a)^2$$

$$= \frac{(-1)^2 \prod_{j=0}^{1}(m+j)}{(ax+b)^{m+2}}(a)^2$$

II. Induction Steps

Suppose for $n = k$, $[(ax+b)^m]^{(k)} = \frac{(-1)^k \prod_{j=0}^{k-1}(m+j)}{(ax+b)^{m+1}}(a)^k$, then

$$[(ax+b)^m]^{(k+1)} = \left[\frac{(-1)^k \prod_{j=0}^{k-1}(m+j)}{(ax+b)^{m+k}}(a)^{k+1}\right]'$$

$$= \left\{(-1)^k \left[\prod_{j=0}^{k-1}(m+j)\right](ax+b)^{-(m+k)}(a)^{k+1}\right\}'$$

$$= (-1)^k \left[\prod_{j=0}^{k-1}(m+j)\right][-(m+k)](ax+b)^{-(m+k)-1}(a)^{k+1}$$

$$= \frac{(-1)^k(-1)\left[\prod_{j=0}^{k-1}(m+j)\right](m+k)}{(ax+b)^{m+k+1}}(a)^{k+1}$$

$$= \frac{(-1)^{k+1} \prod_{j=0}^{(k+1)-1}(m+j)}{(ax+b)^{m+(k+1)}}(a)^{k+1}$$

By the Principle of Mathematical Induction,

$$[(ax+b)^m]^{(n)} = (a)^n \frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}$$

where $n = 1,2,3, \ldots$; $a \in \mathbb{R}$; $m < 0$; and $m \in \mathbb{R}$.

3. Evaluating the Derived Formulas on the Following Series

The derived nth derivative formulas will be applied to Taylor Series and Maclaurin Series since those series involve nth derivatives.

3.1. Taylor Series

The Taylor Series of a real or complex-valued function $f(x)$ that is infinitely differentiable at a real or complex number c is the power series

$$f(x) = f(c) + \frac{f'(c)}{1!}(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots,$$

where $n!$ denotes the factorial of n. In the more compact sigma notation, this can be written as

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

where $f^{(n)}(c)$ denotes the nth derivative of f evaluated at the point c. The Taylor series for any polynomial is the polynomial itself.

3.1.1. Taylor Series Expansion of ax^m if $m > 0$

To evaluate the Taylor Series of ax^m if $m > 0$, we can apply the nth derivative formula $(ax^m)^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n}$.

$$ax^m = ax^m \Big|_{x=c} + \sum_{n=1}^{\infty} \frac{\prod_{j=0}^{n-1}(m-j) \cdot ax^{m-n} \Big|_{x=c}}{n!}(x-c)^n.$$

Using the nth derivative formula for ax^m if $m > 0$, the Taylor Series for ax^m at $x = c$ is

$$ax^m = ax^m \Big|_{x=c} + \sum_{n=1}^{\infty} \frac{(ax^m)^{(n)} \Big|_{x=c}}{n!}(x-c)^n.$$

3.1.2. Taylor Series Expansion of ax^m if $m < 0$

To evaluate the Taylor Series of ax^m if $m < 0$, we can apply the nth derivative formula $(ax^m)^{(n)} = a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{x^{m+n}}$. ax^m where $m < 0$ can also be expressed as $\frac{a}{x^m}$.

$$\frac{a}{x^m} = \frac{a}{x^m}\bigg|_{x=c} + \sum_{n=1}^{\infty} a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{n!\,x^{m+n}}\bigg|_{x=c} \cdot (x-c)^n.$$

Using the nth derivative formula for ax^m if $m < 0$, the Taylor Series for $\frac{a}{x^m}$ at $x = c$ is

$$\frac{a}{x^m} = \frac{a}{x^m}\bigg|_{x=c} + \sum_{n=1}^{\infty} \frac{\left(\frac{a}{x^m}\right)^{(n)} \big|_{x=c}}{n!}\Bigg|_{x=c} \cdot (x-c)^n.$$

3.1.3. Taylor Series Expansion of $(ax+b)^m$ if $m > 0$

To evaluate the Taylor Series of $(ax+b)^m$ if $m > 0$, we can apply the nth derivative formula $[(ax+b)^m]^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right](ax+b)^{m-n}(a)^n$.

$$(ax+b)^m = (ax+b)^m\big|_{x=c} + \sum_{n=1}^{\infty} \frac{(a)^n\left[\prod_{j=0}^{n-1}(m-j)\right]\left[(ax+b)^{m-n}\right]}{n!}(x-c)^n.$$

Using the nth derivative formula for $(ax+b)^m$ if $m > 0$, the Taylor Series for $(ax+b)^m$ at $x = c$ is

$$(ax+b)^m = (ax+b)^m\big|_{x=c} + \sum_{n=1}^{\infty} \frac{[(ax+b)^m]^{(n)}\big|_{x=c}}{n!}(x-c)^n.$$

3.1.4. Taylor Series Expansion of $(ax+b)^m$ if $m<0$

To evaluate the Taylor Series of $(ax+b)^m$ if $m<0$, we can apply the nth derivative formula $[(ax+b)^m]^{(n)} = \frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}(a)^n$. $(ax+b)^m$ where $m<0$ can also be expressed as $\frac{1}{(ax+b)^m}$.

$$\frac{1}{(ax+b)^m} = \frac{1}{(ax+b)^m}\bigg|_{x=c} + \sum_{n=1}^{\infty} \frac{(-1)^n (a)^n \left[\prod_{j=0}^{n-1}(m+j)\right]}{n!\,(ax+b)^{m+n}}\bigg|_{x=c} \cdot (x-c)^n.$$

Using the nth derivative formula for $(ax+b)^m$ if $m<0$, the Taylor Series for $\frac{1}{(ax+b)^m}$ at $x=c$ is

$$\frac{1}{(ax+b)^m} = \frac{1}{(ax+b)^m}\bigg|_{x=c} + \sum_{n=1}^{\infty} \frac{\left[\frac{1}{(ax+b)^m}\right]^{(n)}\Big|_{x=c}}{n!} \cdot (x-c)^n.$$

3.2. Maclaurin Series

A Maclaurin series is a Taylor series expansion of a function about 0, $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$ where $n!$ denotes the factorial of n. In the more compact sigma notation, this can be written as

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

3.2.1. Maclaurin Series of ax^m if $m>0$

To evaluate the Maclaurin Series of ax^m if $m>0$, we can apply the derived nth derivative formula $(ax^m)^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right]ax^{m-n}$.

$$ax^m = ax^m\Big|_{x=0} + \sum_{n=1}^{\infty} \frac{\prod_{j=0}^{n-1}(m-j) \cdot ax^{m-n}\Big|_{x=0}}{n!} x^n.$$

Using the nth derivative formula of ax^m if $m > 0$, the Maclaurin Series for ax^m at $x = 0$ is

$$ax^m = ax^m \mid_{x=0} + \sum_{n=1}^{\infty} \frac{(ax^m)^{(n)} \mid_{x=0}}{n!} x^n.$$

3.2.2. Maclaurin Series of ax^m if $m < 0$

To evaluate the Maclaurin Series of ax^m if $m < 0$, we can apply the nth derivative formula $(ax^m)^{(n)} = a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{x^{m+n}}$. ax^m where $m < 0$ can be expressed as $\frac{a}{x^m}$.

$$\frac{a}{x^m} = \frac{a}{x^m}\Big|_{x=0} + \sum_{n=1}^{\infty} a\frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{n!\, x^{m+n}}\Bigg|_{x=0} \cdot x^n.$$

Using the nth derivative formula for ax^m if $m < 0$, the Maclaurin Series for ax^m if $m < 0$ at $x = c$ is

$$\frac{a}{x^m} = \frac{a}{x^m}\Big|_{x=0} + \sum_{n=1}^{\infty} \frac{\left(\frac{a}{x^m}\right)^{(n)} \mid_{x=c}}{n!}\Bigg|_{x=0} \cdot x^n.$$

3.2.3. Maclaurin Series of $(ax+b)^m$ if $m > 0$

To evaluate the Maclaurin Series of $(ax+b)^m$ if $m > 0$, we can apply the nth derivative formula $[(ax+b)^m]^{(n)} = \left[\prod_{j=0}^{n-1}(m-j)\right](ax+b)^{m-n}(a)^n$.

$$(ax+b)^m = (ax+b)^m \mid_{x=0} + \sum_{n=1}^{\infty} \frac{(a)^n\left[\prod_{j=0}^{n-1}(m-j)\right]\left[(ax+b)^{m-n}\right]\mid_{x=0}}{n!} \cdot x^n.$$

Using the nth derivative formula for $(ax+b)^m$ if $m > 0$, the Maclaurin Series for $(ax+b)^m$ is

$$(ax+b)^m = (ax+b)^m \mid_{x=0} + \sum_{n=1}^{\infty} \frac{[(ax+b)^m]^{(n)} \mid_{x=0}}{n!} x^n.$$

3.1.4. Maclaurin Series of $(ax+b)^m$ if $m<0$

To evaluate the Maclaurin Series of $(ax+b)^m$ if $m<0$, we can apply the nth derivative formula $[(ax+b)^m]^{(n)} = \frac{(-1)^n \prod_{j=0}^{n-1}(m+j)}{(ax+b)^{m+n}}(a)^n$. $(ax+b)^m$ where $m<0$ can also be expressed as $\frac{1}{(ax+b)^m}$.

$$\frac{1}{(ax+b)^m} = \frac{1}{(ax+b)^m}\bigg|_{x=0} + \sum_{n=1}^{\infty} \frac{(-1)^n (a)^n \left[\prod_{j=0}^{n-1}(m+j)\right]}{n!\,(ax+b)^{m+n}}\bigg|_{x=0} \cdot x^n.$$

Using the nth derivative formula for $(ax+b)^m$ if $m<0$, the Maclaurin Series for $f(x) = \frac{1}{(ax+b)^m}$ at $x=c$ is

$$\frac{1}{(ax+b)^m} = \frac{1}{(ax+b)^m}\bigg|_{x=c} + \sum_{n=1}^{\infty} \frac{\left[\frac{1}{(ax+b)^m}\right]^{(n)}\bigg|_{x=0}}{n!} \cdot x^n.$$

Conclusions

The following conclusions were drawn from the results of this study.

1. The same nth derivative is obtained when both using successive differentiation and the nth derivative formula for the function ax^m. If the nth derivative is a very long expression, the result can be written in product notation form. In ax^m, if $m > 0$, the nth derivative is 0 when $n > m$ while in ax^mif $m > 0$, the nth derivative is a constant when $n = m$.
2. The same nth derivative is obtained when both using successive differentiation and the nth derivative formula for the function $(ax+b)^m$. If the nth derivative is a very long expression, the result can be written in product notation form. In $(ax+b)^m$ if $m > 0$, the nth derivative is 0 when $n > m$ while in $(ax+b)^m$ if $m > 0$, the nth derivative is a constant when $n = m$.

3. The nth derivative formula for functions ax^m and $(ax + b)^m$ can be applied in Taylor Series and Maclaurin Series. The same Taylor Series are obtained for functions ax^m and $(ax + b)^m$ when using both conventional method and the application of the nth derivative formula. Also, the same Maclaurin Series are obtained for functions ax^m and $(ax + b)^m$ when using both conventional method and the application of the nth derivative formula.

Recommendations

From the results of this research study, the following recommendations are endorsed.

1. The algorithms and formulas maybe presented to students taking differential calculus. The teachers may use the algorithms and formulas in teaching nth derivative of some functions.
2. Algorithms on the nth derivative of other functions may be developed such as logarithmic, exponential, trigonometric, inverse trigonometric and hyperbolic functions. Derivation of other formulas in getting the nth derivative of other elementary functions maybe explored.
3. Derivative formulas maybe applied in other series like power series, binomial series and infinite series.
4. Concepts like convergence and divergence of series, ratio test, root tests, limit comparison test and alternating series test may be explored in some series that involve nth derivatives.
5. Future researcher may delve on the exploration of application of nth derivatives and series related to it.

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