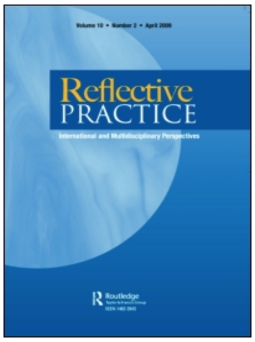

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International and Multidisciplinary Perspectives

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/crep20

Improving learners' productive disposition through realistic mathematics education, a teacher's critical reflection of personal pedagogy

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To cite this article: Von Christopher Chua (2021): Improving learners' productive disposition through realistic mathematics education, a teacher's critical reflection of personal pedagogy, Reflective Practice, DOI: 10.1080/14623943.2021.1974373

To link to this article: https://doi.org/10.1080/14623943.2021.1974373

Published online: 31 Aug 2021.

REFLECTIVE PRACTICE
https://doi.org/10.1080/14623943.2021.1974373

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Improving learners' productive disposition through realistic mathematics education, a teacher's critical reflection of personal pedagogy

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ABSTRACT

This paper reports an action research study I conducted into the mathematical productive disposition of 29 tenth grade learners in the Philippines by analyzing their metaphorical conceptualizations of mathematics in a series of lessons following principles of Realistic Mathematics Education. Analysis of participants' pre-intervention metaphors of mathematics revealed four conceptualizations that capture learners' perceptions of mathematics reflective of their productive disposition: mathematics as a process, mathematics as a puzzle, mathematics as a problem, and mathematics as a difficulty. Insightful changes in the learners' metaphors of mathematics support the argument that making mathematics matter to students can be achieved if teachers make considerable and devoted attempts at making what matters to students mathematical. Several challenges in the implementation of realistic mathematics lessons on Permutation included developing and enabling students' awareness of their social and cultural environment and encouraging critical thinking. Reflecting on my experience, I argue that constant exposure to lessons that present mathematics as contextually and socially relevant can potentially improve students' mathematical productive disposition together with other strands of their mathematical proficiency.

ARTICLE HISTORY
Received 15 May 2021
Accepted 25 August 2021

KEYWORDS
Mathematical proficiency; productive disposition; realistic mathematics education; reflective practice

Introduction: realizing my disregard for productive disposition

In the eight years that I have been teaching Mathematics, I have always considered students' perception of learning mathematics as crucial to their improvement in my class. I encountered students who, due to math anxiety, were unable to meet learning standards despite their ability to perform relatively well in other subjects. Unfortunately, this awareness was not always translated into my practice.

Most interesting in my experience are students who manifest difficulties in dealing with mathematics and yet maintain a positive disposition in class – those who participate in discussions, those who have no inhibitions in raising questions, and those who openly express their learning difficulties. These were the students who I saw struggled with mathematics but not for lack of effort. These students almost always moved past their difficulties as they improved, albeit at a slower pace than their peers who showed better

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proficiency in the subject. By maintaining a positive attitude towards learning mathematics, they demonstrated an important component of mathematical proficiency referred to as mathematical productive disposition.

Mathematical proficiency is theoretically composed of five interdependent, interweaving strands that are proposed to be essential to the task of successfully learning mathematics. Developed by Kilpatrick et al. (2001), this framework recognizes the importance of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

Accurate comprehension of mathematical ideas is called a learner's conceptual understanding. The ability to perform procedures at an acceptable level of flexibility, precision, efficiency, and appropriateness is called procedural fluency. Strategic competence is the learner's ability to formulate, represent, and solve problems. Adaptive reasoning is the capacity to think, logically explain, and reflect on that thinking. Finally, productive disposition is described as the learner's consistent inclination towards sensing mathematics as useful and worthwhile, thus, leading to diligence and belief in one's efficacy. In principle, a mathematics teacher must make certain that all five strands are addressed by how lessons are designed and are evident in the learning experiences of the students.

Guided by this framework, I evaluated the lessons I developed over the past three years. From this analysis, I noted that I focused too much on procedural fluency and conceptual understanding with around sixty percent of the plans I used following either an inductive or deductive approach to teaching mathematics. I also found that most of these plans followed a pattern of (1) articulating the competency I want students to develop; (2) providing a preliminary activity that introduced basic concepts and facilitated activation of prior knowledge; (3) implementing activities that developed students' conceptual understanding and procedural fluency; and (4) assessing students on the competency. Moreover, around three in every ten plans that I created manifested my desire to allow students to develop their methods, representative of adaptive reasoning and strategic competence. These are seen in the forms of project- and performance-based instructional plans that encourage collaboration. Moreover, I often disregarded how students perceived my lessons – whether these helped them sense mathematics as contextually relevant and necessary. This reflection into practice captures what Brookfield (1995) explains as the first of two reasons why reflective practice is important – that such approaches effectively unearth assumptions or idiosyncrasies that we ignore despite these being influential to our work as teachers.

Hence, to develop my ability in designing lessons that explicitly target the improvement of learners' productive disposition, I conducted an action research study with a class of 29 tenth graders in a public school in the Philippines. As I engaged in reflective practice, I was guided by Boud's (2009) description of reflection as practice of engaging in rich and complex situations to make sense of experience. Moreover, I recognize the essentiality of my context, the learners I work with, and their other teachers who encounter similar pedagogical challenges.

Methods

Following an action research design, I began the inquiry by reviewing several lesson plans that I created in the past to determine how often I considered the development of productive disposition. The study subscribed to Reason and Bradbury's (2008) description of action research as a participatory process to developing practical knowledge by bringing together action and reflection, and theory and practice, with the help of others to pursue practical solutions to issues that affect people. In the pursuit of inquiring whether there is an opportunity to develop learners' mathematical productive disposition through realistic mathematics lessons, I recognized the need to engage in a participatory form of research that engaged another teacher and their learners in a process that required a cycle of action and reflection with an effort to connect knowledge from theory and practice. In this process, I embodied a critical reflective stance as I examined my own practice and learned from my experiences working on an intervention.

Loughran (2002) posits that an important element of reflection is the notion of a *problem* and that reflective practice allows one to engage in thinking about what that problem is, and how it should be framed and reframed. To better understand the underlying context of this inquiry, it was necessary that I reflected on my previous practices and used these to inform the changes in my practice.

Identifying weak points in instruction towards improving productive disposition

While I intended to develop the ability to design and implement lessons that build productive disposition, I was not practicing my profession at the time this research was conducted. As a compromise, I coordinated with one of my former colleagues and requested that I be allowed to work with one of her classes.

Miss Lea, not her real name, had twelve years of teaching experience. Since the implementation of the K-12 program in 2012, she had been teaching tenth-grade mathematics for students of the Science, Technology, and Engineering curriculum. As we examined the lesson plans which she crafted for her classes, we noticed that her planning was also inclined towards developing conceptual understanding and procedural fluency like mine. She explained that due to the pressure of helping students perform well in the national achievement test and the implementation of a unified quarterly assessment across all schools in the division, she must ensure that lessons prepared students for the demands of these examinations. Since these tests expect her to cover a predetermined scope of content and competencies, she had to make compromises including choosing simpler, more direct, activities over those that allow students to explore ideas, generate potent preconceptions, and formalize their knowledge through collaborative efforts between her and the class – activities that take more time to execute. This critical introspection has allowed us to discover that our lesson planning practices generally overlooked the development of learners' productive disposition. This resonates with what Brookfield (1995) stated as the second purpose of reflective practice which is to find that our idiosyncratic difficulties are mere examples of a wider structural problem or cultural contradiction.

We identified a need to situate activities in the context of learners so that they can independently establish clear associations between the mathematics they are required to learn and the socio-cultural circumstances they see find themselves in. The goal of ensuring that problems presented in class are more meaningful to learners is suggestive of principles of realistic mathematics education (RME) – a domain-specific theory for mathematics instruction that is characterized by the prominent utility of rich and realistic situations in the learning process (Van den Heuvel-Panhuizen & Drijvers, 2014).

Gauging productive disposition

To capture each student's productive disposition, I asked each student to write their metaphor of mathematics by completing the statement, '*For me, mathematics is like . . . because . . .* ' The first phrase in the prompt leads to what is called a metaphorical vehicle – an object, phenomenon, or activity that the student has a full understanding of which they represent their perception of mathematics and mathematics learning. Requiring the student to continue the response with '*because . . .* ' allows for a description and justification of the association between the vehicle and mathematics. This is called the metaphorical ground. This process of separating a person's metaphorical conceptualization of an abstract idea or a topic into a vehicle and a ground is inspired by the work of Saban (2010) on prospective teachers' metaphorical conceptualizations of the learner. In their work, Saban analyzed metaphors via three elements: the topic, the vehicle, and the ground.

Holman (1980) describes a metaphor as an imaginative analogy from which one can imply how a subject identifies an object with another. A metaphor serves to illuminate a phenomenon of interest through how a person associates it with another phenomenon that he or she already understands (Quale, 2002). In this light, a metaphor is useful when exploring and seeking to understand the esoteric, abstract, novel, or highly speculative (Yob, 2003). In this research that abstract phenomenon in mathematics and central to this paper is the assumption that by allowing students to articulate their perception of mathematics through a metaphor, they can reveal their productive disposition – their sense of how useful, worthwhile, and relevant it is.

The participants were asked to write their metaphors of mathematics twice – once prior to their experience in an intervention and once after. The research compared the pre- and post-intervention metaphors of each learner-participant to represent changes in productive disposition.

The intervention

Consistent with the purpose of the study, the learners participated in five one-hour-per-day math lessons designed following the principles of RME. Van den Heuvel-Panhuizen and Drijvers (2014) explain that six core teaching principles characterize RME: (1) that learners are encouraged to actively participate in the learning process and to perform mathematics; (2) that the lesson should allow learners to apply mathematics in solving meaningful real-life problems; (3) that through the lesson, learners must pass different levels of understanding from constructing informal context-related solutions, then designing schematizations, until they establish the relationship between concept, strategy, and context; (4) that mathematics content is integrated and learners should be able to use their prior understandings to construct new ones; (5) that learning mathematics should be a social activity that foster collaboration; and (6) that the teacher should take a proactive role in the learning process.

A unit of content on permutations was chosen because it was scheduled to be taught to the learners at the time the research was conducted.

Reviewing the relevant literature on productive disposition, RME, and the content and pedagogical considerations of the chosen content unit informed me of the possible challenges, opportunities, and intervening factors that could have affected the intervention.

I also used my knowledge of the school's culture in planning the intervention. In retrospect, I found that my awareness of how people in the school interact and my understanding of the community helped in ensuring that the lessons I planned were realistic to the context of the learners I worked with.

Prior to the intervention, Miss Lea and I worked on identifying opportunities from our experiences and teaching practices relevant to the objective of the research. We also asked the learners to write their pre-intervention metaphors of mathematics. I categorized the metaphorical conceptualizations of the learners to identify those that have less desirable productive dispositions.

Analyzing this pre-intervention data helped me reassess the initial version of the lesson plans I designed and gave me valuable information on how to better implement the lessons. In the process of implementing a plan for a day's lesson, I wrote down reflections right after the class, evaluated our progress, and used what my insights to improve the subsequent plan. This allowed me to capitalize on the opportunities revealed from these reflections.

On the last day of the intervention, I asked the learners to think of their experience in learning the unit on permutations and write another metaphor of mathematics. These were their post-intervention metaphors that I compared with their previous metaphors to determine changes in their productive disposition.

Aside from the learners' metaphors, I also analyzed data from my interviews with Miss Lea, my reflection journal, and observation fieldnotes. To ensure that my personal bias did not affect my interpretation, I made sure to ask the participants to clarify responses that were unclear and confirm my understanding of what these responses implied. Moreover, I had Miss Lea classify the responses of the students according to the categories which I have developed from my own analysis to render the process as credible.

Results and discussion

Four themes of mathematical metaphors

The pre-intervention metaphorical conceptualizations of mathematics of the learners have been analyzed leading to the conceptual themes presented in Table 1.

From the 29 individual metaphors, I present four themes that capture the learners' metaphors that reflect their productive disposition: Mathematics as a Process, Mathematics as a Puzzle, Mathematics as a Problem, and Mathematics as a Difficulty.

Learners who regard mathematics as a process are those who relate learning to developing a skill. They describe this process to come with challenges that they need to overcome, but more importantly that engaging in these challenges are not only necessary but are meaningful and rewarding. Participants' responses under this theme manifest awareness that these challenges are sequential; that the development of one competency is necessary to progress to another level; and that it is not only the knowledge that one achieves in the end which is important, but the process of learning in itself is a meaningful experience. Ultimately, going through the process leads to the attainment of a major goal that justifies the need to encounter the challenges in the process. In the intervention, these learners were often the ones who asked insightful questions about the mathematical concepts brought up in class. They more easily expressed questions about conceptual understanding and in some instances, they were able to explicitly articulate their expectations of the succeeding lessons.

Table 1. Thematic classification of participants' pre-intervention metaphors of mathematics.

Conceptual Theme	Metaphorical Vehicles	Sample metaphor	Frequency (Participants)
Mathematics as a process	riding a bike, learning something new, learning a dance, climbing a tree, planting a tree, climbing Mount Everest, walking the Great Wall of China	*'For me, Mathematics is like* ***climbing a tree****. For you to go to the top, you encounter difficulties along the way. Sometimes you might hurt yourself but when you've finally reached the top, you will slowly see its beauty.'*	**8** (P1, P2, P3, P4, P5, P8, P10, P28)
Mathematics as a puzzle	puzzle, maze, joke, paradox, Rubik's cube, a tangled thread, a doctor's prescription	*'Mathematics is like a* ***paradox*** *because for me at first, it seems like something impossible to solve, explain, understand, but, given time, the answer will show itself in the most logical form possible, you just have to work hard to find it'.*	**10** (P9, P11, P12, P12, P14, P18, P19, P20, P26, P29)
Mathematics as a problem	life problem, obstacle, *pagsubok* (trial),	*'Mathematics is like an* ***obstacle in life****, because whenever there is a problem you need to find a solution to make the problem solved'.*	**4** (P6, P21, P31, P33)
Mathematics as a difficulty	parents, girls, thick book, life, crush, rock, fly	*' Mathematics is like a* ***fly*** *because it is often hard to get'*	**4** (P15, P22, P24, P25)

Those who see mathematics as a puzzle associate learning to a game or worthwhile task that can initially be difficult to understand. Based on their metaphorical conceptualizations, they value consistent effort in dealing with mathematical tasks. Some metaphors classified under this category implied the belief that consistent practice of skills leads to desirable results and gratification. Learners in this category share with the former the ability to see mathematics as useful and fulfilling. The defining characteristic, however, is that learners with this belief perceived mathematics as a set of independent tasks rather than a complex process composed of several interrelated tasks that collectively define an overarching objective.

The third category of learners are those who perceive learning mathematics as a problem – an obstacle to a greater objective. Despite the negative nuance, responses in this theme support the belief that all problems have solutions and that with continuous efforts, these learners are likely to find those solutions. The metaphorical vehicles used by these learners reflect their struggles in dealing with mathematics. In the classroom, these learners were known for taking time in performing tasks and were often reserved with their questions unless asked privately by the teacher.

Finally, students who describe mathematics as a difficulty manifest a disturbing aversion to engaging with mathematics. These are students whose metaphors implied anxiety towards mathematics because their responses did not reflect any sign that they found the exercise of mathematics worthwhile, rewarding, nor even leading to something useful or relevant. Learning the subject is simply hard for them. In class, these learners were known to be quiet and disengaged in the lessons. While they complied with tasks, there were many instances when they easily gave up.

From these descriptions, I posit that learners who regard mathematics as either a process or a puzzle have relatively desirable dispositions that allow them to be more productive in learning Mathematics. Those who view mathematics as a problem may understand the essentiality of learning Mathematics to some extent and are, therefore, able to persevere through the difficulty. However, in performing tasks, they harbor a less desirable attitude towards gaining proficiency in the subject in comparison with the two former categories. A learner who perceives mathematics as a mere difficulty has a weak, detrimental disposition that impedes their success in learning mathematics.

It can be noticed that of the 29 participants, only 26 metaphors are accounted for in the table. The unclassified responses did not fit any of the identified conceptual themes. One of these responses likens mathematics to a relationship, presumably romantic, that has inconsistencies described by the student as '*sometimes … easy to understand, sometimes … complicated*'. Another student used a similar vehicle, that is, '*the presence of someone in your life*' but explaining that this is because a knowledge of mathematics can make life simpler. Lastly, the third uncategorized response simply described mathematics as '*fundamental*'. Regardless, these responses were still used for comparison with post-intervention survey responses.

After the intervention, each learner was asked to write another metaphorical conceptualization of Mathematics reflecting their experiences in the intervention. I classified 23 of the 29 responses into the existing four categories. Six other responses were either too generic that these did not completely reveal the student's disposition or that they were too different to be grouped with others.

The thematic classification of the learners ' post-intervention metaphors is presented in Table 2.

Table 2. Thematic classification of participants' post-intervention metaphors of mathematics.

Conceptual Theme	Metaphorical Vehicles	Sample metaphor	Frequency (Participants)
Mathematics as a process	being in a dance crew; meeting a new person; climbing a tree; planting a tree; aging (growing old); learning to swim	*'Mathematics is like* ***learning how to swim****. You cannot learn it alone or it is hard to learn it without a companion'.*	**6** (P2, P3, P4, P5, P8, P10)
Mathematics as a puzzle	puzzle, maze, joke, paradox, Rubik's cube, a tangled thread	*'Math is like* ***a tangled thread****. It is really hard to untangle the thread but if you think of an easy way to do it, you can easily finish it. Just like math solutions, if you find the problems hard, you must think of an easier way to solve a problem'.*	**13** (P1, P9, P11, P12, P12, P13 P14, P18, P19, P20, P26, P28, P29)
Mathematics as a problem	life problem, obstacle, a fly	*'Mathematics is like an* ***obstacle in life****, because whenever there is a problem you need to find a solution to make the problem solved'.*	**4** (P21, P23, P25, P33)

Notice that the table includes only three of the conceptual themes. Upon examination of the participants' post-intervention metaphors, none of the responses subscribe to the idea of mathematics as a difficulty. In comparison to the pre-intervention metaphors, several learners still implied difficulty in dealing with mathematics but these learners also stated some positive aspect about their experience in learning mathematics during the intervention. As example, I present the responses given by P25,

> [Pre-intervention] *Mathematics is like a fly because it is often hard to get.*

> [Post-intervention] *Math is like a fly because it is hard to get but (the teacher) gave us a trap to get the fly*.

I note the difference in P25's response in that both metaphors began with the same phrase and used the same vehicle, but their post-intervention response reflected a sense of appreciation of the teacher's role is in helping them deal with the difficulty. The added phrase allowed me to recategorize their concept of mathematics from a difficulty to a problem.

Another change in metaphor that I found insightful was that of P15 who stated,

> [Pre-intervention] *Mathematics is like my parent because sometimes they* (sic) *are hard to understand*.

> [Post-intervention] *Mathematics is like my parents – maybe, sometimes they are hard to understand, but if I just listen to them, I will realize it was just easy.*

The post-intervention metaphor does not conform with my understanding of any of the four categories, but it reflects an important change in the learner's thinking – that, like P25, despite believing that mathematics is difficult to understand, trying makes it easier.

Forty-one percent of the students changed an aspect of their metaphors after the intervention with ten being different enough that I decided their post-intervention responses did not anymore reflect the categories they were placed under prior to the intervention.

When participants were asked to reflect on their overall experience in the intervention and identify the features of the lessons that were not common in the math classes they previously experienced, five common responses emerged. The learners appreciated that the lessons centred on discovering and making sense of formulas; that content represented real-life, relatable problem situations; that lessons involved fun and interesting activities that made learning easier; that the process encouraged them to think and analyze; and that activities were collaborative.

Together with the findings from the metaphors and productive dispositions of the students, the learners' comments support the argument that the lessons were, to some extent, successful in helping students see mathematics through a more desirable perspective – one in which they can maintain a kind of disposition that supports productivity in learning mathematics.

Insightful events in the implementation

In this section, I present certain incidents in the implementation because of the insights that I gained from reflecting on them. These are: one during the part where I asked participants to identify some of the social issues that affect them as students and teenagers; one that is representative of the participants' difficulty in expressing their ideas; and one where their understanding of what is socially appropriate determined their answer to a problem.

Students' awareness of social issues that affect them and the role they play

Part of the plan for the second day was to allow students to identify and describe social issues that they believe affect them. The purpose was to use these issues as part of a socially realistic task that would lead the class into discovering the concept of '*how many ways can I prioritize things?*' In the context of the problem, 'things' referred to the social issues.

In my reflective journal, I noted that it took several minutes before one issue was raised. I allowed the class to talk about this issue by asking them two things: (a) how relevant is this to you? and (b) do you see yourself capable of doing something about it? These questions encouraged students to express their opinion, and this led to the identification of several other issues. In all, the class came up with a list of eight social issues, namely, depression, rising cases of social isolation, discrimination (i.e. religious and gender stereotyping), bullying, pornography, parents' unreasonably high expectations, and teenage pregnancy.

I was surprised with the kind of issues the learners identified because I did not anticipate some of these. In retrospect, I may have underestimated the learners' social awareness – how they see themselves as part of their community and the problems in that community. I noticed an increased level of enthusiasm among learners as they worked with the task and they continued to talk about these issues, negotiated ideas, and considered how the mathematics required of them made sense in the context of the problem.

Students' difficulty in expressing ideas

In one of my attempts to facilitate recapitulation at the start of another session, I noticed that the class had difficulty responding to questions. In a post-lesson conference with Miss Lea, she confirmed that many learners have difficulties formally expressing themselves and this was apparent with the way they gave incomplete answers to questions.

The passage below is an actual interaction between me and the class.

Teacher: How do we determine how many possible arrangements or permutations there are?

Student A (in her seat, small voice): factorial

Teacher (addressing student A): Factorial? How does that work?

Student B: Product.

Teacher: Okay . . . product of what, exactly?

Student C: positive integers

Teacher: Yes, these are all valid answers. Can anyone give me a complete statement? Can we put together these important pieces (of information) you're giving me?

Student D: We can use $n_!$ to compute (for permutations) if all (objects are) to be arranged.

Teacher: Correct. And what again does factorial of mean? (calls student A who was acting hesitant)

Student A: Product of all positive integers less than .

Student E: including *n*.

It took the class roughly five minutes to process the first question which I thought was quite basic. I was expecting students to respond faster but noticed some were struggling with how to express their understanding.

A second situation also reflected that some learners were not used to thinking about forms of mathematics that did not follow procedures. On the third day, the class had to work with a series of problems that involved, 'I*n how many ways can people be seated in a bus given some conditions?*' The second question was phrased this way: '*Five seats were unoccupied in the bus. At the next stop, five people got into the bus including a couple who insist on sitting together. In how many ways can all seats be taken?'* This activity included an illustration showing that only two vacant seats were adjacent.

Students struggled because most were forcing the formula to work with the situation given despite the multiple times that I encouraged them to use the diagram as aid. It came to a point that I decided that it was necessary to intervene, and I asked them a series of questions.

Teacher: So, the problem states that there is a couple. If we look at the diagram, which seats can the couple take?

Students point out the only two adjacent seats in the illustration.

Teacher: What does this mean in terms of options?

Student A: There are just two ways?

Teacher: Two ways of what, exactly?

Student A: of the couple sitting (together) because for the first seat any (of the) two of them can sit but only one for what is left.

(Simultaneously, I wrote the numbers 2 and 1 on the boxes representing the two adjacent seats to represent the student's response)

Teacher: Good. Now we're making some progress. I want you to continue from there. What does that tell us about the rest of the available seats?

Students' understanding of 'socially appropriate' reflected in responses to a problem

The problem presented in the previous section of the paper is part of a series of problems that work under the same context but presented varying conditions that supported the development of competencies on permutations. The subsequent problem to this series is similar, but it suggested a different condition: *If one of the six passengers who got in the bus was a senior citizen, and 5 seats were available, in how many ways can all seats be taken?*

The purpose of this problem was twofold: (a) it reveals what learners consider as socially appropriate – albeit arguably contrived – this being their ability to realize the senior citizen's condition in the context of the problem; and (b) consequently, train them in dealing with permutation problems where one object from an array of choices must be selected. Ultimately, the goal in raising this context was to let students see that beliefs can sometimes change the way they deal with mathematics.

The first purpose was accomplished when one student inquired, '*So the senior citizen has to be seated?*' There were several gestures of agreement among the students (e.g. nodding, giving verbal affirmation). With which I responded, '*Does everybody agree? If you think that is appropriate, let's consider that*.'

After this problem, another one was presented: *In the next bus stop, five people got off and two women and five men got in. In how many ways can all five vacant seats be taken?*

The 'senior citizen' problem somehow conditioned learners that this other one also implied the idea of what is 'socially appropriate'. I did not intervene in the small group discussions so learners had the chance talk about what conditions they may or may not include in their solution. Of the eight groups, seven suggested the need to prioritize the women and guarantee them seats in the bus. When probed, learners reasoned that '*We think it's right because they are women*.' However, one group said that while they understand what the other groups meant, they came up with a second answer where the women were not given priority. I allowed them to explain their reasoning to the class and they stated that they needed more information to truly decide including, '*Are the women not old* (sic)?' In this process, another student said, '*How about if one of the male passengers is sick or tired?*'

In dealing with these problems, I noted that more students were working with the diagram. They were negotiating answers, assessing their understanding, and confirming it with others. They were contemplating options, and more interestingly, refining the conditions of the problem to determine a reasonable and valid response. Most students were working with their groupmates. These were clear signs that the class took a legitimate interest in the problem and were thinking mathematically while recognizing the problem's social relevance.

Reflecting on my experience: making math matter means making (what) matters mathematical

The intention of this research was to examine my ability to develop and implement lessons that push productive disposition alongside the other strands of mathematical proficiency. This practice was a departure from what I typically had been doing for the past eight years when I placed more emphasis on conceptual understanding and procedural fluency in how I designed lessons. This intention is captured by one question that propelled the whole inquiry, '*How can I make mathematics matter to students?*'

The inquiry led me to gain insights that made an impact on my understanding of what makes a 'good' mathematics lesson. These insights come from witnessing very specific events such as hearing unexpected student responses and discovering ways by which students think about, discuss, and generalize the processes they go through to solve problems – insights that came with analyzing and reflecting on events independently. However, when I think of the research in its entirety, I had to ask: *What caused learners to take a more active role in the discussion*? *What is it about the kind of problems that they were asked to do that led to some evidence pointing out a more productive mathematical disposition*?

In thinking about these questions, I raise two important points.

First, the approach to learning mathematics matters. By 'approach', I mean how students have been made to think about it, whether the problems have known, pre-established answers that they need to discover or if these problems encourage them to construct understandings and negotiate with peers to determine the most logical conclusion. The former of these two approaches is highly evaluative.

Alro and Johnsen-Hoines (2010) state that mathematics education usually follows a predominantly evaluative approach because the kind of classroom discourse typical to a math class is one where the teacher is in possession of the correct answers and is solely responsible for providing learners with feedback to help them construct understanding. Barnes and Venter (2008) refer to this view of mathematics as consisting of absolute and unchallengeable truths as absolutist.

Learners' awareness of such a perspective leads them to find answers not to satisfy their curiosity but to fulfil what they think the teacher and the subject demands of them. It is in this sense that mathematics maintains a quality of being absolute and definite – a characteristic that does not encourage active participation, does not invoke a sense of responsibility over learning, and does not encourage ownership from learners for their understanding. This evaluative perspective further restricts the attempt to inspire investigations central to social constructivism.

It may be necessary for teachers to become more critical about themselves and their approach to teaching mathematics. From the intervention, I gained a greater appreciation of exposing students to reflective dialogue that I found essential in realistic mathematics pedagogy.

Second, math matters when what matters becomes mathematical. This means that the context of the problems I presented to learners made a lot of difference to developing productive disposition. I now understand that problems should be designed to meaningfully connect competency and context so that content does not appear detached to learners. Planning lessons through a realistic or critical perspective engenders this idea because it allows learners to make direct connections between what they experience, what mathematics they are invited to learn, and how knowing this mathematics can help them widen their understanding of their own experiences. Andersson (2010) explains from the work of Sfard and Prusak (2005) that learning must be the act of closing the gap between the learners' present identity and designated identity; learning is a process of *becoming*. Unless learners establish how learning mathematics affords them to attain their vision of a future self, they may not find the exercise of learning mathematics to be essential.

From a different, but relevant perspective, Rodd (2006) describes any effort to enable low-attaining learners to effectively improve and use a desirable affect towards learning Mathematics to be challenging. This is because learners' attitudes, beliefs, and values may not be aligned with what they are expected to achieve in the curriculum and that attempting to modify learners' affection requires consideration of both pedagogy and social dynamics. In the intervention, learners whose pre-intervention metaphors fell under mathematics as a difficulty were observed to take a more active role in the group interactions compared to how they usually did. While their contribution to the discussions were mostly on the challenge of determining what were socially appropriate given the conditions presented by the problems and they still manifested reservations when their group dealt with the mathematical aspects of the task, this indication of interest in participation may mean that the lessons allowed them to see their beliefs and values to be relevant to the learning goals.

When mathematics lessons are designed to follow a constructive approach by utilizing problems that learners find relevant to what they know and provides the opportunity to know more about what matters to them, we may not only develop learners who know how to do mathematics but know why they need it.

This process of reflecting on my own practice and how the decisions that I make in planning my lessons affect the way that my learners view mathematics has made me realize the importance for teachers to engage in critical reflective practice. As Van Manen (1995) explains, teachers should constantly think about why and what they are doing while they are doing it; that we should continuously seek alternatives to our aims and methods; that we should be prepared to employ changes; that we should reflect on the significance of our students' behavior and consider alternative interpretation of what is happening to them and how their learning affects them socially and psychologically.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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