Examining a Derivation of Hankel's Loop Integral Using Integration

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Abstract

This paper presents a derivation of Hankel's Loop integral using integration through a branch cut. It justifies the derivation of Hankel's loop integral mentioned in references where the Hankel contour is collapsed into the negative real axis. Moreover, it discusses an application to r-Whitney numbers with complex parameters.

Keywords: Analysis, Gamma Function, Hankel contour, Hankel Identity, Hankel's loop integral

Introduction

In the study of the asymptotics of r-Whitney numbers $^{\rm v}$ (C.B.Corcino, R.B. Corcino, and N.G. Acala 2014), the Hankel's loop integral plays an important role. It is through this integral that computations in the case where the parameters are integers, are extended to the case where real parameters are considered. Temme (2014) used the Hankel's loop integral to obtain an asymptotic expansion of the integral

$$G_{\lambda}(z) = \frac{1}{2\pi i} \int_{-\infty}^{(0^{+})} s^{\lambda-1} \, q(s) e^{zs} ds, \tag{1}$$