# Derived Formulas for the nth Derivative of Select Functions

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# Abstract

Calculus is an important subject since it exists in most of university courses. Derivatives is one of the important concepts of calculus which is a precondition topic for most of mathematics courses and other courses in different fields of studies. The main aim of this study was to derive formulas for determining the nth derivative of some standard functions in selected forms. Also, it aimed to evaluate the Taylor Series Expansion and Maclaurin Series of select functions using the derived formulas for the nth derivative. In this study, basic research was employed. Expository method was used in developing the algorithms. Proofs through mathematical induction were presented to guarantee the generalization of the assertion. The following formulas for the nth derivative of selected functions were derived: (𝑎𝑥𝑚)(𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑛, 𝑚 > 0;

𝑗 = 0

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

(𝑎𝑥𝑚)(𝑛) = 𝑎 𝑗 = 0 , 𝑚 < 0 ; [(𝑎𝑥 + 𝑏)𝑚](𝑛) = (𝑎)𝑛[∏𝑛 − 1(𝑚 − 𝑗)](𝑎𝑥 +

𝑥𝑚 + 𝑛

𝑗 = 0

𝑏)𝑚−𝑛, 𝑛 = 1,2,3, … ; 𝑚 > 0; and [(𝑎𝑥 + 𝑏)𝑚](𝑛) =

(𝑎)𝑛(−1)𝑛 ∏𝑛 − 1(𝑚+𝑗)

𝑗 = 0 , 𝑛 = 1,2,3, … ; 𝑚 <

(𝑎𝑥+𝑏)𝑚+𝑛

1. The derived nth derivative formulas were applied to Taylor Series Expansion and Maclaurin Series.

**Keywords**: *derivatives, differential calculus, expository method, nth derivative, maclaurin series, taylor series, Philippines*

# Introduction

Calculus is an important subject since it exists in most of university courses such as economy, engineering, statistics, science, and all mathematical courses like numerical analysis, statistic, differential equation, and operation research.

Calculus is one of the fundamental and underlying branches of mathematics. It was known before as infinitesimal calculus, which is focused on functions, infinite series and sequences, limits, derivatives, and anti-derivatives (Guce, 2013). For most students in mathematics, science, and engineering, calculus is the entry-point to undergraduate mathematics.

Differential calculus is a subfield of calculus concerned with the study of the rates at which quantities change. It is one of the two traditional divisions of calculus, the other being integral calculus, the study of the area beneath a curve. The primary objects of study in differential calculus are the derivative of a function, related notions such as the differential, and their applications. The derivative of a function at a chosen input value describes the rate of change of the function near that input value. The process of finding a derivative is called differentiation.

Finding the nth derivative means to take a few derivatives and examine for a pattern. If one exists, then a formula for the nth derivative can be devised. The usual rules of differentiation to a function will be applied to find each successive derivatives to arrive at the nth. Solving for the nth derivative of some functions is a recurring task especially if successive differentiations will be performed. When solving for the derivative of function one time, the first derivative will be obtained. Differentiating the new function another time will result to the second derivative. Likewise, a third, fourth or fifth application of the rules of differentiation will lead to the third derivative, fourth derivative and fifth derivative, respectively. Having an nth derivative formula

will help the teachers and the students to easily solve for the nth derivative they want to find without performing long and repetitive solutions.

The nth derivative formula can be applied to Taylor Series Expansion and Maclaurin Series since those series involve nth derivatives. An nth derivative formula can be incorporated to Taylor and Maclaurin series, and the result may be expressed in compact form or expanded form. Because of this, the nth term of an expanded Taylor or Maclaurin series can already be determined even without working on lengthy and tedious computations.

The main aim of this study was to derive formulas for determining the nth derivative of some standard functions in selected forms. It also aimed to evaluate the Taylor Series Expansion and Maclaurin Series of select functions using the derived formulas for the nth derivative. The formulas that were generalized were different from existing equations regarding nth derivatives. This study is hoped to be a contribution to the body of mathematical, scientific, and theoretical knowledge as well as to uncover new facts and learn more accurately the characteristics of discovered facts without any direct functional utility. Basic research was conducted with any expected practical use in the end. It primarily aims to generate new knowledge and understand nature and its laws. The new knowledge provides foundation from which the practical applications of it may be drawn.

# Methods

This study utilized the expository method of research. Journals, books and electronic references are the sources of data for the conduct of the study. The researcher read, analyzed, and scrutinized several related studies on nth derivatives to achieve the desired objectives. Pertinent data needed in solving the nth derivatives were gathered from various learning resources which

include but not limited to internet and library. Comparative analysis on the different concepts from different research has been undertaken to consolidate the ideas presented in this paper. Furthermore, consultation with the experts on the field as sought to widen and deepen the understanding and presentation of all the information.

Afterwards, proofs of conjectures were illustrated. Mathematical induction was employed to guarantee the generalization of the assertion. In the principle of mathematical induction, the conjecture satisfied two important properties which are the base case and the inductive hypothesis. Subsequently, algorithms in finding the nth derivative of select functions as well as examples were presented. Finally, the derived formulas were applied to Maclaurin Series and Taylor Series Expansion since those series also involve nth derivatives.

# Results and Discussion

The following formulas for the nth derivative of selected functions were derived. The nth derivative formulas were applied to Taylor Series Expansion and Maclaurin Series.

1. Derived formulas for the nth derivative of the functions 𝑎𝑥𝑚

(𝑎𝑥𝑚)(𝑛) = {

[∏𝑛 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑛, 𝑚 > 0

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

𝑗 = 0

𝑗 = 0

𝑎

𝑥𝑚 + 𝑛

, 𝑚 < 0

* 1. Power Rule on the nth derivative of 𝑎𝑥𝑚 if 𝑚 > 0, 𝑚 𝜖 ℝ

Let 𝑚 > 0, then (𝑎𝑥𝑚)(𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑛, where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 𝜖

𝑗 = 0

ℝ; and 𝑚 < 𝑛.

1. Base Case

For *n* = 1, (𝑎𝑥𝑚)(1) = 𝑎𝑚𝑥𝑚 − 1 = [∏1 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 1

𝑗 = 0

For *n* = 2, (𝑎𝑥𝑚)(2) = 𝑎(𝑚)(𝑚 − 1)𝑥𝑚 – 2 = [∏2 – 1(𝑚 – 𝑗)]𝑎𝑥𝑚 – 2

𝑗 = 0

1

(𝑎𝑥𝑚)(2) = [∏(𝑚 − 𝑗)] 𝑎𝑥𝑚 − 2

𝑗 = 0

1. Induction Steps

Suppose 𝑛 = 𝑘, (𝑎𝑥𝑚)(𝑘) = [∏𝑘 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑘, then (𝑎𝑥𝑚)(𝑘+1) = {[∏𝑘 − 1(𝑚 −

′

] 𝑚 − 𝑘

𝑗) 𝑎𝑥 }

𝑗 = 0

𝑗 = 0

= [∏𝑘 − 1(𝑚 − 𝑗)](𝑚 − 𝑘)𝑎𝑥(𝑚 − 𝑘) − 1

𝑗 = 0

= [∏(𝑘+1) − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − (𝑘 + 1)

𝑗 = 0

By the Principle of Mathematical Induction,

𝑛 − 1

(𝑎𝑥𝑚)(𝑛) = [ ∏(𝑚 − 𝑗)] 𝑎𝑥𝑚 − 𝑛,

𝑗 = 0

where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 > 0; 𝑚 𝜖 ℝ ; and 𝑚 < 𝑛.

* 1. Power Rule on the nth derivative of 𝑎𝑥𝑚 if 𝑚 < 0, 𝑚 𝜖 ℝ

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

Let 𝑚 < 0, then (𝑎𝑥𝑚)(𝑛) = 𝑎 𝑗 = 0 , where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ and 𝑚 𝜖 ℝ.

𝑥𝑚 + 𝑛

1. Base Case

For 𝑛 = 1, (𝑎𝑥𝑚)(1) = −𝑚𝑥− 𝑚 − 1𝑎 = 𝑎

(−1)𝑚

𝑥𝑚 + 1

(−1)1 ∏1 − 1(𝑚+𝑗)

= 𝑎 𝑗 = 0

𝑥𝑚 + 1

For 𝑛 = 2, (𝑎𝑥𝑚)(2) = (−𝑚)(−𝑚 − 1)𝑎𝑥−𝑚−2 = 𝑎 (−1)2(𝑚)(𝑚+1)

𝑥𝑚 + 2

𝑗 = 0

= 𝑎

(−1)2 ∏2 − 1(𝑚 + 𝑗)

𝑥𝑚 + 1

= 𝑎

𝑗 = 0

(−1)2 ∏1 (𝑚 + 𝑗)

𝑥𝑚 + 1

1. Induction Steps

Suppose for 𝑛 = 𝑘, (𝑎𝑥𝑚)(𝑘) = 𝑎

(−1)𝑘 ∏𝑘 − 1(𝑚 + 𝑗)

𝑥𝑚 + 𝑘

𝑗 = 0

, then

(−1)𝑘 ∏𝑘 − 1(𝑚 + 𝑗) ′

= (𝑎𝑥𝑚)(𝑘+ 1) = 𝑎 [

𝑗 = 0

𝑥𝑚 + 𝑘 ]

𝑘 − 1 ′

= 𝑎 {(−1)𝑘 [∏(𝑚 + 𝑗)] 𝑥−(𝑚 + 𝑘)}

𝑗 = 0

𝑘 − 1

= 𝑎(−1)𝑘 [∏(𝑚 + 𝑗)] [−(𝑚 + 𝑘)]𝑥−(𝑚 + 𝑘) − 1

𝑗 = 0

𝑘 − 1

= 𝑎(−1)𝑘 [∏(𝑚 + 𝑗)] [−(𝑚 + 𝑘)]𝑥−𝑚 − 𝑘 − 1

𝑗 = 0

𝑘 − 1

= 𝑎(−1)𝑘 [∏(𝑚 + 𝑗)] [−(𝑚 + 𝑘)]𝑥−𝑚−(𝑘 + 1)

𝑗 = 0

= 𝑎

= 𝑎

(−1)𝑘(−1)[∏𝑘 − 1(𝑚 + 𝑗)](𝑚 + 𝑘)

𝑥𝑚 +( 𝑘 + 1)

(−1)𝑘+1 ∏(𝑘+1) − 1(𝑚 + 𝑗)

𝑗 = 0

𝑗 = 0

𝑥𝑚 + (𝑘 + 1)

By the Principle of Mathematical Induction,

(𝑎𝑥𝑚) = 𝑎

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

𝑥𝑚 + 𝑛

where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 < 0; and 𝑚 𝜖 ℝ.

𝑗 = 0

1. Derived Formulas for the nth Derivative of the Function (𝑎𝑥 + 𝑏)𝑚

[(𝑎𝑥 + 𝑏)𝑚](𝑛) =

(𝑎)𝑛

𝑛 − 1

[ ∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)

𝑗 = 0

𝑚−𝑛

, 𝑛 = 1,2,3, … ; 𝑚 > 0

(𝑎)𝑛(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

𝑗 = 0

(𝑎𝑥 + 𝑏)𝑚+𝑛

{

, 𝑛 = 1,2,3, … ; 𝑚 < 0

* 1. Power Rule on the nth derivative of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, 𝑚 𝜖 ℝ

Let 𝑚 > 0, then [(𝑎𝑥 + 𝑏)𝑚](𝑛) = (𝑎)𝑛[∏𝑛 − 1(𝑚 − 𝑗)](𝑎𝑥 + 𝑏)𝑚−𝑛

𝑗 = 0

where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 𝜖 ℝ; and 𝑚 < 𝑛.

1. Base Case

For 𝑛 = 1, [(𝑎𝑥 + 𝑏)𝑚](1)

1 − 1

= 𝑚(𝑎𝑥 + 𝑏)𝑚−1(𝑎)1 = [∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−1(𝑎)1

𝑗 = 0

0

= [∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−1(𝑎)1

𝑗 = 0

For 𝑛 = 2, [(𝑎𝑥 + 𝑏)𝑚](2)

2− 1

= 𝑚(𝑚 − 1)(𝑎𝑥 + 𝑏)𝑚−2(𝑎)2 = [∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−2(𝑎)2

𝑗 = 0

1

= [∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−2(𝑎)2

𝑗 = 0

1. Induction Steps Suppose 𝑛 = 𝑘,

𝑘 − 1

[(𝑎𝑥 + 𝑏)𝑚](𝑘) = 𝑚(𝑎𝑥 + 𝑏)𝑚 − 𝑘(𝑎)𝑘 = [∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚 − 𝑘(𝑎)𝑘

𝑗 = 0

then [(𝑎𝑥 + 𝑏)𝑚](𝑘+1) = {[∏𝑘 − 1(𝑚 − 𝑗)](𝑎𝑥 + 𝑏)𝑚−𝑘(𝑎)𝑘+1}′

𝑗 = 0

𝑘 − 1

= [∏(𝑚 − 𝑗)] (𝑚 − 𝑘)(𝑎𝑥 + 𝑏)𝑚−𝑘(𝑎)𝑘+1

𝑗 = 0

𝑘 + 1 − 1

= [ ∏ (𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−𝑘(𝑎)𝑘+1

𝑗 = 0

By the Principle of Mathematical Induction,

𝑛 − 1

[(𝑎𝑥 + 𝑏)𝑚](𝑛) = [ ∏(𝑚 − 𝑗)] (𝑎𝑥 + 𝑏)𝑚−𝑛(𝑎)𝑛

𝑗 = 0

where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 > 0; 𝑚 𝜖 ℝ ; and 𝑚 < 𝑛.

* 1. Power Rule on the nth derivative of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0, 𝑚 𝜖 ℝ

Let 𝑚 < 0, then [(𝑎𝑥 + 𝑏)𝑚](𝑛) =

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

𝑗 = 0 (𝑎)𝑛 where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ and

(𝑎𝑥+𝑏)𝑚+𝑛

𝑚 𝜖 ℝ.

1. Base Case

For 𝑛 = 1, [(𝑎𝑥 + 𝑏)𝑚](𝑛) = −𝑚(𝑎𝑥 + 𝑏)− 𝑚 − 1(𝑎)1

(−1)𝑚

= (𝑎𝑥 + 𝑏)𝑚+1

(𝑎)1

(−1)1 ∏1 − 1(𝑚 + 𝑗)

𝑗 = 0

= (𝑎𝑥 + 𝑏)𝑚+1

(𝑎)1

(−1)1 ∏0

𝑗 = 0

=

(𝑚 + 𝑗)

(𝑎)1

(𝑎𝑥 + 𝑏)𝑚+1

For 𝑛 = 2, [(𝑎𝑥 + 𝑏)𝑚](𝑛) = −𝑚(−𝑚 − 1)(𝑎𝑥 + 𝑏)− 𝑚 − 2(𝑎)2

(−1)2(𝑚)(𝑚 + 1)

= (𝑎𝑥 + 𝑏)𝑚+2

(𝑎)2

(−1)2 ∏2 − 1(𝑚 + 𝑗)

𝑗 = 0

= (𝑎𝑥 + 𝑏)𝑚+2

(𝑎)2

(−1)2 ∏1 (𝑚 + 𝑗)

𝑗 = 0

= (𝑎𝑥 + 𝑏)𝑚+2

(𝑎)2

1. Induction Steps

Suppose for 𝑛 = 𝑘, [(𝑎𝑥 + 𝑏)𝑚](𝑘) =

(−1)𝑘 ∏𝑘 − 1(𝑚 + 𝑗)

(−1)𝑘 ∏𝑘 − 1(𝑚 + 𝑗)

𝑗 = 0 (𝑎)𝑘, then

(𝑎𝑥+𝑏)𝑚+1

′

[(𝑎𝑥 + 𝑏)𝑚](𝑘+1) 𝑗 = 0 (𝑎)𝑘+1]

= [ 𝑚+𝑘

(𝑎𝑥 + 𝑏)

𝑘 − 1 ′

= {(−1)𝑘 [∏(𝑚 + 𝑗)] (𝑎𝑥 + 𝑏)−(𝑚 + 𝑘)(𝑎)𝑘+1}

𝑗 = 0

𝑘 − 1

= (−1)𝑘 [∏(𝑚 + 𝑗)] [−(𝑚 + 𝑘)](𝑎𝑥 + 𝑏)−(𝑚 + 𝑘) − 1(𝑎)𝑘+1

𝑗 = 0

(−1)𝑘(−1)[∏𝑘 − 1(𝑚 + 𝑗)](𝑚 + 𝑘)

𝑗 = 0

= (𝑎𝑥 + 𝑏)𝑚 + 𝑘 + 1

(−1)𝑘+1 ∏(𝑘+1) − 1(𝑚 + 𝑗)

(𝑎)𝑘+1

𝑗 = 0

= (𝑎𝑥 + 𝑏)𝑚 +(𝑘 + 1)

(𝑎)𝑘+1

By the Principle of Mathematical Induction,

𝑗 = 0

[(𝑎𝑥 + 𝑏)𝑚](𝑛) = (𝑎)𝑛

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗) (𝑎𝑥 + 𝑏)𝑚+𝑛

where 𝑛 = 1,2,3, … ; 𝑎 𝜖 ℝ; 𝑚 < 0; and 𝑚 𝜖 ℝ.

1. Evaluating the Derived Formulas on the Following Series

The derived nth derivative formulas will be applied to Taylor Series and Maclaurin Series since those series involve nth derivatives.

* 1. Taylor Series

The Taylor Series of a real or complex-valued function 𝑓(𝑥) that is infinitely differentiable at

a real or complex number 𝑐 is the power series

𝑓′(𝑐)

𝑓′′(𝑐)

𝑓′′′(𝑐)

𝑓(𝑥) = 𝑓(𝑐) +

(𝑥 − 𝑐) +

1!

(𝑥 − 𝑐)2 +

2!

(𝑥 − 𝑐)3 + ⋯,

3!

where 𝑛! denotes the factorial of 𝑛. In the more compact sigma notation, this can be written as

∞

𝑓(𝑥) = ∑

𝑛=0

𝑓(𝑛)(𝑐)

𝑛!

(𝑥 − 𝑐)𝑛

where 𝑓(𝑛)(𝑐) denotes the nth derivative of 𝑓 evaluated at the point 𝑐. The Taylor series for any polynomial is the polynomial itself.

* + 1. Taylor Series Expansion of 𝑎𝑥𝑚 if 𝑚 > 0

To evaluate the Taylor Series of 𝑎𝑥𝑚 if 𝑚 > 0, we can apply the nth derivative formula

(𝑎𝑥𝑚)(𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑛.

𝑗 = 0

𝑎𝑥𝑚 = 𝑎𝑥𝑚│

𝑥=𝑐

∞

+ ∑

𝑛=1

∏𝑛−1(𝑚 − 𝑗) ∙ 𝑎𝑥𝑚−𝑛│

𝑛!

𝑗=0

𝑥=𝑐 (𝑥 − 𝑐)𝑛.

Using the nth derivative formula for 𝑎𝑥𝑚 if 𝑚 > 0, the Taylor Series for 𝑎𝑥𝑚 at 𝑥 = 𝑐 is

(𝑎𝑥𝑚)(𝑛)│

∞

𝑎𝑥𝑚 = 𝑎𝑥𝑚│

𝑥=𝑐

+ ∑ 𝑥=𝑐 (𝑥 − 𝑐)𝑛.

𝑛!

𝑛=1

* + 1. Taylor Series Expansion of 𝑎𝑥𝑚 if 𝑚 < 0

To evaluate the Taylor Series of 𝑎𝑥𝑚 if 𝑚 < 0, we can apply the nth derivative formula

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

(𝑎𝑥𝑚 )(𝑛) = 𝑎 𝑗 = 0 . 𝑎𝑥𝑚 where 𝑚 < 0 can also be expressed as

𝑥𝑚 + 𝑛

𝑎 .

𝑥𝑚

𝑎

𝑥𝑚

𝑎

= 𝑥𝑚|

𝑥=𝑐

∞

+ ∑ 𝑎

𝑛=1

(−1)𝑛 ∏𝑛−1(𝑚 + 𝑗)

𝑛! 𝑥𝑚+𝑛 |

𝑗=0

𝑥=𝑐

* (𝑥 − 𝑐)𝑛.

Using the nth derivative formula for 𝑎𝑥𝑚 if 𝑚 < 0, the Taylor Series for 𝑎

𝑥𝑚

at 𝑥 = 𝑐 is

𝑎

𝑥𝑚

𝑎

= 𝑥𝑚|

𝑥=𝑐

∞

+ ∑

𝑛=1

𝑎 (𝑛)

(𝑥𝑚) │

𝑛!

𝑥=𝑐

|

𝑥=𝑐

* (𝑥 − 𝑐)𝑛.
  + 1. Taylor Series Expansion of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0

To evaluate the Taylor Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, we can apply the nth derivative formula

[(𝑎𝑥 + 𝑏)𝑚](𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)](𝑎𝑥 + 𝑏)𝑚−𝑛(𝑎)𝑛.

𝑗 = 0

(𝑎𝑥 + 𝑏)𝑚 = (𝑎𝑥 + 𝑏)𝑚│

𝑥=𝑐

𝑗=0

∞

+ ∑

𝑛=1

(𝑎)𝑛[∏𝑛−1(𝑚 − 𝑗)][(𝑎𝑥 + 𝑏)𝑚−𝑛]

𝑛!

(𝑥 − 𝑐)𝑛.

Using the nth derivative formula for (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, the Taylor Series for (𝑎𝑥 + 𝑏)𝑚

at 𝑥 = 𝑐 is

[(𝑎𝑥 + 𝑏)𝑚](𝑛)│

∞

(𝑎𝑥 + 𝑏)𝑚 = (𝑎𝑥 + 𝑏)𝑚│

𝑥=𝑐

+ ∑ 𝑥=𝑐 (𝑥 − 𝑐)𝑛.

𝑛!

𝑛=1

* + 1. Taylor Series Expansion of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0

To evaluate the Taylor Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0, we can apply the nth derivative formula

[(𝑎𝑥 + 𝑏)𝑚](𝑛) =

(−1)𝑛 ∏𝑛−1(𝑚+𝑗)

𝑗=0 (𝑎)𝑛. (𝑎𝑥 + 𝑏)𝑚 where 𝑚 < 0 can also be expressed as

(𝑎𝑥+𝑏)𝑚+𝑛

1 .

(𝑎𝑥+𝑏)𝑚

1 1 ∞ (−1)𝑛(𝑎)𝑛[∏𝑛 − 1(𝑚 + 𝑗)]

(𝑎𝑥 + 𝑏)𝑚

= (𝑎𝑥 + 𝑏)𝑚|

𝑥=𝑐

𝑗 = 0

𝑛! (𝑎𝑥 + 𝑏)𝑚+𝑛 |

+ ∑

𝑛=1

𝑥=𝑐

* (𝑥 − 𝑐)𝑛.

Using the nth derivative formula for (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0, the Taylor Series for 1 at

(𝑎𝑥+𝑏)𝑚

𝑥 = 𝑐 is

1

(𝑎𝑥 + 𝑏)𝑚

1

= (𝑎𝑥 + 𝑏)𝑚

∞

| 𝑥=𝑐 + ∑

𝑛=1

1 (𝑛)

[(𝑎𝑥 + 𝑏)𝑚]

𝑛!

| 𝑥=𝑐

* (𝑥 − 𝑐)𝑛.
  1. Maclaurin Series

A Maclaurin series is a Taylor series expansion of a function about 0, 𝑓(𝑥) = 𝑓(0) +

𝑓′(0) 𝑥 + 𝑓′′(0) 𝑥2 + 𝑓′′′(0) 𝑥3 + ⋯ + 𝑓(𝑛)(0) 𝑥𝑛 + ⋯ where 𝑛! denotes the factorial of 𝑛. In the more

1! 2! 3! 𝑛!

compact sigma notation, this can be written as

∞

𝑓(𝑥) = ∑

𝑛=0

𝑓(𝑛)(0)

𝑛!

𝑥𝑛.

* + 1. Maclaurin Series of 𝑎𝑥𝑚 if 𝑚 > 0

To evaluate the Maclaurin Series of 𝑎𝑥𝑚 if 𝑚 > 0, we can apply the derived nth derivative formula (𝑎𝑥𝑚)(𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)]𝑎𝑥𝑚 − 𝑛.

𝑗 = 0

𝑎𝑥𝑚 = 𝑎𝑥𝑚│

𝑥=0

∞

+ ∑

𝑛=1

∏𝑛−1(𝑚 − 𝑗) ∙ 𝑎𝑥𝑚−𝑛│

𝑛!

𝑗=0

𝑥=0 𝑥𝑛.

Using the nth derivative formula of 𝑎𝑥𝑚 if 𝑚 > 0, the Maclaurin Series for 𝑎𝑥𝑚 at 𝑥 = 0 is

(𝑎𝑥𝑚)(𝑛)│

∞

𝑎𝑥𝑚 = 𝑎𝑥𝑚│

𝑥=0

+ ∑ 𝑥=0 𝑥𝑛.

𝑛!

𝑛=1

* + 1. Maclaurin Series of 𝑎𝑥𝑚 if 𝑚 < 0

To evaluate the Maclaurin Series of 𝑎𝑥𝑚 if 𝑚 < 0, we can apply the nth derivative formula

(−1)𝑛 ∏𝑛 − 1(𝑚 + 𝑗)

(𝑎𝑥𝑚)(𝑛) = 𝑎 𝑗 = 0 . 𝑎𝑥𝑚 where 𝑚 < 0 can be expressed as

𝑥𝑚 + 𝑛

𝑎 .

𝑥𝑚

𝑎

𝑥𝑚

𝑎

= 𝑥𝑚|

𝑥=0

∞

+ ∑ 𝑎

𝑛=1

(−1)𝑛 ∏𝑛−1(𝑚 + 𝑗)

𝑛! 𝑥𝑚+𝑛 |

𝑗=0

𝑥=0

* 𝑥𝑛.

Using the nth derivative formula for 𝑎𝑥𝑚 if 𝑚 < 0, the Maclaurin Series for 𝑎𝑥𝑚 if 𝑚 < 0

at 𝑥 = 𝑐 is

𝑎

𝑥𝑚

𝑎

= 𝑥𝑚|

𝑥=0

∞

+ ∑

𝑛=1

𝑎 (𝑛)

(𝑥𝑚) │

𝑛!

𝑥=𝑐

|

𝑥=0

* 𝑥𝑛.
  + 1. Maclaurin Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0

To evaluate the Maclaurin Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, we can apply the nth derivative formula [(𝑎𝑥 + 𝑏)𝑚](𝑛) = [∏𝑛 − 1(𝑚 − 𝑗)](𝑎𝑥 + 𝑏)𝑚−𝑛(𝑎)𝑛.

𝑗 = 0

(𝑎𝑥 + 𝑏)𝑚 = (𝑎𝑥 + 𝑏)𝑚│

𝑥=0

∞

+ ∑

𝑛=1

(𝑎)𝑛[∏𝑛−1(𝑚 − 𝑗)][(𝑎𝑥 + 𝑏)𝑚−𝑛]|

𝑛!

𝑗=0

𝑥=0 ∙ 𝑥𝑛.

Using the nth derivative formula for (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, the Maclaurin Series for

(𝑎𝑥 + 𝑏)𝑚 is

[(𝑎𝑥 + 𝑏)𝑚](𝑛)│

∞

(𝑎𝑥 + 𝑏)𝑚 = (𝑎𝑥 + 𝑏)𝑚│

𝑥=0

+ ∑ 𝑥=0 𝑥𝑛.

𝑛!

𝑛=1

* + 1. Maclaurin Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0

To evaluate the Maclaurin Series of (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0, we can apply the nth derivative

formula [(𝑎𝑥 + 𝑏)𝑚](𝑛) =

(−1)𝑛 ∏𝑛−1(𝑚+𝑗)

𝑗=0 (𝑎)𝑛. (𝑎𝑥 + 𝑏)𝑚 where 𝑚 < 0 can also be expressed as

(𝑎𝑥+𝑏)𝑚+𝑛

1 .

(𝑎𝑥+𝑏)𝑚

1 1

(𝑎𝑥 + 𝑏)𝑚 = (𝑎𝑥 + 𝑏)𝑚|

𝑥=0

∞ (−1)𝑛(𝑎)𝑛[∏𝑛 − 1(𝑚 + 𝑗)]

𝑛! (𝑎𝑥 + 𝑏)𝑚+𝑛 |

+ ∑

𝑗 = 0

𝑛=1

𝑥=0

* 𝑥𝑛.

Using the nth derivative formula for (𝑎𝑥 + 𝑏)𝑚 if 𝑚 < 0, the Maclaurin Series for 𝑓(𝑥) =

1 at 𝑥 = 𝑐 is

(𝑎𝑥+𝑏)𝑚

1

(𝑎𝑥 + 𝑏)𝑚

1

= (𝑎𝑥 + 𝑏)𝑚

∞

| 𝑥=𝑐 + ∑

𝑛=1

1 (𝑛)

[(𝑎𝑥 + 𝑏)𝑚]

𝑛!

| 𝑥=0

* 𝑥𝑛.

# Conclusions

The following conclusions were drawn from the results of this study.

* + - 1. The same nth derivative is obtained when both using successive differentiation and the nth derivative formula for the function 𝑎𝑥𝑚. If the nth derivative is a very long expression, the result can be written in product notation form. In 𝑎𝑥𝑚, if 𝑚 > 0, the nth derivative is 0 when

𝑛 > 𝑚 while in 𝑎𝑥𝑚if 𝑚 > 0, the nth derivative is a constant when 𝑛 = 𝑚.

* + - 1. The same nth derivative is obtained when both using successive differentiation and the nth derivative formula for the function (𝑎𝑥 + 𝑏)𝑚. If the nth derivative is a very long expression, the result can be written in product notation form. In (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, the nth derivative is 0 when 𝑛 > 𝑚 while in (𝑎𝑥 + 𝑏)𝑚 if 𝑚 > 0, the nth derivative is a constant when 𝑛 =

𝑚.

* + - 1. The nth derivative formula for functions 𝑎𝑥𝑚 and (𝑎𝑥 + 𝑏)𝑚 can be applied in Taylor Series and Maclaurin Series. The same Taylor Series are obtained for functions 𝑎𝑥𝑚 and (𝑎𝑥 + 𝑏)𝑚 when using both conventional method and the application of the nth derivative formula. Also, the same Maclaurin Series are obtained for functions 𝑎𝑥𝑚 and (𝑎𝑥 + 𝑏)𝑚 when using both conventional method and the application of the nth derivative formula.

# Recommendations

From the results of this research study, the following recommendations are endorsed.

* + - * 1. The algorithms and formulas maybe presented to students taking differential calculus. The teachers may use the algorithms and formulas in teaching nth derivative of some functions.
        2. Algorithms on the nth derivative of other functions may be developed such as logarithmic, exponential, trigonometric, inverse trigonometric and hyperbolic functions. Derivation of other formulas in getting the nth derivative of other elementary functions maybe explored.
        3. Derivative formulas maybe applied in other series like power series, binomial series and infinite series.
        4. Concepts like convergence and divergence of series, ratio test, root tests, limit comparison test and alternating series test may be explored in some series that involve nth derivatives.
        5. Future researcher may delve on the exploration of application of nth derivatives and series related to it.

# Acknowledgment

This precious work will not be realized without these individuals and institutions whose assistance, cooperation, effort, and support made significant impact in the completion of this research paper. With heartfelt appreciation, the researcher sincerely expresses his gratitude to the following:

First and foremost, to Almighty God, the most Divine Source of Life for bestowing the researcher grace, guidance, love and wisdom that gave way to the accomplishment of this dissertation;

To Dr. Tirso A. Ronquillo, President of the Batangas State University, for his leadership in securing quality services and learning experiences rendered by the university;

To Dr. Rowena R. Abrea, Dean of the College of Teacher Education, for her outstanding leadership that serves as an inspiration to the students of the graduate school that motivates them to continue upgrading their professional career as individuals making significant contributions in the field of education;

To Dr. Norrie E. Gayeta, for equipping the researcher the necessary knowledge in writing, organizing and editing research paper and for guiding him in the proper and formal way of conducting research;

To Dr. Emil C. Alcantara, the dissertation adviser, for his insights, ideas, assistance, guidance, and supervision serving as a strong enticement in the pursuance of this study;

To Dr. Charity A. Aldover, Dr. Realiza M. Mame, Dr. Felix M. Panopio and Dr. Ivee K. Guce, for the valuable comments and suggestions they provided to enhance the content of this study;

To Dr. Richard M. Bañez, the technical editor, for meticulously enhancing the readability of this research work;

To the Batangas State University professors who have shaped the researcher to be a persistent, open-minded and smart-working individual enabling him to finish this graduate program;

To the researchers’ friends, colleagues and co-teachers, for believing in the essence and success of this research, and for their support during the busy and hectic times;

And for those people who were not mentioned but somehow served as inspirations to the researcher, thank you.

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