Solution

EXAMPLE 1.5 : The resistance of a certain length of wire is 4.6 Ω at 20 °C and 5.68 Ω at 80 °C. Determine (a) temperature co–efficient of resistance of the material of wire at 0 °C (b) resistance of wire at 60 °C.

SOLUTION :

$R_1 = 5.68\ \Omega$ $\qquad t_1 = 80\ °C$

$R_2 = 4.6\ \Omega$ $\qquad t_2 = 20\ °C$

(a) $$\frac{R_2}{R_1} = \frac{1 + \alpha_o t_2}{1 + \alpha_o t_1}$$

$$\frac{4.6}{5.68} = \frac{1 + \alpha_o (20)}{1 + \alpha_o (80)}$$

$$0.8098 + 64.79\ \alpha_o = 1 + 20\ \alpha_o$$

$$44.79\ \alpha_o = 0.1902$$

$$\alpha_o = 0.00425\ /°C$$

(b) $$\frac{R_2}{R_3} = \frac{1 + \alpha_o t_2}{1 + \alpha_o t_1}$$

$$\frac{4.6}{R_3} = \frac{1 + \alpha_o t_2}{1 + \alpha_o t_3}$$

$$\frac{4.6}{R_3} = \frac{1 + (0.00425)(20)}{1 + (0.00425)(60)}$$

$$R_3 = 4.6 \times \frac{1.255}{1.085}$$

$$R_3 = 5.3\Omega$$

EXAMPLE 1.6 : The resistance of a wire of 3 mm² cross sectional area and 6 m length is 0.15 Ω at 0 °C. When the temperature of the wire is raised to 65 °C the resistance is found to be 0.2 Ω. Calculate the temperature co–efficient of resistance of the wire and its resistivity at 0 °C.

SOLUTION :

$a = 3\ mm^2 = 3 \times 10^{-6}\ m^2$ $\qquad l = 6\ m$

$R_o = 0.15\Omega$

$\therefore\quad R_o = \varrho_o \dfrac{l}{a}$ $\qquad 0.15 = \varrho_o \times \dfrac{6}{3 \times 10^{-6}}$

$$\varrho_o = 7.5 \times 10^{-8}\ \Omega m$$

$$\frac{R_2}{R_1} = \frac{1 + \alpha_o t_2}{1 + \alpha_o t_1}$$

$$\frac{0.15}{0.2} = \frac{1 + (\alpha_o)(0)}{1 + (\alpha_o)(65)}$$

$$0.15\ (1 + 65\ \alpha_o) = 0.2$$

$$\alpha_o = 0.005\ /°C$$

EXAMPLE 1.8 : A copper wire has a resistivity of 1.6×10^{-6} Ω–cm at 0 °C and at 20 °C, the temperature co–efficient of resistance is 1/254.5 °C^{-1}. Find the resistivity and temperature co–efficient of resistance at 60 °C.

SOLUTION :

$$\rho_o = 1.6 \times 10^{-6}\ \Omega\ \text{cm} \qquad \alpha_{20} = 1/254.5\ ^\circ\text{C}^{-1}$$

$$\alpha_{20} = \frac{\alpha_o}{1 + \alpha_o t}$$

$$1/254.5 = \frac{\alpha_o}{1 + 20\alpha_o}$$

$$3.929 \times 10^{-3}\ (1 + 20\ \alpha_o) = \alpha_o$$

$$0.92\ \alpha_o = 3.929 \times 10^{-3}$$

$$\alpha_o = 4.27 \times 10^{-3}$$

$$\alpha_o = 1/234.19\ ^\circ\text{C}^{-1}$$

$$\rho_{60} = \rho_o\ [1 + \alpha_o t]$$

$$= 1.6 \times 10^{-6}\ [1 + 4.27 \times 10^{-3} \times 60]$$

$$= 1.6 \times 10^{-6}\ (1.2562)$$

$$= 2.01 \times 10^{-6}\ \Omega\text{–cm}$$

$$\alpha_{60} = \frac{\alpha_o}{1 + \alpha_o t}$$

$$= \frac{4.27 \times 10^{-3}}{1 + 4.27 \times 10^{-3} \times 60}$$

$$= 3.399 \times 10^{-3}\ ^\circ\text{C}^{-1}$$

$$= \frac{1}{294.2}\ ^\circ\text{C}^{-1}$$

EXAMPLE 1.25 : Find the current supplied by the battery in the network shown in the Fig. E.1.25(a).

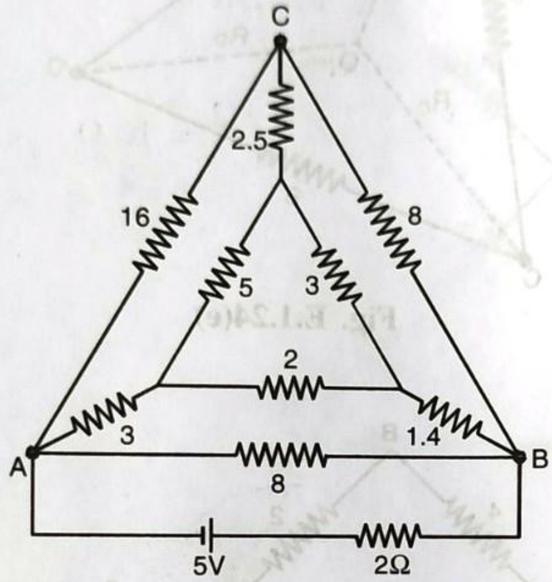
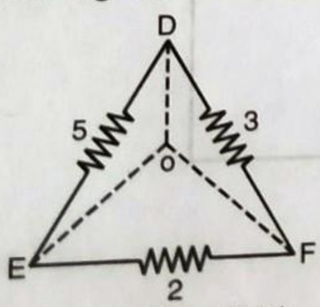


Fig. E.1.25(a)

SOLUTION :

Converting the inner delta to its equivalent star



$$OD = \frac{5 \times 3}{10} = 1.5\ \Omega$$

$$OE = \frac{5 \times 2}{10} = 1\ \Omega$$

$$OF = \frac{3 \times 2}{10} = 0.6\ \Omega$$

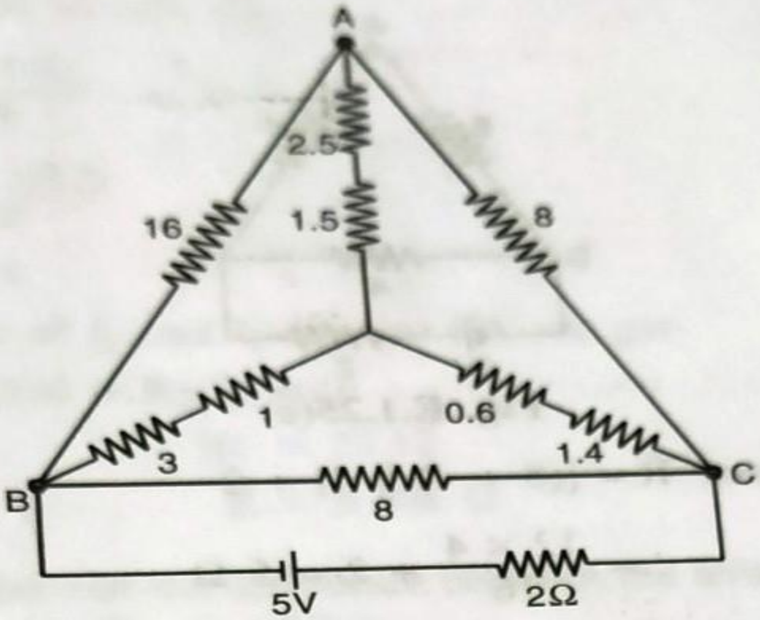


Fig. E.1.25(c)

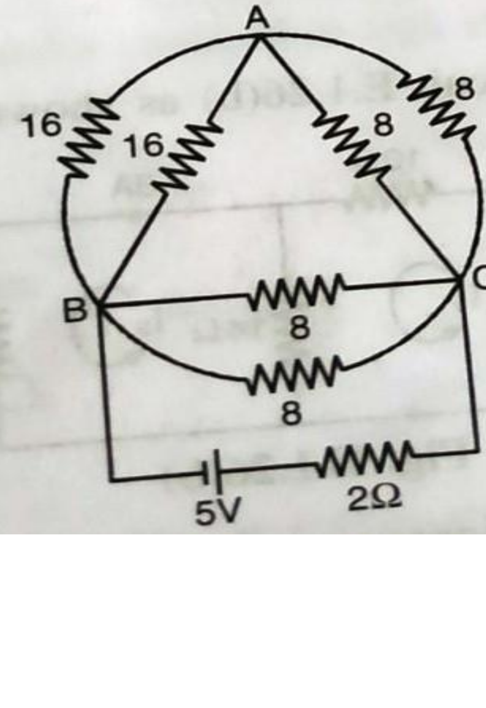
Now converting star ABC to delta

$$AB = \frac{4 \times 4 + 4 \times 2 + 4 \times 2}{2} = 16\ \Omega$$

$$BC = \frac{32}{4} = 8\ \Omega$$

$$CA = \frac{32}{4} = 8\ \Omega$$

Thus, the circuit reduces to



This is further simplified as below :

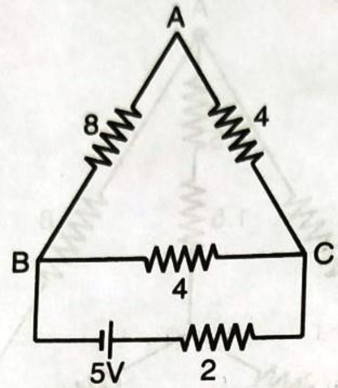


Fig. E.1.25(e)

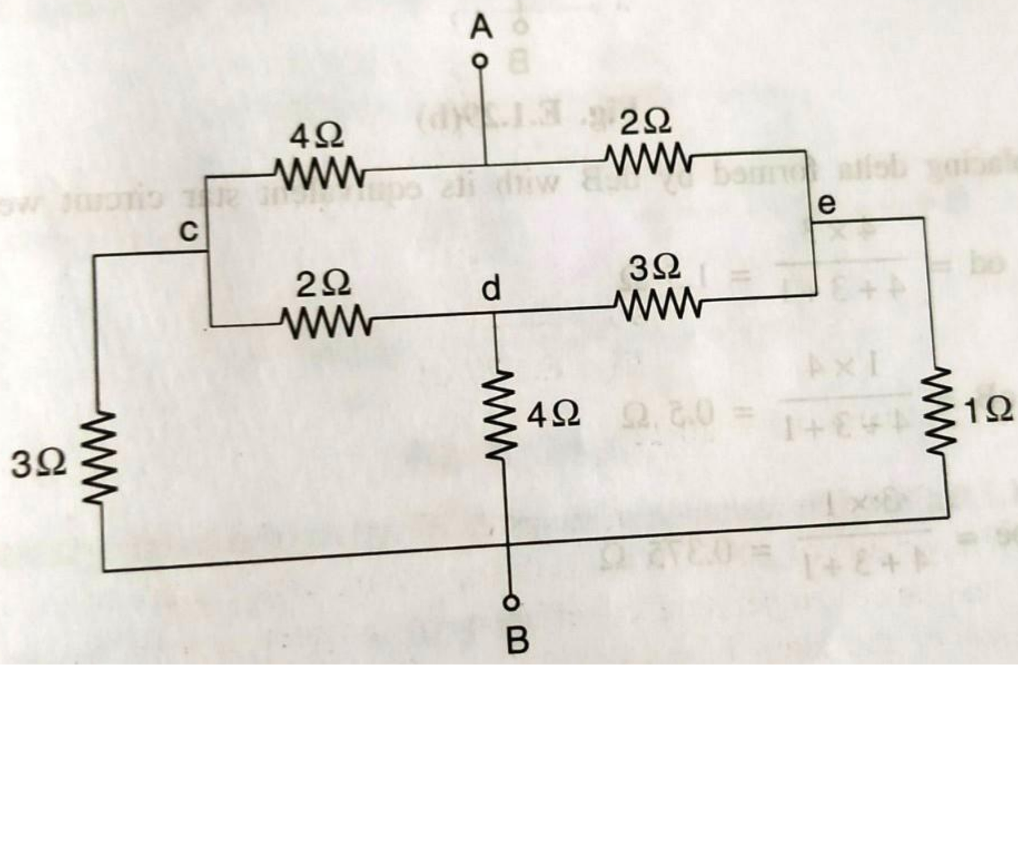
Now total resistance $R = [(8 + 4) \parallel 4] + 2$

$$= \frac{12 \times 4}{16} + 2 = 5\ \Omega$$

$$\therefore \quad I = \frac{5}{5} = 1\text{ A}$$

XAMPLE 1.29 : Determine equivalent resistance between terminals A and B of network shown in fig. E.1.29(a).

(GTU June 2009)



SOLUTION :

Redrawing the given circuit as shown below we get

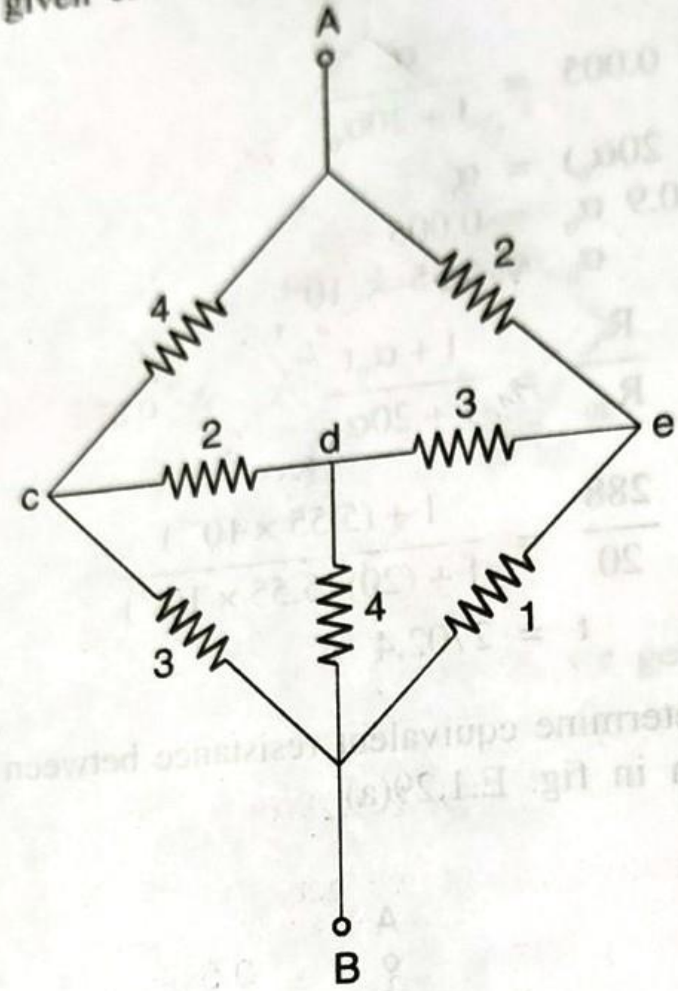


Fig. E.1.29(b)

Replacing delta formed by deB with its equivalent star circuit

$$\text{od} = \frac{4 \times 3}{4+3+1} = 1.5\ \Omega$$

$$\text{oB} = \frac{1 \times 4}{4+3+1} = 0.5\ \Omega$$

$$\text{oe} = \frac{3 \times 1}{4+3+1} = 0.375\ \Omega$$

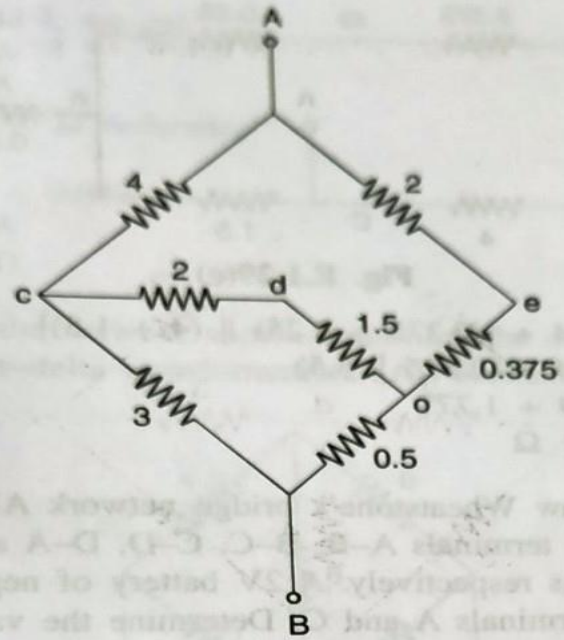


Fig. E.1.29(c)

If we redraw the above circuit we get

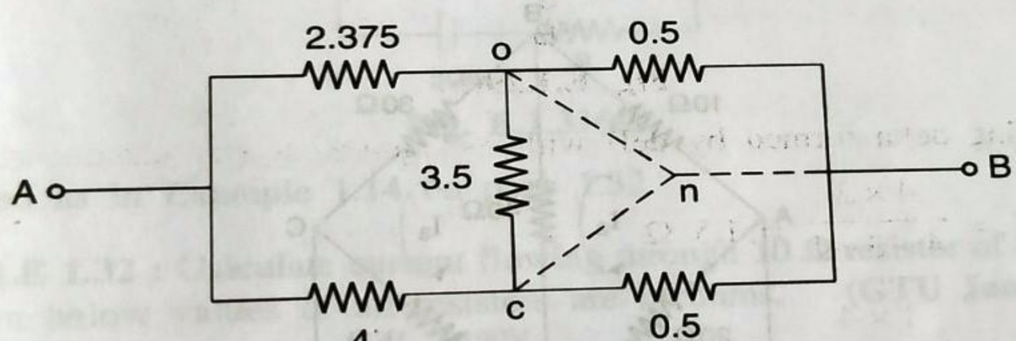


Fig. E.1.29(d)

Replacing delta formed by OCB by its equivalent star we get

$$\text{on} = \frac{3.5 \times 0.5}{3.5 + 0.5 + 3} = 0.25 \ \Omega$$

$$\text{Bn} = \frac{0.5 \times 3}{3.5 + 0.5 + 3} = 0.214 \ \Omega$$

$$\text{Cn} = \frac{3 \times 3.5}{3.5 + 0.5 + 3} = 1.5 \ \Omega$$

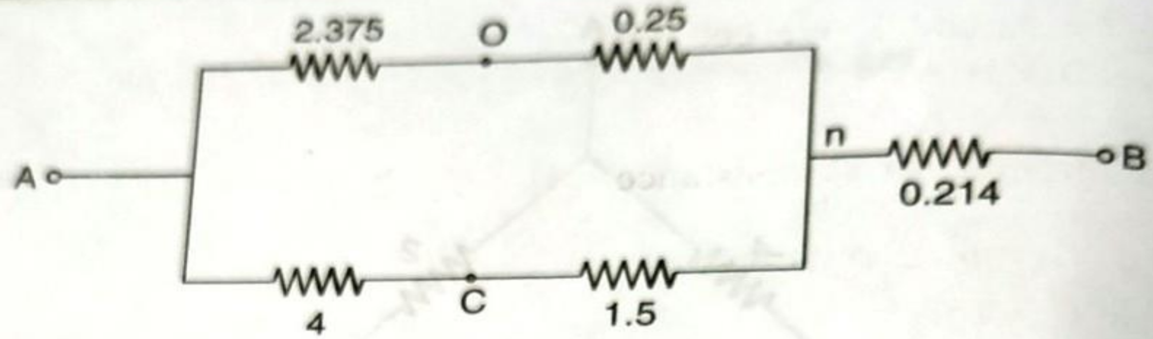
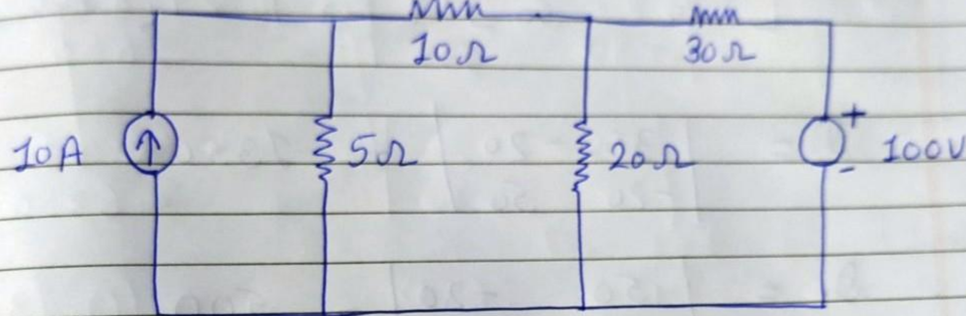


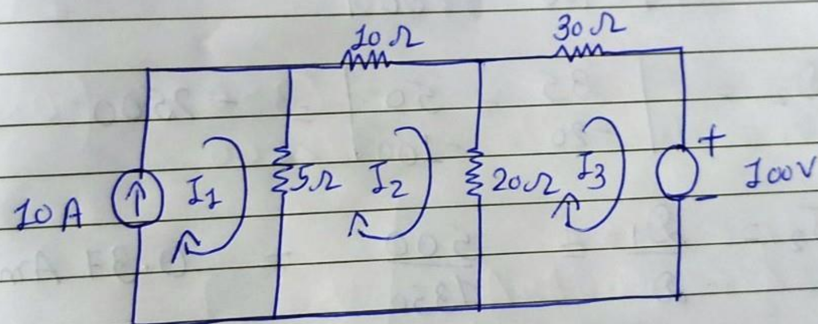
Fig. E.1.29(e)

$$
\begin{aligned}
R_{AB} &= 0.214 + [(2.375 + 0.25) \parallel (4 + 1.5)] \\
&= 0.214 + (2.625 \parallel 5.5) \\
&= 0.214 + 1.777 \\
&= 1.991\ \Omega
\end{aligned}
$$

(7) For the circuit below determine the voltage across the 20 Ω resistor using mesh analysis.



Sol^n



$\therefore \; I_1 = 10 \text{ A}$

Applying KVL in mesh-2

$$0 = -5I_1 + 35I_2 - 20I_3$$

Put $I_1 = 10$ A

$$\therefore \; 35I_2 - 20I_3 = 50 \quad \text{-- -- -- --} \quad (1)$$

Applying KVL in mesh-3

$\therefore 50I_3 - 20I_2 = -100$ --- (2)

Apply Cramer's rule

$$\begin{bmatrix} 35 & -20 \\ -20 & 50 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\therefore D = \begin{vmatrix} 35 & -20 \\ -20 & 50 \end{vmatrix} = 1350$$

$$\therefore D_1 = \begin{vmatrix} 50 & -20 \\ -100 & 50 \end{vmatrix} = 500$$

$$\therefore D_2 = \begin{vmatrix} 35 & 50 \\ -20 & -100 \end{vmatrix} = -2500$$

$$\therefore I_2 = \frac{D_1}{D} = \frac{500}{1350} = 0.37 \text{ Amp}$$

$$\therefore I_3 = \frac{D_2}{D} = \frac{-2500}{1350} = -1.85 \text{ Amp}$$

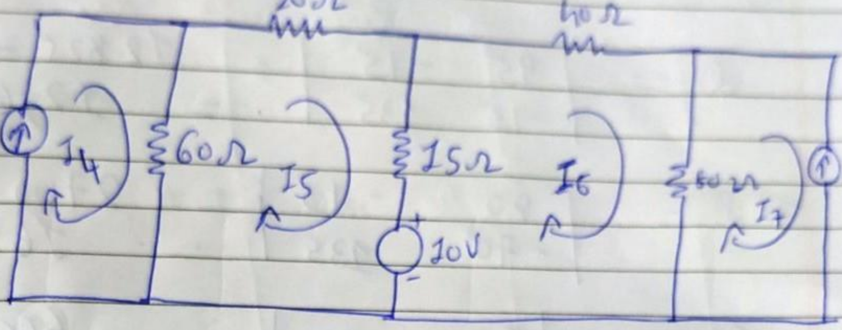
Voltage across 200Ω is

$$V_{200} = (I_2 - I_3)\,20$$

$$= (0.37 + 1.85) \times 20$$

$$\boxed{V_{200} = 44.4\text{ V}}$$

(8) Using mesh analysis find currents I_1, I_2 and I_3 for given circuit in fig. below.



Sol^n

$\therefore \quad I_4 = 0.5\,A \quad \& \quad I_7 = -1\,A$

Apply KVL in mesh-2

$\therefore \quad 20 I_5 + 15(I_5 - I_6) + 60(I_5 - I_4) = (-10)$

$\therefore \quad 20 I_5 + 15(I_5 - I_6) + 60(I_5 - 0.5) = -10$

$\therefore \quad 95 I_5 - 15 I_6 = -10 + 30$

$\therefore \quad 95 I_5 - 15 I_6 = 20 \quad --- (1)$

Apply KVL in mesh-3

$\therefore \quad 40 I_6 + 80(I_6 - I_7) + 15(I_6 - I_5) = 10$

$\therefore \quad 135 I_6 + 80 - 15 I_5 = 10$

$\therefore \quad 135 I_6 - 15 I_5 = -70 \quad --- (2)$

$$\begin{bmatrix} 95 & -15 \\ -15 & 135 \end{bmatrix} \begin{bmatrix} I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 20 \\ -70 \end{bmatrix}$$

$$\mathcal{D} = \begin{vmatrix} 95 & -15 \\ -15 & 135 \end{vmatrix} = 12825 - 225 = 12{,}600$$

$$\mathcal{D}_1 = \begin{vmatrix} 20 & -15 \\ -70 & 135 \end{vmatrix} = 2700 - 1050 = 1650$$

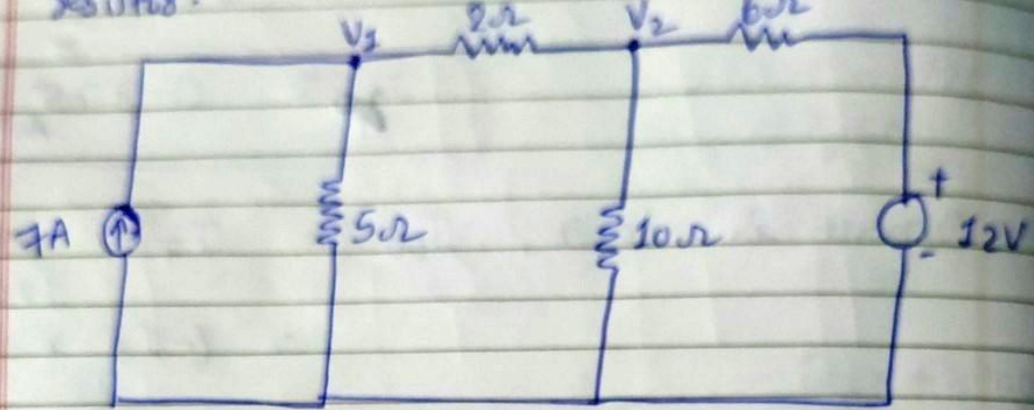
$$\mathcal{D}_2 = \begin{vmatrix} 95 & 20 \\ -15 & -70 \end{vmatrix} = -6650 + 300 = -6350$$

$$I_5 = \frac{\mathcal{D}_1}{\mathcal{D}} = \frac{1650}{12600} = (0.130)\,\text{Amp}$$

$$I_6 = \frac{\mathcal{D}_2}{\mathcal{D}} = \frac{-6350}{12600} = (-0.50)\,\text{Amp}$$

$$\therefore \boxed{\begin{aligned} I_4 &= 0.5\,A \\ I_5 &= 0.130\,A \\ I_6 &= -0.50\,A \\ I_7 &= -1\,A \end{aligned}}$$

(9) For the circuit of fig. below determine the nodal voltages and current through 2Ω resistor.



Sol^n Apply KCL at node-1

$$\frac{V_1}{5} + \frac{V_1 - V_2}{2} = 7 \quad ----$$

$$\therefore \frac{V_1}{5} + \frac{V_1}{2} - \frac{V_2}{2} = 7$$

$$\therefore 0.2 V_1 + 0.5 V_1 - 0.5 V_2 = 7$$

$$\therefore 0.7 V_1 - 0.5 V_2 = 7 \quad ---(1)$$

Apply KCL at node-2

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} + \frac{V_2 - 12}{6} = 0$$

$$\therefore \frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{10} + \frac{V_2}{6} - 2 = 0$$

$$\therefore 0.5V_2 - 0.5V_1 + 0.1V_2 + 0.16V_2 = 2$$

$$\therefore \quad 0.76V_2 - 0.5V_1 = 2 \quad \text{---}(2)$$

$$\therefore \begin{bmatrix} 0.7 & -0.5 \\ -0.5 & 0.76 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

$$\mathcal{D} = \begin{vmatrix} 0.7 & -0.5 \\ -0.5 & 0.76 \end{vmatrix} = 0.532 - 0.25 = 0.282$$

$$\mathcal{D}_1 = \begin{vmatrix} 7 & -0.5 \\ 2 & 0.76 \end{vmatrix}$$

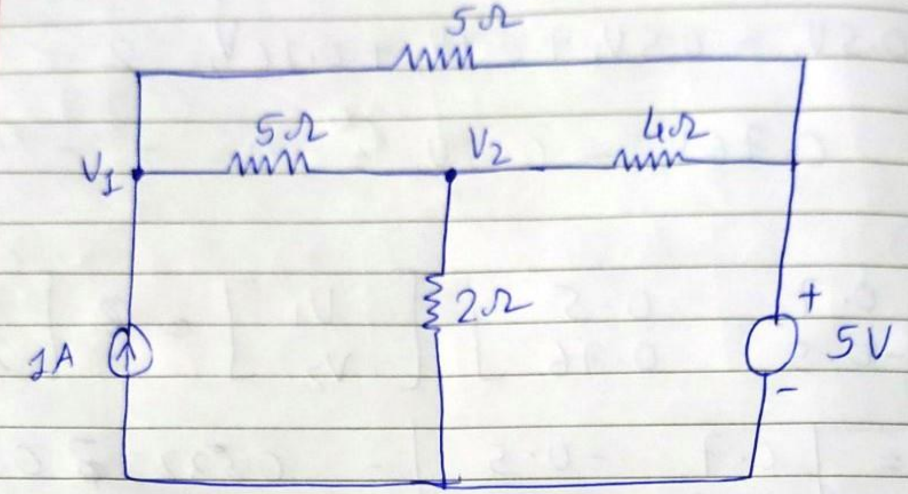
$$= 5.32 + 1 = 6.32$$

$$\mathcal{D}_2 = \begin{vmatrix} 0.7 & 7 \\ -0.5 & 2 \end{vmatrix} = 1.4 + 3.5 = 4.9$$

$$\therefore V_1 = \frac{\mathcal{D}_1}{\mathcal{D}} = \frac{6.32}{0.282} = \underline{\underline{22.41 \text{ Volt}}}$$

$$\therefore V_2 = \frac{\mathcal{D}_2}{\mathcal{D}} = \frac{4.9}{0.282} = \underline{\underline{17.375 \text{ Volt}}}$$

(10) For the circuit of fig below determine the V_1 and V_2 nodal voltages.



Sol^n:- Apply KCL at node-1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - 5}{5} = 1$$

$$\frac{V_1}{5} - \frac{V_2}{5} + \frac{V_1}{5} - 1 = 1$$

$$\therefore \frac{2}{5} V_1 - \frac{V_2}{5} = 2$$

$$\therefore 0.4 V_1 - 0.2 V_2 = 2 \quad \text{--- (i)}$$

Apply KCL at node-2

$$\frac{V_2}{2} + \frac{V_2 - V_1}{5} + \frac{V_2 - 5}{4} = 0$$

$$\frac{V_2}{2} + \frac{V_2}{5} - \frac{V_1}{5} + \frac{V_2}{4} - 1.25 = 0$$

$$0.5 V_2 + 0.2 V_2 - 0.2 V_1 + 0.25 V_2 = 1.25$$

$$0.95 V_2 - 0.2 V_1 = 1.25 \quad \text{--- (2)}$$

$$\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.95 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1.25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.4 & -0.2 \\ -0.2 & 0.95 \end{vmatrix} = 0.38 - 0.04 = 0.34$$

$$\Delta_1 = \begin{vmatrix} 2 & -0.2 \\ 1.25 & 0.95 \end{vmatrix} = 1.90 + 0.25 = 2.15$$

$$\Delta_3 = \begin{vmatrix} 0.4 & 2 \\ -0.2 & 1.25 \end{vmatrix} = 0.5 + 0.4 = 0.9$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{2.15}{0.34} = \underline{\underline{6.32\ V}}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{0.9}{0.34} = \underline{\underline{2.64\ V}}$$

EXAMPLE 2.2 : **Two metal plates of area 100 cm^2 are separated by a dielectric of 2 mm having a relative permittivity of 5. When a dc voltage of 500V is applied across the capacitor plates, find (i) capacitance (ii) charge on the capacitor (iii) electric field strength and (iv) electric flux density.**

SOLUTION :

$$A = 100\ cm^2,\quad d = 2\ mm,\quad \epsilon_r = 5,\quad V = 500\ V$$

(i) $$C = \frac{\epsilon_r \epsilon_o A}{d}$$

$$= \frac{5 \times 8.854 \times 10^{-12} \times 100 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 221.35 \times 10^{-12}\ F$$

$$= 221.35\ pF$$

(ii) $$Q = CV$$

$$= 221.35 \times 10^{-12} \times 500$$

$$= 0.1107 \times 10^{-6}\ C$$

(iii) $$E = \frac{V}{d}$$

$$= \frac{500}{2 \times 10^{-3}}$$

$$= 250 \times 10^3\ V/m$$

$$= 250\ KV/m$$

(iv) $$D = \frac{Q}{A}$$

$$= \frac{0.1107 \times 10^{-6}}{100 \times 10^{-4}}$$

$$= 11.07\ \mu C/m^2$$

EXAMPLE 2.4 : A capacitor is made up of two plates with an area of 11 cm² which are separated by a mica sheet 2 mm thick. If the relative permittivity of mica is 6, find its capacitance. Now, if one plate is moved further to give an air gap 0.5 mm wide between the plate and mica, find the new capacitance. **(G.U. Nov. 2005)**

SOLUTION :

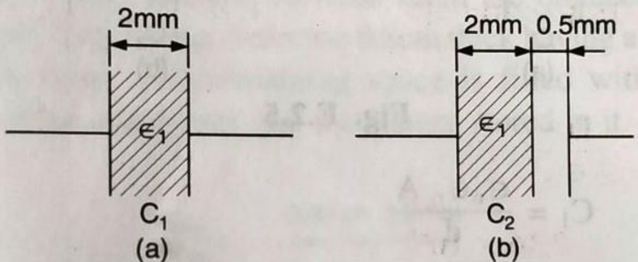


Fig. E.2.4

$A = 11\ cm^2 = 11 \times 10^{-4}\ m^2$, $d = 2\ mm = 2 \times 10^{-3}\ m$, $\in_r = 6$

Refer Fig. E.2.4(a).

$$C = \frac{\in_0 \in_r A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 6 \times 11 \times 10^{-4}}{2 \times 10^{-3}}$$

$$= 29.21 \times 10^{-12}\ F$$

$$= 29.21\ pF$$

$$C = \frac{\in_0 A}{\dfrac{d_1}{\in_{r_1}} + \dfrac{d_2}{\in_{r_2}}}$$

$$= \frac{8.854 \times 10^{-12} \times 11 \times 10^{-4}}{\dfrac{2 \times 10^{-3}}{6} + \dfrac{0.5 \times 10^{-3}}{1}}$$

$$= 11.68 \times 10^{-12}\ F$$

$$= 11.68\ pF$$

EXAMPLE 2.7 : Two plates are kept 1.5 cm apart in air and 1 kV supply is connected across them. Calculate the electric field strength in air when a glass sheet 0.5 cm thick with relative permittivity 3 is introduced between the plates without changing the previous distance between the plates.

SOLUTION :

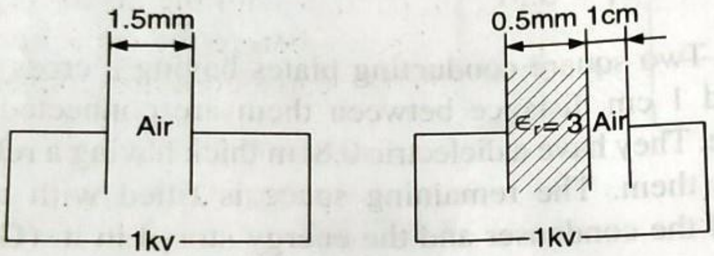


Fig. E.2.7

$d = 1.5$ cm, $V = 1$ kV, $d_g = 0.5$ cm, $\epsilon_{r_g} = 3$, $d_{air} = 1$ cm, $\epsilon_{r_{air}} = 1$

$$C = \frac{\epsilon_o A}{\dfrac{d_{air}}{\epsilon_{r_{air}}} + \dfrac{d_g}{\epsilon_{r_g}}}$$

$$= \frac{8.854 \times 10^{-12} \times A}{1 \times 10^{-12} + \dfrac{0.5 \times 10^{-2}}{3}}$$

$$= 7.589 \times 10^{-10} \times A \quad \text{farads}$$

$$Q = CV$$

$$= 7.589 \times 10^{-10} \times A \times 1000 \text{ C}$$

$$= 7.589 \times 10^{-7} \times A \text{ C}$$

$$D = \frac{Q}{A}$$

$$= \frac{7.589 \times 10^{-7} A}{A}$$

$$= 7.589 \times 10^{-7} \text{ c/m}^2$$

$$E_{air} = \frac{D}{\epsilon_o}$$

$$= \frac{7.589 \times 10^{-7}}{8.854 \times 10^{-12}}$$

$$= 85712.67 \text{ V/m}$$

$$= 85.712 \text{ kV/m}$$