Assignment VI

Adithya, EE17B115

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1 The Laplace Transform

1.1 Question 1

We will first find out X(S) as function of s by substituting the laplace transform of f(t),
Decay factor is 0.5, and we are computing for 50 seconds and plotting.

```
t=np.linspace(0,50,2000)
p1=np.polymul([1,1,2.5],[1,0,2.25])
X1=sp.lti([1,0.5],p1)
t,x1=sp.impulse(X1,None,t)
```

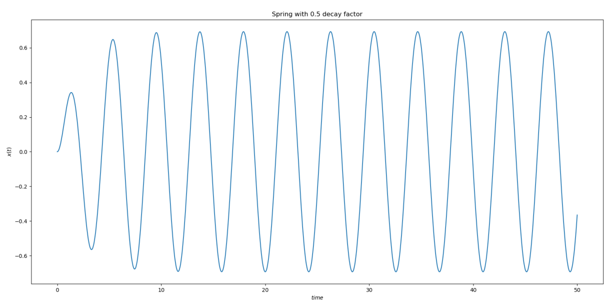

Figure 1: X(t) for decay factor 0.5

1.2 Question 2

We are trying to do the same thing but we reduce the decay factor to 0.05

```
p2=np.polymul([1,0.1,2.2525],[1,0,2.25])
X2=sp.lti([1,0.05],p2)
t,x2=sp.impulse(X2,None,t)
```

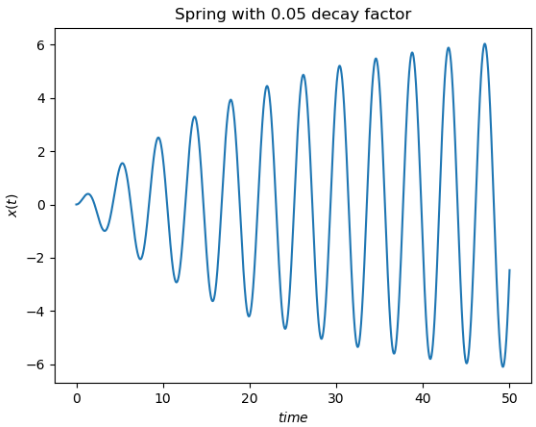

Figure 2: X(t) for decay factor 0.05

1.3 Question 3

Let us plot the same thing (decay factor 0.05) but we change the frequencies of the cosine in f(t) from 1.4 to 1.6.

```
for i in range(5):
    f=1.4+i*0.05
    p=np.polymul([1,0.1,0.0025+f*f],[1,0,2.25])
    X=sp.lti([1,0.05],p)
    t,xx=sp.impulse(X,None,t)
    plt.figure(i+5)
    plt.plot(t,xx)
    plt.title("Spring with 0.05 decay factor and frequency "+str(f))
    plt.xlabel('$time$')
    plt.ylabel('$x(t)$')
    plt.show()
```

We notice that resonance occurs at 1.5 radians/sec, which is the natural frequency of this under-damped spring mass system.

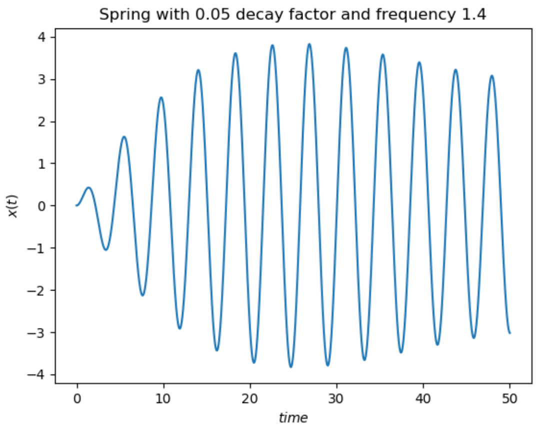

Figure 3: frequency = 1.4

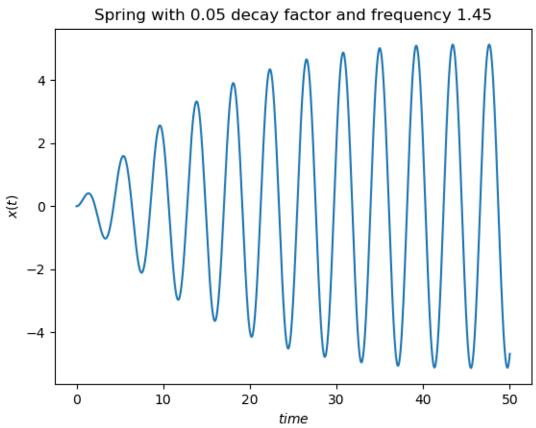

Figure 4: frequency = 1.45

1.4 Question 4

Substitute for y from the first equation into the second and get a fourth order differential equation. Solve for its time evolution, and from it obtain x(t) and y(t) for 0 ¡ t ¡ 20.

```
X=sp.lti([1,0,2],[1,0,3,0])
Y=sp.lti([2],[1,0,3,0])

t=np.linspace(0,20,1000)

t,x=sp.impulse(X,None,t)
t,y=sp.impulse(Y,None,t)
```

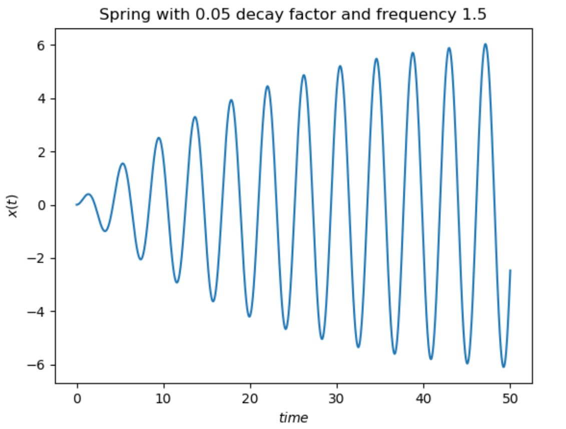

Figure 5: frequency = 1.5

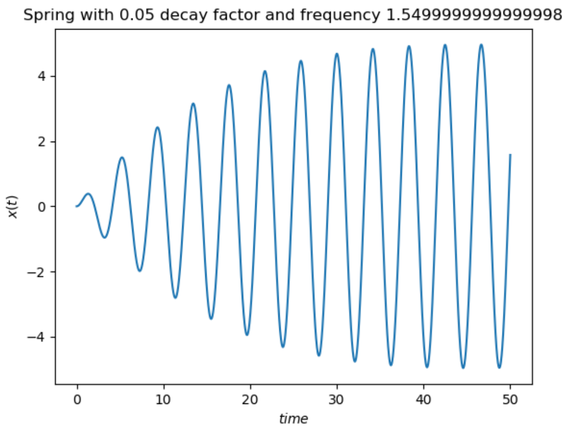

Figure 6: frequency = 1.55

```
plt.figure(10)
plt.plot(t,x)
plt.xlabel('$time$')
plt.ylabel('$x(t)$')
plt.show()

plt.figure(11)
plt.plot(t,y)
plt.xlabel('$time$')
plt.ylabel('$y(t)$')
plt.show()
```

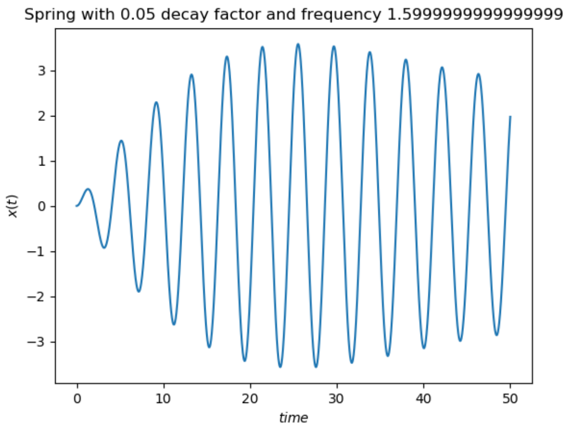

Figure 7: frequency = 1.6

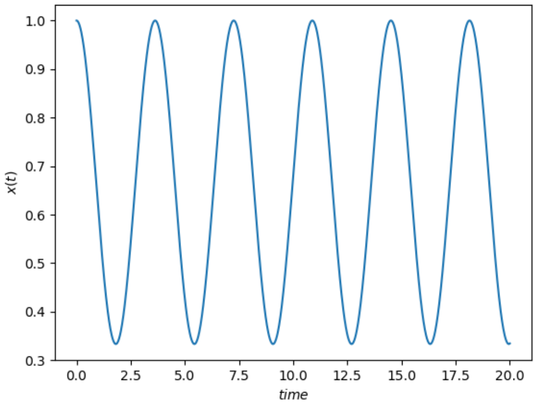

Figure 8: x Vs t

1.5 Question 5

First let us find the transfer function of the given low pass filter, plot its bode plot of magnitude and phase responses.

```
H=sp.lti(1,np.poly1d([1e-12,1e-4,1]))
w,S,phi=H.bode()
plt.figure(0)
plt.subplot(211)
plt.semilogx(w,S)
plt.xlabel('$w$')
plt.ylabel('$magnitude$')
plt.title("Magnitude plot")
```

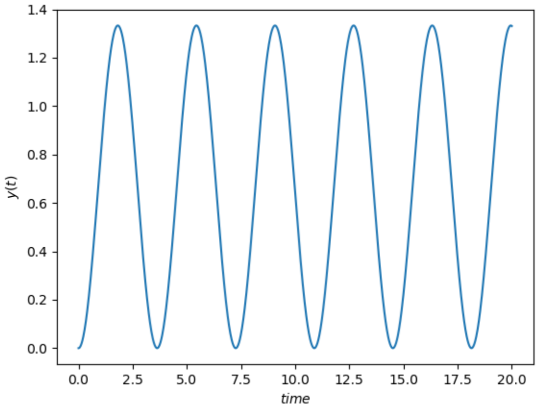

Figure 9: y Vs t

```
plt.subplot(212)
plt.semilogx(w,phi)
plt.xlabel('$w$')
plt.ylabel('$phase$')
plt.title("Phase plot")
plt.show()
```

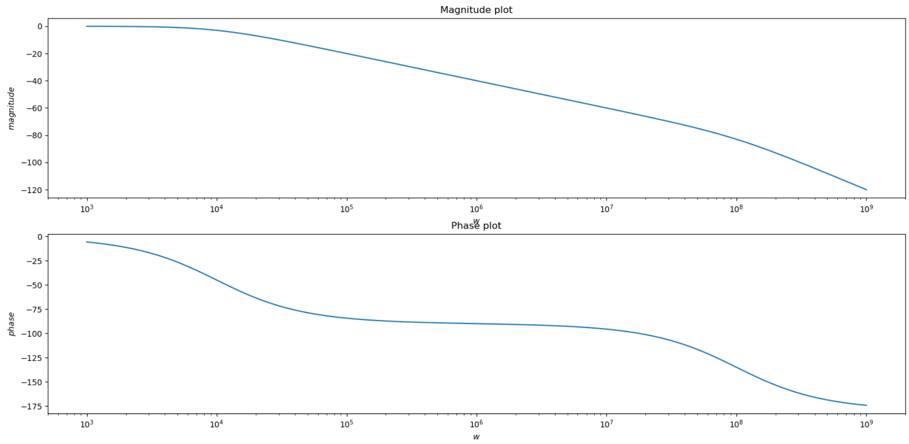

Figure 10: Bode Plot of low pass filter

1.6 Question 6

Transfer function is same as for Question 5, but this time we are giving input.
Using **sp.lsim**, we will calculate the time domain response and plot it in ms and μs time scale

```
t=np.linspace(0,30e-6,1000)
tn=np.linspace(0,30e-3,10000)
f=np.cos(1e3*t)-np.cos(1e6*t)
fn=np.cos(1e3*tn)-np.cos(1e6*tn)
t,y,svec=sp.lsim(H,f,t)
tn,yn,svec=sp.lsim(H,fn,tn)
```

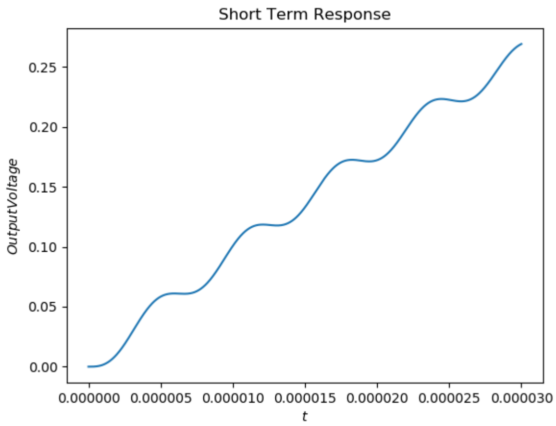

Figure 11: Short term response

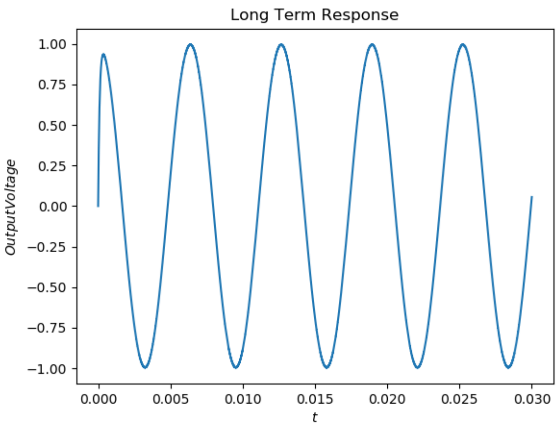

Figure 12: Long term response