# EE17B115_LAB8

April 4, 2019

## 1 Introduction

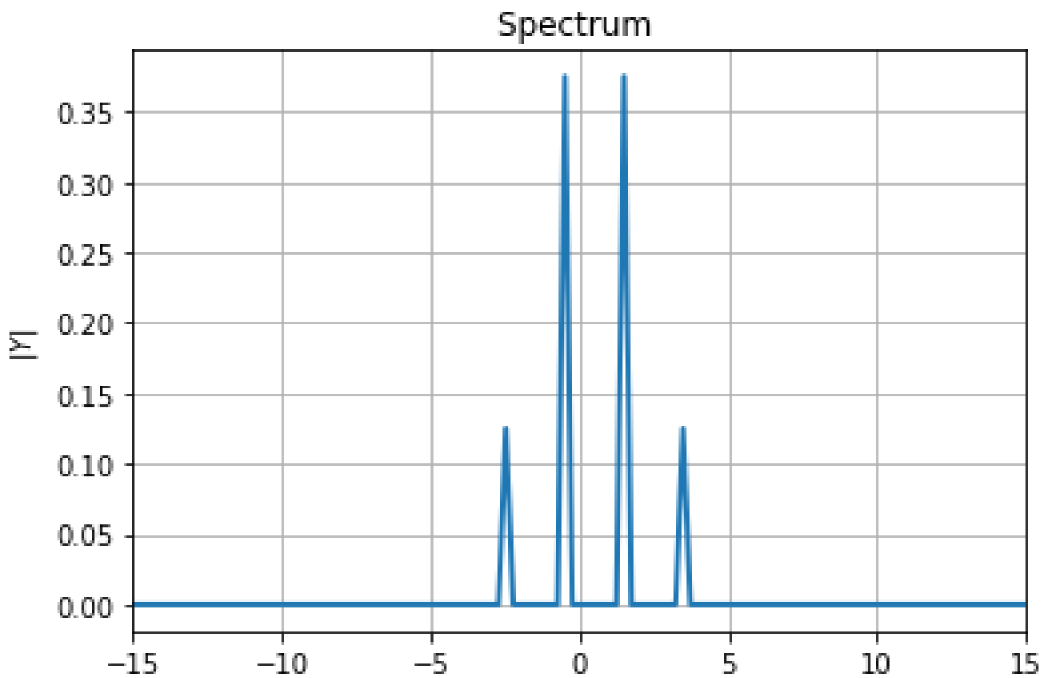
In this assignment, we analyse signals using the Fast Fourier transform. np.fft is a fast implementation of the Discrete Fourier transform. We also attempt to approximate the continuous time fourier transform of a gaussian by windowing and sampling in time domain, and then taking the DFT. We will increase number of samples and size of window till we obtain a required accuracy.

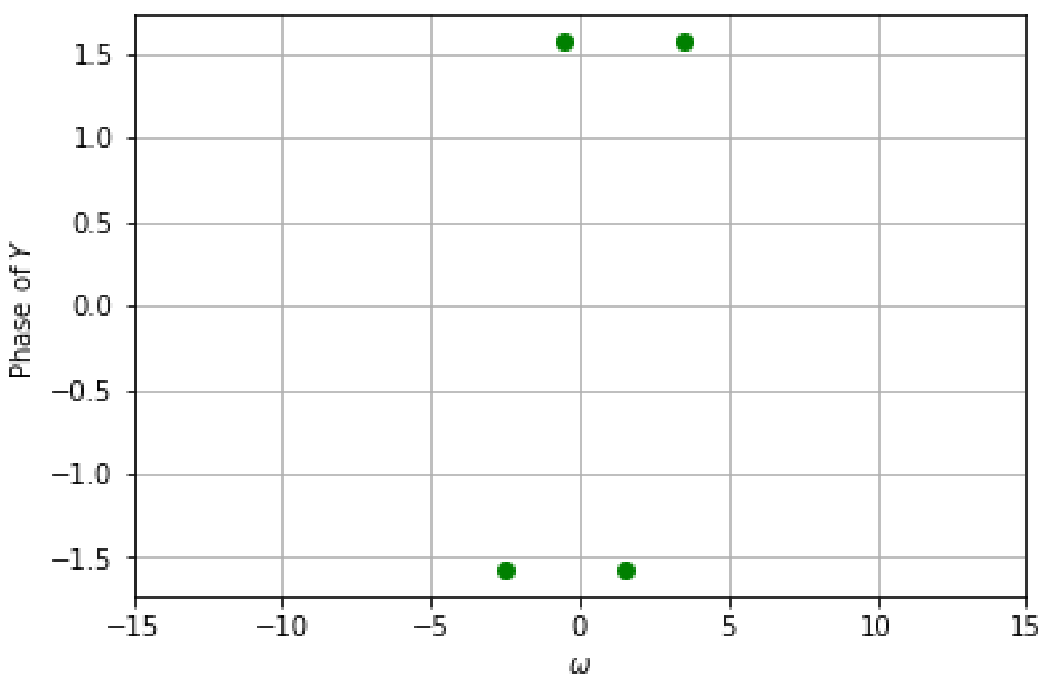
```
In [115]: import numpy as np
          import scipy as sp
          import matplotlib.pyplot as plt
```

## 2 Assignment Questions

### 2.1 Question 2 : Spectrum of $sin^3t$

```
In [121]: t=np.linspace(-4*np.pi,4*np.pi,513)
          t=t[:-1]
          y=(np.sin(t))**3
          Y=np.fft.fftshift(np.fft.fft(y))/512
          w=np.linspace(-63,64,513)
          w=w[:-1]
          plt.figure()
          plt.plot(w,abs(Y),lw=2)
          plt.xlim([-15,15])
          plt.ylabel(r"$|Y|$")
          plt.title(r"Spectrum ")
          plt.grid(True)
          plt.show()
          i=np.where(np.abs(Y)>1e-3)
          plt.plot(w[i],np.angle(Y[i]),'go',lw=2)
          plt.xlim([-15,15])
          plt.ylabel(r"Phase of $Y$")
          plt.xlabel(r"$\omega$")
          plt.grid(True)
          plt.show()
```

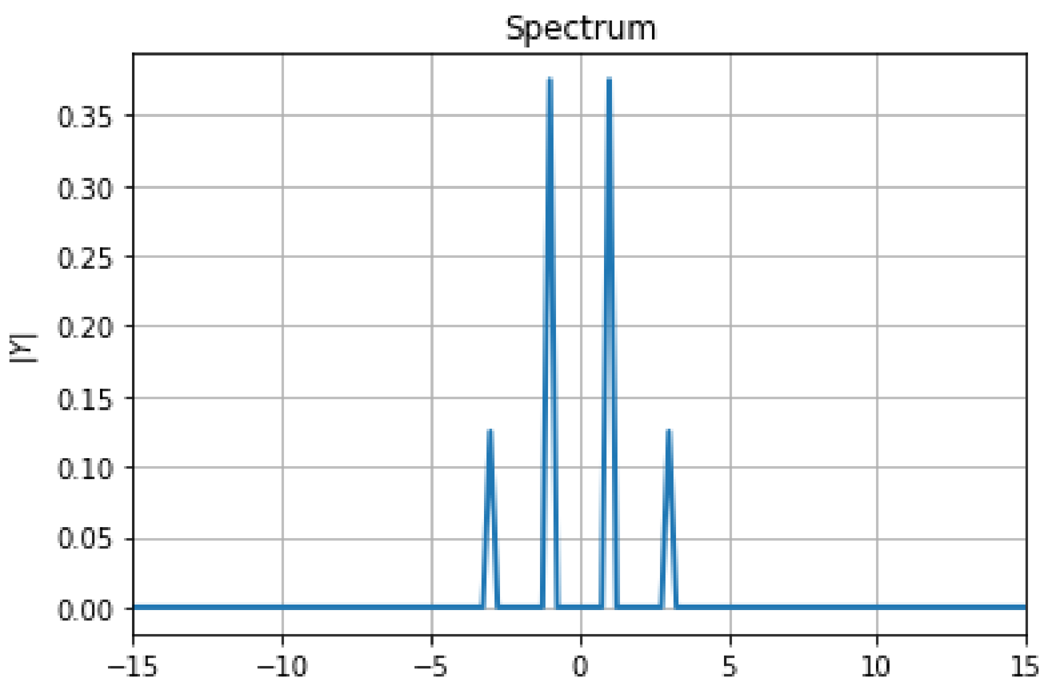


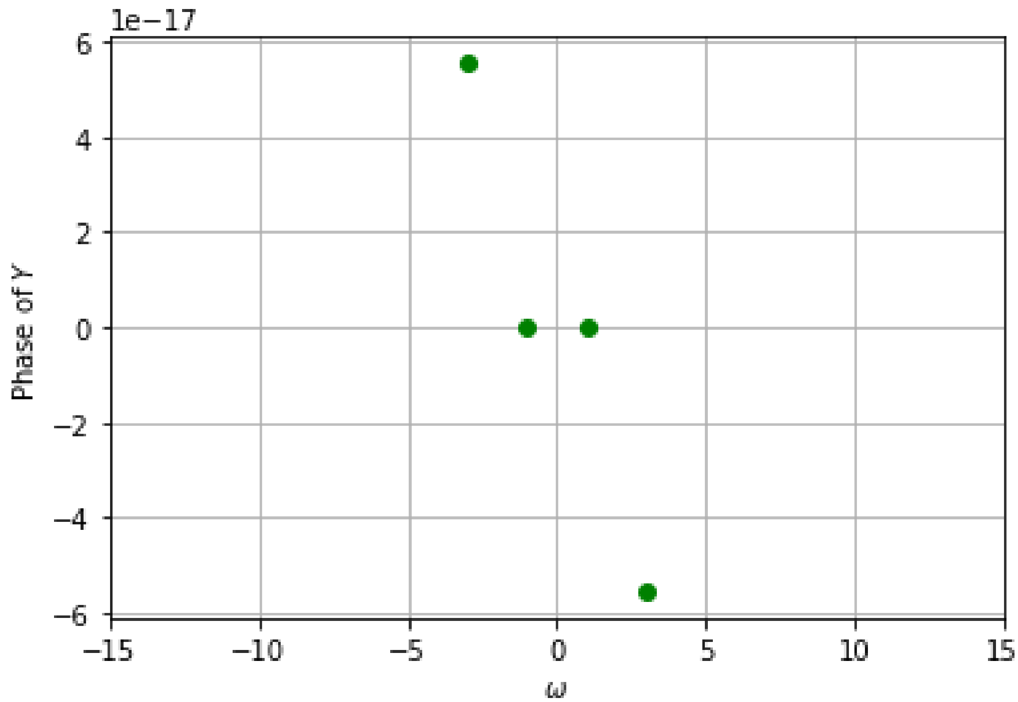




We observe the peaks in the magnitude at the expected frequencies of 1 and 3, along with the amplitudes 0.75 and 0.25 respectively as expected.

### 2.2 Spectrum of $cos^3t$

In [122]:
```
t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=(3*np.cos(t)+np.cos(3*t))/4
Y=np.fft.fftshift(np.fft.fft(y))/512
w=np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.plot(w,abs(Y),lw=2)
plt.xlim([-15,15])
plt.ylabel(r"$|Y|$")
plt.title(r"Spectrum")
plt.grid(True)
plt.show()
i=np.where(np.abs(Y)>1e-3)
plt.plot(w[i],np.angle(Y[i]),'go',lw=2)
plt.xlim([-15,15])
plt.ylabel(r"Phase of $Y$")
plt.xlabel(r"$\omega$")
plt.grid(True)
plt.show()
```





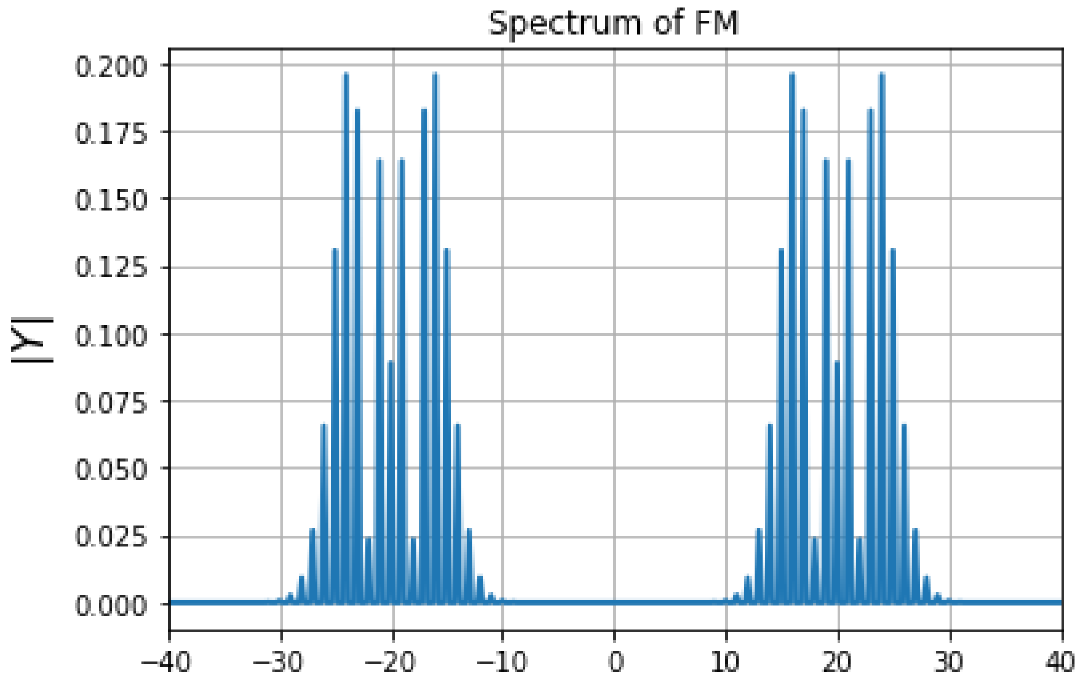

We observe the peaks in the magnitude at the expected frequencies of 1 and 3, along with the amplitudes 0.75 and 0.25 respectively as expected.

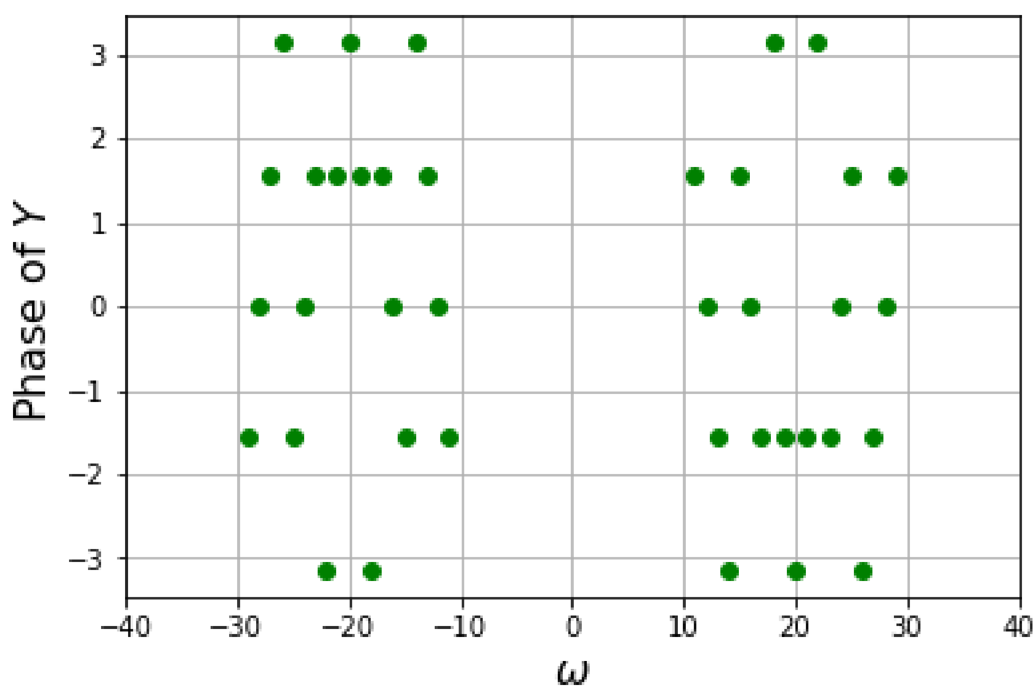
## 3 Question 3: Frequency Modulation

In [118]:
```
t=np.linspace(-4*np.pi,4*np.pi,513)
t=t[:-1]
y=np.cos(20*t + 5*np.cos(t))
Y=np.fft.fftshift(np.fft.fft(y))/512
w=np.linspace(-64,64,513)
w=w[:-1]
plt.figure()
plt.plot(w,abs(Y),lw=2)
plt.xlim([-40,40])
plt.ylabel(r"$|Y|$",size=16)
plt.title(r"Spectrum of FM")
plt.grid(True)
plt.show()

ii=np.where(np.abs(Y)>1e-3)
plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
plt.xlim([-40,40])
plt.ylabel(r"Phase of $Y$",size=16)
plt.xlabel(r"$\omega$",size=16)
```

```
plt.grid(True)
plt.show()
```



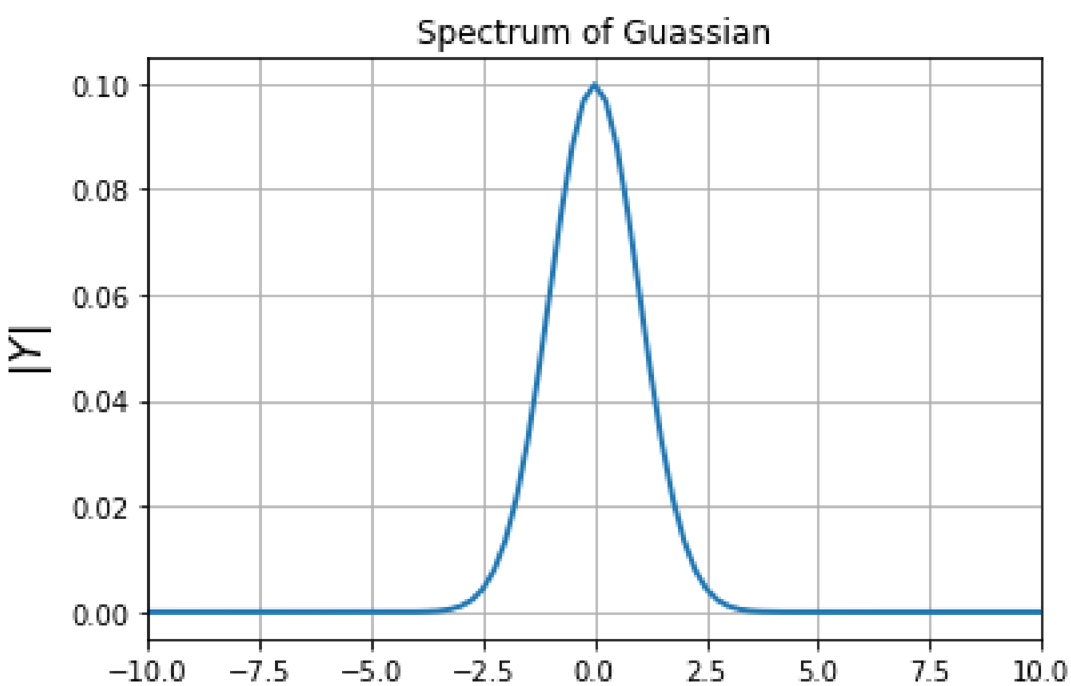
Above plots are the magnitude and phase plots of spectrum in FM case.

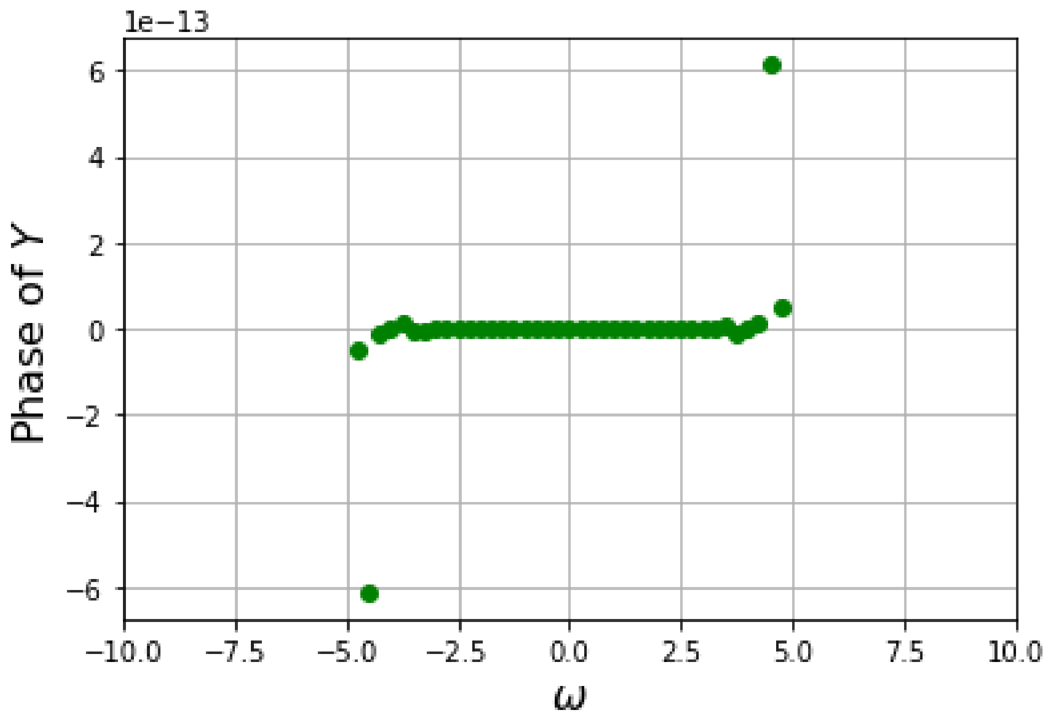
### 3.1 Question 4: Guassian

The Gaussian $e^{-t^2/2}$ is not "bandlimited" in frequency. We want to get its spectrum accurate to 6 digits, by different time ranges, and see what gets us a frequency domain that is so accurate. As the FFT method messes up here due to wrong initial value of t, we use np.fft.iiftshift()

```
In [129]: t=np.linspace(-4*np.pi,4*np.pi,129);t=t[:-1]
          y=np.exp(-t**2/2)
          Y=np.fft.fftshift(np.fft.fft(np.fft.ifftshift(y)))/128
          w=np.linspace(-16,16,129)
          w=w[:-1]
          plt.figure()
          plt.plot(w,abs(Y),lw=2)
          plt.xlim([-10,10])
          plt.ylabel(r"$|Y|$",size=16)
          plt.title(r"Spectrum of Guassian")
          plt.grid(True)
          plt.show()

          ii=np.where(np.abs(Y)>1e-6)
          plt.plot(w[ii],np.angle(Y[ii]),'go',lw=2)
          plt.xlim([-10,10])
          plt.ylabel(r"Phase of $Y$",size=16)
          plt.xlabel(r"$\omega$",size=16)
          plt.grid(True)
          plt.show()
```



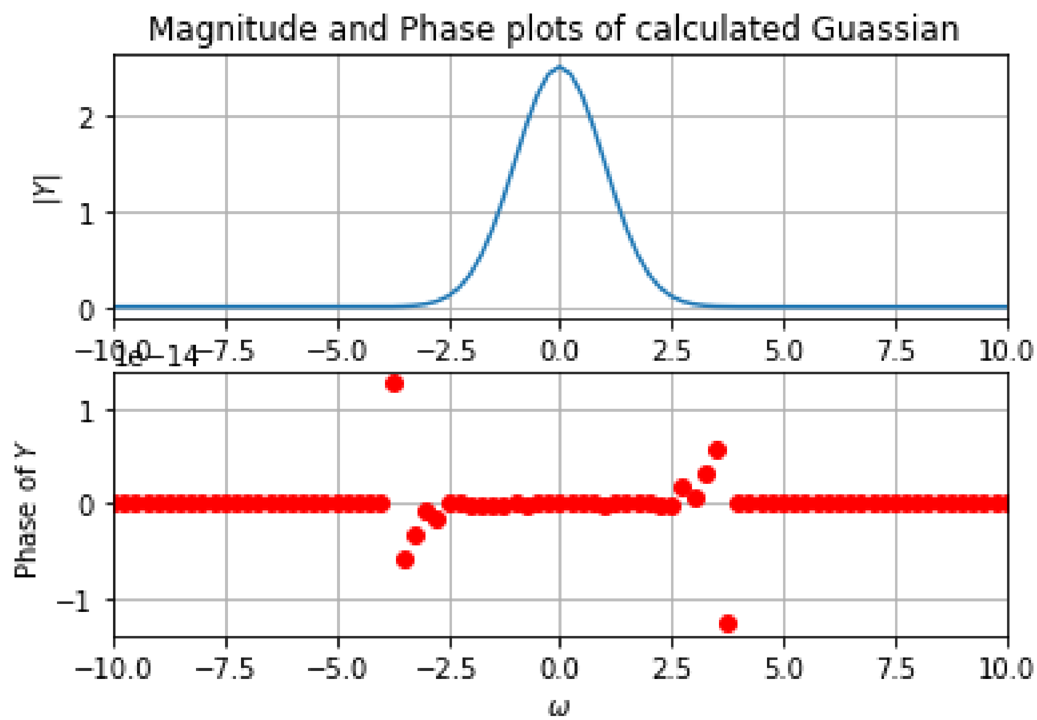


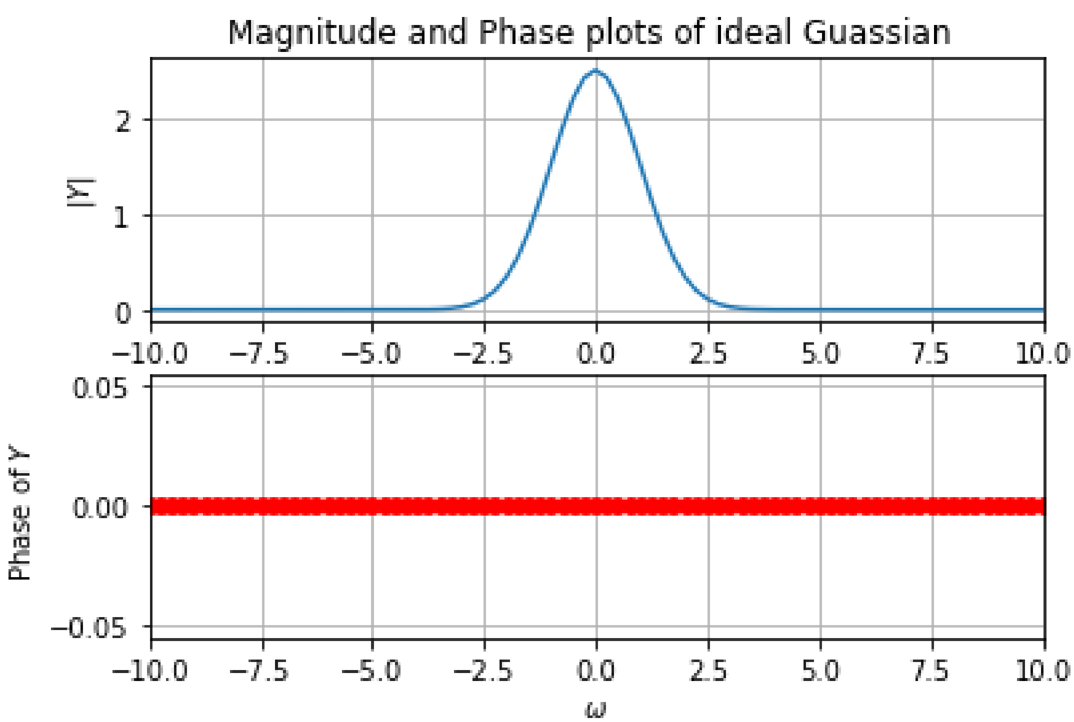

We arrive at above sampling rate and number of samples by checking the accuracy each and every time by calculating error from ideal case in a loop and changing them.

```
In [130]: def ideal(w):
              return (np.sqrt(2*np.pi)) * (np.exp((-w*w)/2))
          T = 8*np.pi
          N = 128
          error = 1
          while error>1e-6:
              t = np.linspace(-T/2,T/2,N+1)[:-1]
              w = np.pi* np.linspace(-N/T,N/T,N+1)[:-1]
              Y1 = (T/N) * np.fft.fftshift(np.fft.fft(np.fft.ifftshift(np.exp(-t*t/2))))
              error = sum(np.abs(Y1-ideal(w)))
              print("Error at N=",N,"is",error)
              N = N*2
              T = T*2
          print("min error =" + str(error))
          print("Values of T,N are",T/2,N/2)
          fig,a = plt.subplots(2)
          a[0].plot(w,abs(Y1)*(1))
          a[0].set_xlim([-10,10])
          a[0].grid(True)
```

```
a[0].set_ylabel(r"$|Y|$")
a[0].set_xlabel(r"$\omega$")
a[0].set_title(r"Magnitude and Phase plots of calculated Guassian ")
ii = np.where(abs(Y1)<10**-3)
phase = np.angle(Y1)
phase[ii] =0
a[1].plot(w,phase,"ro")
a[1].set_xlim([-10,10])
a[1].grid(True)
a[1].set_ylabel(r"Phase of $Y$")
a[1].set_xlabel(r"$\omega$")
plt.show()
fig2,b = plt.subplots(2)
b[0].plot(w,abs(ideal(w)))
b[0].set_xlim([-10,10])
b[0].grid(True)
b[0].set_ylabel(r"$|Y|$")
b[0].set_xlabel(r"$\omega$")
b[0].set_title(r"Magnitude and Phase plots of ideal Guassian ")
b[1].plot(w,np.angle(ideal(w)),"ro")
b[1].set_xlim([-10,10])
b[1].grid(True)
b[1].set_ylabel(r"Phase of $Y$")
b[1].set_xlabel(r"$\omega$")
plt.show()
```

```
Error at N= 128 is 3.5686271076686083e-14
min error =3.5686271076686083e-14
Values of T,N are 25.132741228718345 128.0
```


Magnitude and Phase plots of calculated Guassian
|Y|
1e−14
Phase of Y
ω


Magnitude and Phase plots of ideal Guassian
|Y|
Phase of Y
ω