# EE5121 Convex Optimization Term Paper

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EE17B115

**Title :** Cubic Discrimination

**Reference :** Classification, Section 8.6 of the book "Convex Optimization" by Stephen Boyd and Lieven Vandenberghe (Page No: 422)

**Dataset Link :** 2d datasets (classification)

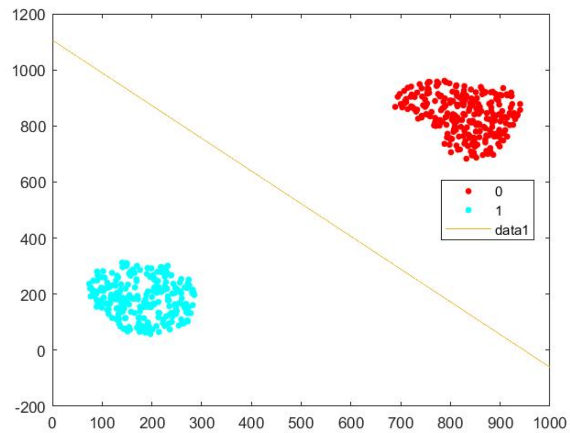
**Contributions :**

- Implemented the idea of Support Vector Machine into Quadratic Discrimination
- Implemented Cubic Discrimination
- Used SVM idea to do Multi-classification (More than 2 classes classification)
- Applied all above classifiers on a kaggle dataset.

# 1 Robust Linear Discrimination

**Contribution :** Applied this to kaggle 2d classification dataset.

```
cvx_begin
    variables p(2) q  t;
    maximize t;
    subject to
    x*p - q >= t;
    y*p - q <= -t;
    norm(p, 2) <= 1;
cvx_end
```

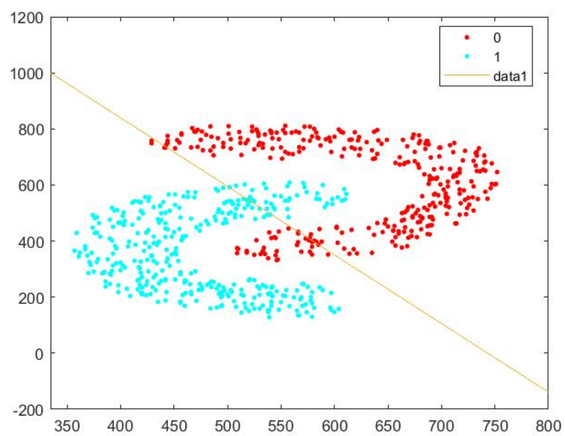



However this thing fails in case of non-linear seperable datasets. We need to use Support Vector Machines.

## 2 Support Vector Machine classifiers

**Contribution :** Applied this to kaggle 2d classification dataset.

```
cvx_begin
    variables u(size(x,1)) v(size(y,1));
    variables p(2) q  ;
    minimize sum(u(:)) + sum(v(:));
    subject to
        x*p - q >= 1 - u;
        y*p - q <= -(1 - v);
        u >= 0;
        v >= 0;
cvx_end
```

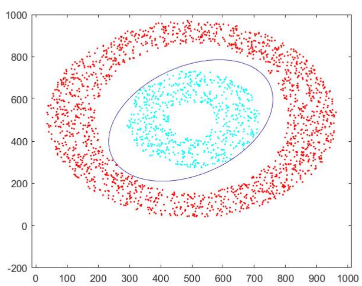


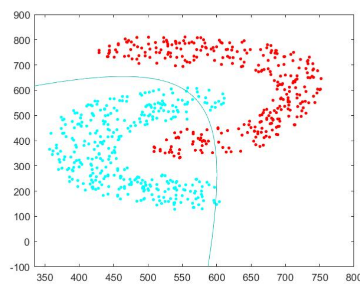

Basically what we are trying to do is to minimize number of misclassified points. But we still couldn't get a nice boundary to classify them and it looks like a linear boundary is not going to be sufficient. So, let's try quadratic discrimination.

## 3 Quadratic Discrimination

**Contribution:** Normal robust quadratic discrimination approach will fail because this dataset may or may not be seperated by a parabola/ ellipse. We need to use the idea of SVM here, i.e, minimize no: of misclassified points.

```
cvx_begin
    variables u(size(x,1)) v(size(y,1));
    variables c d e f g r  ;
    minimize sum(u(:)) + sum(v(:));
    subject to
        c*x(:,1).^2 + d*x(:,2).^2+ e*x(:,1).*(x(:,2))+f*x(:,1)+g*x(:,2) +r
            >= 10-u;
        c*y(:,1).^2 + d*y(:,2).^2+ e*y(:,1).*(y(:,2))+ f*y(:,1)+g*y(:,2) +r
            <= -(10- v);
        u >= 0;
        v >= 0;
cvx_end
```



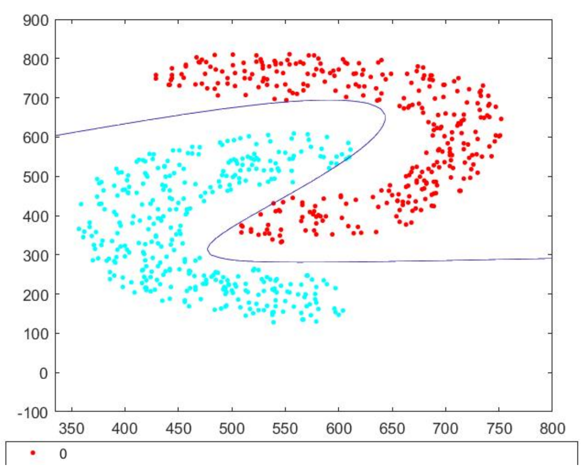




In first case, quadratic discrimination yields a nice boundary. But for second case, still it looks like quadratic boundary is unsufficient as the data can't be seperated by a parabola or ellipse. So, let's try Cubic Discrimination.

# 4 Cubic Discrimination

**Contribution:** Implemented this also similar to how an SVM works and applied it to a 2d classification dataset.

```
cvx_begin
    variables u(size(x,1)) v(size(y,1));
    variables c d e f g r h k p q ;
    minimize sum(u(:)) + sum(v(:));
    subject to
        c * x(:,1).^2 + d * x(:,2).^2+ e*x(:,1).*(x(:,2)) + f *x(:,1) + g * x(:,2) +
            h * x(:,1).^3 + p * x(:,2).^3+ q*x(:,1).*(x(:,2)).*x(:,1) +
            k*x(:,1).*(x(:,2)).*(x(:,2)) + r >= 10 - u;
        c*y(:,1).^2 + d * y(:,2).^2+ e*y(:,1).*(y(:,2)) +  f *y(:,1) + g * y(:,2)
            +h * y(:,1).^3 + p * y(:,2).^3+ q*y(:,1).*(y(:,2)).*y(:,1) +
            k*y(:,1).*(y(:,2)).*(y(:,2)) + r <= -(10- v);
        u >= 0;
        v >= 0;
cvx_end
```



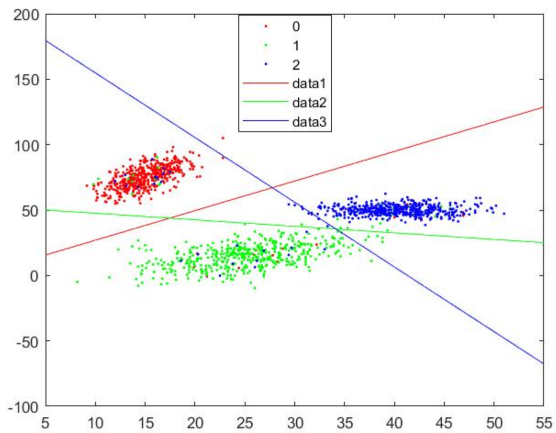
# 5 Multi Classification

**Contribution:** Used one vs all method to classify data into more than 2 classes in CVX. In one-vs-All classification, for the N-class instances dataset, we have to generate the N-binary classifier models. For classifying the object to class 1, we need to linearly seperate class 1 objects and objects of classes other than class 1, similarly N times for N classes.

```
cvx_begin
    variables u(size(x,1)) v(size(y,1)) w(size(z,1));
    variables p(2) q  ;
    minimize sum(u(:)) + sum(v(:)) + sum(w(:));
    subject to
        x*p - q >= 1 - u;
        y*p - q <= -(1 - v);
        z*p - q <= -(1 - w);
        u >= 0;
        v >= 0;
        w >= 0;
cvx_end

cvx_begin
    variables u(size(x,1)) v(size(y,1)) w(size(z,1));
    variables p(2) q  ;
    minimize sum(u(:)) + sum(v(:)) + sum(w(:));
    subject to
        y*p - q >= 1 - v;
        x*p - q <= -(1 - u);
        z*p - q <= -(1 - w);
        u >= 0;
        v >= 0;
        w >= 0;
cvx_end

cvx_begin
    variables u(size(x,1)) v(size(y,1)) w(size(z,1));
    variables p(2) q  ;
    minimize sum(u(:)) + sum(v(:)) + sum(w(:));
    subject to
        z*p - q >= 1 - w;
        x*p - q <= -(1 - u);
        y*p - q <= -(1 - v);
        u >= 0;
        v >= 0;
        w >= 0;
cvx_end
```




Here, blue line seperates blue points from others, red line seperates red points from others and green line seperates green points from others.