IITM-CS6100 : Topics in Design and analysis of Algorithms | Given on: Feb 27

Problem Set #1 | Due on : Mar 14, 23:55 | Evaluation Due on : Mar 20

- Turn in your solutions electronically at the moodle page. The submission should be a pdf file typeset either using LaTeX or any other software that generates pdf. No handwritten solutions are accepted.
- Collaboration is encouraged, but all write-ups must be done individually and independently. For each question, you are required to mention the set of collaborators, if any.
- Submissions will be checked for **plagiarism**. Each case of plagiarism will be reported to the institute disciplinary committee (DISCO).

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1. (10 points) Suppose $p_1, \ldots, p_n \in [0,1]^2$. Let $\mathsf{TSP}(p_1, \ldots, p_n)$ denote the smallest cost of a traveling salesman's tour on $p_1, \ldots, p_n$ with respect to Euclidean distance.
   (a) (6 points) For any $n > 0$, show that $\mathsf{TSP}(p_1, \ldots, p_n) \leq c\sqrt{n}$ for some constant $c > 0$.
   (b) (4 points) for any $n > 0$, show that there are n points $q_1, \ldots, q_n \in [0,1]^2$ such that $\mathsf{TSP}(p_1, \ldots, p_n) \geq c'\sqrt{n}$ for some constant $c' > 0$.
2. (7 points) This exercise to demonstrate the limitations of considering expected running time of an algorithm as a useful measure. For any $n > 0$, describe a function $f : \{0,1\}^n \to \mathbb{N}$ such that
   - $\mathbb{E}[f] = \mathbb{E}_{x \in \{0,1\}^n}[f(x)] = n^c$ for some constant c and
   - $\mathsf{var}[f] = E[f^2] - (E[f])^2 = \Omega(2^n)$.

   I.e., there can be performance measures f which is polynomial in expectation, but variance being exponential. Give formal justification for your answer (i.e., computation of expectation and variance for the function f constructed).
3. (7 points) Obtain profits $p_1, \ldots, p_n$ and weights $w_1, \ldots, w_n$ for the knapsack problem so that $|\mathcal{P}|$ is exponential in n (e.g $2^{\Omega(n)}$). Justify your answer.
4. (6 points) Read the proof of Lemma 3.4 in the notes by Bodo Manthey (in the google drive shared with the class). The proof assumes that $a = (0, \ldots, 0)$ and $b = (\delta, 0, \ldots, 0)$. Why is this assumption without loss of generality? Justify your answer.