Are Markets Random? Evidence from EUR/USD vs Coin Flips

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Abstract

This paper investigates whether financial markets behave like pure randomness by comparing daily movements of the EUR/USD exchange rate with fair coin flips. Using statistical tools such as runs tests, Monte Carlo simulations, transition probabilities, autocorrelation checks, and entropy measures, we examine whether markets follow a "random walk" or display patterns inconsistent with pure chance. The results show that while some aspects of EUR/USD returns resemble randomness, others — such as clustering of volatility and unequal streak probabilities — suggest that markets are not fully random. This project provides a foundation for more advanced modeling of memory, volatility, and predictability in financial time series.

1. Introduction

Financial markets often appear unpredictable: prices rise and fall each day in ways that seem chaotic and difficult to forecast. This perception gave rise to one of the most enduring ideas in finance — the random walk hypothesis (RWH). Originally popularized by economists such as Paul Samuelson (1965) and Burton Malkiel (1973, A Random Walk Down Wall Street), the RWH suggests that asset prices follow a statistical process where each step is independent of the last, much like flipping a fair coin. If true, this would mean tomorrow's price change cannot be predicted from past information, making it virtually impossible to "beat the market" systematically.

Testing the validity of the random walk hypothesis has long been the central theme in both academic finance and practical trading. For market theorists, it is directly tied to the Efficient Market Hypothesis (EMH), which argues that prices fully reflect all available information. For traders and quants, the question of randomness determines whether exploitable patterns exist — or whether all efforts to forecast short-term price movements are futile.

This study focuses on the EUR/USD exchange rate, the most heavily traded currency pair in the world, representing the relative value of the euro against the U.S. dollar. Because of its high liquidity and global importance, EUR/USD is often seen as one of the most "efficient" markets. If any asset were to resemble a true random walk, this would be a prime candidate.

To evaluate whether EUR/USD behaves like a random walk, we compare its sequence of daily up and down movements to simulated coin flips. Specifically, we ask three key questions:

1. Do streaks of ups and downs occur at the same rate as random flips?
2. Does the market show signs of short-term memory, where today's move depends on yesterday's?
3. Are returns entirely structureless, or do patterns such as volatility clustering and long-term persistence emerge?

By addressing these questions, this project explores the foundations of the random walk hypothesis in financial markets. The results not only provide insight into whether EUR/USD behaves like a coin flip but also set the stage for more advanced models of time series forecasting and risk management in later projects.

2. Data

We analyzed daily closing prices of the EUR/USD exchange rate (Yahoo Finance ticker: EURUSD = X) covering the period from January 2010 to January 2025. The EUR/USD currency pair is the most traded in the world, representing the exchange rate between the euro and U.S. dollar, and is widely regarded as a benchmark for global foreign exchange markets.

The dataset consisted of approximately 3,900 observations. Using daily closing prices ensures consistency, as intraday movements can introduce microstructure noise that is not relevant to this study.

To make the data suitable for statistical analysis, we computed daily returns, which measure the percentage change in price from one day to the next:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where P_t is the closing price on day t, and P_{t-1} is the closing price on the previous trading day. Returns are preferred over raw prices because they standardize changes across time and make a series of different scales comparable.

For the purpose of testing randomness in direction only, these returns were reduced to a binary sign sequence:

- 1 = up day ($r_t > 0$)
- 0 = down day ($r_t < 0$)

This simplification allows direct comparison with coin flips, where heads and tails correspond to up and down days. By stripping away the magnitude of returns, we focus purely on the directional predictability of the market. This is consistent with the random walk hypothesis, which states that the direction of future price changes should be unpredictable if markets are efficient.

3. Methodology

To test whether EUR/USD daily movements are statistically indistinguishable from random coin flips, we applied a suite of eight tests, each probing a different aspect of randomness. Together these methods provide a comprehensive framework for evaluating both directional independence and volatility structure.

3.1 Bias Test (Binomial)

The first question is whether EUR/USD shows a directional bias — is it more likely to go up that down? Under the null hypothesis of a fair coin, the proportion of up days should be 50%. A binomial test compares the observed proportion of up days to this benchmark. A significant result would suggest that EUR/USD is tilted toward gains or losses rather than behaving like a fair coin.

3.2 Runs Test (Wald–Wolfowitz)

Even if up and down days occur in the right proportion, their order matters. The runs test counts sequences of consecutive identical outcomes, known as “runs.” For example, an up-up-down-down sequence has three runs. A fair coin produces a predictable average number of runs. Too many runs suggests over-alteration (the series switches too often), while too few runs indicates streak clustering. Both outcomes would imply deviation from randomness.

3.3 Longest Streak Monte Carlo

While the runs test considers the overall frequency of runs, it does not capture extreme streaks. To evaluate extremes, we recorded the longest streak of consecutive ups or downs in the EUR/USD sequence and compared it to simulated coin flips of equal length. By repeating this process thousands of times (Monte Carlo simulation), we obtain the probability of observing such a long streak under pure randomness. This highlights whether extreme persistence in EUR/USD is unusual.

3.4 Markov Chain Memory

Next, we tested for short-term dependence using Markov chain transition probabilities. In a fair coin, the probability of an up day after an up and the probability of an up day after a down should both equal 0.5. If EUR/USD shows $P(up|up) \neq P(up|down)$, it would suggest that yesterday's outcome influences today's — a violation of independence.

3.5 Autocorrelation & Ljung–Box

The runs and Markov tests analyze directional memory, but financial time series often display structure in the magnitude of fluctuations rather than direction. We therefore measure autocorrelation in both raw returns and squared returns.

- Returns: significant autocorrelation would indicate linear predictability.
- Squared returns: correlation here indicates volatility clustering — periods of high or low volatility persisting over time.

To formally test significance, we applied the Ljung-Box test, which checks whether groups of autocorrelations differ from zero.

3.6 Hurst Exponent

To assess long-term memory, we estimated the Hurst exponent using the rescaled range (R/S) method. A value of 0.5 indicates a pure random walk. Values above 0.5 suggest persistence (trending behavior), while values below 0.5 suggest mean reversion (the tendency to revert to an average). This measure complements short-term tests by considering broader temporal structure.

3.7 Entropy

We also calculated Shannon entropy, a measure of uncertainty. For a binary process like up/down days the maximum entropy is 1 bit, corresponding to maximum unpredictability (a perfect fair coin). Lower values indicate that the sequence carries more information, implying predictability or bias.

3.8 Permutation Test (Toy Momentum Rule)

Finally, we performed a permutation test of a simple trading rule: predicting today's direction based on yesterday's. If this "one- step momentum" rule outperforms chance, it would imply exploitable dependence in the series. By shuffling the data many times, we generate a distribution of accuracies under randomness and test whether the actual EUR/USD accuracy is significantly different.

4. Results

4.1 Bias Test

- Up days: 1933/3907 = 0.4948, p-value = 0.5222

Out of 3.907 trading days, EUR/USD closed higher on 1,933 days (49.48%) and lower on 1974 days (50.52%). A binomial test against the null hypothesis of a 50/50 split produced a p-value of 0.52, indicating no significant deviation from fairness.

In other words, EUR/USD shows no directional bias: the chance of an up day is effectively equal to the chance of a down day, consistent with a fair coin flip.

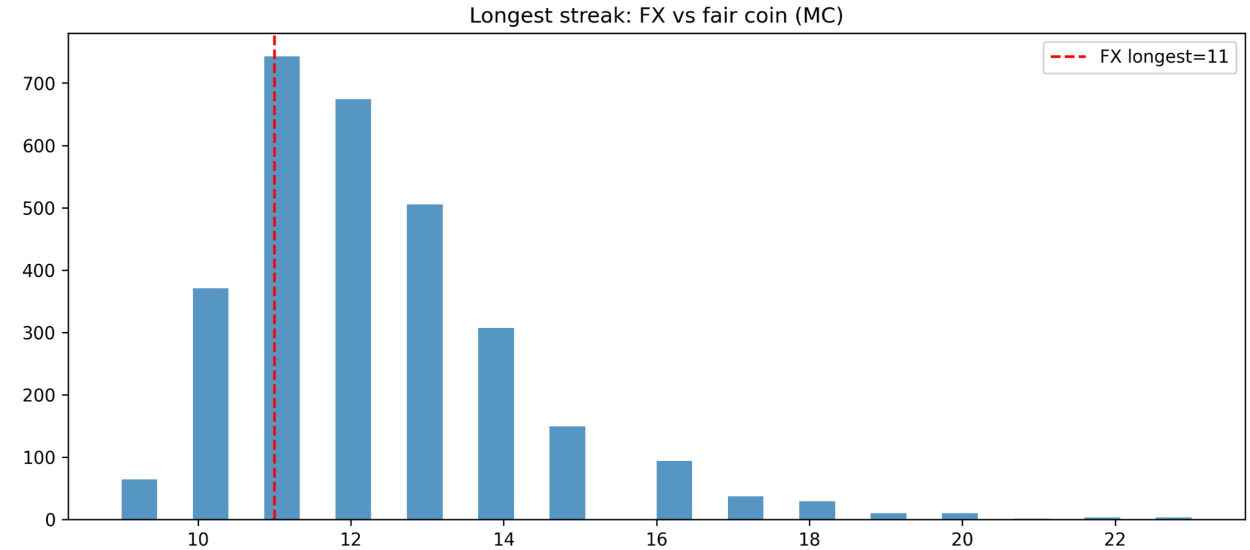
4.2 Runs Test

- R: 1984
- E_mu: 1954.2848733043256
- Var: 976.2848211485384
- z: 0.9510192717791287
- p-value: 0.34159459292114613
- n1: 1933
- n2: 1974

The run test compares the actual number of consecutive up/down streaks to what would be expected from a random coin flip. EUR/USD had 1,984 runs, while the expected number under randomness was 1,954. The difference was small ($z = 0.95$, $p = 0.34$), meaning the null hypothesis of randomness cannot be rejected.

This suggests that the overall frequency of streak in EUR/USD is statistically consistent with a coin flip. In other words, the market does not show obvious express streak or alteration compared to the randomness when viewed in aggregate.

4.3 Longest Streaks

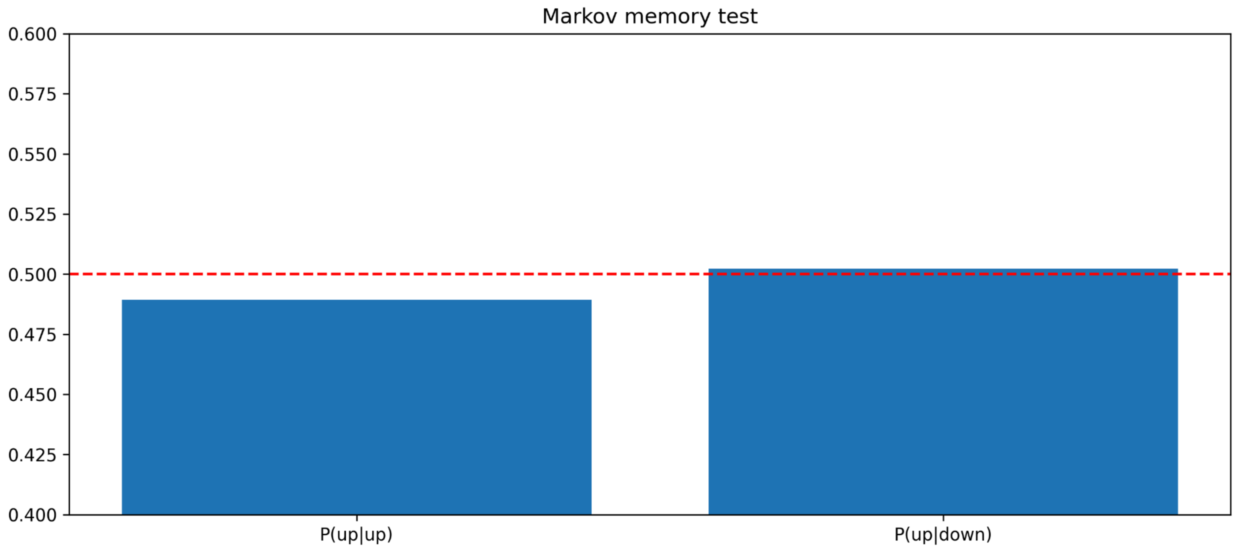


- FX longest run = 11
- MC p-value = 0.8563

The longest observed run in EUR/USD was 11 consecutive days in the same direction. Monte Carlo simulations of 3,000 coin-flip sequences of equal length yielded a distribution of longest streak center around 11-13 days. The observed value fell directly within this range, with a p-value of 0.86.

This suggests that the longest streak in EUR/USD is not unusual compared to a fair coin, supporting the hypothesis that markets at least partially resemble random walks when considering maximum streak.

4.4 Markov Chain Memory

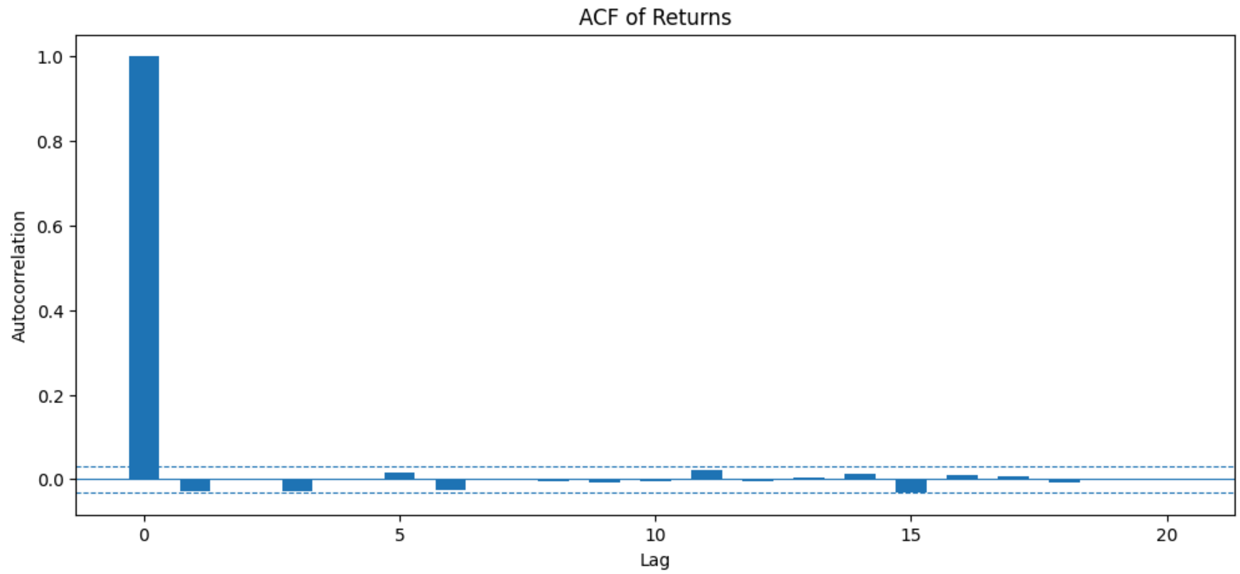


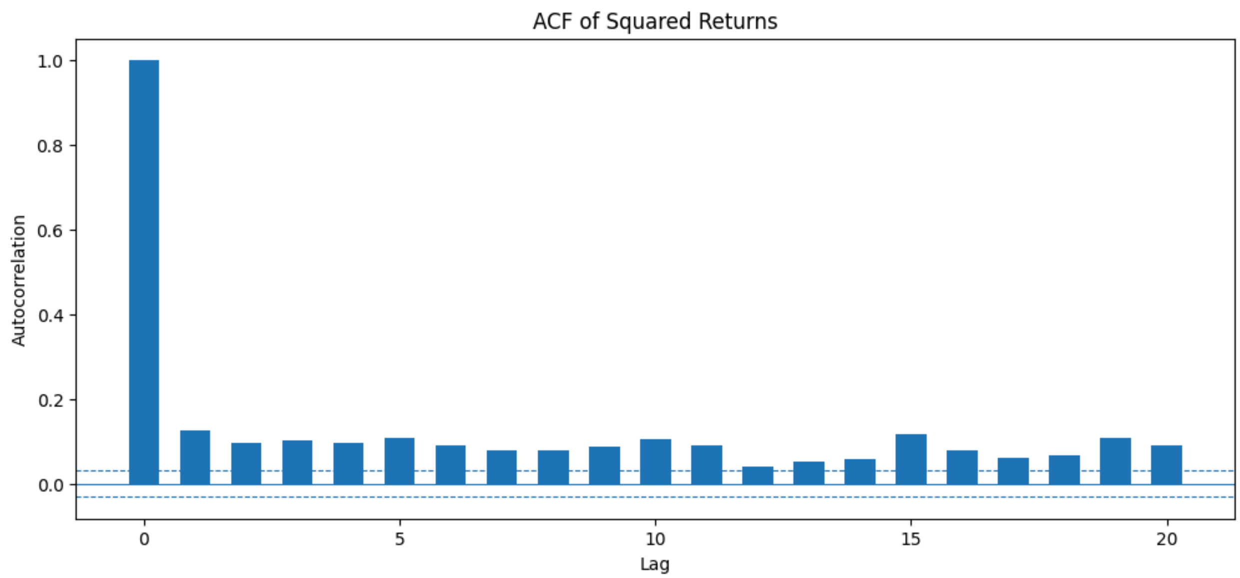
- P(up | up) = 0.4893
- P(up | down) = 0.5023
- p(up | up vs .5) = 0.2554 p(up | down vs .5) = 0.8571

Transition probabilities we estimated to test for short-term dependence in EUR/USD. The probability of an up day following an up day was 48.9%, while the probability of an up day following a down day was 50.2%. Both values are statistically indistinguishable from 50% (p = 0.26 and p = 0.86, respectively).

This indicated that EUR/USD daily returns have no detectable short-term memory in direction. Yesterday's movement provides no useful information about today's direction, consistent with the random walk hypothesis.

4.5 Autocorrelation & Ljung–Box





Ljung-box (returns):

lb_stat	lb_pvalue
10.792768	0.373890
18.153955	0.577267

Ljung-box (squared returns):

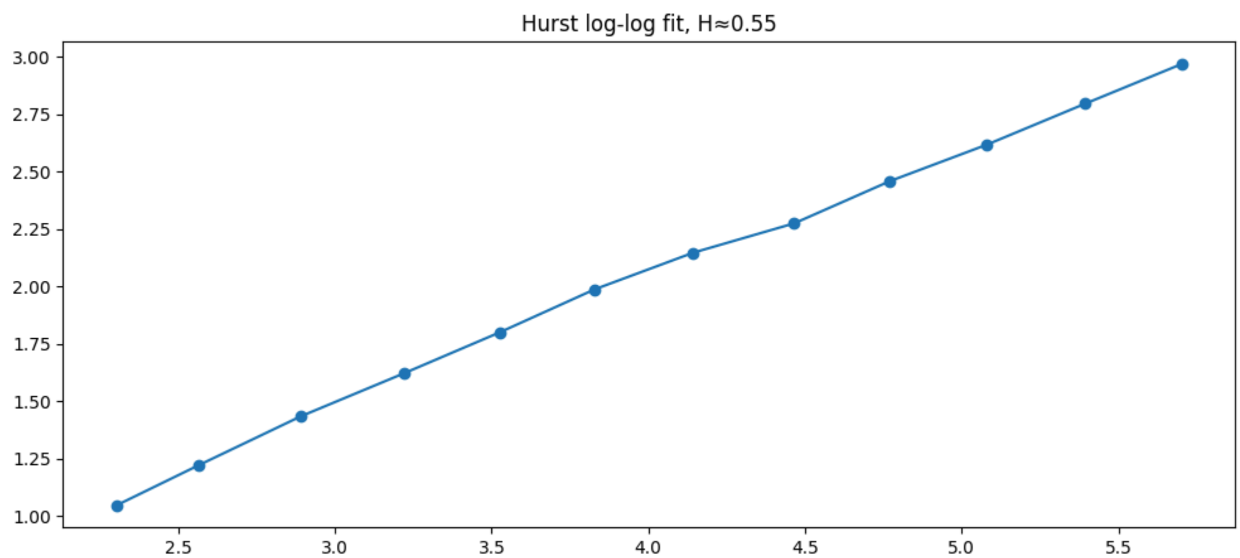
lb_stat	lb_pvalue

374.325016	2.718635e-74
620.526069	1.358391e-118

The autocorrelation function (ACF) of raw returns showed no significant spikes beyond the confidence bands, and the Ljung-Box test returned high p-values (0.37 and 0.58). This indicates that EUR/USD returns are not linearly predictable and are consistent with white noise.

However, the ACF of squared returns revealed clear, persistent autocorrelation, and the Ljung-Box test strongly rejected the null of independence ($p < 10^{-74}$). This confirms the presence of volatility clustering: periods of calm and turbulence tend to group together rather than occur randomly.

4.6 Hurst Exponent



- Hurst exponent ≈ 0.554

Using the rescaled range (R/S) method, the Hurst exponent for EUR/USD was estimated at 0.55. A value of 0.5 corresponds to pure random walk, while values above 0.5 suggest persistence (trending behavior).

The result indicates that EUR/USD exhibits slight persistence, meaning trends may continue more often than expected by chance. However, the deviation from 0.5 is modest, so the series remains much closer to random noise than to strong trend-following behavior.

4.7 Entropy

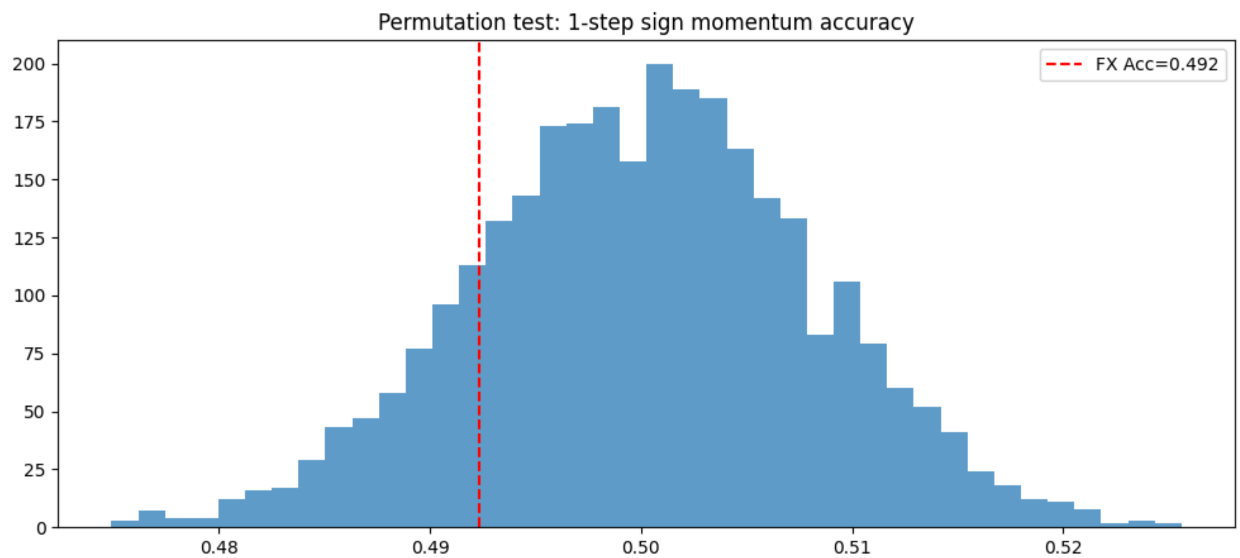
- Entropy (H): ≈ 1.000 bits

- Maximum possible: 1.0 (for a perfectly fair coin)

The Shannon entropy of EUR/USD up/down sequence was estimated at 1.000 bits, the theoretical maximum for a fair coin toss. This result implies that, in terms of directional predictability, EUR/USD is nearly indistinguishable from a perfect random process.

While this supports the random walk hypothesis for daily directions, it does not rule out the presence of other structures, such as volatility clustering or long-term persistence.

4.8 Permutation Test



- FX accuracy = 0.492
- Permutation p ≈ 0.8413

A simple momentum strategy was tested: predicting today's signs based on yesterday's. For EUR/USD, this rule achieved an accuracy of 49.2%, which is slightly below random guessing. A permutation test with 3,000 shuffled sequences confirmed this result was not significant (p = 0.84)

This indicates that short-term directional predictability is absent: yesterday's move provides no useful edge for predicting today.

5. Discussion

The results of multiple statistical tests show that EUR/USD daily returns resemble randomness in many aspects, but not in all.

1. No Directional Bias

   The binomial test found that EUR/USD closed higher in 49.5% of days, almost exactly half. This means the market shows no significant upward or downward tilt over the sample, consistent with a fair coin.

2. Runs and Streaks

   The runs test indicated the number of alternating streaks was consistent with randomness ($p = 0.34$), and the longest observed streak of 11 days was also within the range expected from simulated coins ($p = 0.86$). This suggests that markets do not produce “too many” or “too few” streaks compared to random expectation.

3. No Short-Term Memory

   The Markov chain analysis showed that the probability of an up day after an up (48.9%) or after a down (50.2%) was statistically indistinguishable from 50%. This implies that yesterday’s direction provides no edge in predicting today’s. The permutation test of simple one-step momentum strategy confirmed this: its accuracy (49.2%) was essentially random.

4. Volatility Clustering

   While raw returns were uncorrelated (Ljung-Box p-values > 0.3), squared returns displayed strong autocorrelation ($p < 10^{-70}$). This is clear evidence of volatility clustering: quiet periods are followed by more quiet periods, while turbulent periods cluster together. This is a major departure from coin-like randomness, where squared returns would have no memory.

5. Mild Long-Term Persistence

   The Hurst exponent was estimated at 0.55, slightly above 0.5. This hints at weak persistence, suggesting trends may extend longer than chance alone would predict, though the effect is modest.

6. Entropy and Uncertainty

   Shannon entropy was measured at 1.000 bits — the theoretical maximum for a fair coin. This means that in terms of daily direction alone, EUR/USD is essentially maximally unpredictable.

Taken together, these findings show a dual nature of financial markets:

- In terms of directional movement, EUR/USD looks remarkably similar to a coin flip — no bias, no memory, no reliable short-term edge.
- However, when it comes to risk and volatility, the market is far from random. The clustering of volatility and mild persistence distinguish financial time series from pure randomness.

This duality helps explain why markets are both so difficult to predict and yet still focus on quantitative modeling. Simple directional rules do not work, but advanced models that capture volatility dynamics (such as GARCH) can add value.

6. Conclusion

This study tested whether the EUR/USD exchange rate behaves like a random walk by comparing its daily returns to sequences of fair coin flips. Using a range of statistical tools — including bias and runs tests, Monte Carlo simulations, Markov transitions, autocorrelation analysis, Hurts exponent estimation, entropy, and permutation tests — we found that EUR/USD is random in some aspects, but not in others.

On the one hand, the market showed no directional bias, no excess streaks, no short-term memory, and entropy near its maximum, all of which are consistent with randomness. On the other hand, the data displayed clear volatility clustering and mild long-term persistence, features that a fair coin cannot reproduce.

The overall conclusion is that EUR/USD behaves like “structured noise.” Its daily directions are close to unpredictable, but its volatility and risk dynamics contain patterns that are critical for traders and researchers. These results reinforce the idea that while simple prediction rules fail, advanced statistical models that capture volatility and long memory can extract meaningful insights.

7. Appendix

Data Source

- Yahoo Finance: EUR/USD = X (daily closing prices, January 2010 – January 2025)

Key Python Packages

- yfinance – download FX data
- numpy, pandas – numerical & data handling

- matplotlib, seaborn – charts & visualization
- scipy.stats – statistical tests (binomial, runs, entropy)
- statsmodels – autocorrelation, Ljung–Box, Hurst analysis

Tests performed

1. Bias Test (Binomial): Up vs down days compared to 50/50.
2. Runs Test (Wald-Wolfowitz): Frequency of streaks vs randomness.
3. Longest Streak Monte Carlo: Distribution of maximum runs.
4. Markov Chain Memory: Conditional probabilities (P(up|up)), P((up|down)).
5. Autocorrelation & Ljung–Box: Dependence in returns and squared returns.
6. Hurst Exponent: Long-term persistence vs randomness.
7. Entropy: Information content of directional sequence
8. Permutation Test: Randomization test of a simple momentum rule.

Figures Included

- Run-length histogram
- Longest streak Monte Carlo comparison
- Markov transition probabilities bar chart
- ACF of returns and squared returns
- Hurst log-log fit plot
- Permutation test histogram