2019 8th International Conference on Modeling Simulation and Applied Optimization (ICMSAO)

Control of Axial Active Magnetic Bearing using Reduced Order Model

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***Abstract*—Active Magnetic Bearing (AMB) moves the rotor in the air with the influence of electromagnetic field developed by current carrying coils. The utilization of different active magnetic bearing appliances like Turbo molecular pumps, Gears, Flywheels, Maglev systems etc. are expanding in industrial application due to prompt advancement in the field of electronics and control algorithms. Finite Element method permits numerical analysis for this kind of systems. In this paper, a 2 Dimensional software simulation of an axial AMB system has been performed using COMSOL. The state space system matrices are extracted by creating probe points and a large order system is generated in MATLAB. Then using Krylov subspace based method, the order of the system is reduced. A PID controller is designed using reduced model and applied for full order model. The feasibility of the controller is tested using various kind of signals and it is seen that the controller improves the performance of both systems. Therefore, this paper proposes a different approach for using reduced model instead of large order model for controlling axial active magnetic bearing.**

I. Introduction

In recent years, with the advancement of design in electronics circuits and controller, the application of AMB has been extended in industrial machines. Conventional bearings employ hydraulic oil or another kind of forces to uphold loads, but AMB moves the rotor with the help of active electromagnetic forces developed by feedback controllers. Hence, AMB retains distinct preferences over conventional bearings alike having adaptable stiffness, higher speed, reduced vibration, low bearing losses, lower maintenance cost, lubrication free etc [1].

Due to the advancement in the computing facilities nowadays there is a stiff growth in the numerical simulations. Various well known methods for numerical simulations are Finite difference, Finite element, Finite volume, Boundary element method etc. For modelling and analysis of generated electromagnetic field inside active magnetic bearing Finite element method (FEM) is preferred over other methods [2]-[3]. A comparison of different poles AMB is done via 2 Dimensional FEM method in [4]. The stiffness development of hybrid AMB considering eddy current effects is described in [5].The thermal analysis and loss calculation of AMB embedded machines using FEM are presented in [6]. The FE analysis of the motor for the axial blood flow pump is described in [7]. Authors in [8] presented FE analysis of axial-radial AMB with the help of a non-linear model.

Essentially the FEM method transforms distributed parameter system given by partial differential equations (PDE) to a system of ordinary differential equations (ODE). Normally ODE is easy to solve rather than PDE. In the conversion process large no of ODE is generated and it becomes computationally heavy. Therefore Model Order Reduction (MOR) [9]- [10] is applied to reduce the computational burden and to preserve the system properties. The Reduced Order Model (ROM) not only decreases the simulation time but also makes controller design easier [11].

In this paper, a 2D FEM simulation has been performed using COMSOL. Stationary and time-varying studies have been executed. The state space system matrices are extracted to MATLAB and a large order system of axial AMB is generated. Then the original large order system is reduced to lower order model using Krylov subspace. Then using reduced model a PID controller is obtained and gains are selected. The same controller is also applied to original model. The output performance of both the systems are shown.

II. Modelling of AMB

AMB subsist of mandatory sections alike an electromagnet, axial rotating load, amplifier, a gap sensor, micro-controller etc. The rotating load levitates with the help of the biased current, which also compensate the weight of rotor. During the movement of the rotor, if it is dislocate from the central position then the displacement is recorded by the gap sensor. The controller generates a control signal with the help of measurements of the gap sensor. Then power amplifier boosts the control current and the applied current restores the rotor position. Fig 1 represents the geometry of a simple unidirectional axial AMB. In it, if the axial rotor moves vertically in the z direction with respect to the coil, the electromagnetic control forces that is generated is expected to restore it to its original position [12].

Maxwell's equations govern the electromagnetic fields in any region. Generally, the equations are represented as

$$\nabla \times \mathcal{H} = \mathcal{J} \quad (1)$$

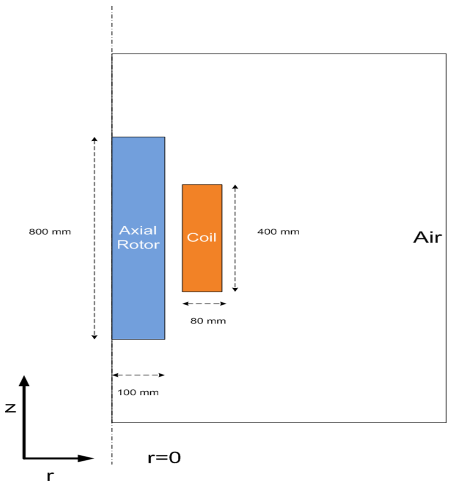

Fig. 1: Schematic of axial AMB (scale 1 : 20).

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{B}}{\partial t} \tag{2}$$

$$\nabla . \mathcal{B} = 0 \tag{3}$$

$$\nabla . \mathcal{D} = \rho_v \tag{4}$$

where $\mathcal{D}$, $\mathcal{B}$, $\mathcal{E}$, $\mathcal{H}$, $\mathcal{D}$, $\mathcal{J}$, ρ_v are the electric flux density, magnetic flux density, electric field intensity, magnetic field intensity, current density and electric charge density respectively. The relation between magnetic vector potential and magnetic flux density is $\mathcal{B} = \nabla \times \mathcal{A}$. By putting the magnetic vector potential in the Maxwell's equations become

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathcal{A} \right) = \sigma \left(\frac{\partial \mathcal{A}}{\partial t} + l(t) \right) \tag{5}$$

where μ, σ and l are permeability, electric conductivity, gradient of potential respectively. The magnetic vector potential follows coulomb gauge condition

$$\nabla \cdot \mathcal{A} = 0 \tag{6}$$

Some part of generated flux density die out on the symmetry boundary, which is assured by giving the boundary condition [13]

$$\mathcal{A} \times n = 0 \tag{7}$$

Maxwell Stress Tensor (MST) method calculates developed electromagnetic force in various moment. For a particular AMB, MST is represented as [14]

$$\mathcal{F} = \int_s \left(\frac{1}{\mu_0} \left(\mathcal{B} \cdot n \right) \mathcal{B} - \frac{1}{2\mu_0} \mathcal{B}^2 n \right) dS \tag{8}$$

The magnetic flux density $\mathcal{B}$ is dependent on control current i_z and displacement z. Therefore, generated force F also becomes a function of i_z and z. The mechanical motion of the axial rotor in z axis is presented as (9), where m denotes the rotor mass and g is the gravitational constant.

$$\mathcal{F} = m \frac{d^2 z}{dt^2} + mg \tag{9}$$

III. Generation of Reduced Model

Simulation and controller design for large order systems are not computationally efficient. Therefore, the large order systems need to be transformed to smaller order systems. The FEM model generated from applying FEM method to (5), (8) and (9) leads to a system of differential algebraic equations (DAE's). Some of the equations in theses DAE's are purely algebraic equations that correspond to parts where there is no conductor. In parts of the domain which contain conducting regions, differential equations arise. The purely algebraic equations can be eliminated without much computational effort by one of the techniques reported in [15] [16]. Therefore, these DAE's can be simplified to a smaller order system of ODE's, given as

$$\bar{E}\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) \tag{10}$$

$$y = \bar{C}x(t) \tag{11}$$

where $\bar{E} \in \mathbb{R}^{n \times n}, \bar{A} \in \mathbb{R}^{n \times n}, \bar{B} \in \mathbb{R}^{n \times m}, \bar{C} \in \mathbb{R}^{p \times n}$. The system has m input, p output and order n, where n denotes number of nodes in the conducting regions. The number n still being in the thousands, the model (10) & (11), referred to as the Full Order Model (FOM), is then reduced using MOR technique.

MOR generates a Reduced Order Model (ROM) of order $r << n$ in such a manner that input-output characteristics and other important information are preserved. Reduced order model is created by projecting the original system onto a dominant subspace which is spanned by a reduced basis of dimension r. The dominant subspace is assumed to be formed by the vectors of an orthogonal matrix $\bar{V}$, where $\bar{V} \in \mathbb{R}^{n \times r}$. Applying a transformation given by (12), reduced order model can be represented as (13) and (14).

$$x(t) = \bar{V} x_r(t) \tag{12}$$

where $x_r \in \mathbb{R}^r$

$$\bar{E}_r \dot{x_r}(t) = \bar{A}_r x_r(t) + \bar{B}_r u(t) \tag{13}$$

$$y = \bar{C}_r x_r(t) \tag{14}$$

where $\bar{E}_r \in \mathbb{R}^{r \times r}, \bar{A}_r \in \mathbb{R}^{r \times r}, \bar{B}_r \in \mathbb{R}^{r \times m}, \bar{C}_r \in \mathbb{R}^{p \times r}$ such that, $\bar{E}_r = \bar{V}^T \bar{E} \bar{V}, \bar{A}_r = \bar{V}^T \bar{A} \bar{V}, \bar{B}_r = \bar{V}^T \bar{B}, \bar{C}_r = \bar{C} \bar{V}$.

The axial AMB is treated as a linear problem, so a linear model order technique has been applied. Krylov subspace based techniques can be applied to systems with order as large as millions [17]- [18]. Therefore, Krylov based techniques are used for reduction process as the order of the original model is very huge. Two sided Krylov based Arnoldi algorithm is used for generation of reduced order model [19].

IV. Results and Discussion

COMSOL Multiphysics is used for the simulation of 2D axial AMB. There are two types of simulation is considered here i.e motionless and time varying. The generation of flux density in motionless simulation is given by Fig. 2, where rotor is not moving in any direction. The whole time in time varying simulation is 0.25 s. The step time for time varying simulation is chosen as 0.005 s. Figure 3 represents the surface flux density at time t=0.1575 s, where rotor is displaced downwards due to an axial motion.

The original system of AMB has 5545 nodes i.e n equals 5545. After applying Arnoldi algorithm system is reduced to r = 15. As the number of states reduced, the total simulation time is also reduced. Time taken for the original COMSOL model to simulate is 15 minutes. To generate linear original model in MATLAB after extraction of the matrices, takes 1.2 minute. Then computation of reduced model takes 3 seconds. Figure 4 demonstrates the unit step response of the original large order model and reduced order model. In the figure, it can be seen that the original system and the reduced system curves are matching except for some points. Figure 5 shows the sine response of the original large order model and reduced order model. It can be seen that the graphs are matching expect some peak points. Figure 6 shows frequency response of the original and reduced order model of sizes (r=20,18,15,12). It is clearly seen that the curves are exactly matching for reduced model of size 15. Other reduced models of size 12,18,20 differs from the original model curve in amplitude as well as phase plot. From Figure 6, it is clearly seen that MOR successfully works till reduced order model of size 15. Reduced order model of size 12 differs in amplitude and phase plot from frequency $10^1 rad/s$. Therefore, reduced model of size 15 is selected for reduction procedure. The poles of reduced order system are at s = -194.32, -39.41, -13.0583, -7.05, -3.78, -2.63, -1.7944, -1.42, -1.11, -0.91, -0.75, -0.70, -0.66, -0.29, -0.33. A PID controller is obtained by considering the reduced order model and the gains are selected as K_p=1505.16, K_i=1113.36 and K_d=642.37. Then the same controller is used for the full order model. Figure 7 explains the structure for closed loop control for axial AMB system. The performances of PID controller with step and sine input are shown. Figure 8 and 9 gives the response of reduced and original model with step input for 10s simulation. In both the cases with PID control the output matches with reference signal. Figure 10 and 11 gives the response of reduced and original model with sine input for 10s simulation. For both kind of input signal the output follows the reference signal after 1.3s. In the application of both types of the reference signals, without the controller, fail to achieve the desired results. The controller has been tested on 15 different models out of which only one has been reported here. In these cases, the controller is working smoothly for the full order model and it is likely to work in other cases as well. However, a proper theoretical analysis that the controller design for reduced order model will work for the full order model is still required but it is outside the scope of this paper.

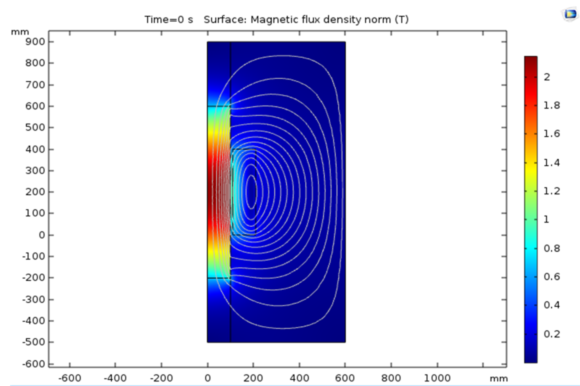

Fig. 2: Static simulation surface flux density.

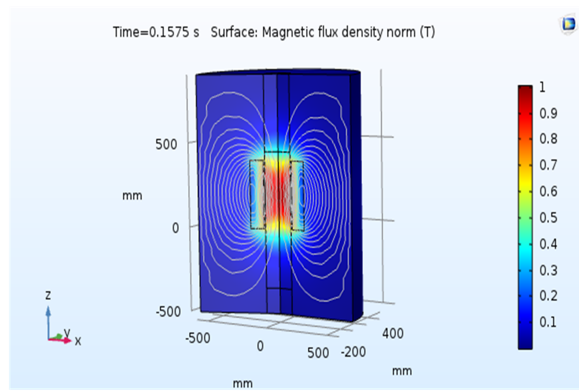

Fig. 3: Time varying flux density at t=0.1575 s.

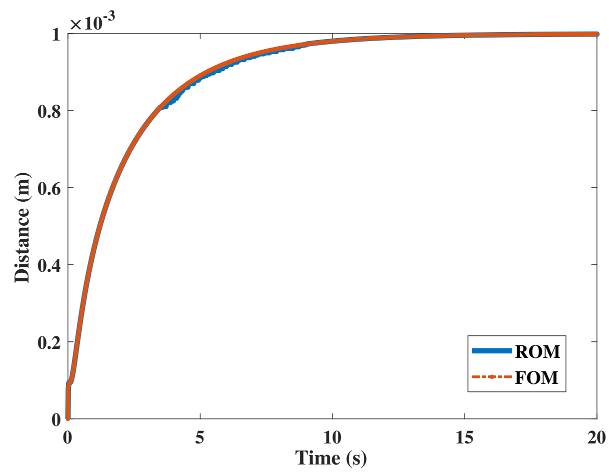

Fig. 4: Step response of original and reduced model.

V. Conclusion

This paper presents an alternative approach to design a PID controller for axial AMB using reduced order model rather than the original model. A 2 Dimensional FEM computer simulation has been accomplished for the axial AMB using COMSOL software. Stationary and time varying studies are executed for the extraction of system matrices to MATLAB. In the extraction process a high order system is generated. Krylov subspace based reduction method is applied for the reduction process. A PID controller is used and the performances are evaluated with various kind of reference signals. Therefore, this paper describes an alternative to control of axial AMB by using reduced order model instead of large model.

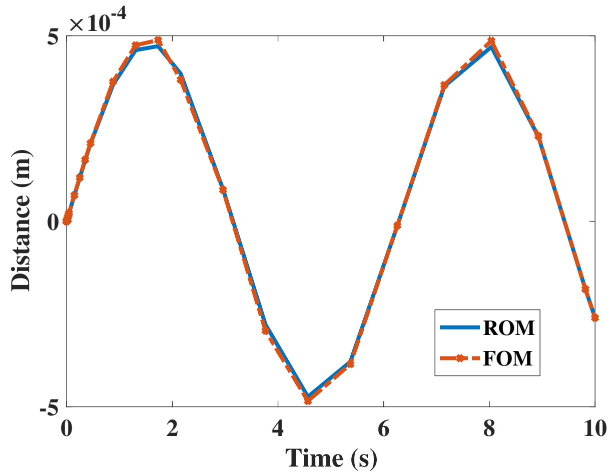

Fig. 5: Sine response of original and reduced model.

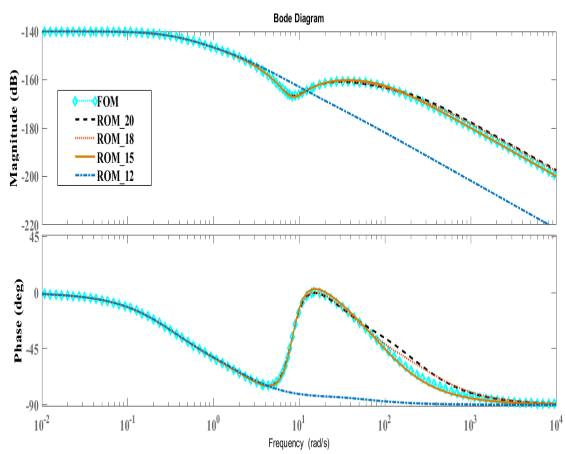

Fig. 6: Bode plot of original and different sizes of reduced model.

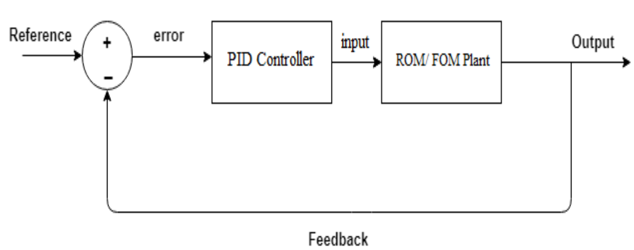

Fig. 7: Block diagram for control system.

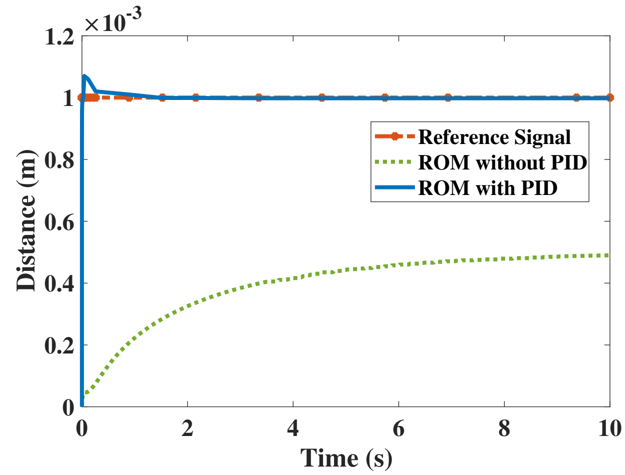

Fig. 8: O/P response of reduced model with step input.

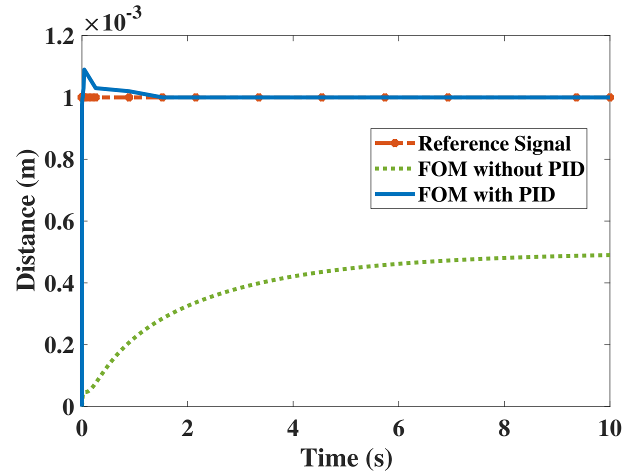

Fig. 9: O/P response of original model with step input.

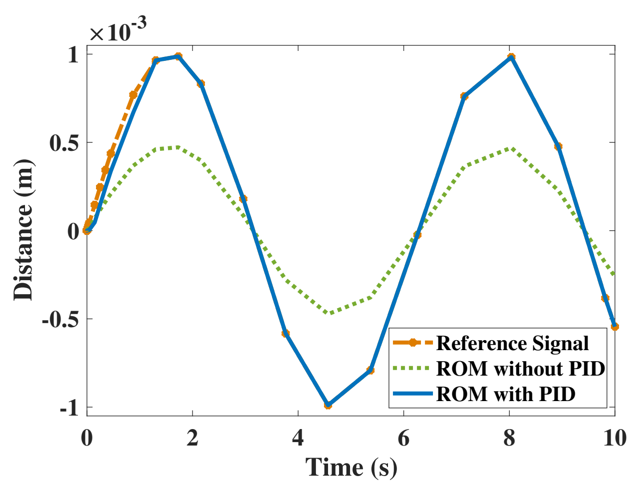

Fig. 10: O/P response of reduced model with sine input.

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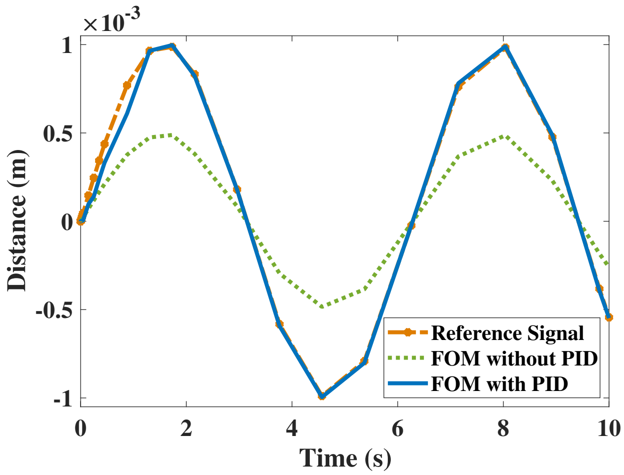

Fig. 11: O/P response of original model with sine input.

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