UNIT-4

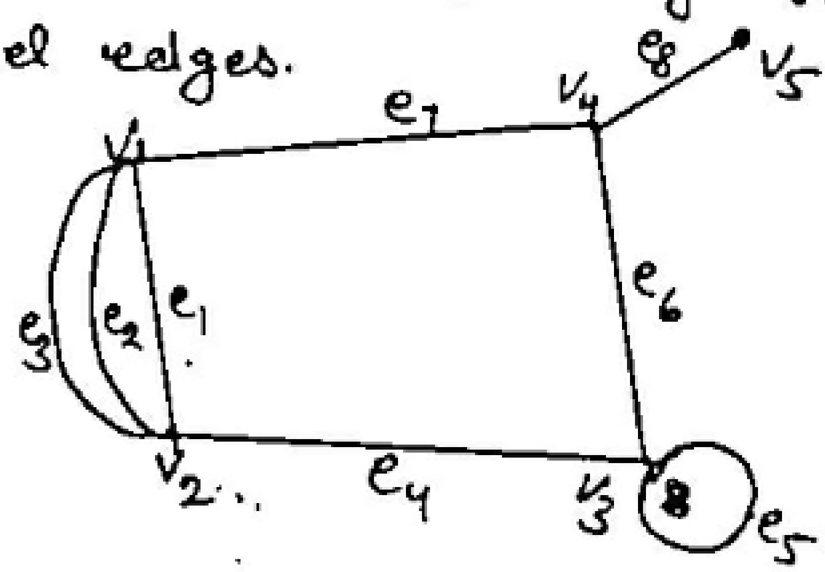
Graph Theory

Definition.

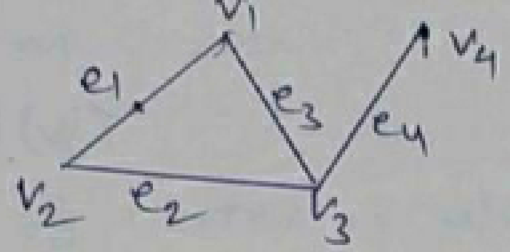
Graph :- A graph $G=(V,E)$ consists of a set of objects $V=\{v_1, v_2, \ldots\}$ whose elements are called vertices (or points or nodes) and another set $E=\{e_1, e_2 \ldots\}$ whose elements are called edges or lines or branches

Self loop (or loop) -> An edge is said to be a self loop if its both end vertices are same

Parallel edges or Multiple edges :- If there are two or more than two edges having the same pair of end vertices the such edges are called parallel edges in the following fig. e_1, e_2, e_3 are parallel edges.

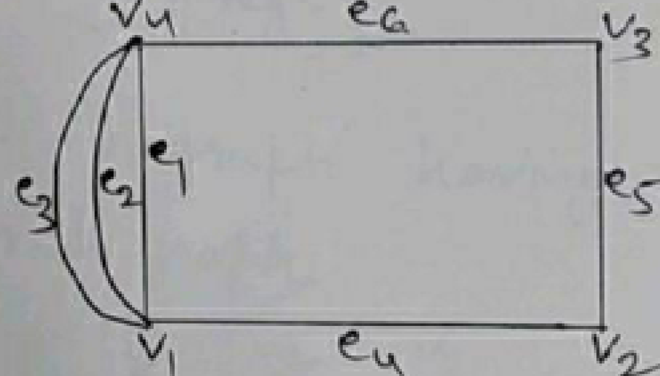


Simple graph :- A graph that have neither self loops nor parallel edges is called a simple graph.



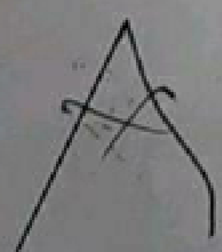
Multigraph :- A graph having some parallel edges is called multigraph.

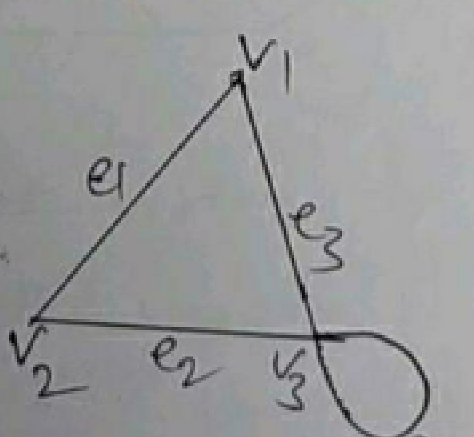
In multigraph no self loops are allowed

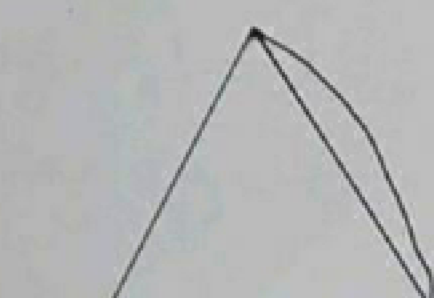


Pseudograph or General Graph - A graph having some self loop is called pseudograph.

In pseudograph parallel edges are allowed.







Degree of vertex :- The degree of vertex is the number of edges incident on a vertex v. the self loop is counted twice.

The degree of vertex V_i is denoted by. $\deg(v_i)$ or $d(v_i)$

Degree of vertex is always a non -ve.

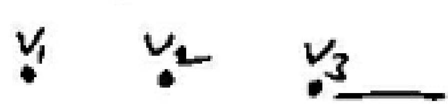
If $\deg(v) = 0$ then v is called an isolated vertex

if $\deg(v) = 1$ then v is called an end vertex of pendant Vertex or a leaf.

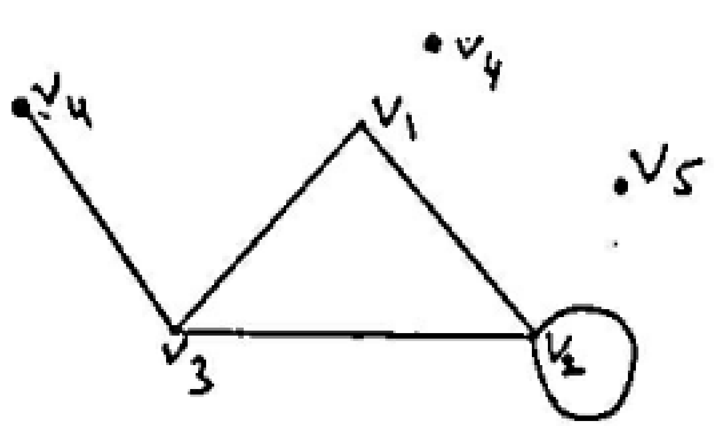
$\delta(G) = \min.\{\deg v\}$

$\Delta(G) = \max\{\deg v\}$

Null graph :- A graph having no edges is called null graph



isolated vertex



$\deg(v_1) = 2$

$\deg(v_2) = 4$

$\deg(v_3) = 3$

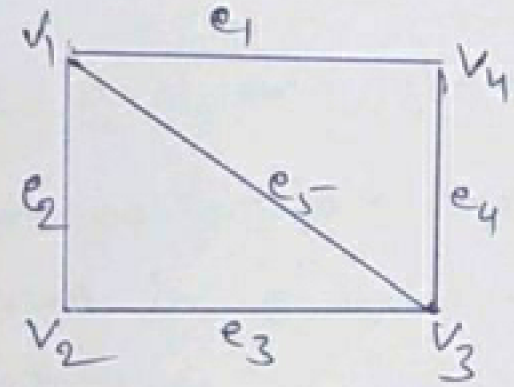
$\deg(V_4) = 1$

$\deg(v_5) = 0$

V_4 = Pendant vertex

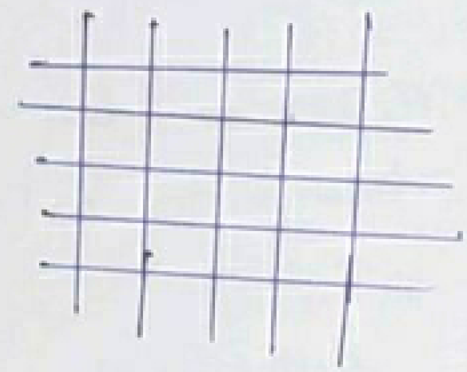
V_5 = isolated vertex

Finite and Infinite graph :-



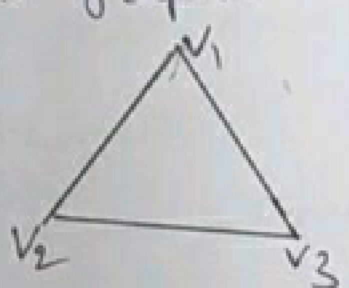
finite graph.

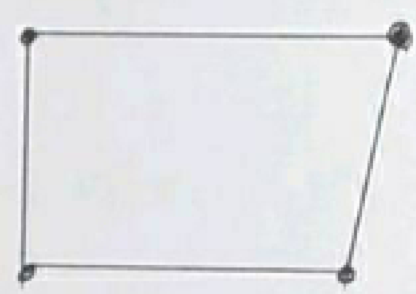
finite no. of vertices and finite no. of edge.



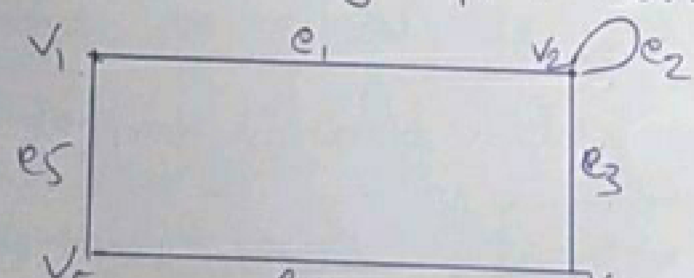
(Infinite graph)

Regular Graph :- A graph G in which all vertices are of equal degree is called a regular graph.





Walk :- A walk in a graph is a alternating sequence of vertices and edges which begins and end with vertices such that no edge repeated However a vertex may apper more than once



① $v_1 - e_1 - v_2 - e_2 - v_2$ ② $v_1 - e_1 - v_2 - e_3 - v_3$ ③ $v_1 - e_5 - v_5 - e_4 - v_3$

In a walk begins and ends vertices are said to be terminal vertices.

There are two type of walk

(a.) open walk :- If the terminal vertices are differen in a walk, is called open walk

For example

$\underline{V_1} - V_5 - V_4 - e_4 - \underline{V_3}$ is open walk

(b) closed walk :- If the terminal vertices are same same then walk is called closed walk

$\underline{V_1} - e_1 - V_2 - e_3 - V_3 - e_4 - V_5 - \underline{V_1}$

Path :- An open walk in which no vertex appears more than once is called path or simple path

for example

in given figure $V_1 - e_1 - V_2 - e_3 - V_3 - e_4 - V_4$

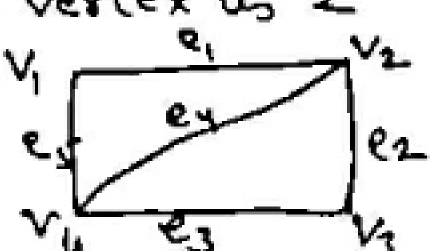
The length of the path is the no. of edges in the path

Hence the length of the path is ___

Circuit or cycle :- A circuit is a closed walk. in which all vertices are distinct. except the terminal vertices (initial & final)

Clearly degree of each vertex is 2

circuit. $V_2 - e_2 - V_3 - e_3 - V_4 - e_4 - V_2$



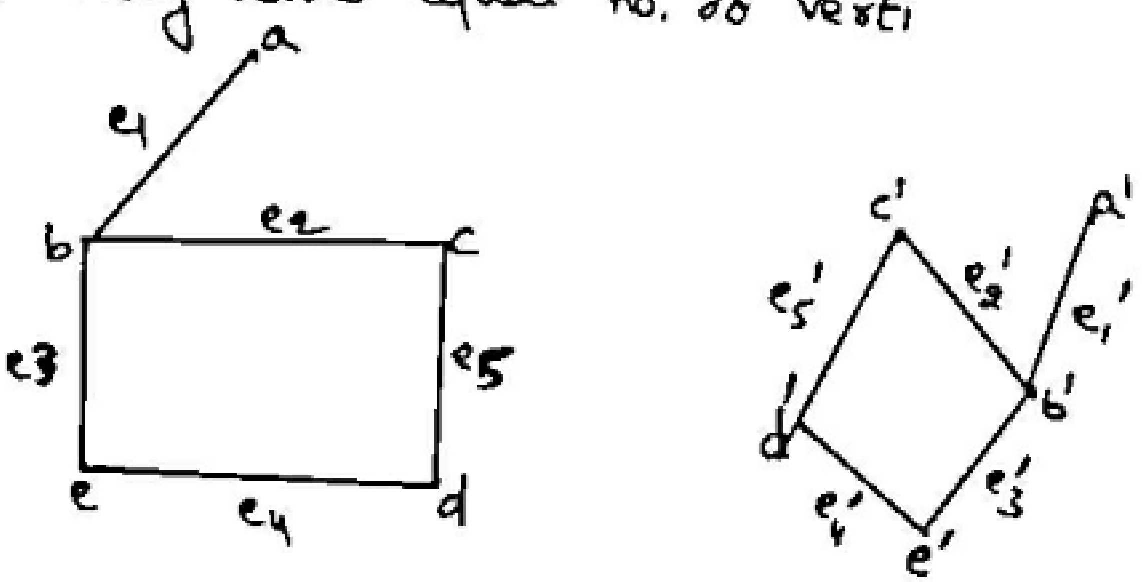
Isomorphic graphs.

Two graph $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are isomorphic, if there is one-to-one correspondence b/w vertices of the graph G_1 and G_2 and b/w their edge such that their incidence relationship is preserved

Thus two graphs are isomorphic to each other if.

(i.) They have same no. of edge and vertices

ii) They have equal no. of verti



Here we observe that

No. of edge and vertices are same in a graph i.e 5

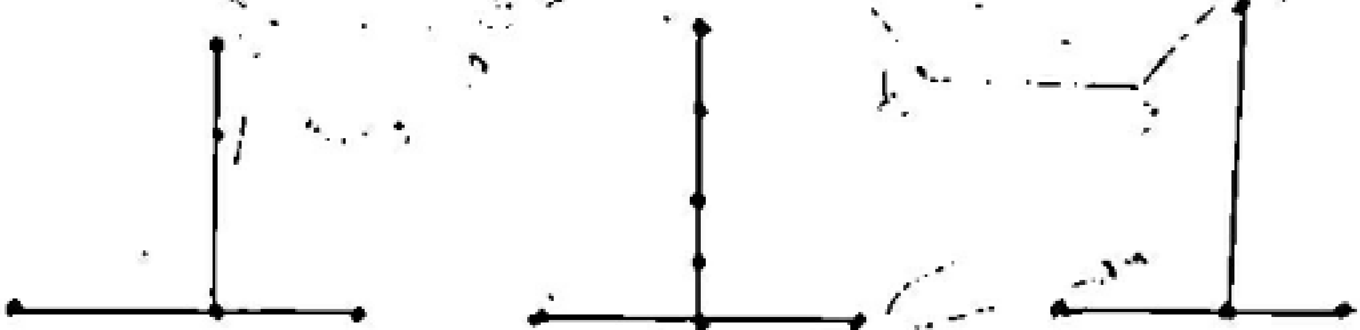
$deg(a) = deg(a') = 1$

$deg(b) = deg(b') = 3$

$deg(c) = deg(c') = 2$ .....

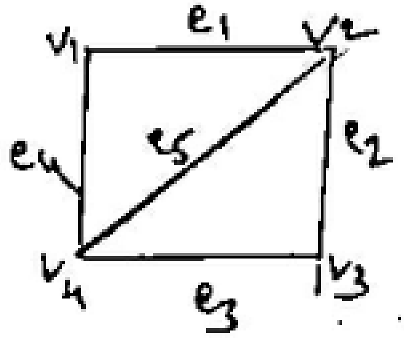
Homeomorphic Graphs

Two graph are said be homomorphic to each other if one is obtained from the other by merger of two edges incident on a vertex or by inspection of a vertex of degree 2. whose resulting in creation of an edge in series.

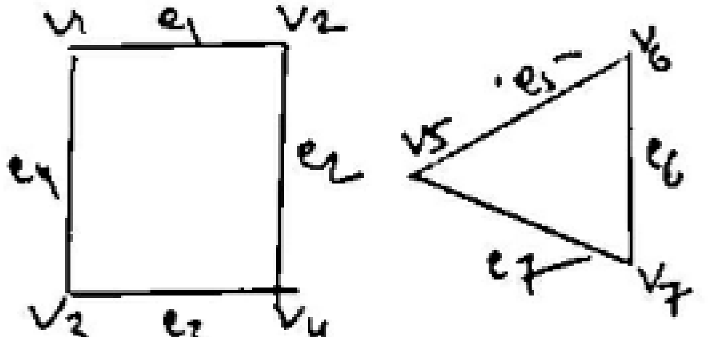


Connected and disconnected graph

A graph G is called connected if there is at least one path b/w every pair of vertices in the graph G otherwise the graph G is called disconnected.

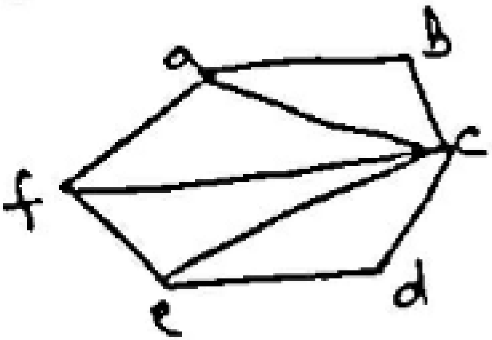


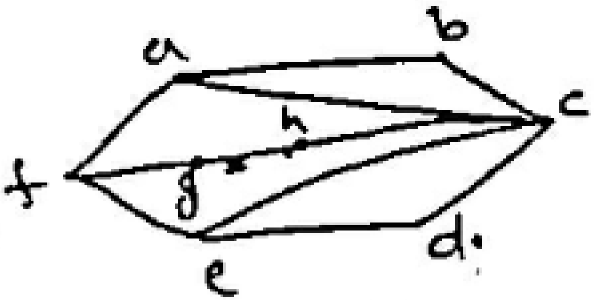
Connected



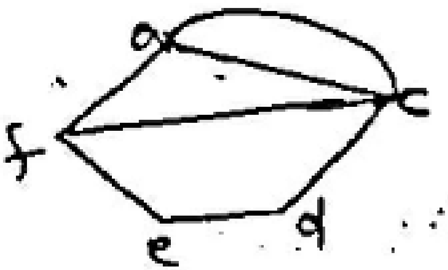
Disconnected

Two graph are said to be homomorphic if graph can be obtained from the other by creation of edges or merger of edges in Series.





creations



Theorem.

Hand shaking theorem / sum of degree theorem.

Theorem - The sum of the degree of all vertices in a graph is equal to twice the no. of edges.

i.e. $\sum deg(v) = 2e$

Proof :- Let $G=(V,E)$ be a graph and let no. of edges in $G = e$. Then we have to prove that $\sum_{v \in V} deg(v) = 2e$.

This theorem prove by Induction.

Step-1 If no. of edge in G is zero i.e $e=0$
in this case degree of each vertex is zero

$$\sum_{v \in V} deg(v) = 0 \Rightarrow 2 \times 0 = 2e.$$

Step-2 if $e=1$ i.e if there is only one edge in G
In this case G has only two vertex and degree of each vertex is one

$$\sum_{v \in V} deg(v) = 1+1 = 2 = 2 \times 1 = 2e$$

$\therefore$ the theorem is true in this case

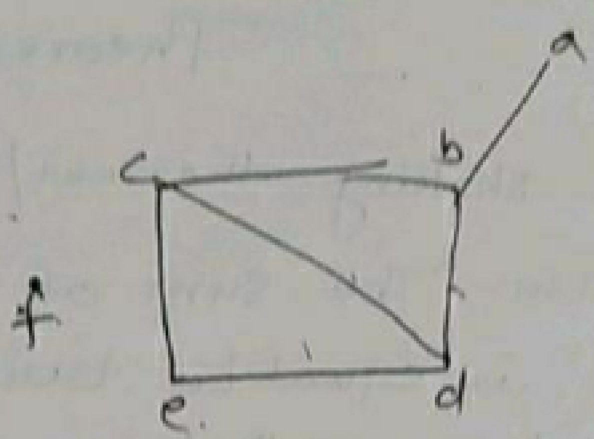
Step-3 Now assume that the theorem is true for all graph having $e-1$ edges

$$\sum deg(v) = 2(e-1)$$

Now if we add one edge $e' = (a,b)$ to obtain the graph G which have total no. of e edges then the degree of each of vertices a & b will be Increased by one (Since an edge contributes 2 degree) $\therefore$ $\sum_{v \in V} deg(v) = 2(e-1)+2 = 2e$

Hence theorem is completely proved //

$$\sum_{i=1}^{n} dv_i = 2e$$



if we add one vertex f and connect with edge than it increase two degree.

each edge contribute 2 degree.

so sum of degree of all vertices is equal to no. of edges.

$a = 1$
$b = 3$
$c = 3$
$d = 3$
$e = 2$

12

$= 2 \times 6$ (no. of edges)

Theorem-2 prove that the number of vertices of odd degree in a graph is always even.

i.e $\sum_{i=odd} d(v_i) =$ An even number.

Proof we know that

$$\sum_{i=1}^{n} d(v_i) = 2e \quad \text{--- (1)}$$

we can write eqn (1) as.

$$\sum_{i=1}^{n} d(v_i) = \sum_{i=even} d(v_i) + \sum_{i=odd} d(v_i)$$

i.e $$\sum_{i=odd} d(v_i) = \sum_{i=1}^{n} d(v_i) - \sum_{i=even} d(v_i) \quad \text{--- (2)}$$

we know that

$$\sum_{i=1}^{n} d(v_i) = 2e \text{ (even)}$$

$$\sum_{i=even} d(v_i) = \text{even number} \quad (\text{even} + \text{even} = \text{even})$$

Hence from eqn 2

$$\sum_{i=odd} d(v_i) = \text{even no.} - \text{even no.}$$

$$= \text{an even number}$$

(because the difference of two even no. is also even)

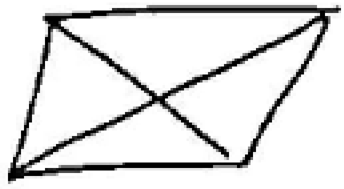
Hence proved.

Different Types of graphs.

(1) Complete Graph :- If in a simple graph there exist an edge b/w each & every point pair of vertices then the graph is called complete graph.

or

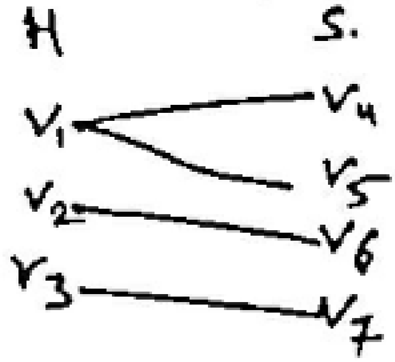
Let $G = (V, E)$ be a graph the graph G is called complete graph if every vertex in G is connected to every other vertex in G. A complete graph with n vertex is usually denoted by K_n.



Note. The total no. of edges in a complete graphs with n vertices is $\frac{n(n-1)}{2}$.

Bipartite Graph → A Graph $G(V, E)$ is called Bipartite if its vertex set $V(G)$ can be devided in two non-empty disjoint subset H and S in such a way that each edge $e \in E(G)$ has.

its one end point in H & other in. S.



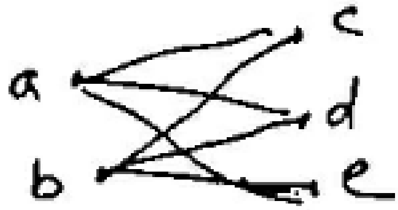
Bipartite.

(it is denoted by $k_{m,n}$)

m = no. of vertices in H

n = no of. vertices in S.

Complete Bipartite :- If each vertex of H is connected to each vertex of S. then. such a graph is called complete bipartite graph.



Prove that :- The maximum number of edges in a simple graph with 'n' vertices is equal to $n(n-1)/2$

Proof :- Let (V,E) be a simple graph with n vertices. & e

So let $V = \{v_1, v_2, v_3, --- v_n\}$

$E = \{e_1, e_2, e_3, - - -\}$

Let v_1 be any arbitary vertex

* if we join v_1 with all remaing vertices the no. of edges will be atmost $(n-1)$

v_1 can not join with v_1 because graph is simple ∴ it does not contain self loop.

Similarly if we join v_2 with remaining vertices except v_1 then no. of edges will be atmost $n-2$

v_2 can not join with v_1 because v_1 already join with v_2 and graph is simple graph so no parallel edge allowed.

Proceeding in this manner the vertex v_{n-1} will join only with v_n and gives only one edge.

Hence maximum no. of edges in G.

$$= (n-1) + (n-2) + (n-3) + --- + 3 + 2 + 1$$

$$= \frac{1}{2} n(n-1)$$

Hence proved

Show that the total number of edges in a complete graphs with n vertices is $\frac{n(n-1)}{2}$

proof

we know that

$$\sum_{i=1}^{n} d(v_i) = 2e$$

where e is the no. of edges with n vertices in G

$$\Rightarrow d(v_1) + d(v_2) + d(v_3) + \dots + d(v_n) = 2e \quad \text{—(1)}$$

Since we know that in a complete graph G, degree of each vertex has $(n-1)$ edges

$\therefore$ (1) becomes

$$(n-1) + (n-1) + (n-1) + \dots \; n \text{ times} = 2e$$

$$\Rightarrow n(n-1) = 2e$$

$$\Rightarrow e = \frac{n(n-1)}{2}$$

Hence no. of edges in a complete graph with n vertices is $n(n-1)/2$

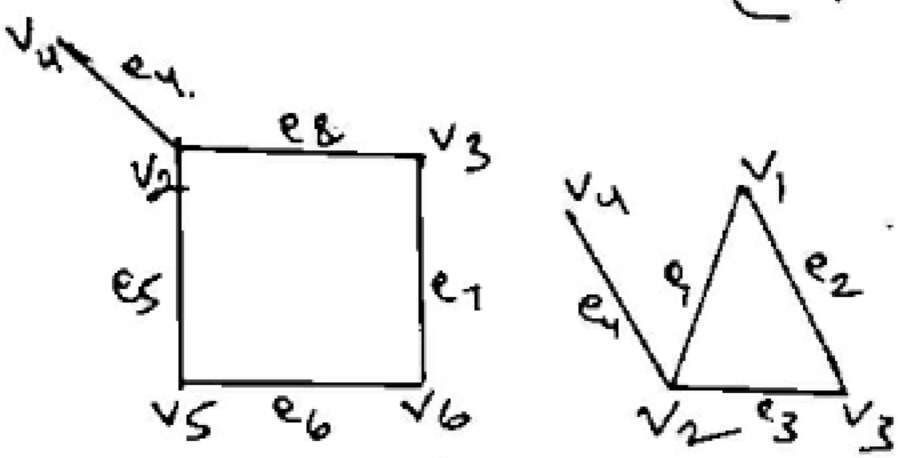
Operation on Graphs.

(i) Union of two graphs :- The union of $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ is a graph.

$G = (G_1 \cup G_2)$ whose vertex set $V = \{V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$

i.e:

$$G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$$



$V_1 = \{v_2, v_3, v_4, v_5, v_6\}$

$E_1 = \{e_4, e_5, e_6, e_7, e_8\}$

$V_2 = \{v_1, v_2, v_3, v_4\}$

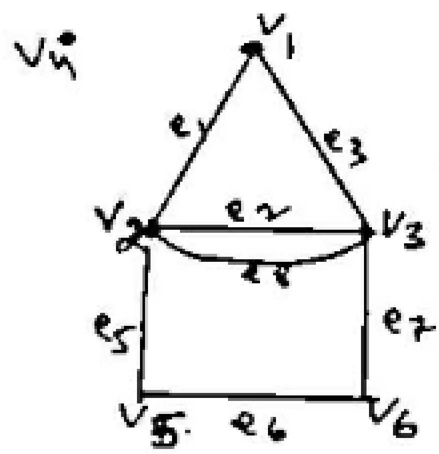
$E_2 = \{e_1, e_2, e_3, e_4\}$

Intersection of two graphs:-

The intersection of two graphs $G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$ is a graph G denoted by $G_1 \cap G_2$ whose $V = V_1 \cap V_2$ and $E = E_1 \cap E_2$

$$\therefore \quad G = G_1 \cap G_2 = (V_1 \cap V_2 \; , E_1 \cap E_2)$$

Intersection.



Ring sum of two graphs.

Ring Sum of two graphs:-

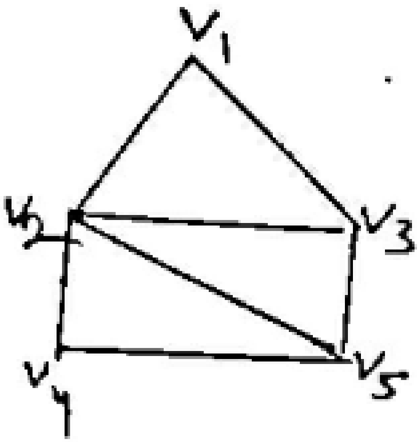
Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The ring sum of G_1 & G_2 is a graph denoted by $G_1 \oplus$ and is defined by

$$G_1 \oplus G_2 = (V_1 \cup V_2 \; , E_1 \cup E_2 - E_1 \cap E_2)$$

Thus. $G_1 \oplus G_2$ consists all vertices of G_1 and G_2 But consists of only those edges that are either in G_1 or G_2 but not both.

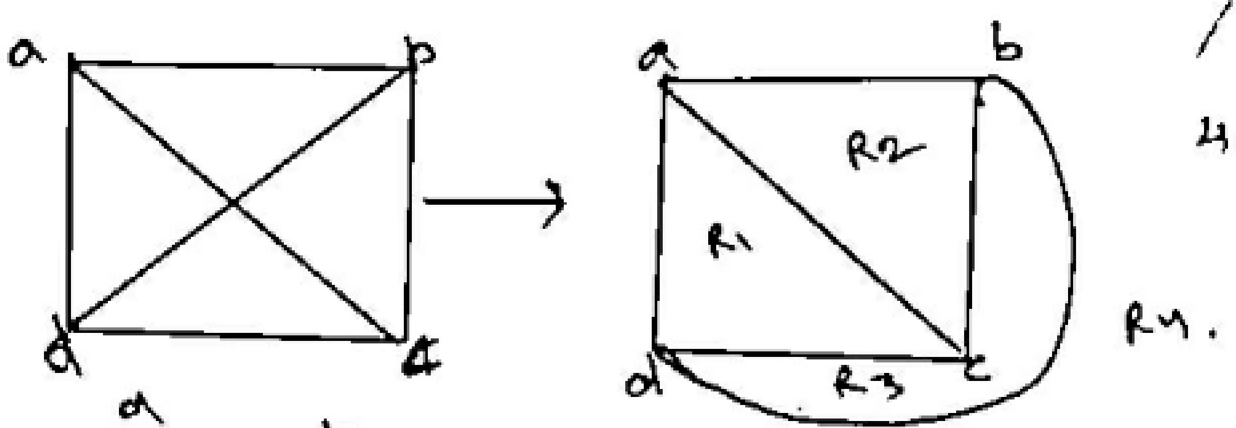
Decomposition of graphs :- Let G be a graph. The graph G is called to be decomposed into two subgraph G_1 and G_2 if

$$G_1 \cup G_2 = G \quad \text{and} \quad G_1 \cap G_2 = \phi \text{ (a null graph}$$

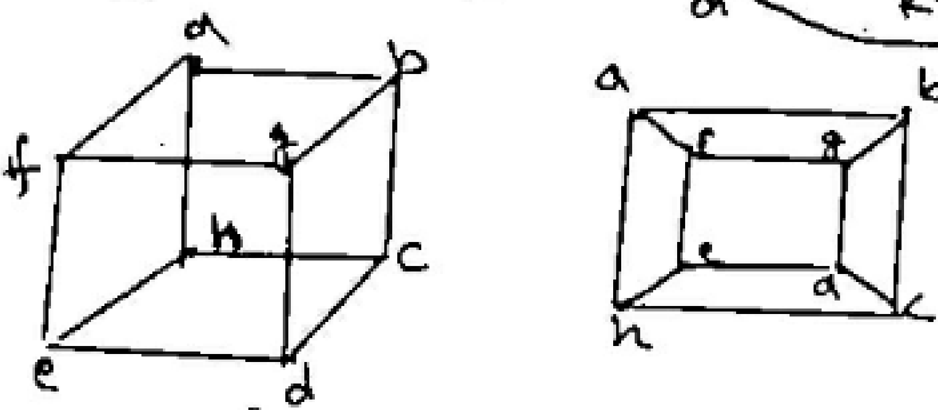


Planar Graph

Planar Graph:- A graph is called planer if it can be drawn in the plane without any edges crossing to each other and a graph that can-not be drawn on a plane without a crossing b/w its edges is called non planar graph



4 Region

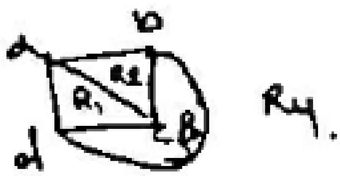


Note :- (1) 1,2,3,4 vertices area complete graph is always planar.

(2) More than 5 vertices complete graphs always non planar.

Region of the graph.

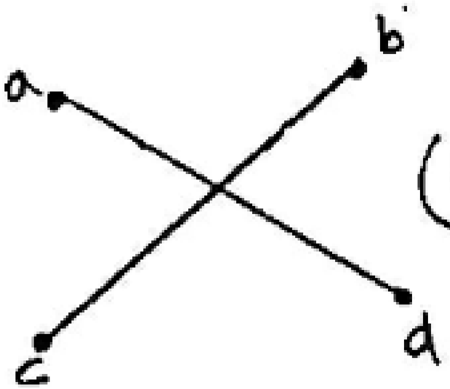
Any planar graph G partitions the plane into of disjoint regions. in which exactly one is infinite region (unbounded region) and other are finite region (bounded region)



4 Region.

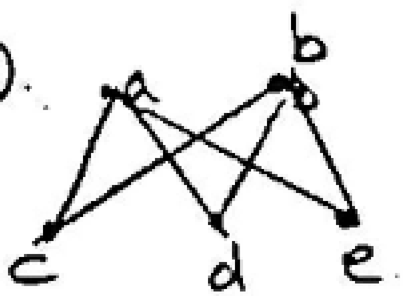
Q. Show that $K_{2,2}$, $K_{2,3}$ and $K_{2,4}$ are Plannar.

(a)



$(K_{2,2})$

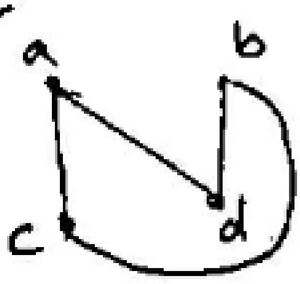
(b)

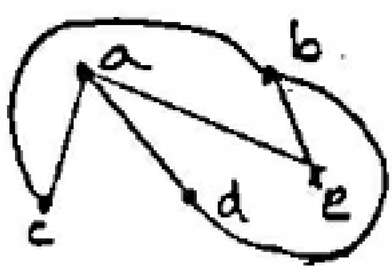


$K_{2,4}$ draw and solve yourself.

Ans

(a)

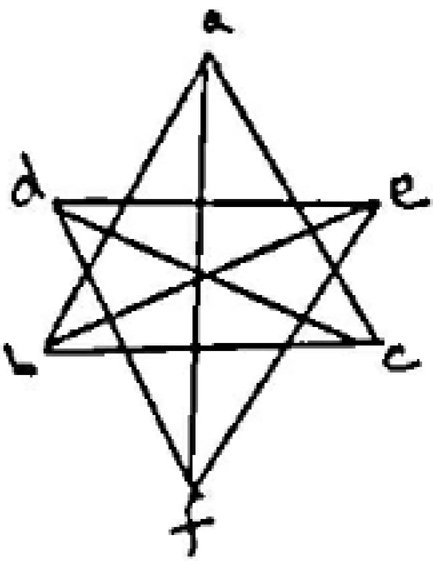




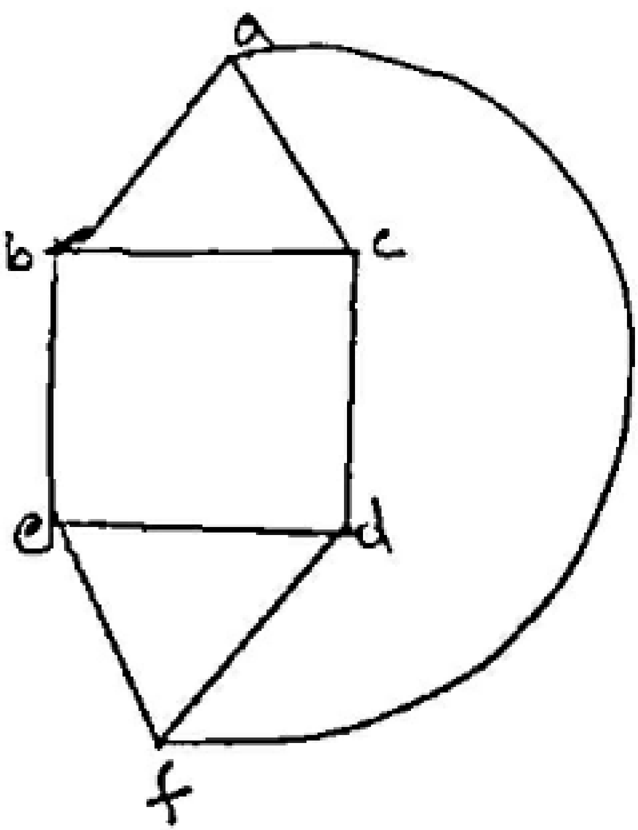
Show that the following graph is planar.

or

Draw a planar representation of graph given below.



Ans



Euler's formula.

State and prove Euler's Formula

Boaafe. ga

Statement :- If G is a connected planner. graph then.

$r = e - v + 2$ where v denotes the number of vertkes, e → no of edges and r denotes the no. of regions in G

Proof :- we shall prove the theorem by mathematical Induction on e of G

Basic step. if $e=0$ then. G has $v=1$ and one infinite region. i.e $v=1$ $r=1$

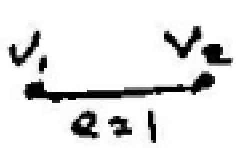
So $r = e - v + 2$ —— ①

$1 = 0 - 1 + 2$ ⇒ $1 = 1$ True.

Suppose $e=1$ then r may be 1 or 2

In case $e=1$ and $v=2$ then. $r = 1 - 2 + 2 = 1$

Clearly graph has one Infinite region.



$v=2$.

Again In case $e=1$ $v=1$ then.

$r = 1 - 1 + 2 = 2$. The graph has two region r_1 and r_2

Hence the result is true for $e=1$

② Induction step:- Suppose that the result is true for ~~k=k~~. $e=k$ edge.

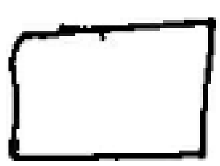
$$r_k = e_k - v_k + 2 \text{ is true.}$$

③ Verification step:- $r_k = e_k - v_k + 2$ —— ②

if it is true for $n=k$ then it is true for $n=k+1$

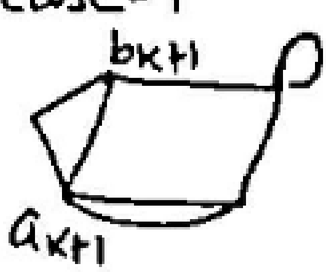
Case -1

Let (a_{k+1}, b_{k+1}) be the edge that is added to G_k make G_{k+1}



G_{k+1} then we add 1 edge.

case-1



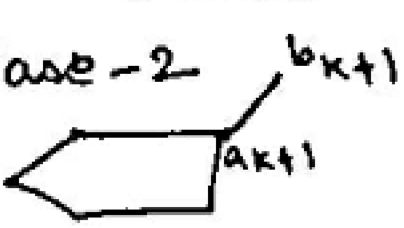
$$r_{k+1} = r_k + 1$$
$$e_{k+1} = e_k + 1$$
$$v_{k+1} = v_k$$

Thus $r_{k+1} = e_{k+1} - v_{k+1} + 2$

$$r_k \cancel{+1} = e_k \cancel{+1} - v_k + 2$$
$$r_k = e_k - v_k + 2.$$

Hence the result is true

Case-2



$$r_{k+1} = r_k, \quad e_{k+1} = e_k + 1$$
$$v_{k+1} = v_k + 1$$
$$r_k = e_k \cancel{+1} - v_k \cancel{+1} + 2$$
$$r_k = e_k - v_k + 2$$

Hence Proved

Condition for a graph to be planar.

Theorem:- If G is a connected simple planar graph with $n \geq 3$ vertices and e-edges then to show that $e \leq 3n-6$

Proof:- Let G be a simple connected planar graph

$\Rightarrow$ each region of G is bounded by atleast three edges and each edge belongs exactly to two region. So that we have

$$2e \geq 3r$$

$$r \leq \frac{2}{3}e$$

$$n+r \leq n+\tfrac{2}{3}e \quad \text{—— (1)}$$

r = is no. of region
e = no of edges

By Euler's formula.

$$n-e+r=2$$

$$n+r=e+2 \quad \text{—— (2)}$$

from (1) and (2)

$$e+2 \leq n+\tfrac{2}{3}e$$

$$3e+6 \leq 3n+2e$$

$$e \leq 3n-6$$

Hence proved

Theorem:- 2 If G is connected simple graph with $n(\geq 3)$ vertices and e edges and no circuits of length 3 then $e \leq 2n-4$.

Proof Since G is planar so degree of each region is at least 4.

$\Rightarrow$ total no. of edges around all the regions is $\geq 4r$.

Again as every edge borders two regions

$\therefore$ total no. of edges which borders all the region is 2e

Hence $4r \leq 2e$

$$r \leq \frac{2}{4}e$$

$$r \leq \frac{1}{2}e$$

$$n+r \leq n+\frac{1}{2}e \quad \text{— (1)}$$

By Euler's formula

$$r = e-n+2$$

$$r+n = e+2 \quad \text{— (2)}$$

from (1) and (2)

$$e+2 \leq n+\frac{e}{2}$$

$$2e+4 \leq 2n+e$$

$$2e-e \leq 2n-4$$

$$e \leq 2n-4$$

Hence proved

Q. A connected plane graph has 10 vertices each of degree 3. Into how many regions does a representation of this planar graph split the plane?

Solution. Given no. of vertices $V = 10$

Given degree of each $v = 3$

$\therefore \sum \deg v =$ degree of vertex $\times$ no. of vertex

$$3 \times 10$$
$$= 30$$

But we know that $\sum \deg v = 2e$

$$2e = 30$$
$$e = 30/2 = 15$$

By Euler's formula.

$$v - e + r = 2$$
$$10 - 15 + r = 2$$
$$r = 7$$

Hence no. of region $= 7$.

Q. Prove that $K_{3,3}$ is non planar.

Solution.

The graph $K_{3,3}$ is bipartite graph and has no circuits of length 3

$\therefore$ Here $V = 6$

$$e = 3 \times 3 = 9$$

Now $2V - 4 = 2 \times 6 - 4$
$$= 12 - 4$$
$$= 8$$

$8 < 9$

$\Rightarrow 2V - 4 < e$

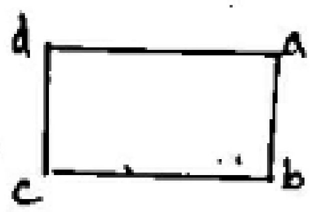
which is contradiction to $e \leq 2n - 4$

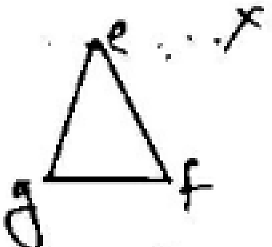
Hence $K_{3,3}$ is non-planar.

Eulerian Path and Circuits

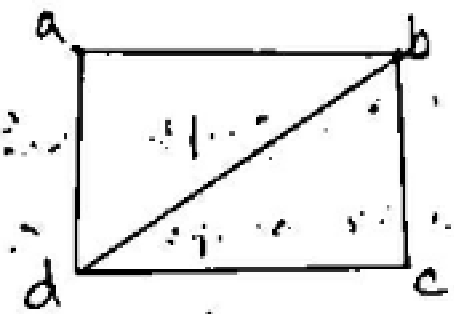
Eulerian path :- Eulerian path In graph $G = (V, E)$ is defined as a path which traverses each edge In the graph G once and only, once.

Eularian circuit/cycle - ① it is a closed walk
② and walk visit every edge of a graph exactly once.

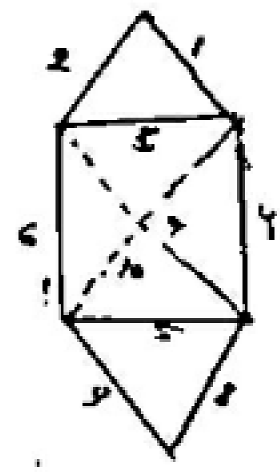




Not a Eular graph.



Not a Eular graph



Eularian graph :- A graph G Which contain a Eular circuit is called Eulariangraph or Eular graph.

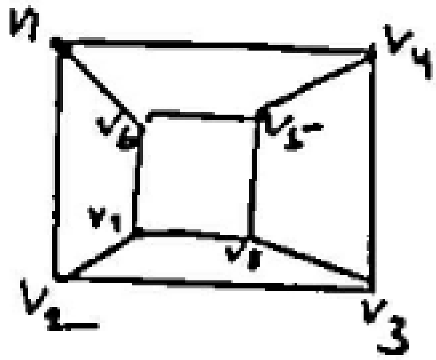
open Eular walk :- A open walk which visit every edgo of the graph exactly once.

Open Eular graph :- A graph which contain a. open Eular walk is called open Eular graph. (semi Eular graph)

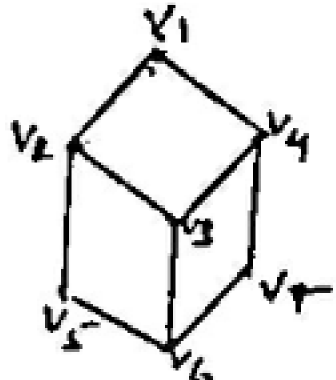
Hamiltonian Graph.

Hamiltonian Circuit:- In a connected graph $G=(V,E)$ a Hamiltonian circuit is defined to be a closed walk which traverses every vertex of G exactly once except the starting vertex

Hamiltonian graph:- A graph G is said to be Hamiltonian graph if it contains a Hamiltonian circuit.



Yes it is a Hamiltonian.

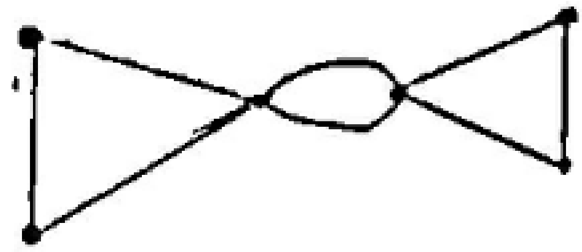


Not a Hamiltonian.

Hamiltonian path:- A path that passes through each of the vertices in a graph G exactly once is called Hamiltonian path

if remove any one edge from Hamiltonian circuit a Hamiltonian path is obtained.

Draw a graph with six vertices containing an Eularian circuit but not Hamiltonian circuit.

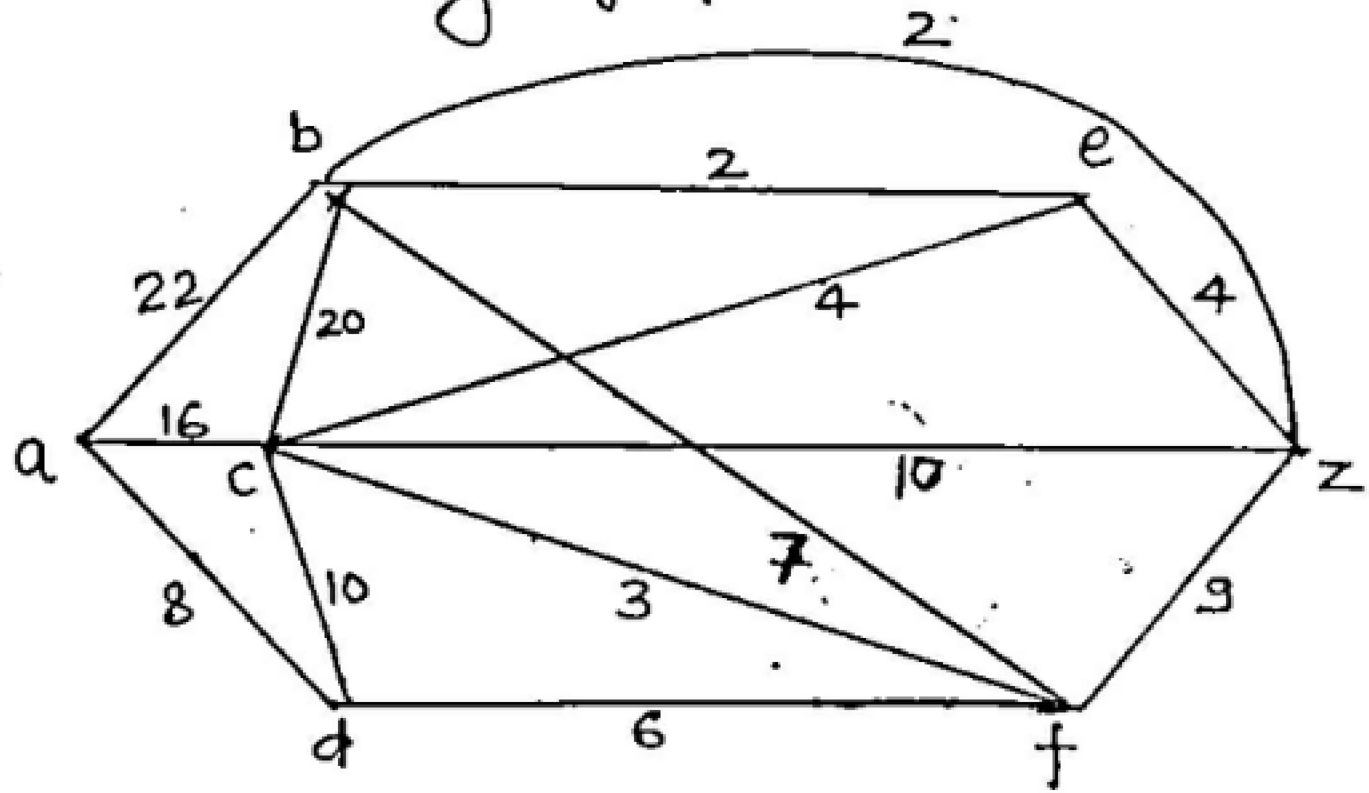


Weighted Graph

Weighted graph: A weighted graph is a graph in which a non-negative real number $w(e)$ is assigned to its each edge e. The number $w(e)$ is called the weight or length of the edge e.

Weight of a path :- Let G be a weighted graph. The weight of a path in the weighted graph G is equal to the sum of the weights of edges in the path

Find the shortest path b/w a and z for the following graph.



Solution. → To find shortest path from a to z.

Let $G = (V, E)$ denoted the graph in given figure

Here $V = \{a, b, c, d, e, f, z\}$

Step-1 Taking $P_1 = \{a\}$ $T_1 = \{b, c, d, e, f, z\}$

$l(b) = 22$, $l(c) = 16$, $l(d) = 8$ $l(e) = l(f) = l(z) = \infty$

Here $d \in T_1$ has the minimum index 8

Step-2 Taking $P_2 = \{a, d\}$ $T_2 = \{b, c, e, f, z\}$

$$l(b) = \min\{22, 8+\infty\} = 22$$
$$l(c) = \min\{16, 8+10\} = 16$$
$$l(e) = \min\{\infty, 8+\infty\} = \infty$$
$$l(f) = \min\{\infty, 8+6\} = 14$$
$$l(z) = \min\{\infty, 8+\infty\} = \infty.$$

Hence the minimum index label. $L(f) = 14$.

Step-3 Taking $P_3 = \{a, d, f\}$ $T_3 = \{b, c, e, z\}$

minimum length. $= 14$.

$$l(b) = \min\{22, 14+7\} = 21$$
$$l(c) = \min\{16, 14+3\} = 16$$
$$l(e) = \min\{\infty, 14+\infty\} = \infty$$
$$l(z) = \min\{\infty, 14+9\} = 23$$

Hence minimum index 16 it has. $c \in T_3$

step-4 Taking $P_4 = \{a, d, f, c\}$ $T_4 = \{b, e, z\}$

$$l(b) = \min\{21, 16+20\} = 21$$
$$l(e) = \min\{\infty, 16+4\} = 20$$
$$l(z) = \min\{23, 16+10\} = 23$$

$\therefore$ minimum length (index label) = 20
and vertex is e.

Step-5 Taking $P_5 = \{a, d, f, c, e\}$ $T_5 = \{b, z\}$

$$l(b) = \min\{21, 20+2\} = 21$$
$$l(z) = \min\{23, 20+4\} = 23$$

Hence minimum index label = 21
and vertex is b

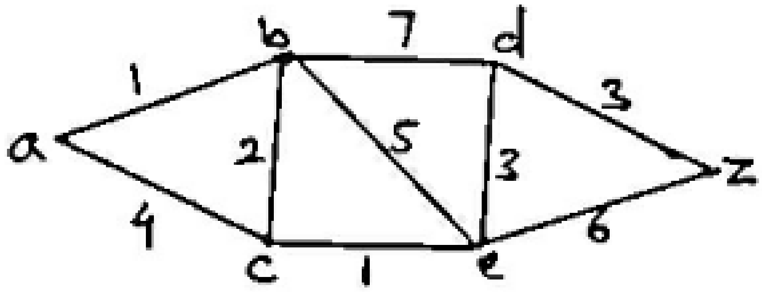
step-6. $P_6 = \{a, d, f, c, e, b\}$. $T_6 = \{z\}$

$$l(z) = \min\{23, 21+2\} = \underline{23}$$

Thus the shortest length b/w $a \to z = 23$.

The shortest path is $a \to d \to f \to z$.

Write an algorithm for shortest path in weighted graph and use it to find shortest path from a to z in the graph shown in following figure. where numbers. associated with the edges are weights.



Solution

Let $G=(V,E)$ denote be the graph.

Here $V=\{a,b,c,d,e,z\}$

Step-I Taking $P_1=\{a\}$ $T_1=\{b,c,d,e,z\}$ Now

$l(b)=1$ $l(c)=4$ $l(d)=\infty=l(e)=l(z)$

Thus $b\in T_1$ has the minimum index 1

Step-II Taking $P_2=\{a,b\}$ $T_2=\{c,d,e,z\}$

$l(c)=\min\{4,1+2\}=3$

$l(d)=\min\{\infty,1+7\}=8$

$l(e)=\min\{\infty,1+5\}=6$

$l(z)=\min\{\infty,1+\infty\}=\infty$

Thus $c\in T_2$ has minimum index 3

Step- III Taking $P_3 = \{a, b, c\}$ $T_3 = \{d, e, z\}$

$$l(d) = \min\{8, 3+\infty\} = 8$$

$$l(e) = \min\{6, 3+1\} = 4$$

$$l(z) = \min\{\infty, 3+\infty\} = \infty$$

Thus $e \in T_3$ has minimum index 4

Step- IV $P_4 = \{a, b, c, e\}$ $T_4 = \{d, z\}$

$$l(d) = \{8, 4+3\} = 7$$
$$l(z) = \{\infty, 4+6\} = 10$$

Thus $d \in T_4$ has minimum index 7

Step-5 $P_5 = \{a, b, c, e, d\}$ $T_5 = \{z\}$

$$l(z) = \min\{10, 7+3\} = 10$$

Hence the length of the shortest path from a to z is 10. The shortest path in this graph is

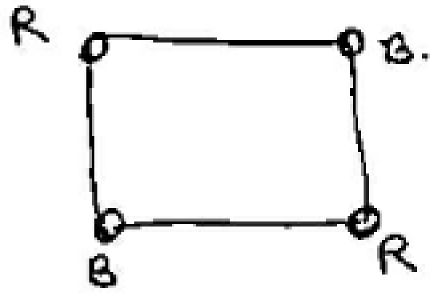
$$a \to b \to c \to e \to d \to z.$$

Graph Coloring and Chromatic number

Graph Coloring :- A coloring of a graph G is an assignment of colors to its vertices so that no two adjacent vertices have the same color.

This type of coloring is called proper coloring or vertex coloring.

proper coloring means color a graph with minimum no. of colors.

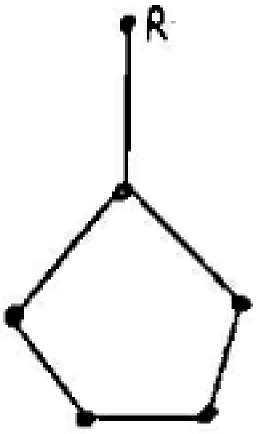


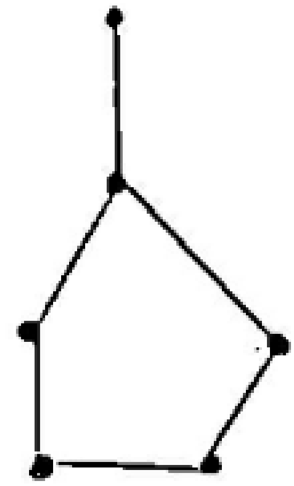
$\chi(G) = 2$

2 ~~coloring graph~~.

Chromatic number :- The chromatic number of a graph G is the minimum number of colors required for proper coloring the vertices of G and is denoted by $\chi(G)$

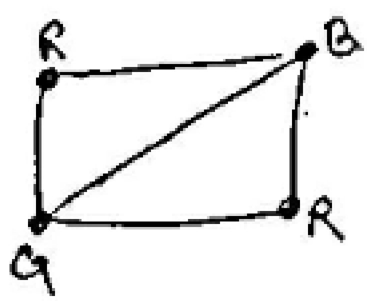
Thus if a graph G requires n different colors (but not less than n) for its proper coloring then $\chi(G) = n$ and G is called n-chromatic graph.





3-colors graph.

$\chi(G) = 3$.



$\chi(G) = 3$

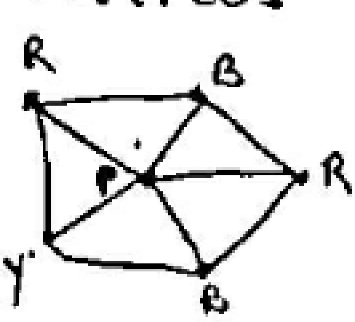
Note. ① Every Bipartite graph is 2-chromatic number: $\chi(K_{m,n}) = 2$

② A complete graph K_n of n vertices is n-chromatic i.e $\chi(K_n) = n$.

③ If G is a null graph or isolated graph then. $\chi(G) = 1$

④ A graph with only one edge is 2 chromatic

⑤ If W_n represent a wheel graph with n vertices then $\chi(W_n) = 3$ if n is ~~even~~ ~~odd~~ even
$\chi(W_n) = 4$ if n is ~~odd~~ odd ~~even~~.



$\chi(W_5) = 4$.

Matrix Representation of Graphs.

there are two way to Represent a graph in matrix.

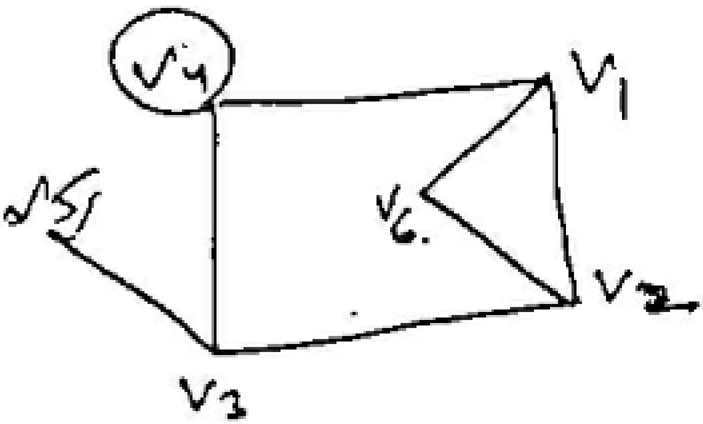
① By Adjacency matrix
② By Incidence matrix.

① Adjacency matrix (for undirected graph)

Let G be graph with n vertices. and no parallel. edge (self loops are allowed) then. the adjacency matrix G is an n×n symmetric matrix $a = [a_{ij}]$ defined by.

$$a_{ij} = \begin{cases} 1, & \text{if there is a edge b/w } i^{th} \,\&\, j^{th} \text{ vertex} \\ 0 & \text{if there is no edge b/w them} \end{cases}$$

Q. Write the adjacency matrix for the graph.



Ans

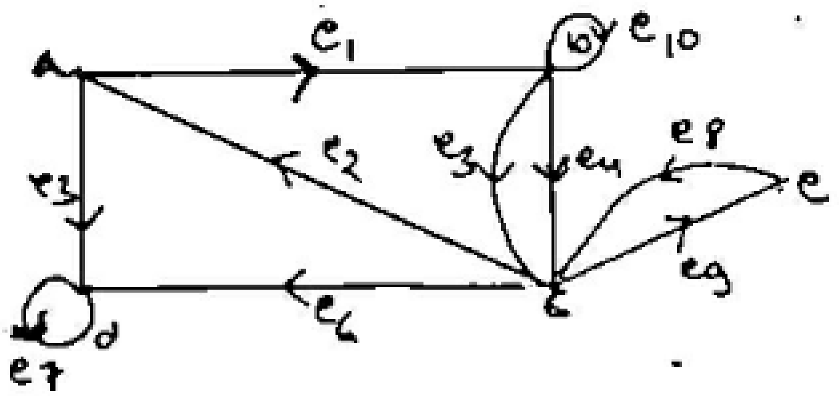
$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & & & & & \\ 1 & & & & & \\ 0 & & & & & \\ 1 & & & & & \end{bmatrix} \end{array}$$

Adjacency matrix for Directed Graph.

The adjacency matrix :- Let G be a digraph with n vertices $v_1, v_2, \dots v_n$ then the adjacency matrix of G is an $n \times n$ matrix $A = [a_{ij}]$ defined by .

$$a_{ij} = \begin{cases} = r & \text{if } r \text{ edges are directed from } i\text{th vertex } v_i \text{ to } j\text{th vertex } v_j \\ = 0 & \text{otherwise} \end{cases}$$

$r = 1$ if digraph has no parallel edge .



Solution.

$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right] \end{array}$$