Unit 3
AC Circuits

GENERATION OF AC VOLTAGE

What is Alternating Current (AC)?

- AC is an electric current that periodically reverses direction.
- Unlike direct current (DC), where electrons flow in one direction, in AC the flow of electrons changes direction periodically.

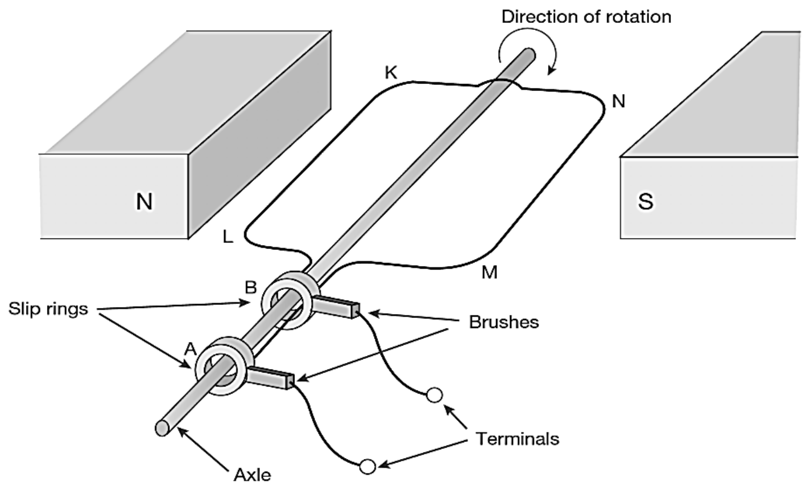
Principle of AC Generation

AC voltage is generated by electromagnetic induction — the phenomenon where a changing magnetic field induces an electromotive force (emf) in a conductor.

Simple AC Generator

A basic AC generator consists of:

- A rotating coil placed between magnetic poles.
- Slip rings and brushes for current collection.



Working:

Let a coil of N turns rotate in a uniform magnetic field B with an angular velocity ω:

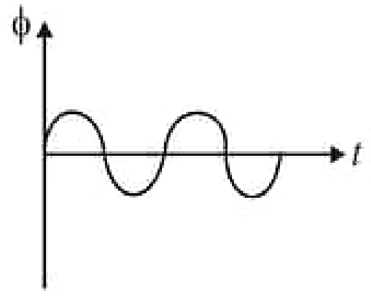
- At time t, the angle between magnetic field and coil is $\theta = \omega t$.
- The magnetic flux through coil:

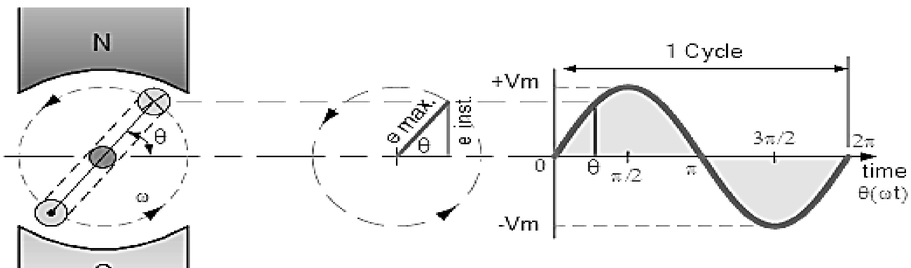
$$\phi = B \cdot A \cdot cos(\omega t)$$

- The induced emf (e), by Faraday's Law:

$$e = -\frac{d\phi}{dt} = N\ A\ B\ \omega\ sin(\omega t)$$

This equation gives a sinusoidal waveform — alternating voltage.





Instantaneous Value

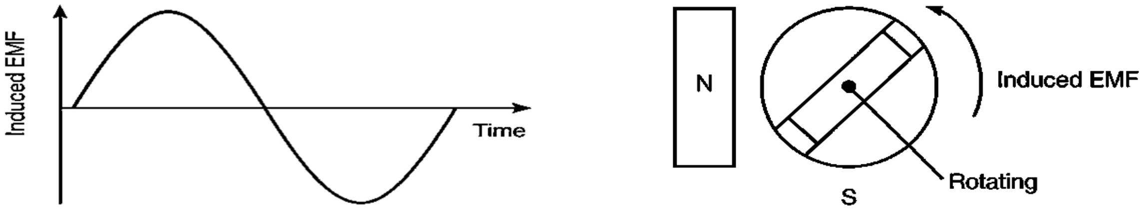
EMF EQUATION

When a conductor (coil) rotates in a magnetic field, the flux linking with it changes continuously. According to Faraday’s Law of Electromagnetic Induction:

$$e = -N\frac{d\phi}{dt}$$

Where:

- e = Instantaneous EMF induced
- N = Number of turns in the coil
- ϕ = Magnetic flux linking the coil (Wb)



(a) Sinusoidal EMF waveform

(b) Flux variation and rotor rotation

Flux linking the coil at any time t (assuming sinusoidal variation):

$$\phi(t) = \phi_m \, sin(\omega t)$$

Where ϕ_m is the maximum flux per pole

By Faraday's Law:

$$e(t) = -N\frac{d\phi}{dt} = -N\frac{d}{dt}[\phi_m \, sin(\omega t)\,]$$

$$e(t) = -N\,\phi_m\,\omega\,cos(\omega t) = -N\,\phi_m\,\omega\,sin(\omega t + 90°)$$

Thus, the induced EMF is sinusoidal and can be written as:

$$e(t) = E_m \, sin(\omega t + \phi)$$

Where:

- $E_m = N\phi_m\omega$ is the maximum EMF (amplitude)
- ϕ is the phase angle

In practical alternators:

- The winding is distributed and short-pitched.
- Therefore, two factors reduce the total EMF:
  - Distribution Factor K_d
  - Pitch Factor K_p

So, the total induced EMF per phase (maximum) becomes:

$$E_{max} = 2\pi f\phi T K_p K_d \qquad [\because \omega = 2\pi f]$$

Where:

- T = Total number of turns per phase
- f = Frequency
- ϕ = Flux per pole
- K_p = Pitch factor
- K_d = Distribution factor

Since $E_{rms} = \frac{E_{max}}{\sqrt{2}}$:

$$E_{rms} = \frac{2\pi f\phi T K_p K_d}{\sqrt{2}}$$

Let $E = \ RMS$ value per phase. Then:

$$E = 4.44 f\phi T K_p K_d$$

This is the final EMF Equation of an Alternator, expressing the RMS voltage per phase.

AVERAGE, RMS, AND EFFECTIVE VALUES

What is an AC Signal?

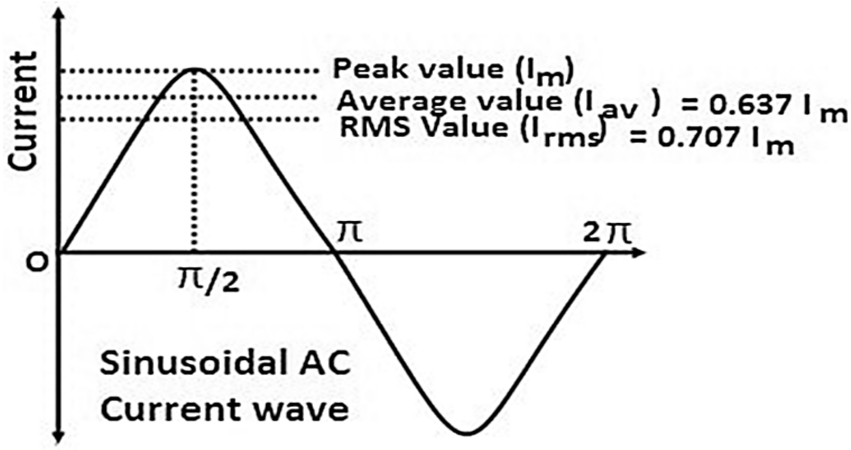
An Alternating Current (AC) is one that changes its magnitude and direction periodically. The most common AC waveform is sinusoidal.

Let the instantaneous value of AC be:

$$v(t) = V_m \, sin(\omega t)$$

where:

- V_m = maximum or peak voltage
- $\omega = 2\pi f$ = angular frequency
- t = time



(a) Average Value of AC (Over Half Cycle)

Definition: The average value of an AC waveform over a half cycle is the arithmetic mean of all instantaneous values.

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \, sin(\theta)\, d\theta = \frac{2V_m}{\pi}$$

Formula:

$$V_{avg} = \frac{2}{\pi} V_m \approx 0.637V_m$$

Note: The average value of a full AC cycle is zero, so only the half cycle is considered.

(b) RMS (Root Mean Square) Value

Definition: It is the square root of the average of the squares of all instantaneous values over one complete cycle. This value gives the equivalent DC value that would produce the same heating effect.

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \, sin^2(\theta) d\theta} = \frac{V_m}{\sqrt{2}}$$

Formula:

$$V_{rms} = \frac{V_m}{\sqrt{2}} \approx 0.707V_m$$

(c) Form Factor and Peak Factor

Form Factor = Ratio of RMS value to Average value

$$Form\ Factor = \frac{V_{rms}}{V_{avg}} = \frac{0.707V_m}{0.637V_m} \approx 1.11$$

Peak Factor = Ratio of Peak value to RMS value

$$Peak\ Factor = \frac{V_m}{V_{rms}} = \frac{V_m}{0.637V_m} \approx 1.414$$

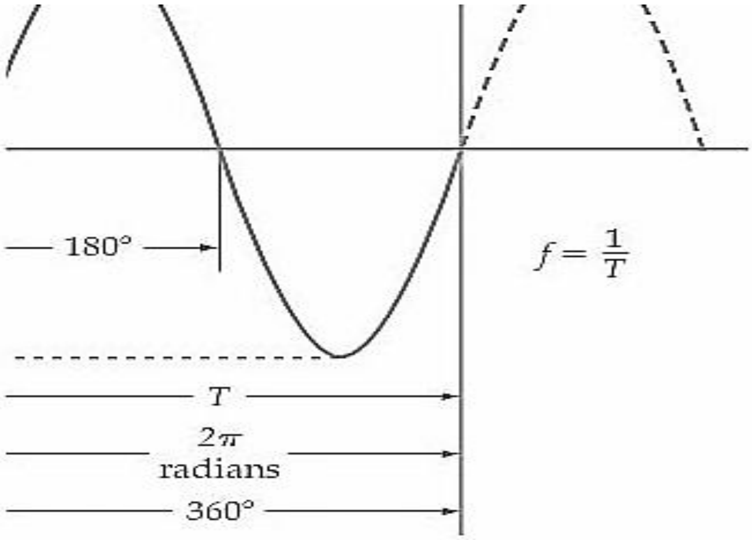
AC QUANTITIES

AC Voltage or Current

- Time-dependent and represented as:

$$v(t) = V_m \sin(\omega t + \phi) \quad or \quad i(t) = I_m \sin(\omega t + \phi)$$

- ϕ is the phase angle showing the time shift.



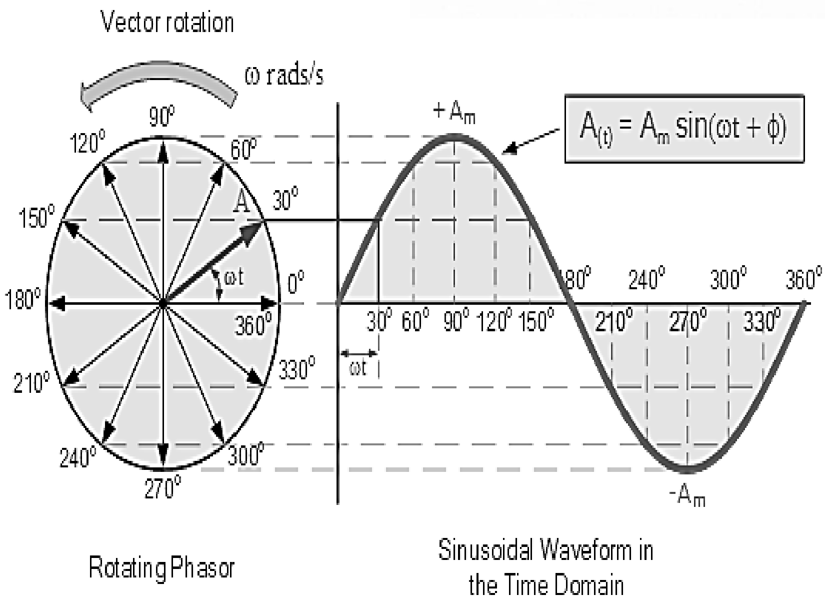
Key Characteristics of AC

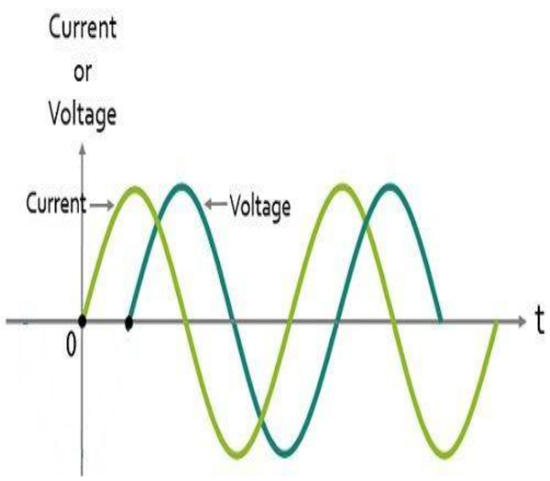
Parameter	Description
Amplitude	Maximum value reached by the wave (peak value)
Frequency (f)	Number of cycles per second, measured in Hz (Hertz)
Time Period (T)	Time taken to complete one full cycle, T=1/f
Angular Frequency	ω=2πf
Phase	Indicates the shift between two waveforms

Phasor Representation

- A phasor is a rotating vector representing AC quantities with magnitude and angle.
- Example:

$$V = V_m \angle\phi$$



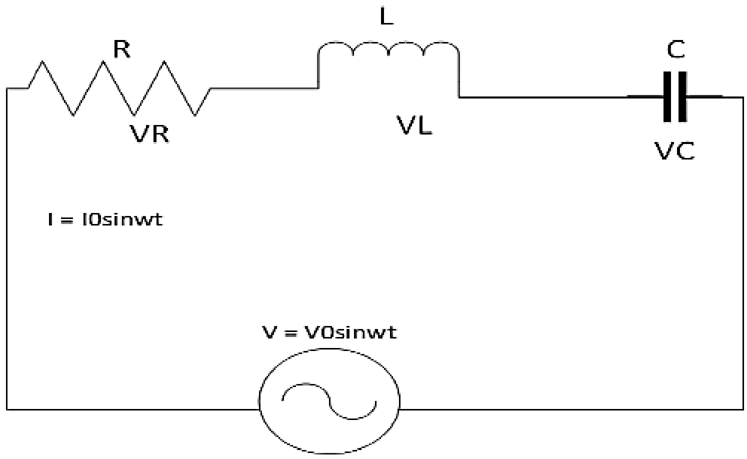


where ϕ is phase angle with reference to origin.

RLC SERIES CIRCUIT

Circuit Description

- Consists of a Resistor (R), Inductor (L), and Capacitor (C) connected in series with an AC voltage source.



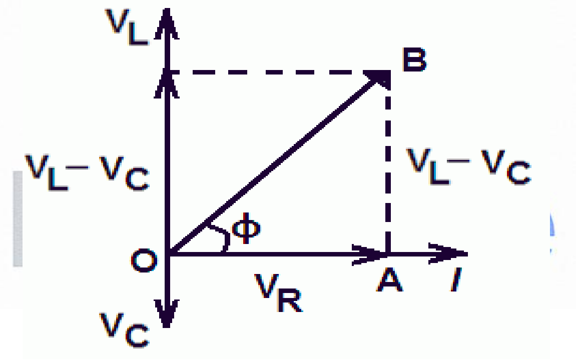
Voltage Drops Across Elements

Let the supply be:

$$V(t) = V_m \, sin(\omega t)$$

Then:

- Voltage across Resistor, $V_R = IR$ (in phase with current)
- Voltage across Inductor, $V_L = IX_L = I\omega L$ (leads current by 90°)
- Voltage across Capacitor, $V_C = IX_C = \frac{I}{\omega C}$ (lags current by 90°)



Impedance of RLC Circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Where:

- $XL = \omega L$ is Inductive Reactance
- $X_C = \frac{1}{\omega C}$ is Capacitive Reactance

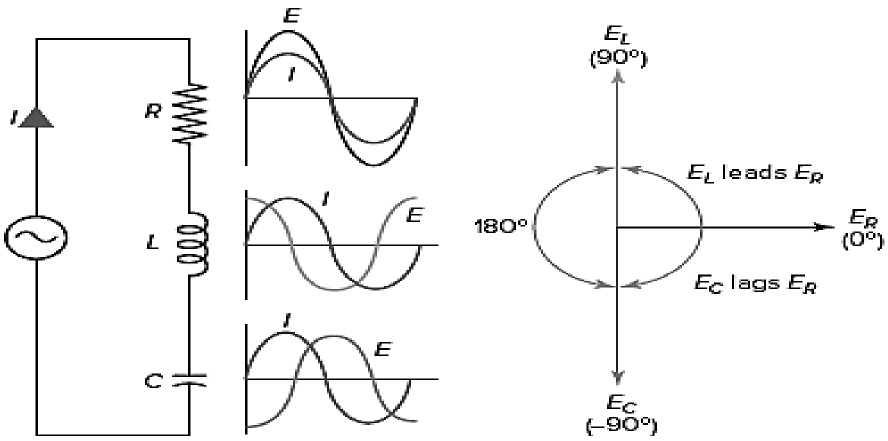
Current in Circuit

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Phase Angle (Φ)

$$tan(\phi) = \frac{X_L - X_C}{R} \quad \Rightarrow \quad \phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

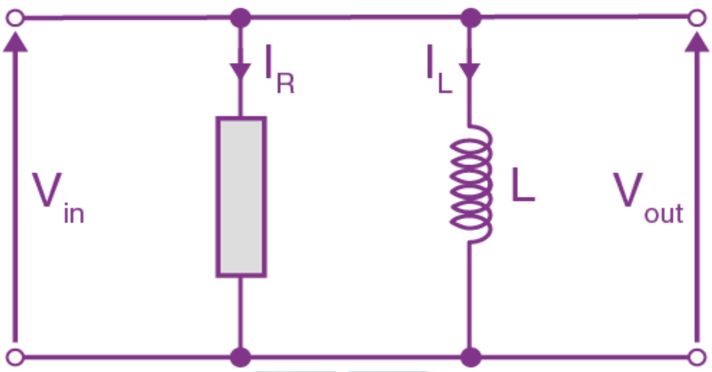
- If $X_L > X_C$, circuit is inductive, current lags voltage.
- If $X_L < X_C$, circuit is capacitive, current leads voltage.
- If $X_L = X_C$, circuit is resonant and purely resistive.



PARALLEL AND SERIES-PARALLEL CIRCUITS

Parallel AC Circuits

- In a parallel AC circuit, the voltage across each branch is the same, but the current may differ depending on the impedance of each branch.



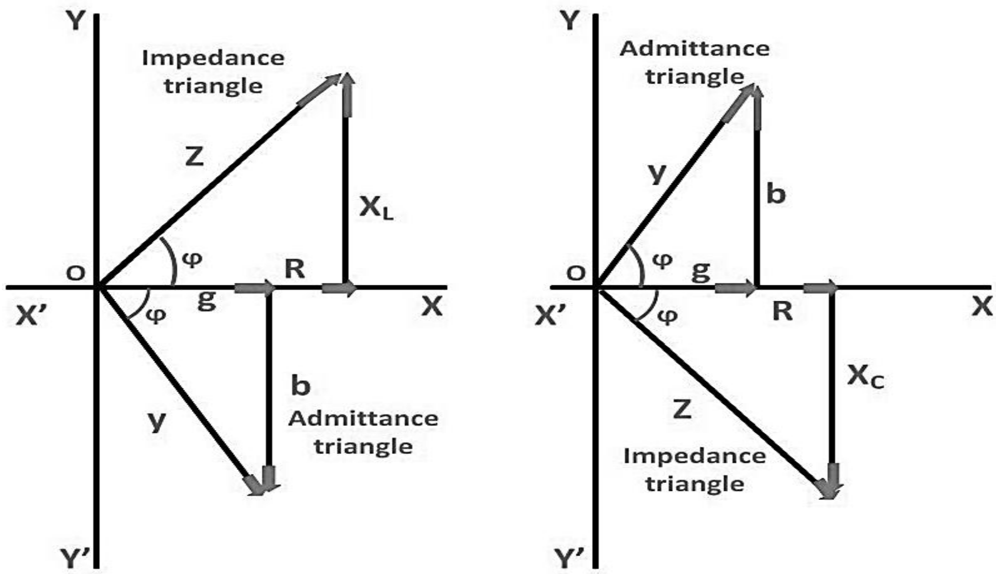
Key Concepts:

- The total current is the phasor sum of individual branch currents.
- Use admittance (Y): reciprocal of impedance.

$$Y = \frac{1}{Z} = G + jB$$

where:

- GGG is conductance (real part)
- BBB is susceptance (imaginary part)



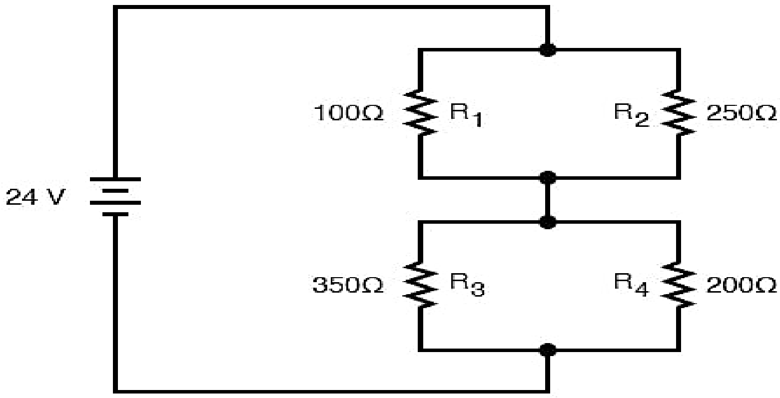
Example: Two Branches

- One branch with R, one with L:

$$I_1 = \frac{V}{R}, \qquad I_2 = \frac{V}{j\omega L}$$
$$I_{total} = I_1 + I_2$$

Series-Parallel AC Circuits

- Combination of series and parallel elements.
- Analyse using:
  - Phasor diagrams
  - Impedance combinations
  - Ohm's law
- Convert complex circuits into simpler equivalent circuits.



COMPLEX AND PHASOR REPRESENTATION

Phasor Concept

- A phasor is a rotating vector representing a sinusoidal function.
- Converts time-varying sinusoids into complex numbers.

Phasor Conversion:

- Given: $v(t) = V_m \sin(\omega t + \phi)$
- Phasor form: $\vec{V} = V_m \angle\phi$

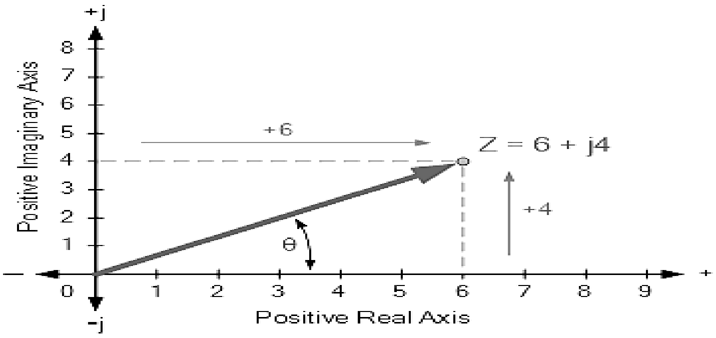
Complex Representation

$$V = V_m(\cos\phi + j\sin\phi) = V_m e^{j\phi}$$

Operations:

- Addition/Subtraction: In rectangular form
- Multiplication/Division: In polar form
- Conversion:

$$Rectangular \leftrightarrow Polar \leftrightarrow Exponential$$



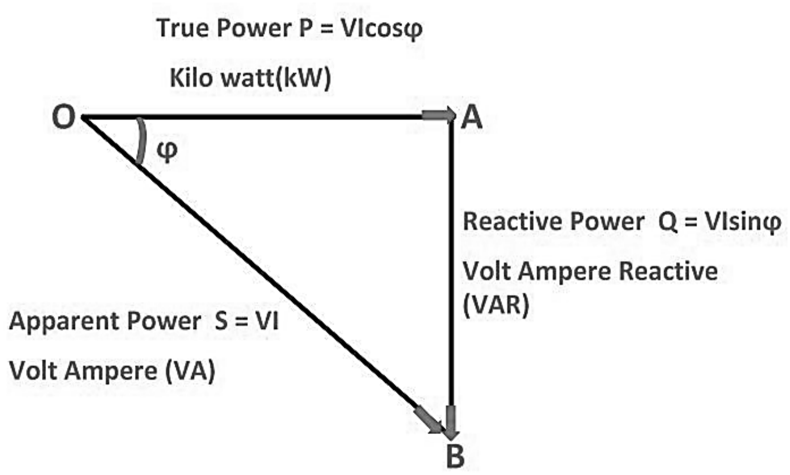
POWER AND POWER FACTOR

Power in AC Circuits

Unlike DC circuits where power is simply $P = VI$, in AC circuits, voltage and current vary sinusoidally and may not be in phase, especially when reactive components like inductors and capacitors are involved.

So, AC power must be divided into 3 components:

- Active (Real) Power
- Reactive Power
- Apparent Power



Types of Power in AC Circuits:

(a) Active Power (P)

- Also called True Power or Real Power.
- Represents the actual power consumed or utilized in the circuit.
- It performs useful work like lighting, heating, running motors, etc.

Formula:

$$P = VIcos\phi \qquad (in\,watts,\ W)$$

Where:

- V = RMS Voltage
- I = RMS Current
- ϕ = Phase angle between voltage and current
- cos ϕ = Power factor

(b) Reactive Power (Q)

- Power stored and released periodically by reactive components (inductors/capacitors).
- Does no useful work, but contributes to the total current.

Formula:

$$Q = VIsin\phi \qquad (in\ volt - amperes\ reactive,\ VAR)$$

(c) Apparent Power (S)

- The product of the RMS voltage and current.
- It is the total power that appears to be transferred.

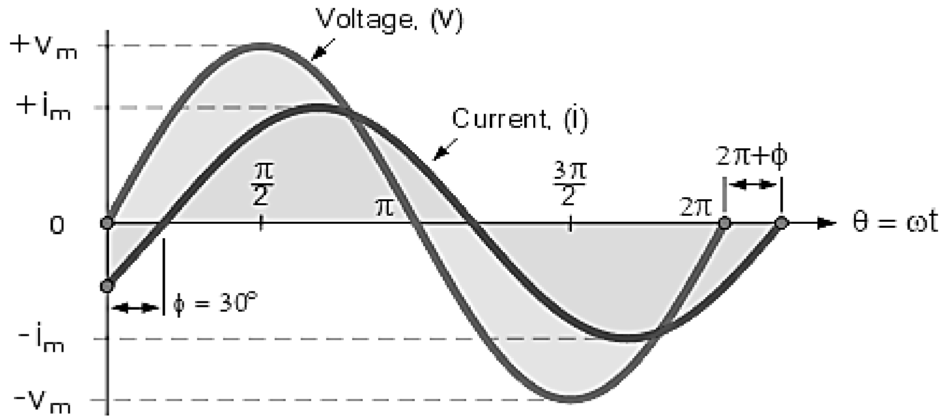
Formula:

$$S = VI \qquad (in\ volt - amperes,\ VA)$$

Power Factor (PF)

Power Factor is the cosine of the angle between the voltage and current phasors in an AC circuit.

$$Power\ Factor = cos\phi = \frac{Active\ Power\ (P)}{Apparent\ Power\ (S)}$$



Types of Power Factor:

Type	Value	Condition
Unity PF	$cos\phi = 1$	Purely resistive load
Lagging PF	$cos\phi < 1$	Inductive load (current lags)
Leading PF	$cos\phi < 1$	Capacitive load (current leads)

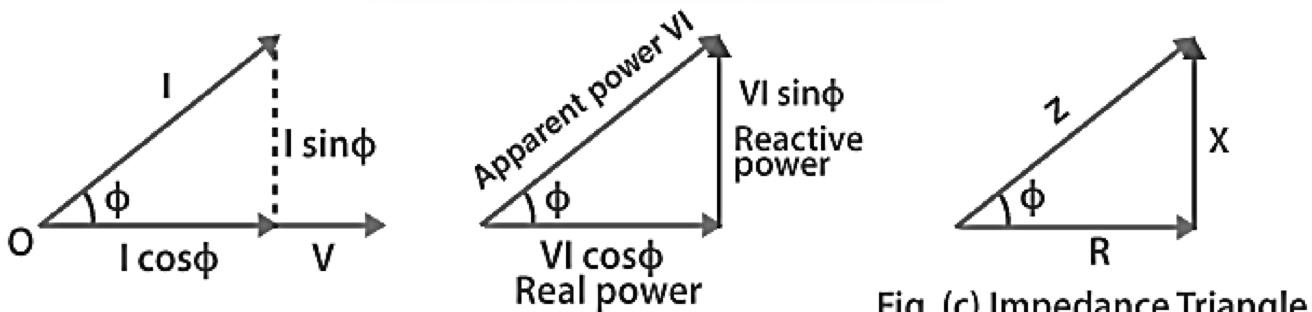


Fig. (a) Power Factor

Fig. (b) Power Triangle

Fig. (c) Impedance Triangle

Effects of Low Power Factor:

1. Increases the current drawn → more I^2R losses
2. Requires larger size transformers, cables
3. Increases electricity bills
4. Poor voltage regulation
5. Reduces system efficiency

Methods to Improve Power Factor:

Method	Description
Capacitor banks	Provide leading current
Synchronous condensers	Over-excited synchronous motors act like capacitors
Phase advancers	Improve PF in induction motors

SINGLE PHASE & THREE PHASE AC CIRCUITS

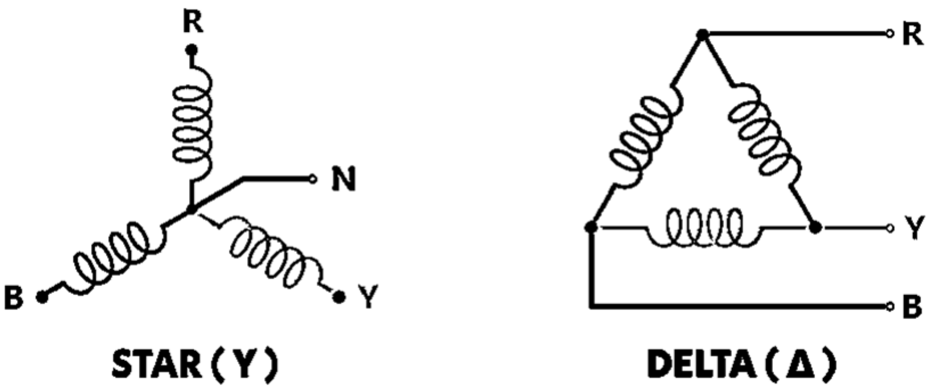
Star And Delta Connection

Star (Y) Connection:

- In Star configuration, one end of each of the 3 windings is connected to form a neutral point.
- The other ends are connected to the line conductors.

Delta (Δ) Connection:

- In Delta configuration, the ends of the windings are connected in a closed loop.



Line & Phase Quantities

Parameter	Star Connection	Delta Connection
Line Voltage (VL)	√3 × Phase Voltage (VP)	Line Voltage = Phase Voltage
Line Current (IL)	Line Current = Phase Current	√3 × Phase Current
Neutral Point	Present	Absent

In STAR:

- $VP = VL / \sqrt{3}$
- $IL = IP$

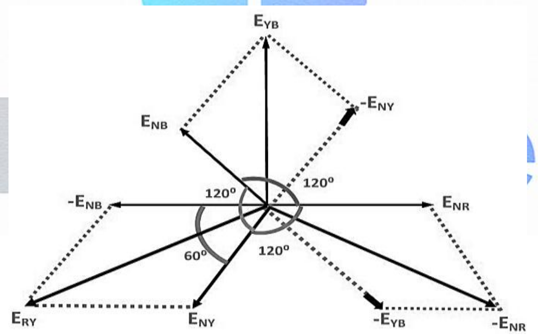
In DELTA:

- $VL = VP$
- $IL = \sqrt{3} \times IP$

Phasor Diagram

Phasor for STAR Connection:

- Line voltages are 120° apart.
- Each phase voltage lags or leads the corresponding line voltage by 30°.

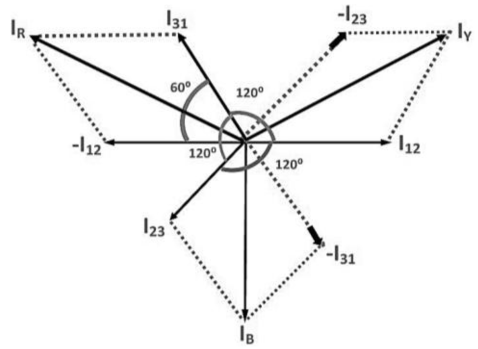


Description:

- Draw three vectors VR, VY, and VB at 120° apart for line voltages.
- Phase voltages are shorter and lag behind line voltages by 30°.

Phasor for DELTA Connection:

- Line voltage equals phase voltage.
- Phase and line currents differ in magnitude and phase.



Measurement Of Power In 3-Phase Balanced Circuits

Total Power in 3-Phase System:

$$P = \sqrt{3} \times VL \times IL \times cos(\varphi)$$

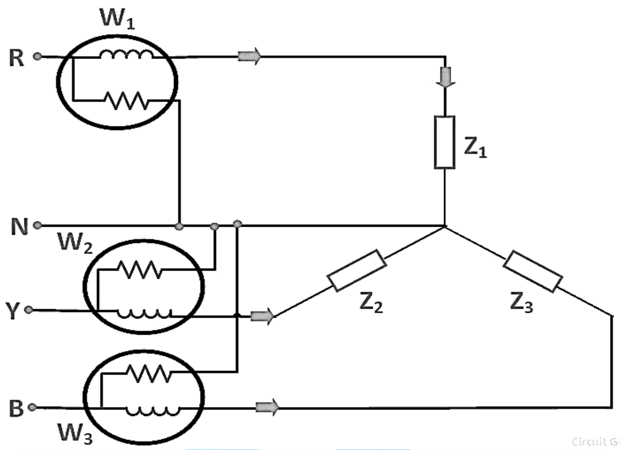
Where:

- VL = Line voltage
- IL = Line current
- $cos(\varphi)$ = Power factor

Methods to Measure Power:

1. Three Wattmeter Method:

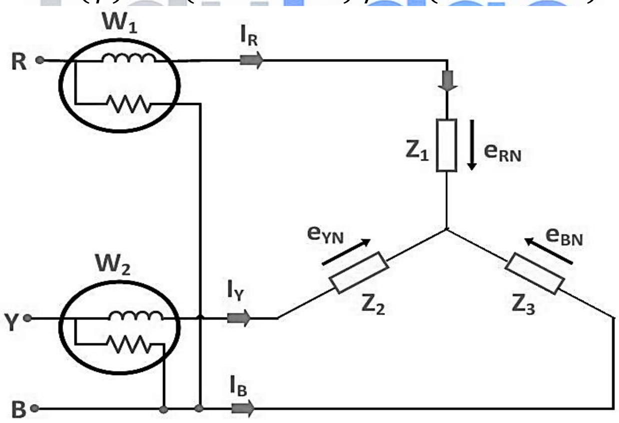
- 3-watt meters connected in each phase.
- Accurate but costly.



2. Two Wattmeter Method (Most Used):

- Requires only 2-watt meters.
- Used in both balanced and unbalanced systems.
- Total power: $P = W1 + W2$
- Power factor can be found by:

$$cos(\varphi) = (W1 + W2) / \sqrt{3}(W1 - W2)$$



3. One Wattmeter Method:

- Can be used only in balanced loads.
- Used for economic measurement in symmetrical systems.