Algorithmic and Computational Mathematics

"Assignment Brief

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| Descriptive details of Assignment:  For each task the following **should be provided/demonstrated**:  - the mathematical concept used for solving the problem, with definitions and formulas;  - the algorithm (with the results at intermediate steps if necessary);  - the numerical (and analytical, if asked) results.  what I want is the assignment to be writing using Latex, purely mathematically derivative/solution, also the code in python, and C++.  If needed, the problems should be solved with the help of a programming code. Code can be included in the report, but should be commented explaining the above points. In any case, a clear description of the algorithm of the code should be provided. The same applies to the output of the code. |

Task 1

The tasks are specific numerical problems that should be solved providing necessary explanations.

For the system of nonlinear equations

{sin(𝑥1 + 1.5) − 𝑥2 + 2.9 = 0 cos(𝑥2 − 2) + 𝑥1 = 0

1. Localise the roots of the system graphically.
2. Give the description of the Newton’s method of solving systems of nonlinear equations.
3. Solve the system using Newton’s method with the accuracy 𝜀 = 10−6.
4. Give the description of the gradient descent method of solving systems of nonlinear equations.
5. Solve the system using the gradient descent method with the accuracy 𝜀 = 10−6.

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Task 2

The tasks are specific numerical problems that should be solved providing necessary explanations. The specific requirements on what is expected in the report describing the solutions are given in the accompanied "Assignment Brief" document.

For the initial value Cauchy problem

𝑥𝑦′ + 𝑦 = 2𝑦2 ln 𝑥 , 𝑦(1) = 1.5, 𝑥 ∈ [1,3]

1. Find the optimal integration step for the fourth-order Runge–Kutta method with the accuracy 10−4.
2. Find the solution using the fourth-order Runge–Kutta method with the accuracy 10−4 and plot it.
3. Find the solution using the Euler method and the optimal step found above and add its plot to the graph.
4. Find the exact solution of the problem, calculate the maximal absolute deviation from the approximate solutions at the integration points.
5. Put the results of calculations in a table and on the graph, compare them.