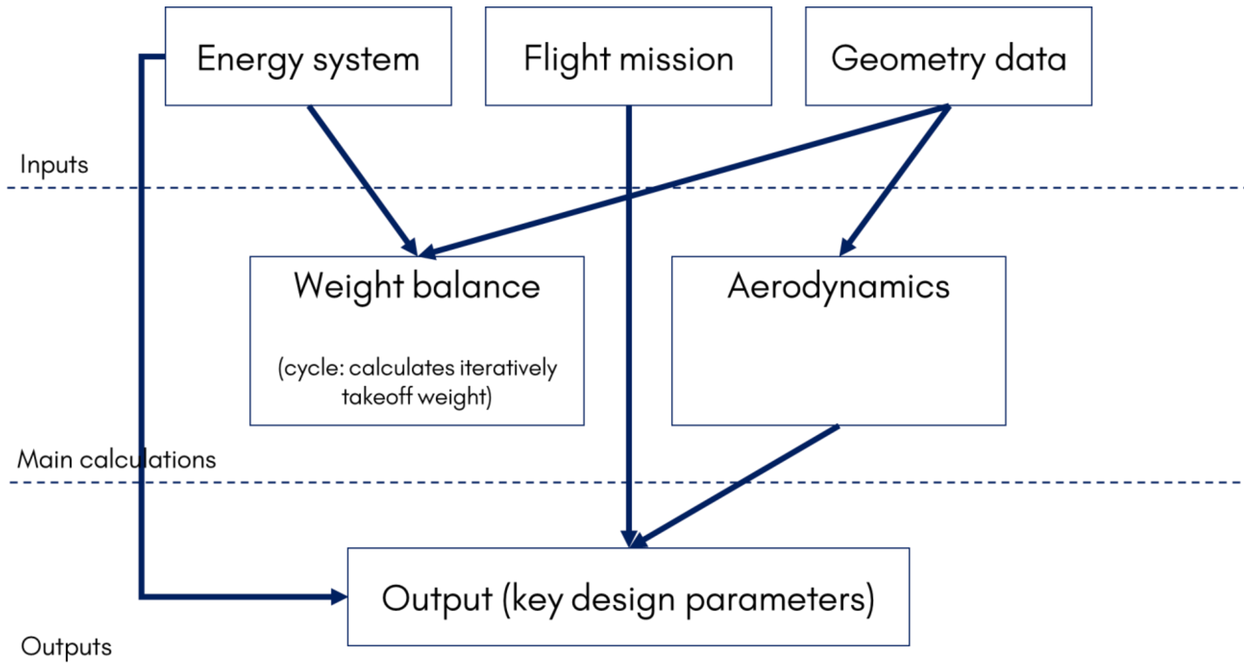
GENERAL EQUATIONS FOR AN AIRCRAFT DESIGN

Contents

1. MODEL STRUCTURE



Pic.1 Model structure

2. MISSION DESIGN

Pic.2 Flight plan

2.1 Takeoff

$$V_0 - takeoff\ velocity\ [\frac{m}{s}]$$

$$V_{takeoff} = V_0 * 0.7 - average\ takeoff\ velocity\ [\frac{m}{s}]$$

$$f_{friction_{to}} = 0.04 - friction\ coeffiient$$

2.2 Climb

$\theta_{climb} - climb\ angle\ [deg]$

$$l_{climb} = \frac{H}{sin\left(\theta_{climb} * \frac{\pi}{180}\right)} - climb\ diagonal\ distance[m]$$

$$d_{climb} = \frac{H}{tan\left(\theta_{climb} * \frac{\pi}{180}\right)} - climb\ horizontal\ distance[m]$$

2.3 Cruise flight

$H - flight\ altitude\ [m]$

$P_{hf}\ -\ Flight\ altitude\ pressure\ [Pa]$

$\rho_{cruise}\ -\ Flight\ altitude\ air\ density\ [kg/m3]$

$V_{cruise} -\ Cruise\ speed\ [m/s]$

$M_{cruise} -\ Cruise\ Mach\ number$

$q_{cruise}\ =\ 0.5 * ro * V_{cruise}^2 -\ Cruise\ dynamic\ pressure\ [Pa]$

$g\ =\ 9.81 -\ Gravitational\ acceleration\ [m/s2]$

$\mu -\ kinematic\ viscosity\ parameter\ according\ to\ ISA - 76\ [\frac{m2}{s}]$

2.4 Descend

$\theta_{descend} - descend\ angle\ [deg]$

$$l_{descend} = \frac{H}{sin\left(\theta_{descend} * \frac{\pi}{180}\right)} - descend\ diagonal\ distance[m]$$

$$d_{descend} = \frac{H}{tan\left(\theta_{descend} * \frac{\pi}{180}\right)} - descend\ horizontal\ distance[m]$$

2.5 Landing

$V_{landing} - landing\ velocity\ [\frac{m}{s}]$

$f_{friction_{land}} = 0.3 - landing\ friction\ coefficient$

2.6 Load factors

$P_{cabin} - Cabin\ pressure\ (equal\ to\ pressure\ \ at\ 2000\ m\ altitude)\ [Pa]$

$P_{delta} - cabin\ pressure\ differential, [Pa]$

$n = 3 - Limit\ load\ factor$

$$N_z = 1.5 * limit\ load\ factor\ (3 - 4\ for\ normal\ category\ aircraft)$$
$$- ultimate\ load\ factor$$

$N_l = 1.5 * n_{landing\ gear} - ultimate\ landing\ load\ factor$

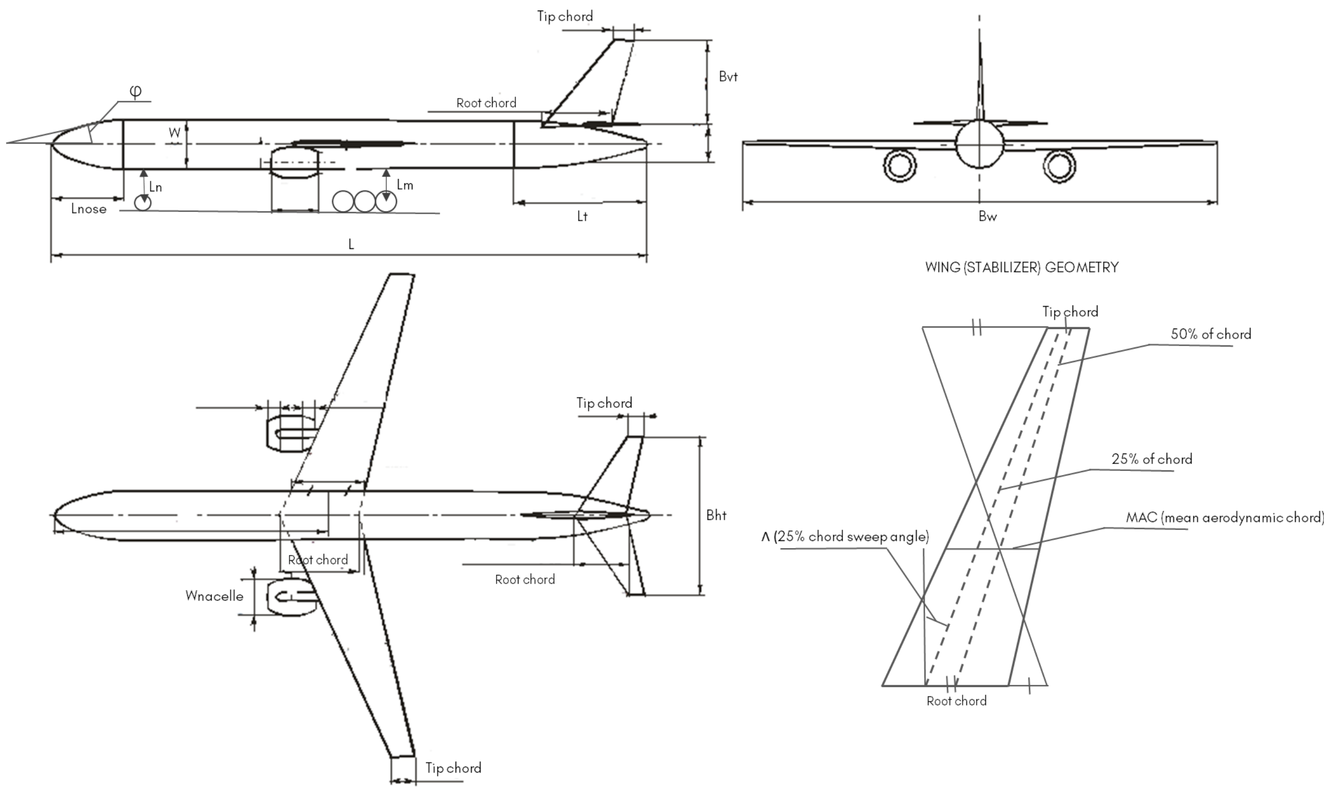
2.7 Payload

$N_p - number\ of\ personnel\ onboard\ (crew\ and\ passengers)$

$W_{payload} - Payload\ weight\ [kg]$

$P_{delta} - cabin\ pressure\ differential, [Pa]$

3. GEOMETRY DATA



Pic.3 Aircraft and its geometry parameters

3.1 Fuselage (nacelle)

$V_{pr} - volume\ of\ pressurized\ section, [m^3]$

$S_f - fuselage\ wetted\ area, [m^2]$

$L_t - tail\ length, [m]$

$L_{nose} - nose\ part\ length\ [m]$

$L - fuselage\ structural\ length, [m]$

$D - fuselage\ structural\ depth, [m]$

$W - total\ fuselage\ structural\ width, [m]$

$$S_{fus} = W^2 * \frac{pi}{4} - crossectional\ area\ [m2]$$

$$r_{relative} = \frac{nose\ radius}{fuselage\ radius} - nose\ relative\ radius$$

$$\varphi - nose\ semipart\ angle\ [rad]$$

$$D_{fus} = sqrt\left(4 * \frac{S_{fus}}{pi}\right) - fuselage\ equivalent\ diametr\ [m]$$

$$AR_{fus} = \frac{L}{W} - fuselage\ aspect\ ratio$$

$$AR_{nose} = \frac{L_{nose}}{W} - nose\ part\ aspect\ ratio$$

$$AR_{aft} = \frac{L_{aft}}{W} - rear\ part\ aspect\ ratio$$

3.2 Wing (or stabilizer or fin)

$$k_{A_{wing}} - airfoil\ type\ coefficient\ (supercritical\ (0.97)\ or\ conventional\ (0.88))$$

$$B_w - wing\ span, [m]$$

$$TR_w = \frac{tip\ chord}{root\ chord} - wing\ taper\ ratio$$

$$AR_w = \frac{B_w^2}{S_w} - wing\ aspect\ ratio$$

$$t/c - thickness - to - chord - ratio$$

$$S_w - trapezoidal\ wing\ area, [m^2]$$

$$\Lambda - \text{wing sweep at 25\% MAC}$$

$$mac = \frac{S_w}{B_w} - Mean\ aerodynamic\ chord\ [m]$$

$S_{wf} \ = \ W * mac - wing\ area\ intersected\ with\ fuselage\ [m^2]$

$\frac{H_t}{H_v} = 0\ if\ tail\ is\ conventional, 1\ if\ tail\ is\ T - shaped$

3.3 Landing gear

$W_l - landing\ design\ gross\ weight, [kg]$

$L_m - extended\ lentgh\ of\ main\ landing\ gear, [m]$

$L_n - extended\ lentgh\ of\ nose\ landing\ gear, [m]$

4. ENERGY SYSTEM

Conventional propulsion (piston or turboprop engine)

$PW \ \approx 150 - 350 - power - to - weight\ ratio, \left[\frac{Wt}{kg}\right]$

$N_{en} - number\ of\ engines$

$\eta_{prop} - propeller\ efficiency\ ({\sim}0.8\ for\ cruise\ flight\ and\ {\sim}0.6 - 0.7\ for\ takeoff\ and\ landing)$

$N_t - number\ of\ fuel\ tanks$

$W_{fw} - weight\ of\ fuel\ in\ wing, [kg]$

$W_{fuel} - fuel\ weight, [kg]$

$V_t - total\ fuel\ volume, [m^3]$

$V_i - intergral\ tanks\ volume, [m^3]$

5. WEIGHT BALANCE

5.1 Propulsion system weight calculation

$$W_{installed\ engine\ (total)} = 2.421 * W_{en}^{0.922} * N_{en}$$

$$W_{en} = 0.5638 * P_{en\ max}^{0.91}$$

$W_{en} - engine\ weight\ [kg]$

$P_{en\ max} - engine\ takeoff\ power\ [kW]$

5.2 Structures weight calculation

5.2.1 Fuselage weight calculation

$$W_{fuselage} = 0.23 * S_f^{1.086} * \left(N_z * W_{dg}\right)^{0.177} * L_t^{-0.051} * (L/D)^{-0.072} * q^{0.241} + W_{press}$$

$$W_{press} = 1.2926 * \left(V_{pr} * P_{delta}\right)^{0.271}$$

$W_{press} - weight\ penalty\ due\ to\ pressurization, [kg]$

$W_{dg} - flight\ design\ gross\ weight, [kg]$

5.2.2 Wing weight calculation

$$W_{wing} = 0.13817 * S_w^{0.758} * W_{fw}^{0.0035} * \left(\frac{AR_w}{cos^2\Lambda}\right)^{0.6} * q^{0.006} * TR_w^{0.04} * \left(\frac{100t/c}{cos\Lambda}\right)^{-0.3} * \left(N_z * W_{dg}\right)^{0.49}$$

$W_{fw} - weight\ of\ fuel\ in\ wing, [kg]$

5.2.3 Empennage weight calculation

$$W_{ht} = 0.01917 * (N_z * W_{dg})^{0.414} * q^{0.168} * S_{ht}^{0.896} * \left(\frac{100t/c}{cos\Lambda_{ht}}\right)^{-0.12} * \left(\frac{AR_{ht}}{cos^2\Lambda_{ht}}\right)^{0.043} * TR_{ht}{}^{-0.02}$$

$$W_{vt} = 0.12 * \left(1 + 0.02 * \frac{H_t}{H_v}\right) * (N_z * W_{dg})^{0.376} * q^{0.122} * S_{vt}^{0.873} * \left(\frac{100t/c}{cos\Lambda_{vt}}\right)^{-0.49} * \left(\frac{AR_{vt}}{cos^2\Lambda_{vt}}\right)^{0.357} * TR_{vt}{}^{0.039}$$

5.2.4 Landing gear weight calculation

$$W_{main\ landing\ gear} = 0.12855 * (N_l * W_l)^{0.768} * (L_m)^{0.409}$$

$$W_{nose\ landing\ gear} = 0.242 * (N_l * W_l)^{0.566} * (L_n)^{0.845}$$

5.3 Equipment weight calculation

$$W_{fuel\ system} = 64.7374 * V_t^{0.726} * \left(\frac{1}{1 + {}^{V_i}/_{V_t}}\right)^{0.363} * N_t^{0.242} * N_{en}^{0.157}$$

$$W_{flight\ controls} = 0.43613 * L^{1.536} * B_w^{0.371} * (N_z * W_{dg} * 10^{-4})^{0.8}$$

$$W_{hydraulics} = 1.1734 * K_h * W^{0.8} * M^{0.5}$$

$$K_h - 0.05\ (low\ subsonic), 0.11\ (medium\ subsonic), 0.12\ (high\ subsonic)$$

$$W_{electrical} = 8.533 * (W_{fuel\ system} + W_{avionics})^{0.51}$$

$$W_{avionics} = 2 * W_{uav}^{0.933}$$

$W_{uav} - uninstalled\ avionics\ weight\ (350 - 650\ kg), [kg]$

$$W_{air\ conditioning\ and\ anti-ice} = 0.2074 * W_{dg}^{0.52} * N_p^{0.68} * W_{avionics}^{0.17} * M^{0.08}$$

$$W_{furnishings} = 0.0582 * W_{dg} - 29.48$$

6. AERODYNAMICS

6.1 Polar coefficient and lift coefficient

$$e = \frac{1}{1 + 0.025 * AR_w} - Oswald\ coefficient$$

$$A = \frac{1}{pi * AR_w * e} - polar\ coefficient$$

$$C_{Lcruise} = 0.71 * sqrt(pi * AR_w * C_{D0}) - optimal\ lift\ coefficient$$

6.2 Drag coefficient

$$C_D = C_{D0} + A * C_L^2 - optimal\ C\_lcruise$$

$$K_{int} \sim 0.2 - 0.25 - interference\ coefficient\ (depends\ on\ position\ of\ wing)$$

$$\nu_\mu = -0.04 * M^2 - 0.03 * M + 1 - flow\ compressibility\ coefficient$$

6.2.1 Fuselage (nacelle) drag

$$Re_{fus} = V_{cruise} * \frac{L}{\mu} - Reynolds\ number$$

$$C_{f_{fus}} = 0.0454 * Re_{fus}^{-0.189} - flat\ plate\ friction\ coefficient$$

$$\nu_{lambda_{fus}} = 1.7564 * AR_{fus}^{-0.225} - fuselage\ shape\ coefficient$$

$$C_{D0_{fus_{friction}}} = 2 * C_{f_{fus}} * \nu_\mu * \nu_{lambda_{fus}} * \frac{S_f}{2 * S_{fus}} - friction\ drag\ coefficient$$

$$C_{D0_{par}} = (1.0699 * M^3 - 2.2393 * M^2 + 1.6016 * M - 0.3859) - 0.01 * (AR_{nose} - 2) - parabolic\ nose\ drag\ coefficient$$

$$delta_{C_{D_{dump}}} = 1.6667 * M^3 - 2.1786 * M^2 + 0.8512 * M - 0.0386 - dumping\ coefficient$$

$$C_{D0_{nose}} = C_{D0_{par}} * \left(1 - r_{relative}^2 * \left(cos(\varphi)\right)^2 * \left(3.1 - 1.4 * r_{relative} * cos(\varphi) - 0.7 * r_{relative}^2 * \left(cos(\varphi)\right)^2\right)\right) + delta_{C_{D_{dump}}} * r_{relative}^2 - nose\ drag\ coefficient$$

$$C_{D0_{aft}} = (0.5455 * M^2 - 0.6764 * M + 0.2698) - 0.013 * \left(AR_{aft} - 2\right) - aft\ part\ drag\ coefficient$$

$$C_{D0_{fus_{pressure}}} = C_{D0_{nose}} + C_{D0_{aft}} - pressure\ drag\ coefficient$$

$$C_{D0_{fuselage}} = C_{D0_{fus_{friction}}} + C_{D0_{fus_{pressure}}} - fuselage\ drag\ coefficient$$

6.2.2 Wing (or stabilizer or fin) drag

$$K1 = 2 - \frac{S_{with_{eng}}}{S_w} - wing - nacelles\ interference\ coefficient$$

$$\nu_{c_{wing}} = 1 + 1.5 * t/c - airfoil\ thickness\ coefficient$$

$$Re_{wing} = V_{cruise} * \frac{mac}{\mu} - Reynolds\ number$$

$$C_{f_{wing}} = 0.0454 * Re_{wing}.^{-0.189} - flat\ plate\ friction\ coefficient$$

$$C_{D0_{form_{wing}}} = 0.925 * K1.* C_{f_{wing}} * \nu_{\mu} * \nu_{c_{wing}} - form\ drag\ coefficient$$

$$M_{DD_{wing}} = \frac{k_{A_{wing}} - W_{total} * \frac{\frac{g}{q_{cruise} * S_w}}{10 * \left(cos(\Lambda)\right)^2} - \frac{t/c}{cos(\Lambda)}}{cos(\Lambda)}$$

$$- drag\ divergence\ Mach\ number$$

$$M_{cr_{wing}} = M_{DD_{wing}} - 0.108 - critical\ Mach\ number$$

$$C_{D0_{wave_{wing}}} = 20 * \left(M - M_{cr_{wing}}\right).^4 - wave\ drag\ coefficient$$

$$C_{D0_{prime_{wing}}} = C_{D0_{form_{wing}}} + C_{D0_{wave_{wing}}} - clean\ wing\ drag\ coefficient$$

$$C_{D0_{wing}} = C_{D0_{prime_{wing}}} + K_{int}.* C_{D0_{form_{wing}}}.* \frac{S_{wf}}{S_w} - wing\ drag\ coefficient$$

6.3 Zero drag coefficient

$$C_{D0} = 1 * \left(C_{D0_{fuselage}}.* \frac{S_{fus}}{S_w} + C_{D0_{wing}} + C_{D0_{ht1}} * \frac{S_{ht1}}{S_w} + C_{D0_{ht2}} * \frac{S_{ht2}}{S_w} + C_{D0_{vt}} * \frac{S_{vt}}{S_w} + n_{eng} * C_{D0_{nacelle}} * \frac{S_{nac}}{S_w}\right) - zero\ drag\ coefficient$$

7. OUTPUTS

7.1 Takeoff

$$Cl_{takeoff} = W_{total} * \frac{g}{S_w * 0.5 * 1.225 * V_{takeoff}^2} - Takeoff\ lift\ coefficient$$

$$Cd_{takeoff} = C_{D0} + A * Cl_{takeoff}^2 - Takeoff\ drag\ coefficient$$

$$X_{takeoff} = Cd_{takeoff} * V_{takeoff}^2 * 1.225 * 0.5 * S_w - Drag\ force\ [N]$$

$$R_{takeoff} = P_{en_{max}} * N_{en} * 1000 * 0.0155 - Thrust\ [N]$$

$$F_{takeoff} = f_{friction_{to}} * \left(W_{total} * g - Cl_{takeoff} * V_{takeoff}^2 * 0.5 * 1.225 * S_w\right)$$

$$- Friction\ force\ [N]$$

$$a_{takeoff} = g * \frac{\frac{R_{takeoff} - X_{takeoff} - F_{takeoff}}{W_{total}}}{10} - Acceleration\ [m/s2]$$

$$L_{takeoff} = V_0^2 * \frac{0.5}{a_{takeoff}} - Takeoff\ distance\ [m]$$

$$W_{fuel_{takeoff}} = 2 * SFC_{eng} * R_{takeoff} * \frac{L_{takeoff}}{Prop_{eff}} - Takeoff\ consumed\ fuel\ [kg]$$

7.2 Landing

$$Cl_{landing} = W_{total} * \frac{g}{S_w * 0.5 * 1.225 * V_{landing}^2} - Landing\ lift\ coefficient$$

$$Cd_{landing} = C_{D0} * 5 + A * Cl_{landing}^2 - Landing\ drag\ coefficient$$

$$a_{landing} = g * \left(2 * \frac{f_{friction_{land}}}{3} + \frac{1}{3} * \frac{Cd_{landing}}{Cl_{landing}}\right) - Acceleration\ [m/s2]$$

$$L_{landing} = V_{landing}^2 * \frac{0.5}{a_{landing}} - Landing\ distance\ [m]$$

$$W_{fuel_{landing}} = 2 * SFC_{eng} * 0.06 * R_{takeoff} * \frac{L_{takeoff}}{Prop_{eff}} - Landing\ consumed\ fuel\ [kg]$$

7.3 Climb

$$W_{fuel_{climb}} = W_{total} - exp\left(log(W_{total}) - SFC_{eng} * 0.001 * \frac{0.000277}{Prop_{eff}} * g * \left(\frac{cos(phi_{climb})}{19} + sin(phi_{climb})\right) * l_{climb}\right) - Climb\ consumed\ fuel\ weight\ [kg]$$

7.4 Descend

$$W_{fuel_{descend}} = W_{total} - exp\left(log(W_{total}) - SFC_{eng} * 0.001 * \frac{0.000277}{Prop_{eff}} * g * \left(\frac{cos(phi_{descend})}{10} - sin(phi_{descend})\right) * l_{descend}\right) - Descend\ consumed\ fuel\ weight\ [kg]$$

7.5 Cruise flight

$$L_{D_{cruise}} = g * \left(W_{total} - m_{fuel_{cruise}}\right) * q_{cruise} * \frac{S_w}{C_{D0} * q_{cruise}^2 * S_w^2 + A * g^2 * \left(W_{total} - m_{fuel_{cruise}}\right).^2} - Cruise\ lift - to - drag\ ratio$$

$$fL\_cruise = Prop\_eff * L\_D\_cruise./(SFC\_eng * 0.001 * 0.000277 * (W\_total - m\_fuel\_cruise) * g);$$

$$L_{cruise} = \int fL_{cruise} * dm_{fuel_{cruise}} - Cruise\ flight\ distance$$

$$Flight_{distance} = L_{cruise} + d_{climb} + d_{descend} - Total\ flight\ distance$$

8. EXAMPLE

Reference case – Piaggio Avanti P.180 aircraft

Pic.4 Piaggio P.180 Avanti

Parameter	P.180	Math.model	Error
Takeoff weight [kg]	5489	5615	2.3%
Flight distance [km]	2592	2321	10.45%
Takeoff distance [m]	972	1066	9.7%
Landing distance [m]	1000	979	2.1%