Mathematical Methods in Engineering and Applied Science
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Problem Set 7. Due on Nov. 23 at 23:59.

(1) Solve the following LP problem both in Matlab using **linprog** (or elsewhere using an analog of **linprog**) and directly by plotting the required regions:

$$\begin{aligned} \text{minimize } f(x) &= x_1 + x_2 \\ \text{subject to } 2x_1 + x_2 &\geq 2 \\ 2x_1 + 2x_2 &\geq 3 \\ x_1 + 2x_2 &\geq 2 \\ 3x_1 - x_2 &\leq 6 \\ 3x_2 - x_1 &\leq 6. \end{aligned}$$

(2) Consider the data:
$x = 1 : 24$
$y = [75, 77, 76, 73, 69, 68, 63, 59, 57, 55, 54, 52, 50, 50, 49, 49, 49, 50, 54, 56, 59, 63, 67, 72]$.
with the cubic, $y = Ax^3 + Bx^2 + Cx + D$, chosen to fit them.

(a) Find the best fit by using **fminsearch** starting with the initial condition $(1, 1, 1, 60)$.
(b) Set up the normal equation and find the least-squares fit by solving it.
(c) Find the fit by using the genetic algorithm starting with the same initial condition as in (a). If there is no convergence, or it is too slow, try other initial conditions.
(d) Compare the computation times by the genetic algorithm with the **fminsearch** method, for example using **tic** and **toc** commands in Matlab. The minima in both cases have to agree with each other within 1%. You should use the same initial conditions in both methods.

(3) Find the Fourier cosine series of $\sin x$ and sine series of $\cos x$ on $[0,\ \pi]$. Which one does a better job of representing its function and why? Make plots of 2-term as well as 10-term approximations together with the original functions.

(4) Find the complex Fourier series $u = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ of $u = \operatorname{sgn}(x)$ on $[-\pi,\ \pi]$. Plot the magnitude of the coefficients, $|c_n|^2$, as a function of n. What is $\sum_{n=-\infty}^{\infty} |c_n|^2$?

(5) A harmonic oscillator is hit with the force $f(t) = (-1)^k, \quad t \in [k\pi, (k+1)\pi),\ k = 0, 1, 2....$ The equation of motion is $\ddot{x} + \omega_0^2 x = f(t)$ and the initial conditions are $x(0) = \dot{x}(0) = 0$.

(a) Expand $x(t)$ in the Fourier series in t, plug it into the equation and determine the solution. Assume that ω_0 is not an integer. Plot the solution $x(t)$ over $t \in [0, 10\pi]$ at $\omega_0 = \sqrt{2}$ including sufficient number of terms in the series.
(b) (*Optional, for extra credit*). Redo part (a) for $\omega_0 = 1$.

(6) Write down the Fourier matrix F_8 and decompose it into three factors containing F_4. Let $f = \text{zeros}(8, 1)$ be a zero vector.

(a) Modify f so that $f(1) = 1$, find the Fourier transform $\hat{f}$ of f and plot $\left|\hat{f}\right|^2$.

(b) Now let $f\left(1:2\right)=1$ leaving the other components 0, and plot $\left|\hat{f}\right|^2$.

(c) Do the same with $f\left(1:3\right)=1$. Explain your observations.

(7) (*Optional, for extra credit*). Given the data:
$x=1:10,\ y=[0.78, 1.27, 1.33, 1.69, 1.96, 1.67, 2.07, 2.11, 1.91, 1.92]$,
determine the least squares fit of the form $y=a\left(1-\exp\left(-bx\right)\right)$ by setting up a nonlinear system of equations for a and b and solving it with Newton's method (that you should implement yourself).