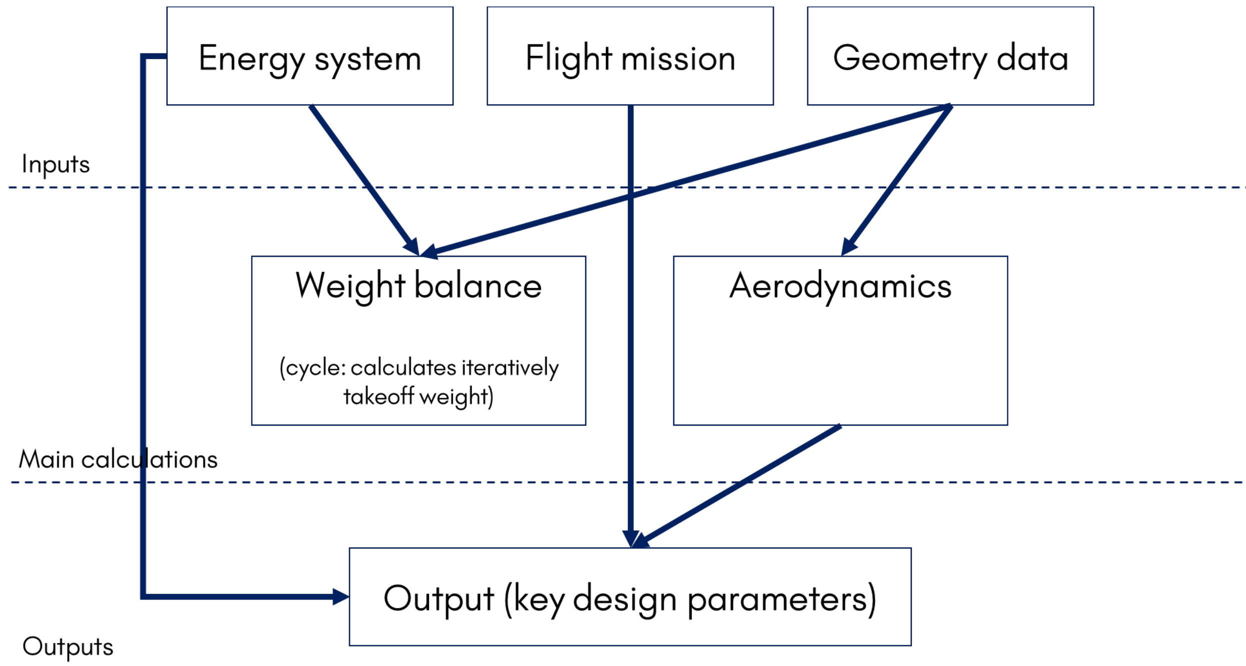
GENERAL EQUATIONS FOR AN AIRCRAFT DESIGN

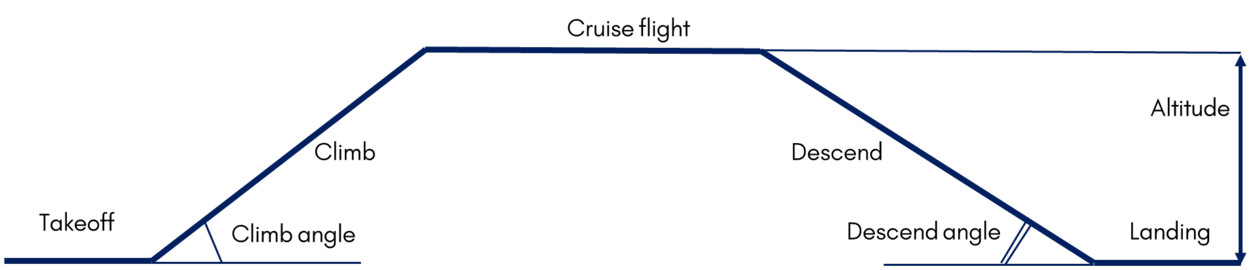
Contents

1. MODEL STRUCTURE



Pic.1 Model structure

2. MISSION DESIGN



Pic.2 Flight plan

2.1 Takeoff

$V_0 - preliminary\ takeoff\ velocity\ [\frac{m}{s}]$

$V_{0\ final} - final\ takeoff\ velocity\ [\frac{m}{s}]$

$V_{takeoff} = V_{0\ final} * 0.7 - average\ takeoff\ velocity\ [\frac{m}{s}]$

$f_{friction_{to}} = 0.04 - friction\ coeffiient$

2.2 Climb

$\theta_{climb} - climb\ angle\ [deg]$

$$l_{climb} = \frac{H}{sin\left(\theta_{climb} * \frac{\pi}{180}\right)} - climb\ diagonal\ distance[m]$$

$$d_{climb} = \frac{H}{tan\left(\theta_{climb} * \frac{\pi}{180}\right)} - climb\ horizontal\ distance[m]$$

2.3 Cruise flight

$H - flight\ altitude\ [m]$

$P_{hf} - Flight\ altitude\ pressure\ [Pa]$

$\rho_{cruise} - Flight\ altitude\ air\ density\ [kg/m3]$

$V_{cruise} - Cruise\ speed\ [m/s]$

$M_{cruise} - Cruise\ Mach\ number$

$q_{cruise} = 0.5 * ro * V_{cruise}^2 - Cruise\ dynamic\ pressure\ [Pa]$

$g = 9.81 - Gravitational\ acceleration\ [m/s2]$

$\mu - kinematic\ viscosity\ parameter\ according\ to\ ISA - 76\ [\frac{m2}{s}]$

2.4 Descend

$\theta_{descend} - descend\ angle\ [deg]$

$$l_{descend} = \frac{H}{sin\left(\theta_{descend} * \frac{\pi}{180}\right)} - descend\ diagonal\ distance[m]$$

$$d_{descend} = \frac{H}{tan\left(\theta_{descend} * \frac{\pi}{180}\right)} - descend\ horizontal\ distance[m]$$

2.5 Landing

$V_{landing} - preliminary\ landing\ velocity\ \left[\frac{m}{s}\right]$

$V_{landing\ final} - final\ landing\ velocity\ [\frac{m}{s}]$

$f_{friction_{land}} =\ 0.3 - landing\ friction\ coefficient$

2.6 Load factors

$P_{cabin} - Cabin\ pressure\ (equal\ to\ pressure\ \ at\ 2000\ m\ altitude)\ [Pa]$

$P_{delta} - cabin\ pressure\ differential, [Pa]$

$n\ =\ 3\ -\ Limit\ load\ factor$

$N_z = 1.5 * limit\ load\ factor\ (3 - 4\ for\ normal\ category\ aircraft) - ultimate\ load\ factor$

$N_l = 1.5 * n_{landing\ gear} - ultimate\ landing\ load\ factor$

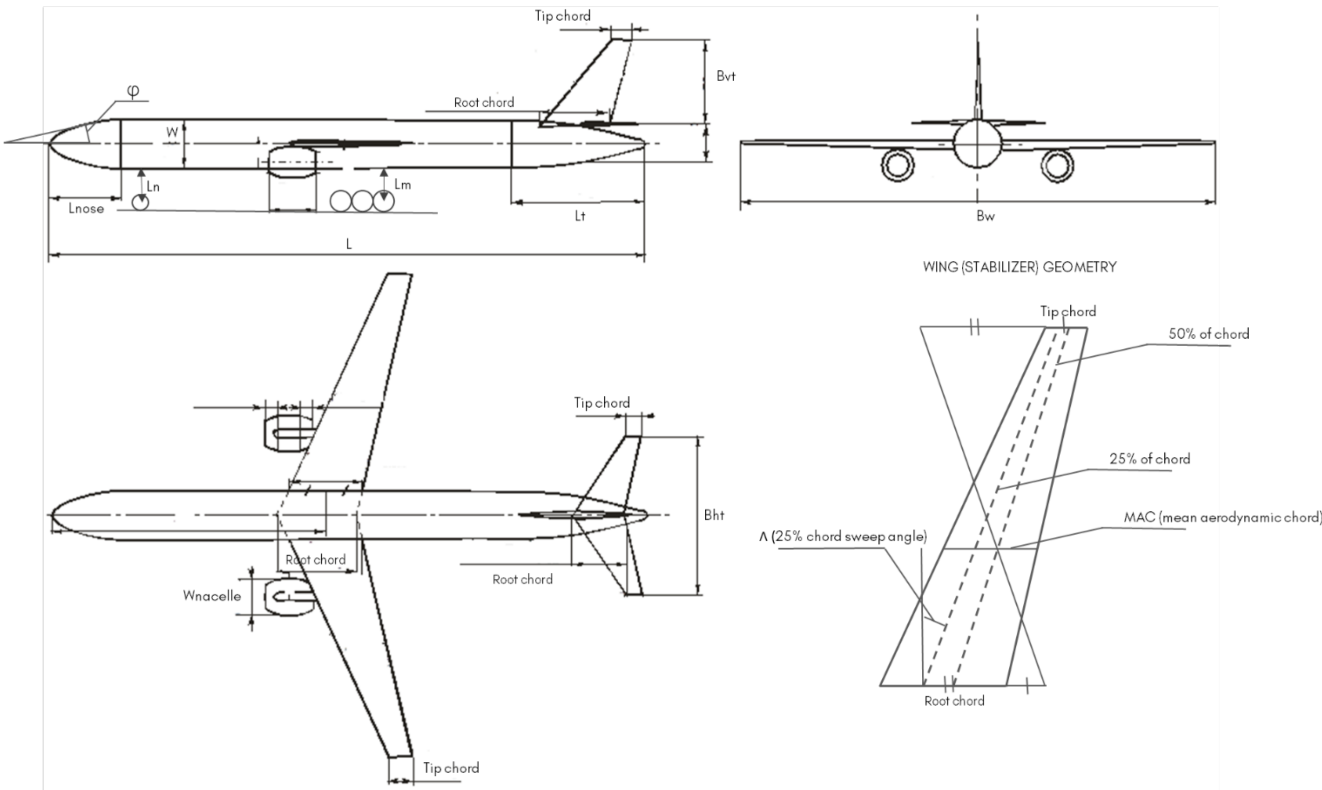
2.7 Payload

$N_p - number\ of\ personnel\ onboard\ (crew\ and\ passengers)$

$W_{payload} - Payload\ weight\ [kg]$

$P_{delta} - cabin\ pressure\ differential, [Pa]$

3. GEOMETRY DATA



Pic.3 Aircraft and its geometry parameters

3.1 Fuselage (nacelle)

$V_{pr} - volume\ of\ pressurized\ section, [m^3]$

$S_f - fuselage\ wetted\ area, [m^2]$

$L_t - tail\ length, [m]$

$L_{nose} - nose\ part\ length\ [m]$

$L - fuselage\ structural\ length, [m]$

$D - fuselage\ structural\ depth, [m]$

$W - total\ fuselage\ structural\ width, [m]$

$$S_{fus} = W^2 * \frac{pi}{4} - crossectional\ area\ [m2]$$

$$r_{relative} = \frac{nose\ radius}{fuselage\ radius} - nose\ relative\ radius$$

$$\varphi - nose\ semipart\ angle\ [rad]$$

$$D_{fus} = sqrt\left(4 * \frac{S_{fus}}{pi}\right) - fuselage\ equivalent\ diametr\ [m]$$

$$AR_{fus} = \frac{L}{W} - fuselage\ aspect\ ratio$$

$$AR_{nose} = \frac{L_{nose}}{W} - nose\ part\ aspect\ ratio$$

$$AR_{aft} = \frac{L_t}{W} - rear\ part\ aspect\ ratio$$

3.2 Wing (or stabilizer or fin)

$$k_{A_{wing}} - airfoil\ type\ coefficient\ (supercritical\ (0.97)\ or\ conventional\ (0.88))$$

$$B_w - wing\ span, [m]$$

$$TR_w = \frac{tip\ chord}{root\ chord} - wing\ taper\ ratio$$

$$AR_w = \frac{B_w^2}{S_w} - wing\ aspect\ ratio$$

$$t/c - thickness - to - chord - ratio$$

$$S_w - trapezoidal\ wing\ area, [m^2]$$

$$\Lambda - \text{wing sweep at 25\% MAC}$$

$$mac = \frac{S_w}{B_w} - Mean\ aerodynamic\ chord\ [m]$$

$S_{wf} \ = \ W * mac - wing\ area\ intersected\ with\ fuselage\ [m^2]$

$\frac{H_t}{H_v} = 0\ if\ tail\ is\ conventional, 1\ if\ tail\ is\ T - shaped$

$\overline{mac_f} \approx 0.2 - flap\ chord\ related\ to\ wing\ MAC$

$\overline{b_f} = [0 \ldots 1] - flap\ span\ related\ to\ winspan$

$S_0 = [0 \ldots S_w] - propellerblown\ wing\ area, [m^2]$

$TV_{ht} \approx 1.2 - horizontal\ stabilizer\ tail\ volume$

$TV_{vt} \approx 0.1 - vertical\ stabilizer\ tail\ volume$

LA_{ht}
$= [0.4 * L \ldots 0.6L]$
$- distance\ between\ horizontal\ stabilizer\ and\ center\ of\ gravity, [m]$

LA_{vt}
$= [0.4 * L \ldots 0.6L]$
$- distance\ between\ horizontal\ stabilizer\ and\ center\ of\ gravity, [m]$

$S_{ht} = \frac{TV_{ht} * mac_w * S_w}{LA_{ht}} - horizontal\ stabilizer\ area, [m^2]$

$S_{ht} = \frac{TV_{vt} * mac_w * S_w}{LA_{vt}} - verticalal\ stabilizer\ area, [m^2]$

3.3 Landing gear

$W_l - landing\ design\ gross\ weight, [kg]$

$L_m - extended\ lentgh\ of\ main\ landing\ gear, [m]$

$L_n - extended\ lentgh\ of\ nose\ landing\ gear, [m]$

4. ENERGY SYSTEM

Conventional propulsion (piston or turboprop engine)

$PW \ \approx 150 - 350 - power - to - weight\ ratio, \left[\frac{Wt}{kg}\right]$

$$SFC_{eng} = 0.408 - specific\ fuel\ consumption, \left[\frac{kg}{kW * h}\right]$$

$$SFC_{ground}$$
$$= 2.2 * SFC_{eng}$$
$$- \ specific\ fuel\ consupmtion\ before\ takeoff\ and\ after\ landing, [\frac{kg}{kW * h}]$$

$N_{en} - number\ of\ engines$

$\eta_{eng} \sim 0.85 - electric\ engine\ efficiency$

$N_t - number\ of\ fuel\ tanks$

$W_{fw} - weight\ of\ fuel\ in\ wing, [kg]$

$W_{fuel} - fuel\ weight, [kg]$

$V_t - total\ fuel\ volume, [m^3]$

$V_i - intergral\ tanks\ volume, [m^3]$

$P_{en\,max} = PW * W_{total} - engine\ takeoff\ power\ [kW]$

$SW_{eng} \approx 0.2 - electric\ engine\ weight - to - power\ ratio, [\frac{kg}{kW}]$

$p_{elec} \approx 1.05 - electric\ systems\ power\ penalty\ coefficient$

$t_{ground} = 0.25\ [h] - preparation + start + taxi\ in + taxi\ out\ time$

$$E_{ground} = 0.06 * P_{en\,max} * t_{ground} - energy\ required\ for\ preparation + start + taxi\ in + taxi\ out, [Wh]$$

5. PROPELLER

$$D_{prop} = \frac{B_w - W}{N_{en}} - \Delta_{prop} * N_{en} - propeller\ diameter$$

$$A_{prop} = \frac{\pi * D_{prop}^2}{4} - propeller\ disk\ area, [m^2]$$

$$\Delta_{prop} - distance\ between\ propellers$$

$$F_{prop} = \frac{4 * P_{en\ max}}{\pi * D_{prop}^2} - propeller\ loading, [\frac{Wt}{m^2}]$$

$$Thrust_{max} = 3.21 * P_{en\ max} * F_{prop}^{0.33} - maximum\ engine\ thrust, [N]$$

$$B = \frac{2 * Thrust_{max}}{\rho * V_0^2 * A_{prop}} - loading\ factor\ first\ iteration$$

$$\eta_{prop} - propeller\ efficiency\ (\sim 0.8\ for\ cruise\ flight\ and\ \sim 0.6 - 0.7\ for\ takeoff\ and\ landing)$$

6. WEIGHT BALANCE

$$W_{dg} - flight\ design\ gross\ weight\ (preliminary\ takeoff\ weight), [kg]$$

6.1 Propulsion system weight calculation

$$W_{installed\ engine\ (total)} = 2.421 * W_{en}^{0.922} * N_{en} - installed\ engine\ weight, [kg]$$

$$W_{en} = SW_{eng} * P_{en\ max} - engine\ weight, [kg]$$

6.2 Structures weight calculation

$$W_{structure} = W_{fuselage} + W_{wing} + W_{ht} + W_{main\ landing\ gear} + W_{nose\ landing\ gear} - structure\ weight, [kg]$$

6.2.1 Fuselage weight calculation

$$W_{fuselage} = 0.23 * S_f^{1.086} * \left(N_z * W_{dg}\right)^{0.177} * L_t^{-0.051} * \left(\frac{L}{D}\right)^{-0.072} * q^{0.241} + W_{press} - fuselage\ weight, [kg]$$

$$W_{press} = 1.2926 * \left(V_{pr} * P_{delta}\right)^{0.271} - weight\ penalty\ due\ to\ pressurization, [kg]$$

6.2.2 Wing weight calculation

$$W_{wing} = 0.13817 * S_w^{0.758} * W_{fw}^{0.0035} * \left(\frac{AR_w}{cos^2\Lambda}\right)^{0.6} * q^{0.006} * TR_w^{0.04} * \left(\frac{\frac{100t}{c}}{cos\Lambda}\right)^{-0.3} * \left(N_z * W_{dg}\right)^{0.49} - wing\ weight, [kg]$$

$$W_{fw} - weight\ of\ fuel\ in\ wing, [kg]$$

6.2.3 Empennage weight calculation

$$W_{ht} = 0.01917 * \left(N_z * W_{dg}\right)^{0.414} * q^{0.168} * S_{ht}^{0.896} * \left(\frac{\frac{100t}{c}}{cos\Lambda_{ht}}\right)^{-0.12} * \left(\frac{AR_{ht}}{cos^2\Lambda_{ht}}\right)^{0.043} * TR_{ht}{}^{-0.02} - horizontal\ stabilizer\ weight$$

$$W_{vt} = 0.12 * \left(1 + 0.02 * \frac{H_t}{H_v}\right) * \left(N_z * W_{dg}\right)^{0.376} * q^{0.122} * S_{vt}^{0.873} * \left(\frac{\frac{100t}{c}}{cos\Lambda_{vt}}\right)^{-0.49} * \left(\frac{AR_{vt}}{cos^2\Lambda_{vt}}\right)^{0.357} * TR_{vt}{}^{0.039}$$

$$- vertical\ stabilizer\ weight$$

6.2.4 Landing gear weight calculation

$$W_{main\ landing\ gear} = 0.12855 * (N_l * W_l)^{0.768} * (L_m)^{0.409}$$

$$- main\ landing\ gear\ weight, [kg]$$

$$W_{nose\ landing\ gear} = 0.242 * (N_l * W_l)^{0.566} * (L_n)^{0.845}$$

$$- nose\ landing\ gear\ weight, [kg]$$

6.3 Equipment weight calculation

$$W_{fuel\ system} = 64.7374 * V_t^{0.726} * \left(\frac{1}{1 + {V_i}/{V_t}}\right)^{0.363} * N_t^{0.242} * N_{en}^{0.157}$$

$$- fuel\ system\ weight, [kg]$$

$$W_{flight\ controls} = 0.43613 * L^{1.536} * B_w^{0.371} * \left(N_z * W_{dg} * 10^{-4}\right)^{0.8}$$

$$- flight\ controls\ weight, [kg]$$

$$W_{hydraulics} = 1.1734 * K_h * W^{0.8} * M^{0.5} - hydraulics\ weight, [kg]$$

$K_h - 0.05\ (low\ subsonic), 0.11\ (medium\ subsonic), 0.12\ (high\ subsonic)$
$- hydraulics\ weight\ coefficient$

$W_{electrical} = 8.533 * \left(W_{fuel\ system} + W_{avionics}\right)^{0.51}$
$- electrical\ system\ weight, [kg]$

$W_{avionics} = 2 * W_{uav}^{0.933} - avionics\ weight, [kg]$

$W_{uav} - uninstalled\ avionics\ weight\ (350 - 650\ kg), [kg]$

$W_{air\ conditioning\ and\ anti-ice} = 0.2074 * W_{dg}^{0.52} * N_p^{0.68} * W_{avionics}^{0.17} * M^{0.08}$

$W_{furnishings} = 0.0582 * W_{dg} - 29.48 - furnishings\ weight, [kg]$

$W_{equipment} = W_{fuel\ system} + W_{flight\ controls} + W_{hydraulics} + W_{electrical} +$
$W_{avionics} + W_{uav} + W_{air\ conditioning\ and\ anti-ice} + W_{furnishings} -$
$equipment\ weight, [kg]$

$W_{total} = W_{fuel} + W_{structure} + W_{equipment} - final\ total\ takeoff\ weight, [kg]$

7. AERODYNAMICS

7.1 Polar coefficient and lift coefficient

$$e = \frac{1}{1 + 0.025 * AR_w} - Oswald\ coefficient$$

$$A = \frac{1}{pi * AR_w * e} - polar\ coefficient$$

$C_{Lcruise} = 0.71 * sqrt(pi * AR_w * C_{D0}) - optimal\ lift\ coefficient$

7.2 Drag coefficient

$C_D = C_{D0} + A * C_L^2 - drag\ coeffcient$

$K_{int} \sim 0.2 - 0.25 - interference\ coefficient\ (depends\ on\ position\ of\ wing)$

$\nu_{\mu} = -0.04 * M^2 - 0.03 * M + 1 - flow\ compressibility\ coefficient$

7.2.1 Fuselage (nacelle) drag

$$Re_{fus} = V_{cruise} * \frac{L}{\mu} - Reynolds\ number$$

$$C_{f_{fus}} = 0.0454 * Re_{fus}^{-0.189} - flat\ plate\ friction\ coefficient$$

$$\nu_{lambda_{fus}} = 1.7564 * AR_{fus}^{-0.225} - fuselage\ shape\ coefficient$$

$$C_{D0_{fus_{friction}}} = 2 * C_{f_{fus}} * \nu_{\mu} * \nu_{lambda_{fus}} * \frac{S_f}{2 * S_{fus}} - friction\ drag\ coefficient$$

$$C_{D0_{par}} = (1.0699 * M^3 - 2.2393 * M^2 + 1.6016 * M - 0.3859) - 0.01 * (AR_{nose} - 2) - parabolic\ nose\ drag\ coefficient$$

$$delta_{C_{D_{dump}}} = 1.6667 * M^3 - 2.1786 * M^2 + 0.8512 * M - 0.0386 - dumping\ coefficient$$

$$C_{D0_{nose}} = C_{D0_{par}} * \left(1 - r_{relative}^2 * \left(cos(\varphi)\right)^2 * \left(3.1 - 1.4 * r_{relative} * cos(\varphi) - 0.7 * r_{relative}^2 * \left(cos(\varphi)\right)^2\right)\right) + delta_{C_{D_{dump}}} * r_{relative}^2 - nose\ drag\ coefficient$$

$$C_{D0_{aft}} = (0.5455 * M^2 - 0.6764 * M + 0.2698) - 0.013 * \left(AR_{aft} - 2\right) - aft\ part\ drag\ coefficient$$

$$C_{D0_{fus_{pressure}}} = C_{D0_{nose}} + C_{D0_{aft}} - pressure\ drag\ coefficient$$

$$C_{D0_{fuselage}} = C_{D0_{fus_{friction}}} + C_{D0_{fus_{pressure}}} - fuselage\ drag\ coefficient$$

7.2.2 Wing (or stabilizer or fin) drag

$$K1 = 2 - \frac{S_{with_{eng}}}{S_w} - wing - nacelles\ interference\ coefficient$$

$$\nu_{c_{wing}} = 1 + 1.5 * t/c - airfoil\ thickness\ coefficient$$

$$Re_{wing} = V_{cruise} * \frac{mac}{\mu} - Reynolds\ number$$

$$C_{f_{wing}} = 0.0454 * Re_{wing}.^{-0.189} - flat\ plate\ friction\ coefficient$$

$$C_{D0_{form_{wing}}} = 0.925 * K1.* C_{f_{wing}} * \nu_{\mu} * \nu_{c_{wing}} - form\ drag\ coefficient$$

$$M_{DD_{wing}} = \frac{k_{A_{wing}} - W_{total} * \frac{\overline{\frac{g}{q_{cruise} * S_w}}}{10 * \left(cos(\Lambda)\right)^2} - \frac{t/c}{cos(\Lambda)}}{cos(\Lambda)}$$
$$- drag\ divergence\ Mach\ number$$

$$M_{cr_{wing}} = M_{DD_{wing}} - 0.108 - critical\ Mach\ number$$

$$C_{D0_{wave_{wing}}} = 20 * \left(M - M_{cr_{wing}}\right).^4 - wave\ drag\ coefficient$$

$$C_{D0_{prime_{wing}}} = C_{D0_{form_{wing}}} + C_{D0_{wave_{wing}}} - clean\ wing\ drag\ coefficient$$

$$C_{D0_{wing}} = C_{D0_{prime_{wing}}} + K_{int}.* C_{D0_{form_{wing}}}.* \frac{S_{wf}}{S_w} - wing\ drag\ coefficient$$

7.2.3 Zero drag coefficient

$$C_{D0} = 1 * \left(C_{D0_{fuselage}}.* \frac{S_{fus}}{S_w} + C_{D0_{wing}} + C_{D0_{ht1}} * \frac{S_{ht1}}{S_w} + C_{D0_{ht2}} * \frac{S_{ht2}}{S_w} + C_{D0_{vt}} * \frac{S_{vt}}{S_w} + n_{eng} * C_{D0_{nacelle}} * \frac{S_{nac}}{S_w}\right) - zero\ drag\ coefficient$$

7.3 Lift coefficient

$\alpha_{takeoff} = [0 \dots 3] - takeoff\ angle\ of\ attack, [deg]$

$\alpha_{landing} = [3 \dots 8] - landing\ angle\ of\ attack, [deg]$

$$C_L^\alpha = 2 * \pi^2 * \frac{AR_w * cos\Lambda_w}{180 * (2 + AR_w)} - lift\ coefficient\ derivative, [\frac{1}{deg}]$$

$C_{L\,\max airfoil} = [1.6 \dots 1.8] - airfoil\ maximum\ lift\ coefficient$

$$C_{L\,\max 0} = 0.5 * k_\eta * C_{L\,\max airfoil} * (1 + cos\Lambda_w)$$
$$- aircraft\ default\ maximum\ lift\ coefficient$$

$$k_\eta = -0.2135 * TR_w^2 + 0.2398 * TR_w + 0.8737 - taper - ratio\ coefficient$$

$$\Delta C_{L\,flap} = k_\delta * \overline{mac_f} * \ \overline{b_f} * \frac{mac}{S_w} * C_L^\alpha * 57.3 * cos\Lambda_w$$
$$- additional\ flap\ lift\ coefficient$$

$k_\delta - flap\ coefficient\ (see\ table\ below)$

Flap type	k_δ
Conventional (takeoff)	0.34
Conventional (landing)	0.52
Fowler flap (takeoff)	0.7
Fowler (landing)	1.33
Double-slotted Fowler flap (takeoff)	0.79
Double-slotted Fowler flap (landing)	1.49

$$C_{L\,max\ without\ prop} = C_{L\,\max 0} + \Delta C_{L\,flap}$$
$$- maximum\ aircraft\ lift\ coefficient\ excluding\ propeller\ effect$$

$$C_{L\,max\ final} = C_{L\,max\ without\ prop} - aircraft\ lift\ coefficient\ first\ iteration$$

$$V_{\max takeoff} = V_0 - maximum\ takeoff\ lift\ coefficient\ velocity\ first\ iteration, [\frac{m}{s}]$$

$V_{\max landing}$
$= V_{landing}$
$-$ $maximum\ landing\ lift\ coefficient\ velocity\ first\ iteration, [\frac{m}{s}]$

$B_{takeoff} = B$ - $takeoff\ loading\ factor\ first\ iteration$

$B_{landing} - landing\ loading\ factor\ first\ iteration$

Takeoff lift coefficient loop

$Lift_{\max 0} = 0.68 * C_{L\ max\ final} * \frac{(\rho * V_{max})^2}{2} * S_w -$
$maximum\ takeoff\ lift\ force, [N]$

$$Drag_{max\ 0} = \left(C_{D0} + A * C_{L\ max\ final}^2\right) * \frac{\left(\rho * V_{\max takeoff}\right)^2}{2} * S_w$$
$- maximum\ takeoff\ drag\ force, [N]$

$$\theta_{eff} = arctg\left(\frac{Lift_{\max 0}}{Thrust_{max} - Drag_{max\ 0}}\right)$$
$- effective\ flow\ deflection\ angle, [deg]$

$$C_\mu = B * \frac{A_{prop}}{S_0} - propeller\ coefficient$$

ΔC_L^α
$$= \pi * \frac{1.152 * \sqrt{C_\mu} + 1.106 * C_\mu + 0.051 * C_\mu * \sqrt{C_\mu}}{180}$$
$- derivative\ of\ a\ lift\ coefficient\ increment\ by\ the\ angle\ of\ attack, [\frac{1}{deg}]$

ΔC_L^θ
$$= \pi * \frac{-0.19356 * C_\mu^2 + 2.87645 * C_\mu + 6.2 * \bar{b_f}}{180}$$
$- derivative\ of\ a\ lift\ coefficient\ increment\ by\ the\ angle\ of\ a\ flow\ deflection, [\frac{1}{deg}]$

$$\Delta C_{L\,prop} = \left(\Delta C_L^{\theta} * sin\theta_{eff} + \Delta C_L^{\alpha} * \alpha_{takeoff} - N_{en} * B * \frac{A_{prop}}{S_0} * sin\theta_{eff}\right) * \frac{S_0}{S_w} * \left(1 + \frac{t}{c}\right) * \frac{C_L^{\alpha}}{2 * \pi} + N_{en} * B * \frac{A_{prop}}{S_w} * sin\theta_{eff}$$

$- additional\ lift\ coefficient\ caused\ py\ propeller$

$$C_{L\,max\,final\,takeoff} = C_{L\,max\,without\,prop} + \Delta C_{L\,prop}$$

$- final\ aircraft\ takeoff\ maximum\ lift\ coefficient$

$$V_{\max takeoff} = \sqrt{\frac{2 * W_{total}}{0.68 * C_{L\,max\,final\,takeoff} * \rho * S_w}}$$

$- \ final\ takeoff\ maximum\ lift\ coefficient\ velocity$

$B_{takeoff} = \frac{2 * Thrust_{max}}{\rho * V_{\max takeoff}^2 * A_{prop}} - final\ takeoff\ loading\ factor$

Landing lift coefficient loop

$Lift_{\max landing} = \ 0.68 * C_{L\,max\,final} * \frac{(\rho * V_{\max landing})^2}{2} * S_w - maximum\ landing\ lift\ force, [N]$

$$Drag_{max\,landing} = \left(C_{D0} + A * C_{L\,max\,final}^2\right) * \frac{\left(\rho * V_{\max landing}\right)^2}{2} * S_w$$

$- maximum\ landing\ drag\ force, [N]$

$$\theta_{eff} = arctg\left(\frac{Lift_{\max landing}}{Thrust_{max} - Drag_{max\,landing}}\right)$$

$- effective\ flow\ deflection\ angle, [deg]$

$C_{\mu} = B * \frac{A_{prop}}{S_0} - propeller\ coefficient$

ΔC_L^{α}
$= \pi * \frac{1.152 * \sqrt{C_\mu} + 1.106 * C_\mu + 0.051 * C_\mu * \sqrt{C_\mu}}{180}$
$- derivative\ of\ a\ lift\ coefficient\ increment\ by\ the\ angle\ of\ attack, [\frac{1}{deg}]$

ΔC_L^{θ}
$= \pi * \frac{-0.19356 * C_\mu^2 + 2.87645 * C_\mu + 6.2 * \bar{b_f}}{180}$
$- derivative\ of\ a\ lift\ coefficient\ increment\ by\ the\ angle\ of\ a\ flow\ deflection, [\frac{1}{deg}]$

$\Delta C_{L\,prop} = \left(\Delta C_L^{\theta} * sin\theta_{eff} + \Delta C_L^{\alpha} * \alpha_{takeoff} - N_{en} * B * \frac{A_{prop}}{S_0} * sin\theta_{eff}\right) * \frac{S_0}{S_w}$
$* \left(1 + \frac{t}{c}\right) * \frac{C_L^{\alpha}}{2 * \pi} + N_{en} * B * \frac{A_{prop}}{S_w} * sin\theta_{eff}$
$- additional\ lift\ coefficient\ caused\ py\ propeller$

$C_{L\,max\,final\,landing}$
$= C_{L\,max\,without\,prop} + \Delta C_{L\,prop}$
$- final\ aircraft\ landing\ maximum\ lift\ coefficient$

$V_{\max landing} = \sqrt{\frac{2 * W_{total}}{0.68 * C_{L\,max\,final} * \rho * S_w}}$
$- \ final\ landing\ maximum\ lift\ coefficient\ velocity, [\frac{m}{s}]$

$B_{landing} = \frac{2 * Thrust_{max}}{\rho * V_{\max landing}^2 * A_{prop}} - final\ landing\ loading\ factor$

8. OUTPUTS

8.1 Takeoff

$F_{takeoff} = f_{friction_{to}}$
$* \left(W_{total} * g\ - C_{L\,max\,final\,takeoff} * V_{takeoff}^2 * 0.5 * \rho * S_w\right)$
$- Friction\ force, [N]$

$$a_{takeoff} = g * \frac{\frac{Thrust_{prop} - Drag_{max\,0} - F_{takeoff}}{W_{total}}}{10} - Acceleration\ [m/s2]$$

$$L_{takeoff} = V_{takeoff}^2 * \frac{0.5}{a_{takeoff}} - Takeoff\ distance\ [m]$$

$$W_{fuel_{takeoff}} = SFC_{eng} * 0.001 * 0.000277 * Thrust_{prop} * \frac{L_{takeoff}}{\eta_{eng} * \eta_{prop}} * p_{elec}$$

$$- Takeoff\ consumed\ fuel\ [kg]$$

8.2 Landing

$$Cd_{landing} = C_{D0} + A * C_{L\,max\,final\,landing}^2 - Landing\ drag\ coefficient$$

$$a_{landing} = g * \left(2 * \frac{f_{friction_{land}}}{3} + \frac{1}{3} * \frac{Cd_{landing}}{C_{L\,max\,final\,landing}}\right)$$

$$- Acceleration, [m/s2]$$

$$L_{landing} = V_{\max landing}^2 * \frac{0.5}{a_{landing}} - Landing\ distance, [m]$$

$$W_{fuel_{landing}} = SFC_{eng} * 0.001 * 0.000277 * 0.06 * Thrust_{prop} * \frac{L_{landing}}{\eta_{eng} * \eta_{prop}}$$

$$* p_{elec} - Landing\ consumed\ fuel, [kg]$$

8.3 Climb

$$W_{fuel_{climb}} = W_{total}$$

$$- exp\left(log(W_{total}) - SFC_{eng} * 0.001 * \frac{0.000277}{\eta_{eng} * \eta_{prop}} * g * p_{elec}\right.$$

$$\left. * \left(\frac{cos(phi_{climb})}{15} + sin(phi_{climb})\right) * l_{climb}\right)$$

$$- Climb\ consumed\ fuel\ weight\ [kg]$$

8.4 Descend

$$W_{fuel_{descend}} = W_{total} - exp\left(log(W_{total}) - SFC_{eng} * 0.001 * \frac{0.000277}{\eta_{eng} * \eta_{prop}} * p_{elec} * g * \left(\left(\frac{cos(phi_{descend})}{15} - sin(phi_{descend})\right) * l_{descend}\right)\right.$$

$$- Descend\ consumed\ fuel\ weight\ [kg]$$

8.5 Cruise flight

$$L_{D_{cruise}} = g * \left(W_{total} - m_{fuel_{cruise}}\right) * q_{cruise} * \frac{S_w}{C_{D0} * q_{cruise}^2 * S_w^2 + A * g^2 * \left(W_{total} - m_{fuel_{cruise}}\right).^2}$$

$$- Cruise\ lift - to - drag\ ratio$$

$$m_{fuel\ cruise} = \mathrm{W_{fuel}} - W_{fuel_{takeoff}} - W_{fuel_{landing}} - W_{fuel_{climb}} - W_{fuel_{descend}} - \frac{SFC_{ground}*0.001}{E_{ground}} - cruise\ fuel\ mass, [kg]$$

$$fL_{cruise} = \eta_{eng} * \eta_{prop} * p_{elec} * L/D_{cruise}./(SFC_{eng} * 0.001 * 0.000277 * (W_{total} - m_{fuel\ cruise}) * g)$$

$$L_{cruise} = \int_0^{m_{fuel\ cruise}} fL_{cruise} * dm_{fuel_{cruise}} - Cruise\ flight\ distance$$

$$Flight_{distance} = L_{cruise} + d_{climb} + d_{descend} - Total\ flight\ distance$$

9. EXAMPLE

Reference case – Piaggio Avanti P.180 aircraft

Pic.4 Piaggio P.180 Avanti

Parameter	P.180	Math.model	Error
Takeoff weight [kg]	5489	5615	2.3%
Flight distance [km]	2592	2321	10.45%
Takeoff distance [m]	972	1066	9.7%
Landing distance [m]	1000	979	2.1%