12.2

i) $pV = f(T) \Rightarrow p = \frac{1}{V} f(T), \quad V = \frac{1}{p} f(T)$

$$\Rightarrow \left.\frac{\partial p}{\partial T}\right|_V = \frac{1}{V}\frac{\partial f}{\partial T}, \quad \left.\frac{\partial V}{\partial T}\right|_p = \frac{1}{p}\frac{\partial f}{\partial T}$$

ii) from $\left.\frac{\partial Q}{\partial T}\right|_p = C_p, \quad \left.\frac{\partial Q}{\partial T}\right|_V = C_V$

$$\Rightarrow \left.\frac{\partial Q}{\partial V}\right|_p = \left.\frac{\partial Q}{\partial T}\frac{\partial T}{\partial V}\right|_p = C_p \left.\frac{\partial T}{\partial V}\right|_p$$

$$\left.\frac{\partial Q}{\partial p}\right|_V = \left.\frac{\partial Q}{\partial T}\frac{\partial T}{\partial p}\right|_V = C_V \left.\frac{\partial T}{\partial p}\right|_V$$

iii) adiabatic change means $dQ = 0$

$$\therefore \left.\frac{\partial Q}{\partial p}\right|_V dp + \left.\frac{\partial Q}{\partial V}\right|_p dV = 0 \Rightarrow \frac{\partial p}{\partial V} = -\left.\frac{\partial Q}{\partial V}\right|_p \Big/ \left.\frac{\partial Q}{\partial p}\right|_V$$

$$\Rightarrow \frac{\partial p}{\partial V} = \frac{-C_p \left(\frac{\partial T}{\partial V}\right)_p}{C_V \left.\frac{\partial T}{\partial p}\right|_V} = -\frac{C_p}{C_V}\frac{\frac{p}{\frac{\partial f}{\partial T}}}{\frac{V}{\frac{\partial f}{\partial T}}} = -\gamma\frac{p}{V}$$

$$\Rightarrow \frac{\partial p}{p} = -\gamma\frac{\partial V}{V} \Rightarrow \ln p = C \ln\frac{1}{V^\gamma} \Rightarrow pV^\gamma = k : \text{const.}$$

12.3

as we know $\left.\frac{\partial Q}{\partial T}\right|_p = C_p$ , $\left.\frac{\partial Q}{\partial T}\right|_V = C_V$

we can ~~wrt~~ write $đQ = \left.\frac{\partial Q}{\partial T}\right|_p dT + \left.\frac{\partial Q}{\partial p}\right|_T dp$

or $đQ = \left.\frac{\partial Q}{\partial T}\right|_V dT + \left.\frac{\partial Q}{\partial V}\right|_T dV$

$\therefore$ if we make $A = \left.\frac{\partial Q}{\partial p}\right|_T$ & $B = \left.\frac{\partial Q}{\partial V}\right|_T$

then $đQ = C_p dT + A dp$ or $đQ = C_V dT + B dV$

$\Rightarrow (C_p - C_V) dT = B dV - A dp$

at constant T, $dT = 0$ $\Rightarrow$ $\left(\frac{\partial p}{\partial V}\right)_T = \frac{B}{A}$

adiabatic change : $đQ = 0$

$\therefore dp = -\frac{C_p}{A} dT$ , $dV = -\frac{C_V}{B} dT$

i) $\left.\frac{\partial p}{\partial V}\right|_{\text{adiabatic}} = \frac{C_p}{C_V} \frac{B}{A} = \gamma \left.\frac{\partial p}{\partial V}\right|_T$

ii) $\left.\frac{\partial V}{\partial T}\right|_{\text{adiabatic}} = -\frac{C_V}{B} = -C_V \frac{1}{C_p - C_V} \left.\frac{\partial V}{\partial T}\right|_p$

$= \frac{1}{1-\gamma} \left.\frac{\partial V}{\partial T}\right|_p$

iii) $\left.\frac{\partial p}{\partial T}\right|_{\text{adiabatic}} = -\frac{C_p}{A} = +C_p \frac{1}{C_p - C_V} \left.\frac{\partial p}{\partial T}\right|_V$

$= \frac{\gamma}{\gamma - 1} \left.\frac{\partial p}{\partial T}\right|_V$

12.4

eq (12.36) : $\left.\frac{\partial p}{\partial V}\right|_{adiabatic} = \gamma \left(\frac{\partial p}{\partial V}\right)_T$

isothermal

$\frac{\partial p}{\partial V}$ is a slope on p-V diagram

$\therefore$ adiabats is more steeper than isotherms by $\gamma$

13.4

~~i) $Q_1 = C_p (T$~~

$$
\begin{array}{ccc}
(p_1, V_1, T_1) & \xrightarrow[\text{(i)}]{\text{isobaric}} & (p_1, V_2, T_2) \\
\text{adiabatic} \ \nwarrow \text{(iii)} & & \text{(ii)} \swarrow \ \text{isochoric} \\
 & (p_2, V_2, T_3) &
\end{array}
$$

heat transferred during process

(i) $Q_1 = C_p (T_1 - T_2) = \gamma C_v (T_1 - T_2)$

(ii) $Q_2 = C_v (T_3 - T_2)$

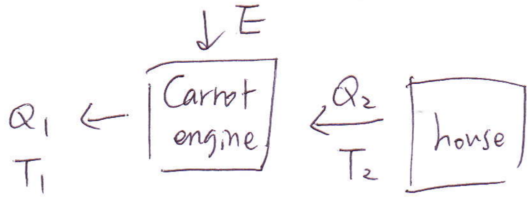
in the case of ideal gas, $pV \propto T$,

$$\therefore \eta = \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{Q_1}{Q_2} = 1 - \gamma \frac{T_1 - T_2}{T_3 - T_2} = 1 - \gamma \frac{p_1 V_1 - p_1 V_2}{p_2 V_2 - p_1 V_2}$$

$$= 1 - \gamma \frac{(V_1/V_2) - 1}{(p_2/p_1) - 1}$$

13.6

steady state




leakage $Q = A(T_1 - T_2)$

i) $Q_2 = Q$ (steady state)

ii) $Q_1 = E + Q_2 \quad = E + A(T_1 - T_2)$

iii) $\Delta S = 0 \quad \therefore -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0 \Rightarrow Q_1 = \frac{T_1}{T_2} Q_2 = Q\frac{T_1}{T_2}$

$$\Rightarrow \frac{T_1}{T_2} A(T_1 - T_2) = E + A(T_1 - T_2)$$

$$\Rightarrow AT_1^2 - AT_1T_2 = ET_2 + AT_1T_2 - AT_2^2$$

$$\Rightarrow T_2^2 - \left(2T_1 + \frac{E}{A}\right) T_2 + T_1^2 = 0$$

$$\therefore T_2 = T_1 + \frac{E}{2A} \pm \sqrt{\left(\cancel{A}T_1 + \frac{E}{2A}\right)^2 - T_1^2}$$

$$= T_1 + \frac{E}{2A} \pm \sqrt{\left(\frac{E}{2A}\right)^2 + \frac{T_1E}{A}}$$

$$E = A(T_1 - T_2)\left(\frac{T_1}{T_2} - 1\right) = \frac{A}{T_2}(T_1 - T_2)^2$$

$$\therefore 0.3E = \frac{A}{293} 10^2 \quad \& \quad E_{max} = \frac{A}{\cancel{29}293}(\Delta T)^2$$

$$\therefore 0.3 = \frac{10^2}{(\Delta T)^2} \Rightarrow (\Delta T)^2 = \frac{10^2}{0.3} \simeq 333.3\cdots$$

$$\therefore \Delta T \simeq 18.26$$

$\therefore$ the highest outside temperature is $38.26^\circ C$

1b.2

(a) (i) $\left(\frac{\partial T}{\partial V}\right)_U = \left(\frac{\partial T}{\partial U}\right)_V \left(\frac{\partial U}{\partial V}\right)_T = -\frac{1}{C_V}\left[T\left(\frac{\partial S}{\partial V}\right)_T - p\right]$

$= -\frac{1}{C_V}\left[T\left(\frac{\partial p}{\partial T}\right)_V - p\right]$ $\quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$

: Joule

(ii) $\left(\frac{\partial T}{\partial V}\right)_S = -\underbrace{\left(\frac{\partial T}{\partial S}\right)_V}_{\frac{T}{C_V}} \underbrace{\left(\frac{\partial S}{\partial V}\right)_T}_{\left(\frac{\partial p}{\partial T}\right)_V} = -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V$

: adiabatic

(iii) $\left(\frac{\partial T}{\partial p}\right)_H = -\left(\frac{\partial T}{\partial H}\right)_p \left(\frac{\partial H}{\partial p}\right)_T = -\frac{1}{C_p}\left[T\left(\frac{\partial S}{\partial p}\right)_T + V\right]$ $\quad \left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$

$= \frac{1}{C_p}\left[T\left(\frac{\partial V}{\partial T}\right)_p - V\right]$

: Joule-kelvin

(b) $pV = nRT$

$\left(\frac{\partial T}{\partial V}\right)_U = 0$ & $\left(\frac{\partial T}{\partial p}\right)_H = 0$ are trivial

$\left(\frac{\partial T}{\partial V}\right)_S = \frac{P}{nR} = -\frac{T}{C_V}\frac{nR}{V} \Rightarrow \frac{dT}{T} = -\frac{nR}{C_V}\frac{dV}{V} = -\frac{C_p - C_V}{C_V}\frac{dV}{V}$

$\Rightarrow T \propto V^{1-\gamma} \Rightarrow V^{-\gamma}(1-\gamma) \propto \frac{P}{C_p - C_V}$

$\therefore pV^{\gamma} = \text{const}$.

1b.3

1st law $dU = dW + dQ = -pdV + dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

$\therefore dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$

$\Rightarrow C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{C_V} + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] \underbrace{\left(\frac{\partial V}{\partial T}\right)_p}_{V\beta_p}$

$\therefore \frac{C_p - C_V}{V\beta_p} - p = \left(\frac{\partial U}{\partial V}\right)_T$

16.5

$$\cancel{dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV}$$

$$dS = \frac{dQ}{T} = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V dT + \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$$ ! eq in 16.3

$$\Rightarrow \left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{T}\left(\frac{\partial U}{\partial T}\right)_V , \quad \left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]$$

as we know $dS$ is the exact differential

$\Rightarrow \frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$ then we can get

$$p = T\left(\frac{\partial p}{\partial T}\right)_V - \left(\frac{\partial U}{\partial V}\right)_T$$