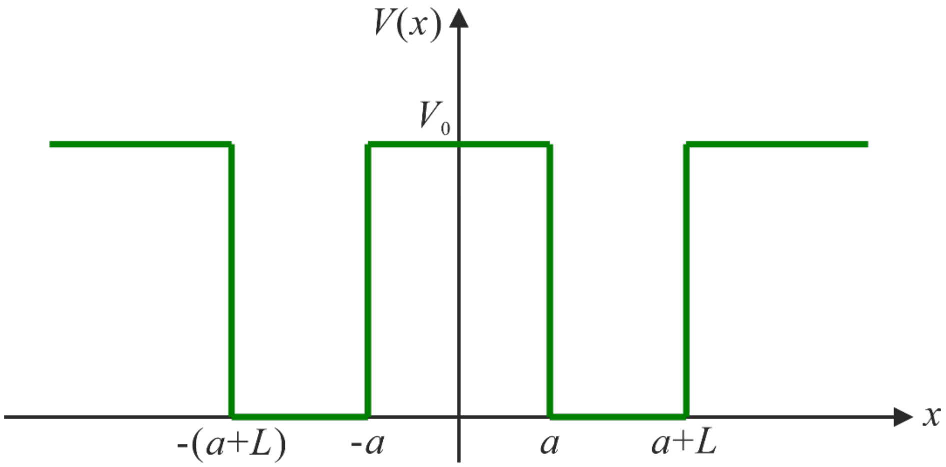
Homework #3 (due date: Thursday, April 4)

1. Let's consider a particle of mass *m* bound in the double potential well depicted in the figure below.



(a) It was mentioned in the class that the eigenstates should have either even parity or odd parity. Prove that the energy eigenvalues ($E<V_0$) satisfy the following equation:

$$\tan k_1 L = \frac{k_1 k_2 \left[1+\left(\tanh k_2 a\right)^{\pm 1}\right]}{k_1^2 - k_2^2 \left(\tanh k_2 a\right)^{\pm 1}} \qquad \left(k_1 = \sqrt{\frac{2mE}{\hbar^2}},\ k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}\right),$$

where (+) and (-) signs correspond to the even and odd states, respectively.

(b) Show that the above condition approaches that for a single potential well as the two wells get closer to each other. (I hope you solved Exercise 5.2.6 of Shankar successfully.)

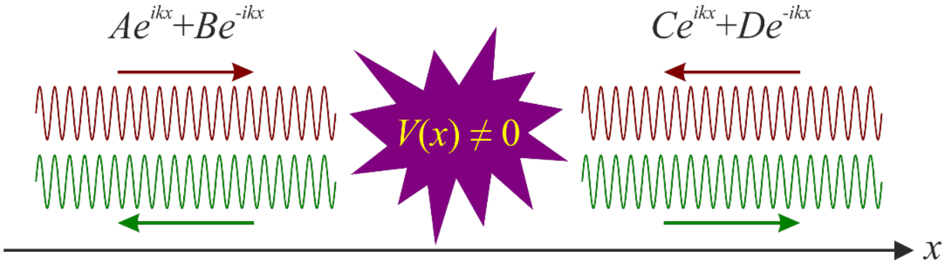
(c) It was also mentioned in the class that the energy eigenvalues for even and odd states are not the same (e.g. inversion doubling of an NH_3 molecule). Obtain the energy difference between the lowest-order even and odd states in the case where the two wells are very far from each other. Check that the energy difference approaches 0 as the separation between the two wells increases.

2. Solve the following exercises in Shankar.

Exercises 5.3.2, 5.3.4, 5.4.2

3. Do the same calculation as Exercise 5.4.2(b), assuming that the energy is *greater* than V_0.

4. In the class, we discussed several 1D scattering problems in which a (quasi-) plane wave is incident from x=–∞. Now, let's modify the situation. Two plane waves are coming to a region of $V(x)\neq 0$. One is incident from x=–∞ and the other from x=+∞ (See the figure below).



(a) We can define a *scattering matrix* S which relates the scattered waves (green waves C & B) to the incident waves (red waves A & D) as follows:

$$\begin{bmatrix} C \\ B \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix}$$

Show that the scattering matrix S is unitary using the concept of probability current.

(b) Obtain the scattering matrices for the three cases you solved (hopefully) above: (i) Shankar 5.4.2(a), (ii) Shankar 5.4.2(b) and (iii) Shankar 5.4.2(b) with $E>V_0$. Are the matrices indeed unitary?