Exercice 1 question 4 Concours général 2023

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Montrer que :

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$f(x) = \left(1 + \frac{1}{x}\right)^x$$

$$\ln(f(x)) = \ln\left(\left(1 + \frac{1}{x}\right)^x\right)$$

$$\ln(f(x)) = x \ln\left(\frac{x+1}{x}\right)$$

$$\ln(f(x)) = x(\ln(x+1) - \ln(x))$$

$$\ln(f(x)) = x \int_x^{x+1} \frac{1}{t}\, dt$$

$$\frac{x+1-x}{x+1} \leq \int_x^{x+1} \frac{1}{t}\, dt \leq \frac{x+1-x}{x}$$

$$\frac{1}{x+1} \leq \int_x^{x+1} \frac{1}{t}\, dt \leq \frac{1}{x}$$

$$\frac{x}{x+1} \leq x \int_x^{x+1} \frac{1}{t}\, dt \leq \frac{x}{x}$$

$$\frac{x+1-1}{x+1} \leq x \int_x^{x+1} \frac{1}{t}\, dt \leq \frac{x}{x}$$

$$\frac{x+1-1}{x+1} \leq x \int_x^{x+1} \frac{1}{t}\, dt \leq \frac{x}{x}$$

$$1 - \frac{1}{x+1} \leq x \int_x^{x+1} \frac{1}{t}\, dt \leq 1$$