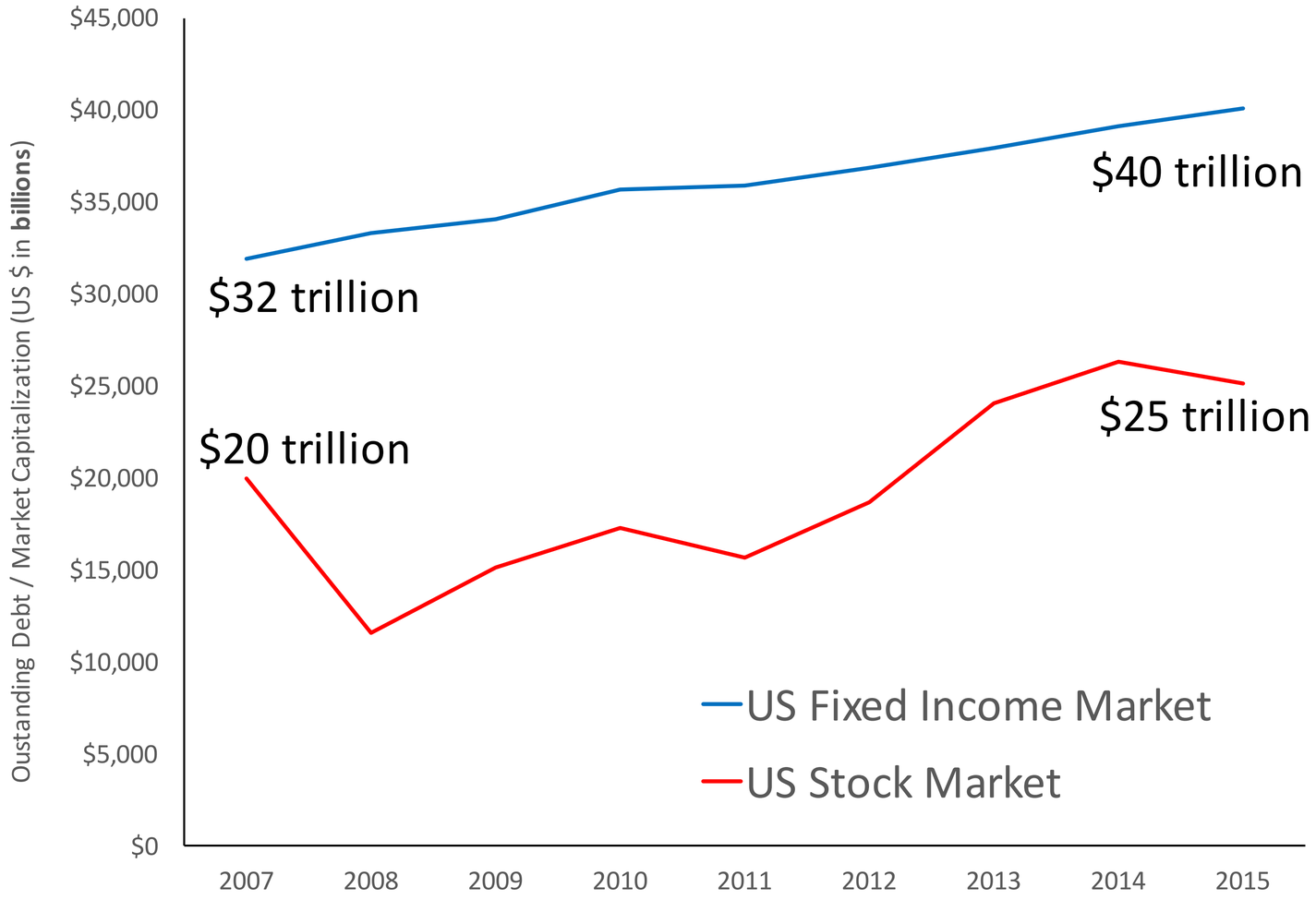
Bond Valuation and Analysis

The Fixed Income Market Is Large...



Sources: Bond data from the Securities Industry and Financial Markets Association; Stock data from the World Bank.

Layout of the Course

- **Chapter 1:** Bond Valuation
- **Chapter 2:** Estimating Yield to Maturity
- **Chapter 3:** Duration and Convexity
- **Chapter 4:** Comprehensive Example

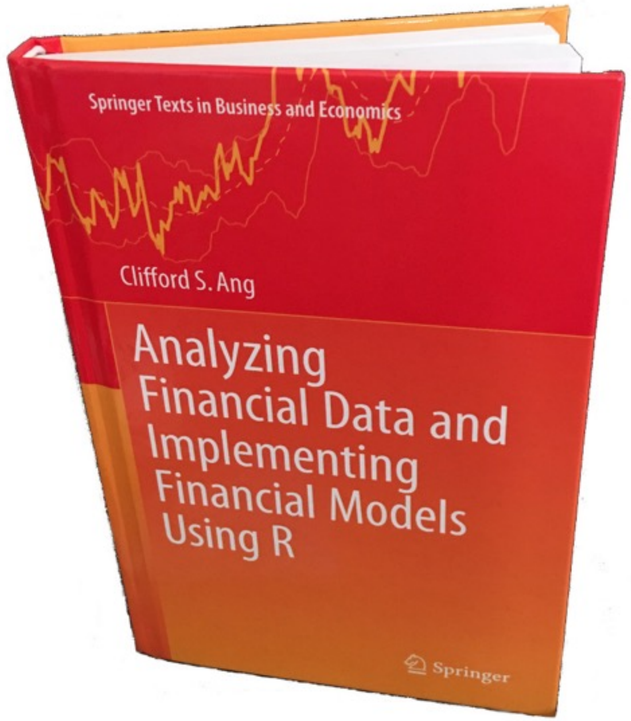
What You Should Know

- Introduction to R
- Intermediate R

- No prior experience with financial analysis necessary!

About me

- Advise clients on valuation and other financial issues primarily related to litigation
- Previously taught investments, investment management, and corporate finance
- Author of *Analyzing Financial Data and Implementing Financial Models Using R*



BOND VALUATION AND ANALYSIS

See you in the course!

BOND VALUATION AND ANALYSIS

Welcome to the Course!

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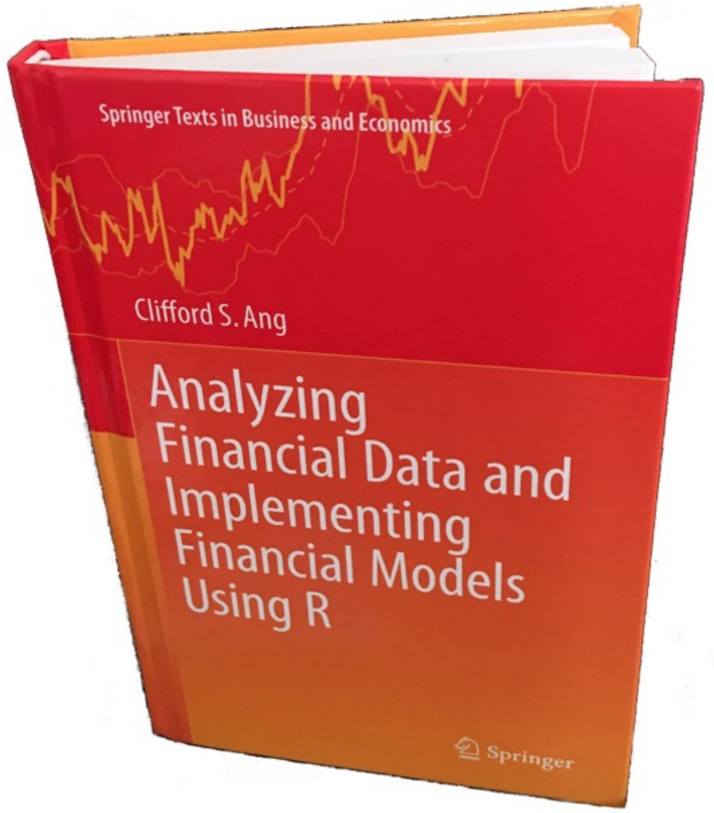
- Author of *Analyzing Financial Data and Implementing Financial Models Using R*



Bonds

- Debt instrument
- Repay borrowed amount + interest
- Allows us to focus on fundamental concepts of bond valuation

Characteristics of a Bond - I

- **Issuer:** The entity that borrows the money
  - Corporations
  - Governments
  - Municipalities
- **Principal:** The amount borrowed
  - Also called **par value** or **face value**

Characteristics of a Bond - II

- **Coupon Rate:** The amount of interest issuer agrees to pay
  - Annually, semi-annually, or quarterly
  - Fixed or floating rate
- **Maturity Date:** Date when principal amount is returned to investor
  - Some bonds do not mature

Characteristics of a Bond - III

- **Embedded Options**
  - Could affect bond's cash flow profile - i.e., can change amount and timing of cash flow
  - For example, **callable bond**
    - Issuer can buyback bond earlier than maturity at a pre-agreed price
  - More complex analysis required

The Bond We Will Use

- Annual coupons
- Fixed rate
- Fixed maturity
- No embedded options

Price vs. Value

- We will use the terms “price” and “value” interchangeably, but there are distinctions:
  - **Price:** Amount paid to acquire asset
  - **Value:** How much the asset is worth
- For actively traded assets, price *may* be considered the best estimate of value

BOND VALUATION AND ANALYSIS

Let’s practice!

BOND VALUATION AND ANALYSIS

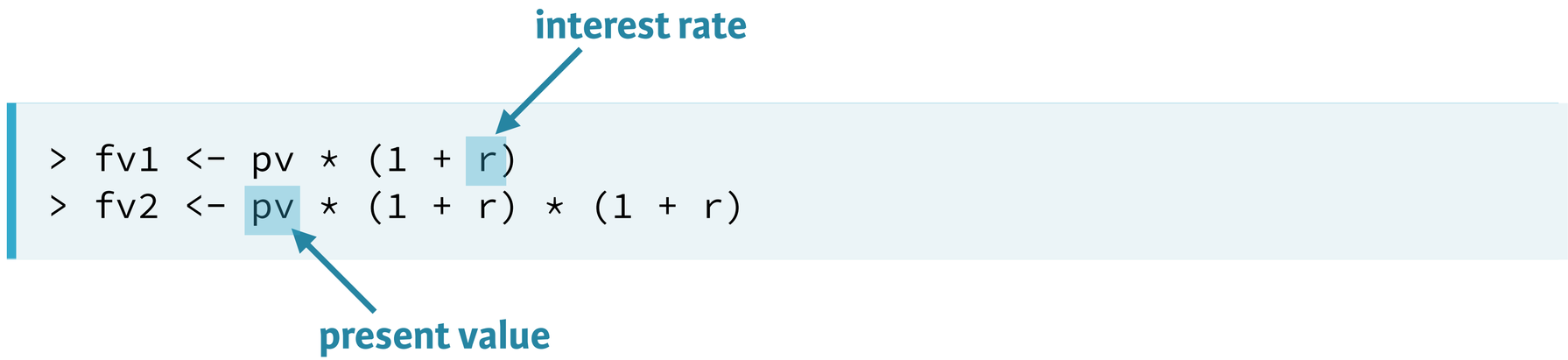
Time Value of Money

Time Value of Money (TVM)

- $1 today is worth more than $1 tomorrow
- Suppose you won $10,000 in a game, what would you choose?
  - Receive the $10,000 today?
  - Receive the $10,000 one year from now?

Future Value

- The future value is the value of $1 at some point in the future
- Prefer $1 today, so **would have to be compensated** to agree to receive the cash flow in the future
- Future value (`fv`) one and two years from now can be calculated as:



Present Value

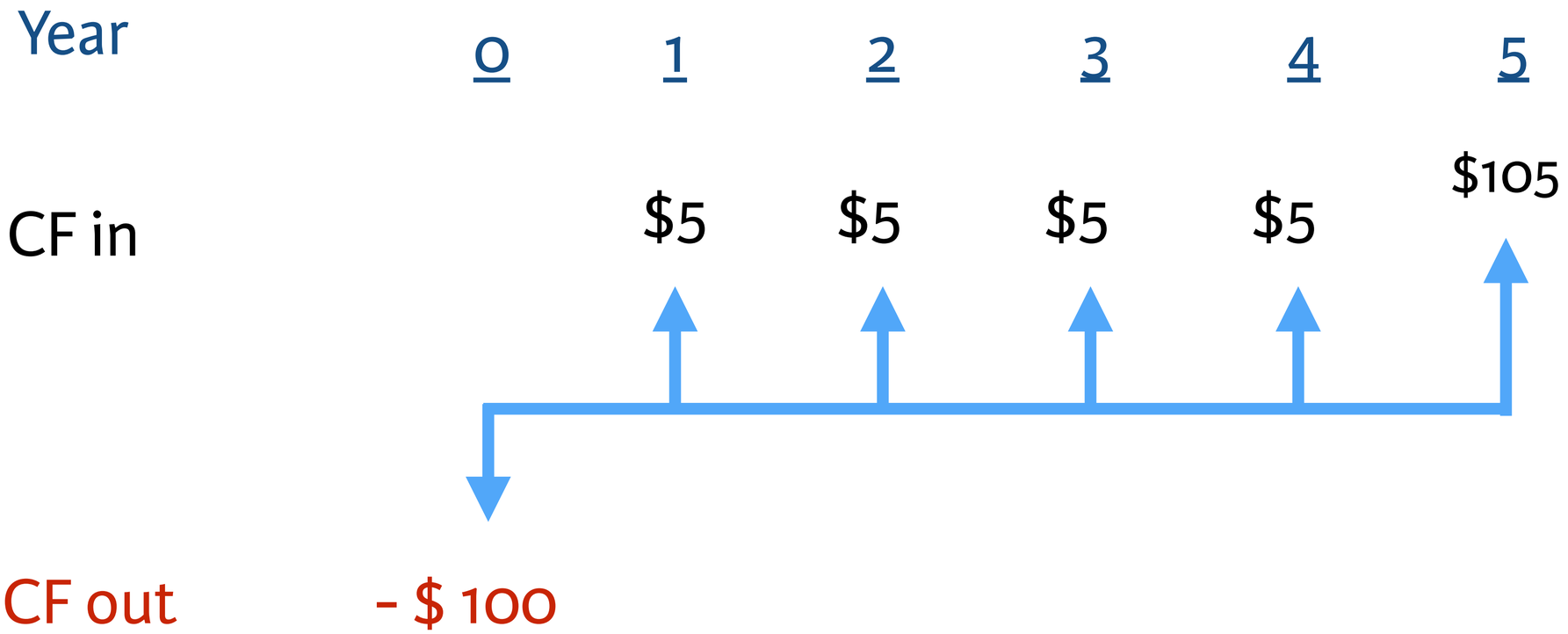
- Reverse logic of future values
- The value of $1 in the future is worth less today
- So you will be willing to take less than $1 today instead of waiting to receive $1 one or two years from now
- This can be calculated as follows:

```
> pv <- fv1 / (1 + r)
> pv <- fv2 / ((1 + r) * (1 + r))
```

TVM Applied To Bonds

- We can apply this Time Value of Money concept to bonds
- Example:
  - $100 par value, 5% coupon rate (= $5),
    5 years to maturity
  - Price = $100 today

Bond Investors' Trade-Off

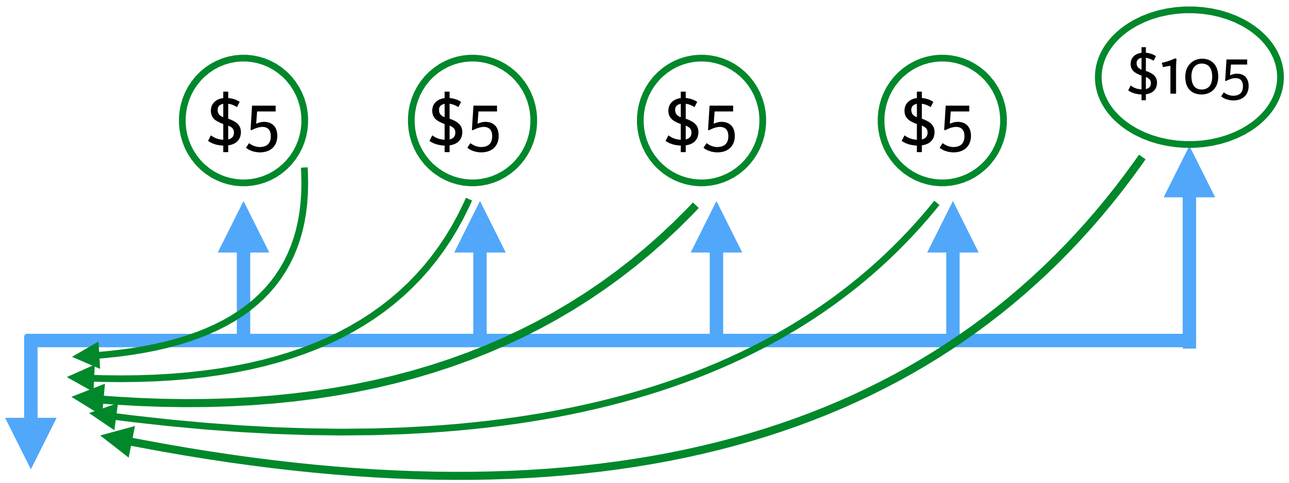


Comparing Cash Flows

Year

0 1 2 3 4 5

CF in



CF out - $ 100

BOND VALUATION AND ANALYSIS

Let’s practice!

BOND VALUATION AND ANALYSIS

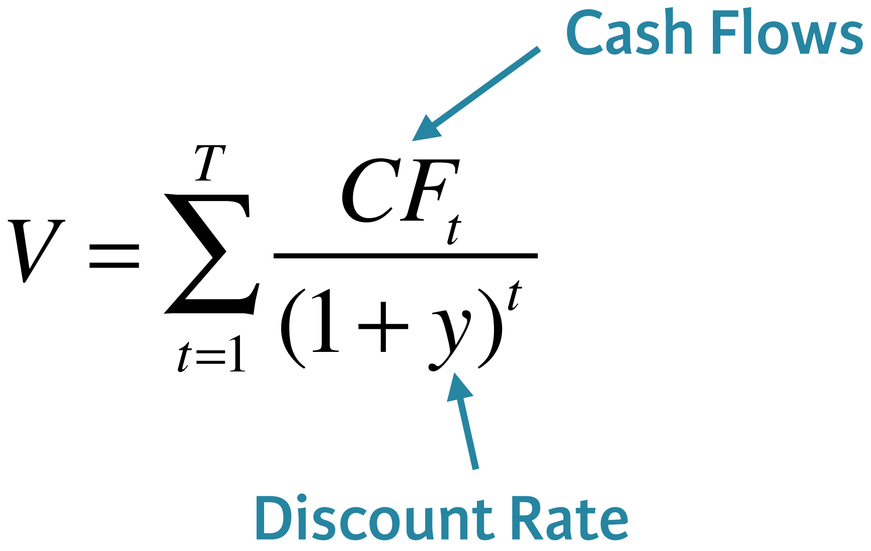
Bond Valuation

Bond Valuation

- In this course, we will consider the following simple bond:
  - Fixed Annual Coupon Rate
  - Fixed Maturity Date
  - Option-free

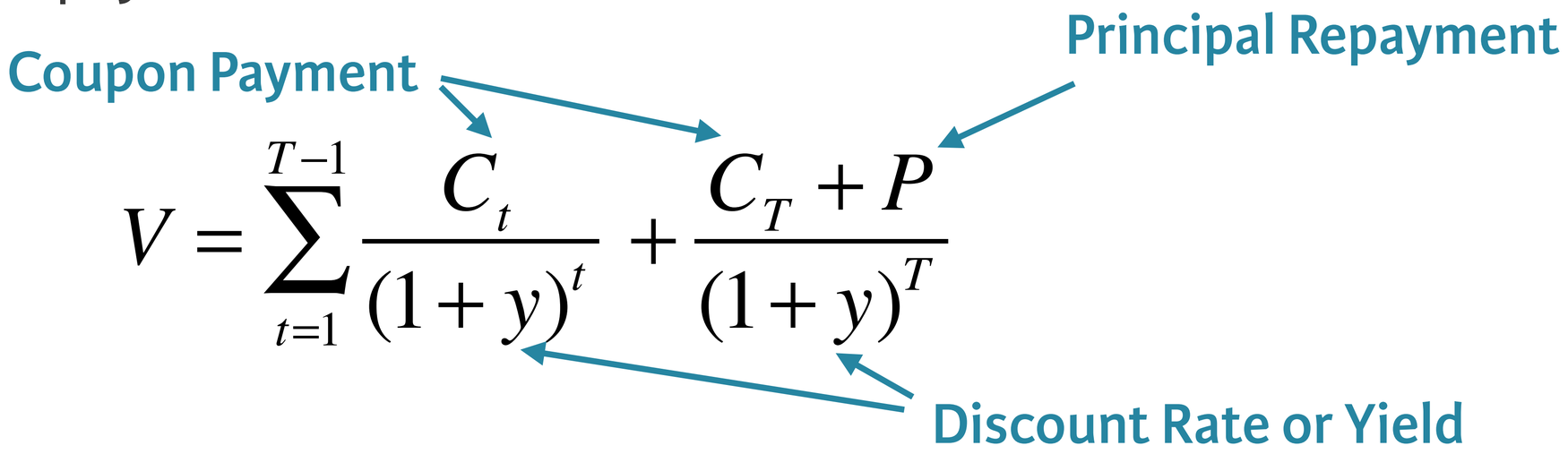
Value of an Asset

- The value of an asset = present value of expected future cash flows
- Cash flows: discounted at the appropriate risk-adjusted discount rate



Laying Out a Bond's Cash Flows

- Prior to maturity, the investor receives coupon payments



- At maturity, the investor receives the last coupon payment and the par value

Creating a Cash Flow Vector

$$V = \sum_{t=1}^{T-1} \frac{C_t}{(1+y)^t} + \frac{C_T + P}{(1+y)^T}$$

```
> cf <- c(c1, c2, c3, c4, c5, . . . )
```

coupon payment

coupon + principal

Converting to Data Frame

- So we can add additional columns, we need to convert the cash flow vector into a data frame
- Use the `data.frame()` command

```
> cf <- data.frame(cf)
```

Creating a Time Index

- Each cash flow occurs at a certain period of time
  - The unit of the periods will be in years
- We create a variable that creates a time index

```
> cf$t <- c(1, 2, 3, 4, 5, . . . )
```

Calculating the PV Factors

- To discount the cash flows, we need a “discount rate”
  - For bonds, the discount rate is called a “yield”
- We create a present value factor used for discounting

```
> cf$pv_factor <- 1 / (1 + y)^cf$t
```

```
> pv_factor <- 1 / (1 + .10)^2
> pv_factor
[1] 0.8264463
```

PV of Cash Flows

- We calculate each cash flow's present value

```
> cf$pv <- cf$cf * cf$pv_factor
```

- The sum of the present values of the bond's cash flow is equal to the bond's value

```
> sum(cf$pv)
```

BOND VALUATION AND ANALYSIS

Let’s practice!

BOND VALUATION AND ANALYSIS

Converting Your Code Into Function

Bond Valuation Function

- We will value many bonds in this course
- Steps described in prior chapter will be repeated
- We will create the `bondprc()` function to simplify calculations

Steps in Bond Valuation - I

- Generalize these inputs:
  - p for par value,
  - r for coupon rate,
  - ttm for time to maturity,
  - y for yield
- We also make some of the code more generic

Steps in Bond Valuation - II

```
> cf <- c(rep(p * r, ttm - 1), p * (1 + r))
```

- `rep(x, y)` - repeats y times the value of x
  - `x = p * r` = coupon payment
  - `y = ttm - 1` = bond's time to maturity minus one year
- `p * (1 + r)` = principal + final coupon payment

Steps in Bond Valuation - III

```
> cf <- data.frame(cf)
```

- Convert to data frame so we can add variables to the data (same as last section)

```
> cf$t <- as.numeric(rownames(cf))
```

- Create time index used for discounting
  - `rownames()` of "cf" vector is equal to 1, 2, 3, 4, until the "ttm" of bond
  - `as.numeric()` needed to ensure values are read as numbers

Steps in Bond Valuation - IV

```
> cf$pv_factor <- 1 / (1 + y)^cf$t
```

- Calculate PV Factor

```
> cf$pv <- cf$cf * cf$pv_factor
```

- Calculate PV of each cash flow

```
> sum(cf$pv)
```

- Sum PV to arrive at bond's value

Wrap the Code

- Create the `bondprc()` function
- This will take as inputs p, r, ttm, and y

```
bondprc <- function(p, r, ttm, y){
  cf <- c(rep(p * r, ttm - 1), p * (1 + r))
  cf <- data.frame(cf)
  cf$t <- as.numeric(rownames(cf))
  cf$pv_factor <- 1 / (1 + y)^cf$t
  cf$pv <- cf$cf * cf$pv_factor
  sum(cf$pv)
}
```

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Let’s practice!