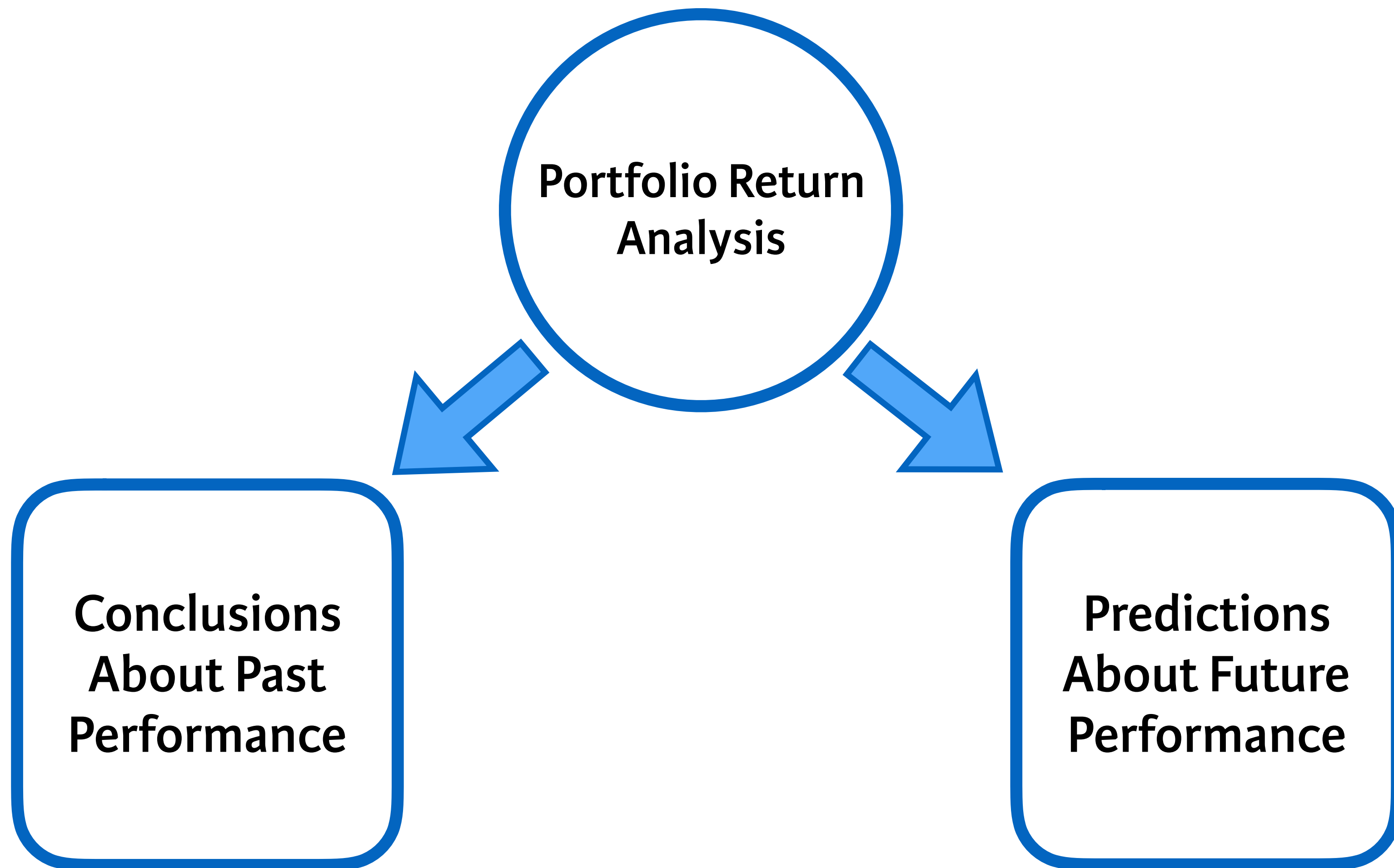




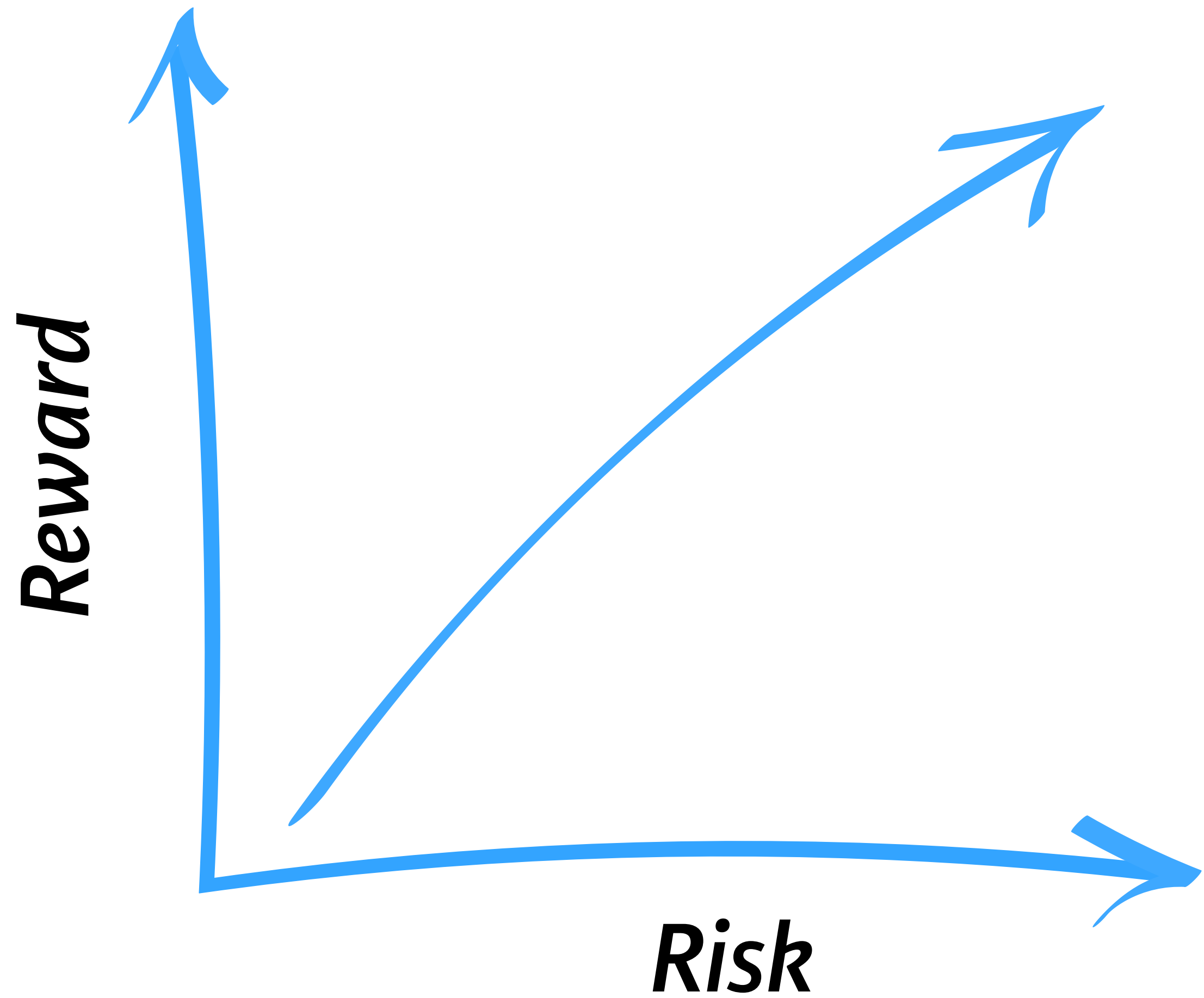
INTRODUCTION TO PORTFOLIO ANALYSIS

Dimensions of Portfolio Performance

Interpretation of Portfolio Returns

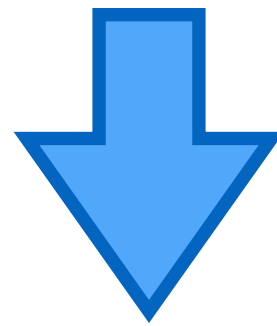


Risk vs. Reward

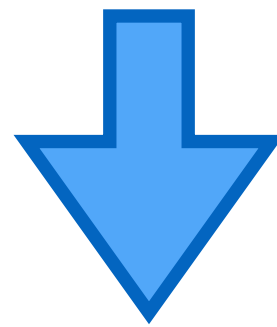


Need For Performance Measure

Portfolio Returns



Performance & Risk Measures
Reward → portfolio mean return
Risk → portfolio volatility



Interpretation

Arithmetic Mean Return

- Assume a sample of T portfolio return observations:

$$R_1, R_2, \dots, R_T$$

- Reward Measurement: Arithmetic mean return is given:

$$\hat{\mu} = \frac{R_1 + R_2 + \dots + R_T}{T}$$

- It shows how large the portfolio return is on average

Risk: Portfolio Volatility

- De-meaned return

$$R_i - \hat{\mu}$$

- Variance of the portfolio

$$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \dots + (R_T - \hat{\mu})^2}{T - 1}$$

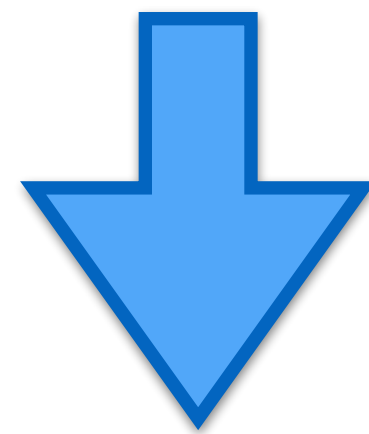
- Portfolio Volatility:

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

No Linear Compensation In Return

- Mismatch between average return and effective return

$$\text{final value} = \text{initial value} * (1 + 0.5) * (1 - 0.5) = 0.75 * \text{initial value}$$



$$\text{Average Return} = (0.5 - 0.5) / 2 = 0$$

Geometric Mean Return

- Formula for *Geometric Mean* for a sample of T portfolio return observations R_1, R_2, \dots, R_T :

$$\text{Geometric mean} = [(1 + R_1) * (1 + R_2) * \dots (1 + R_T)]^{\frac{1}{T}} - 1$$

- Example: +50% & -50% return

$$\begin{aligned}\text{Geometric mean} &= [(1 + 0.50) * (1 - 0.50)]^{\frac{1}{2}} - 1 \\ &= 0.75^{\frac{1}{2}} - 1 \\ &= -13.4\%\end{aligned}$$

Application to the S&P 500

S & P 500



dextergonzaga.info



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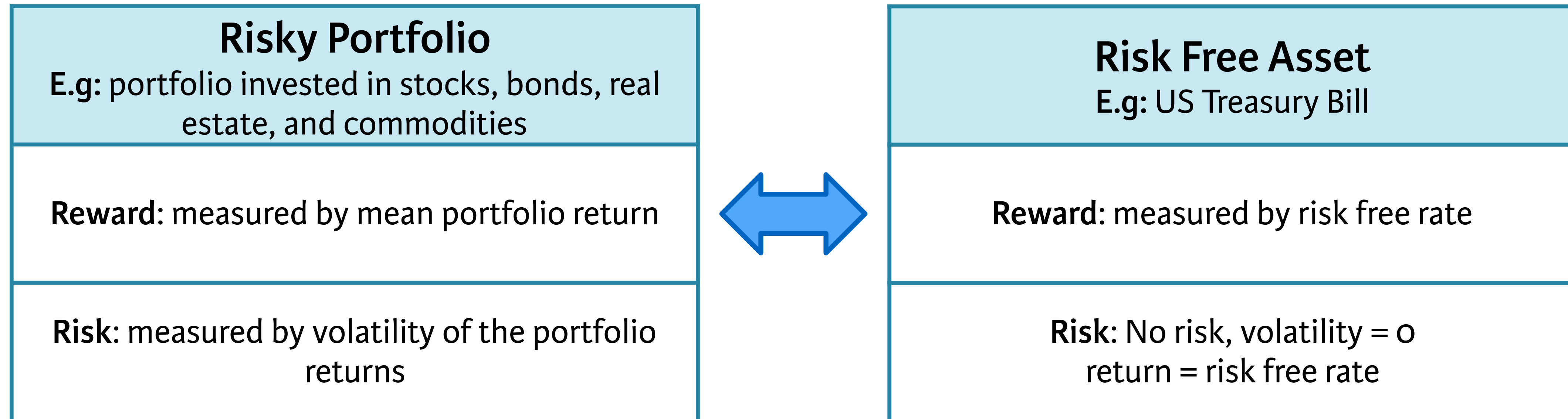
Let's practice!



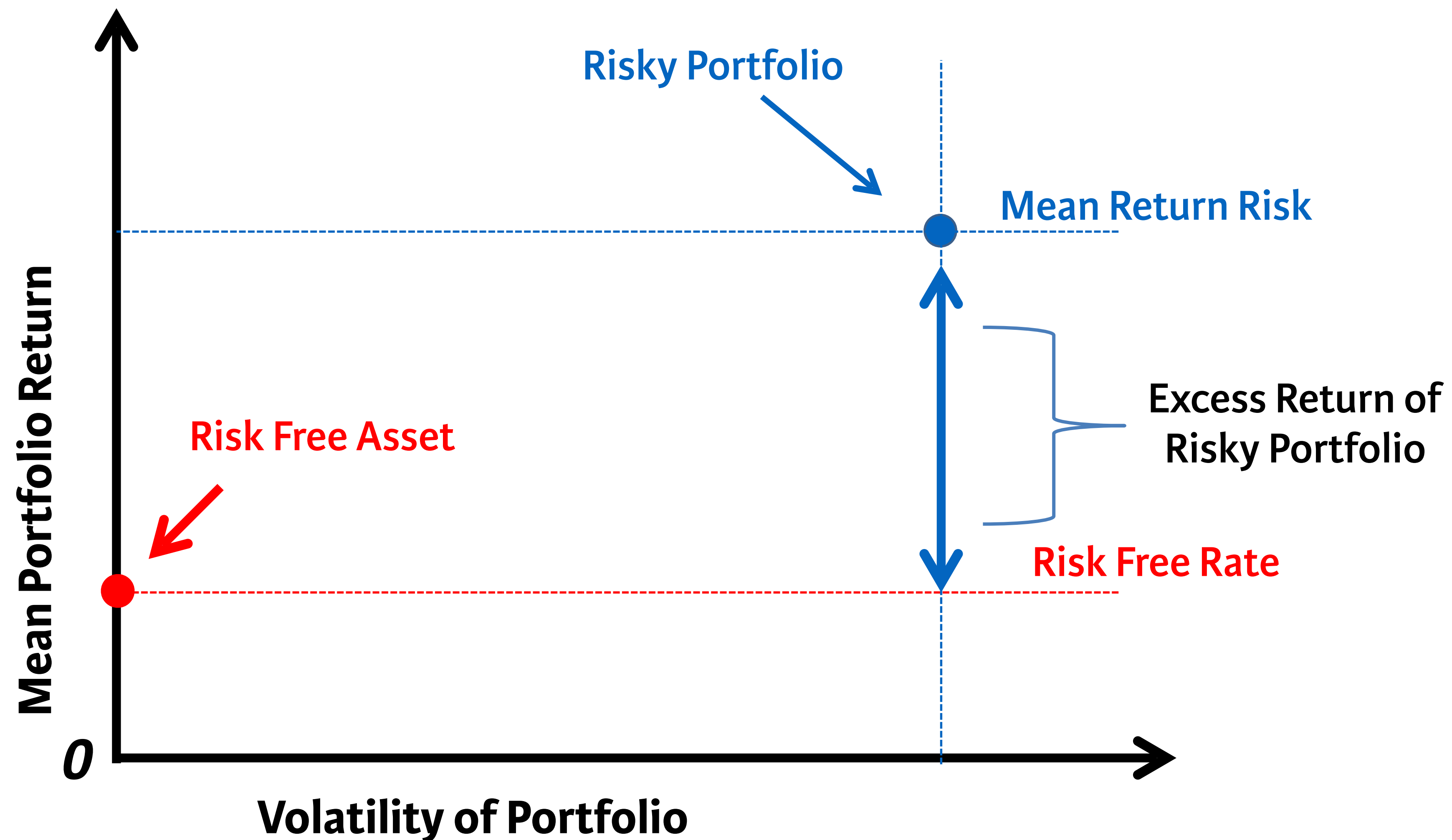
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The (Annualized) Sharpe Ratio

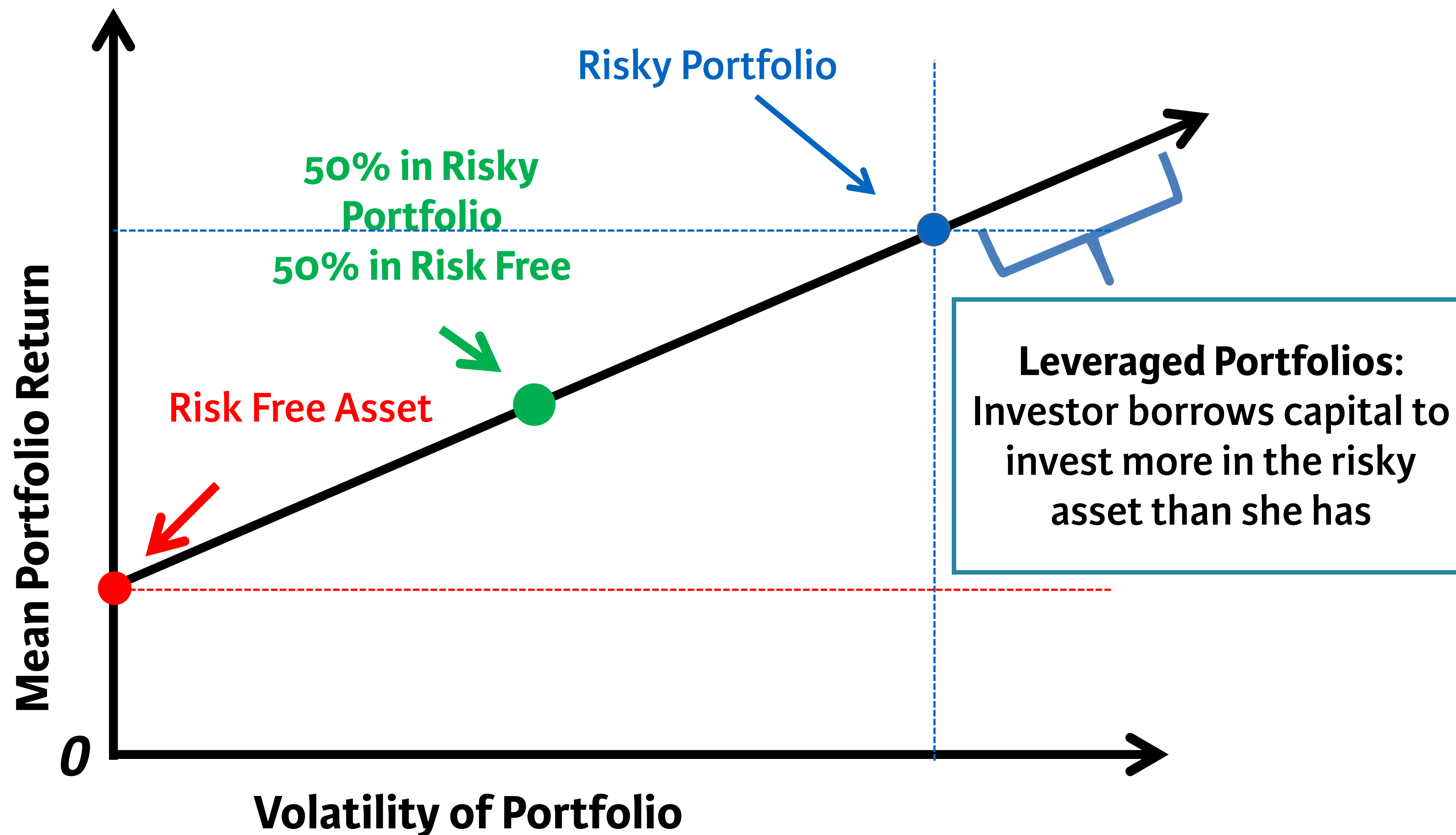
Benchmarking Performance



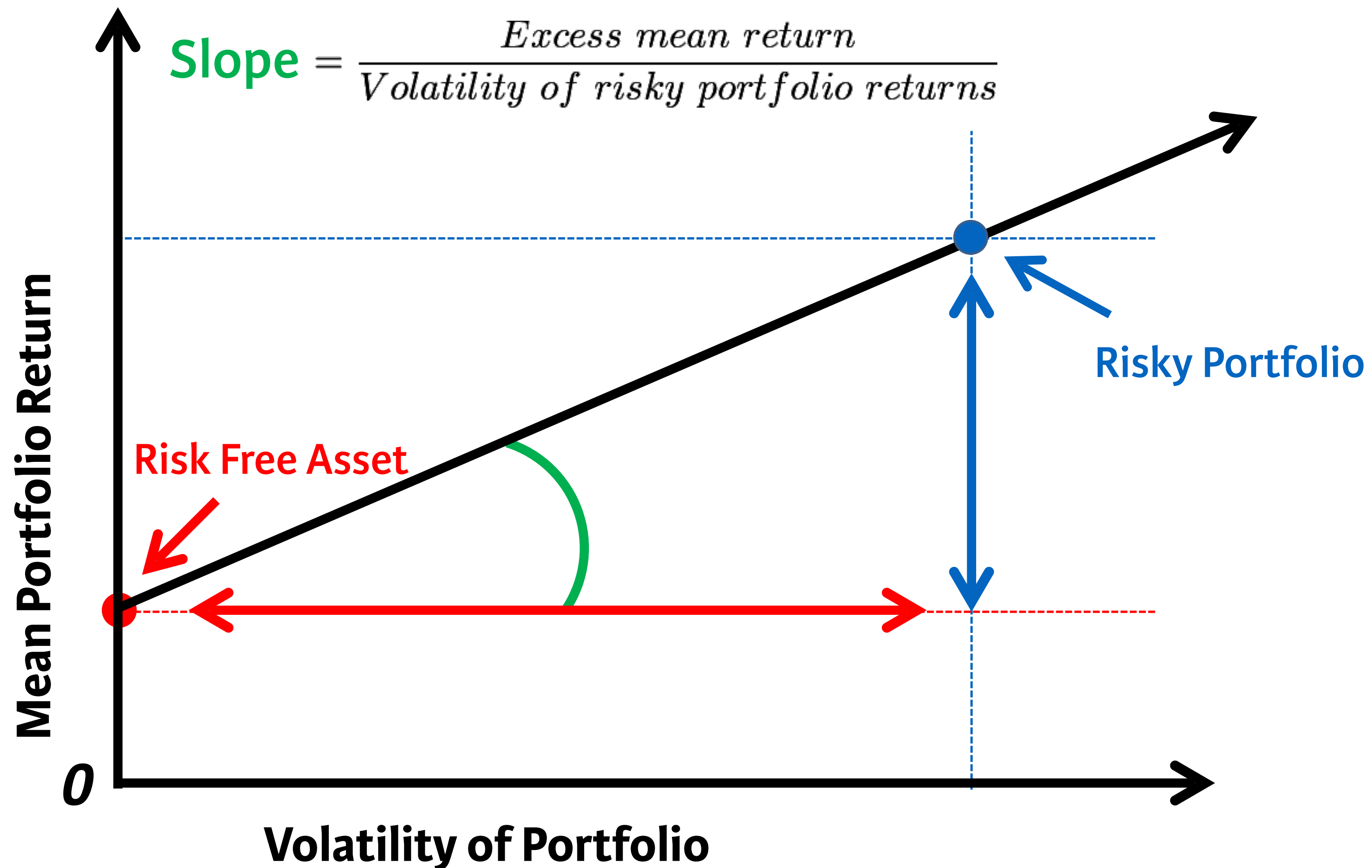
Risk-Return Trade-Off



Capital Allocation Line



The Sharpe Ratio



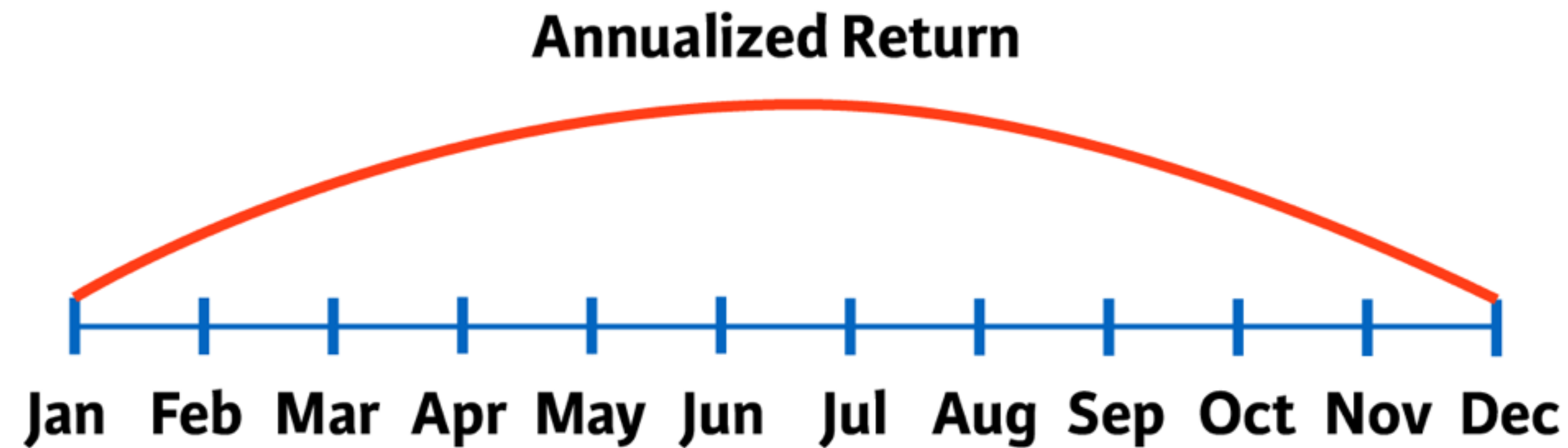
Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c(-0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> (mean(sample_returns) - 0.004)/StdDev(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	0.4035897

Annualize Monthly Performance



Arithmetic mean: monthly mean * 12

Geometric mean, when R_i are monthly returns:

$$[(1 + R_1) * (1 + R_2) * \dots * (1 + R_T)]^{\frac{12}{T}} - 1$$

Volatility: monthly volatility * sqrt(12)

Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> Return.annualized(sample_returns, scale = 12) /
Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * \dots * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897	sqrt(12)	1.398076



INTRODUCTION TO PORTFOLIO ANALYSIS

Let's practice!



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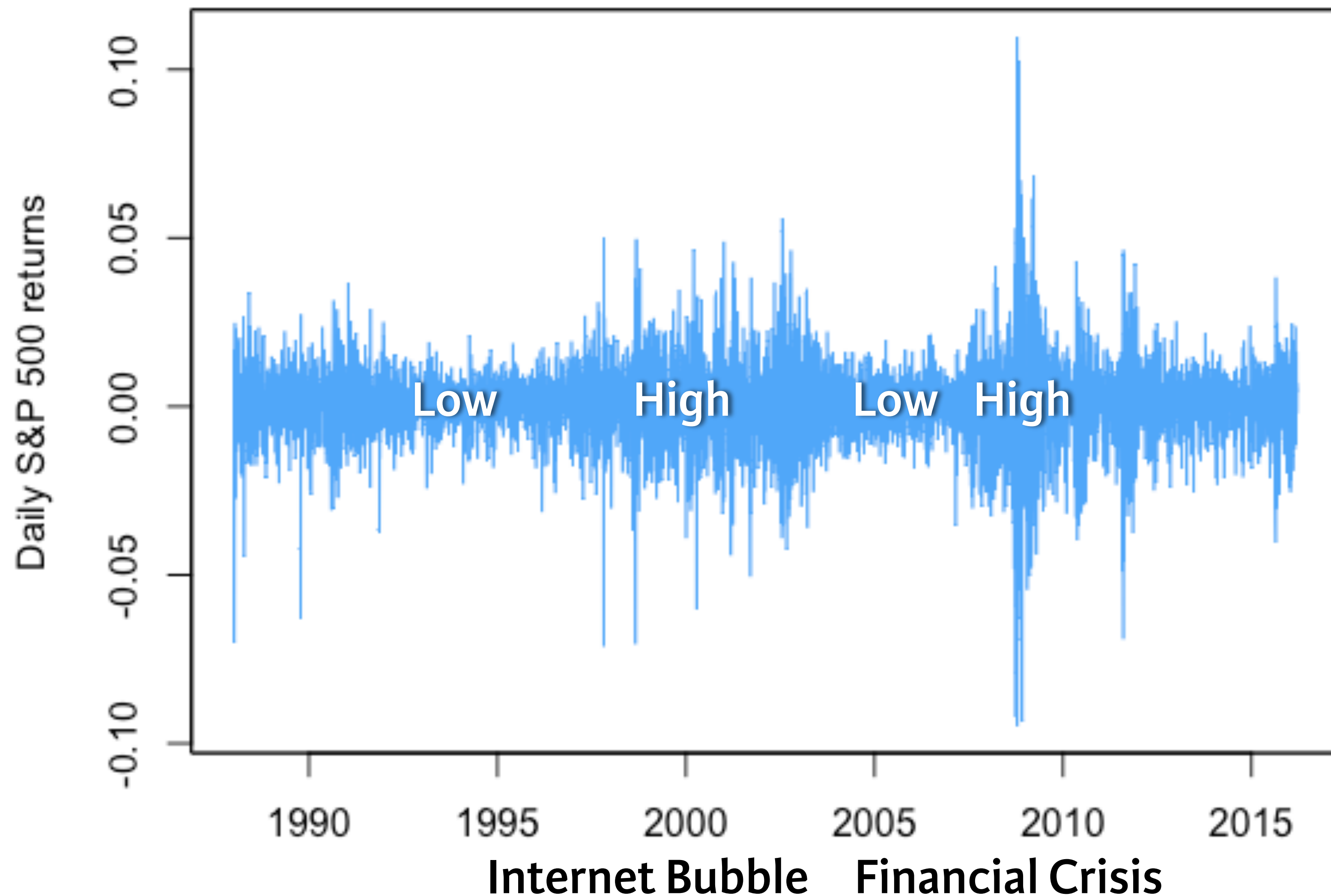
Time-Variation In Portfolio Performance

Bulls & Bears

- Business cycle, news, and swings in the market psychology affect the market

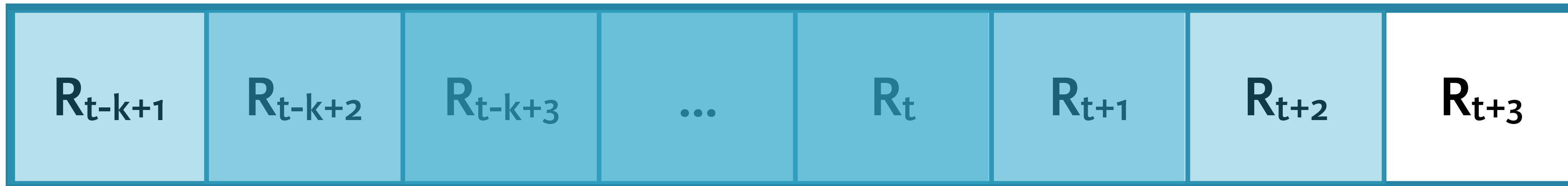


Clusters of High & Low Volatility

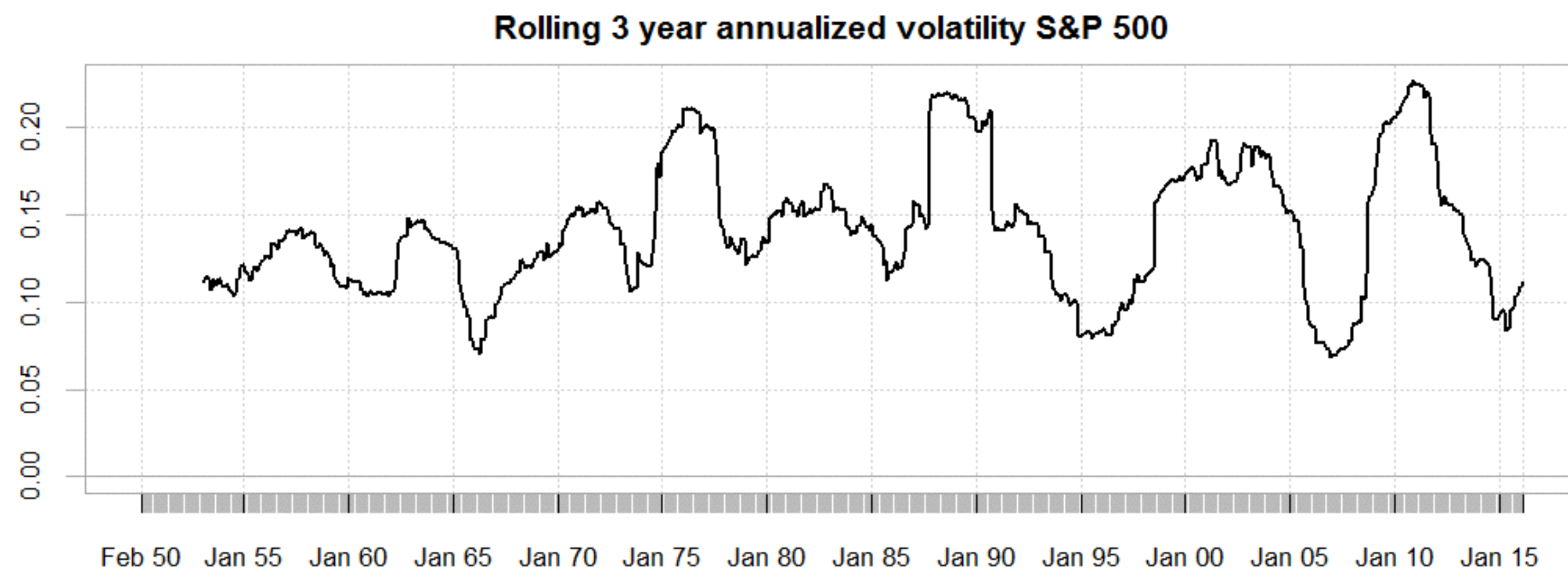
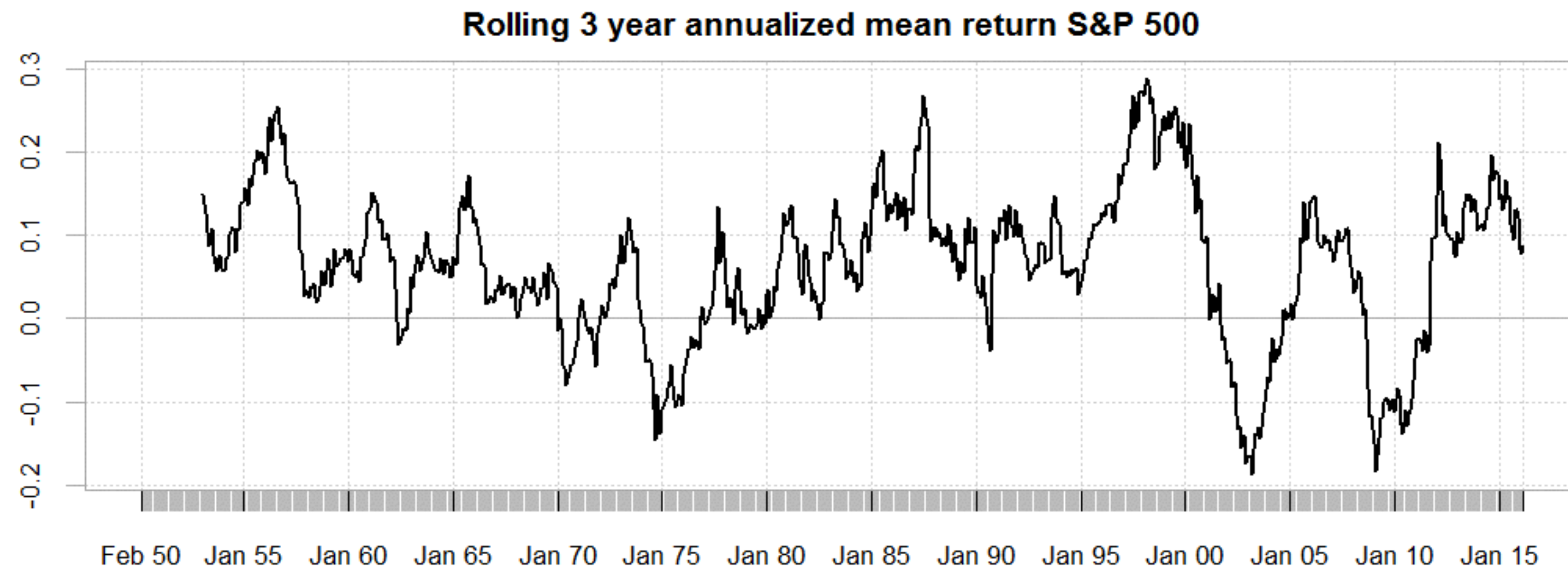


Rolling Estimation Samples

- Rolling samples of K observations:
 - Discard the most distant and include the most recent



Rolling Performance Calculation



Choosing Window Length

- Need to balance noise (long samples) with recency (shorter samples)
- Longer sub-periods smooth highs and lows
- Shorter sub-periods provide more information on recent observations



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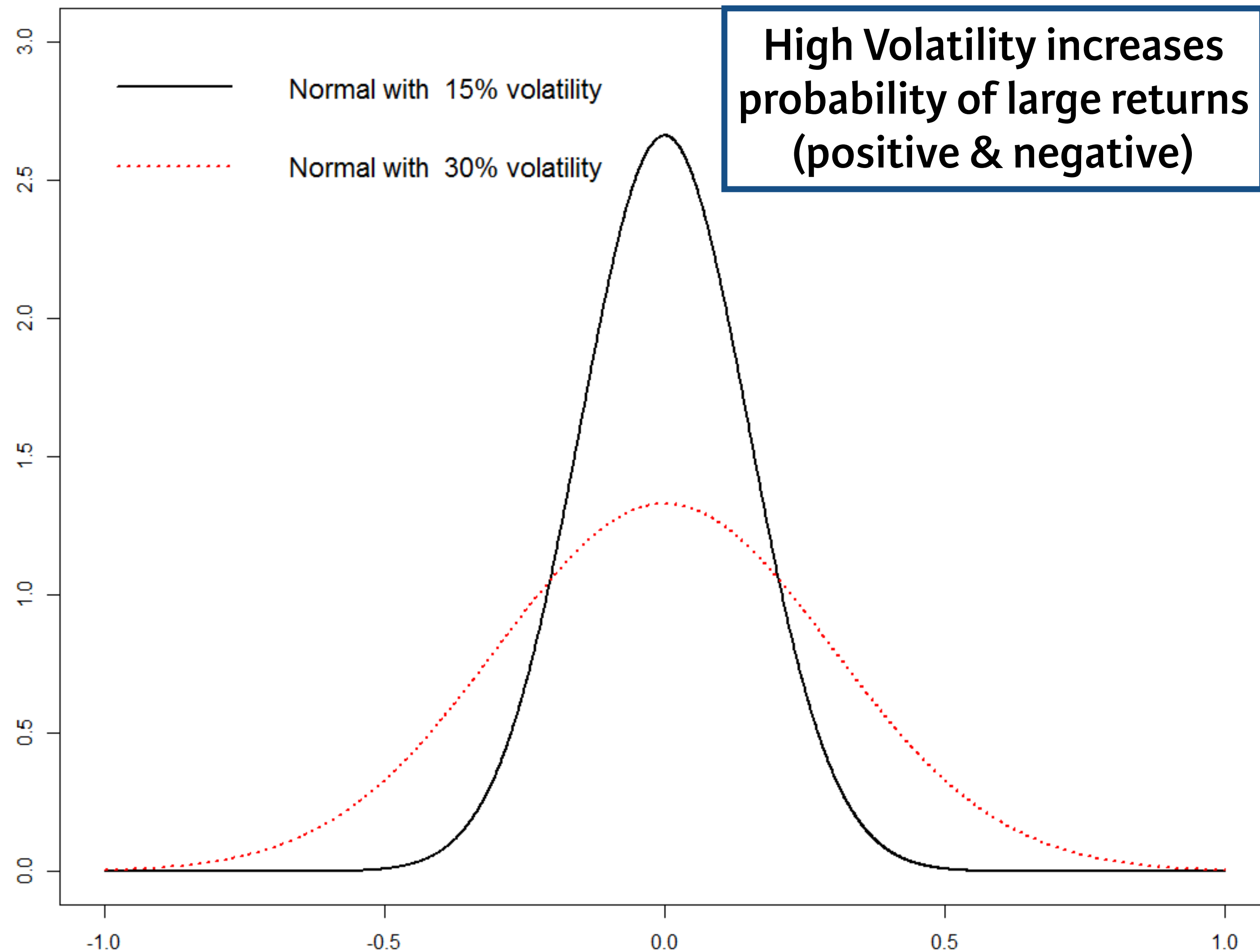
Let's practice!



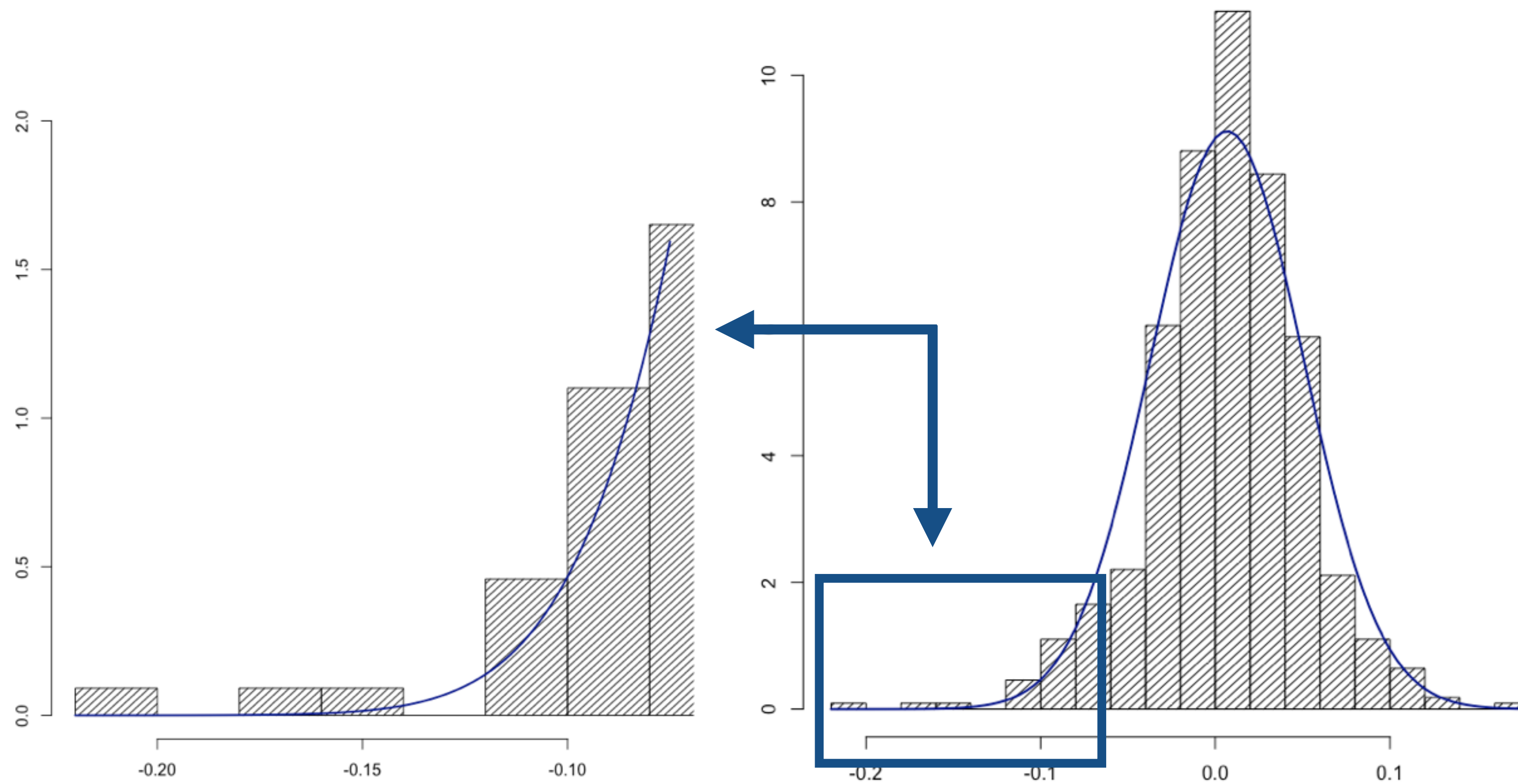
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Non-Normality of the Return Distribution

Volatility Describes “Normal” Risk



Non-Normality of Return



Portfolio Return Semi-Deviation

- Standard Deviation of Portfolio Returns:
 - Take the *full sample* of returns

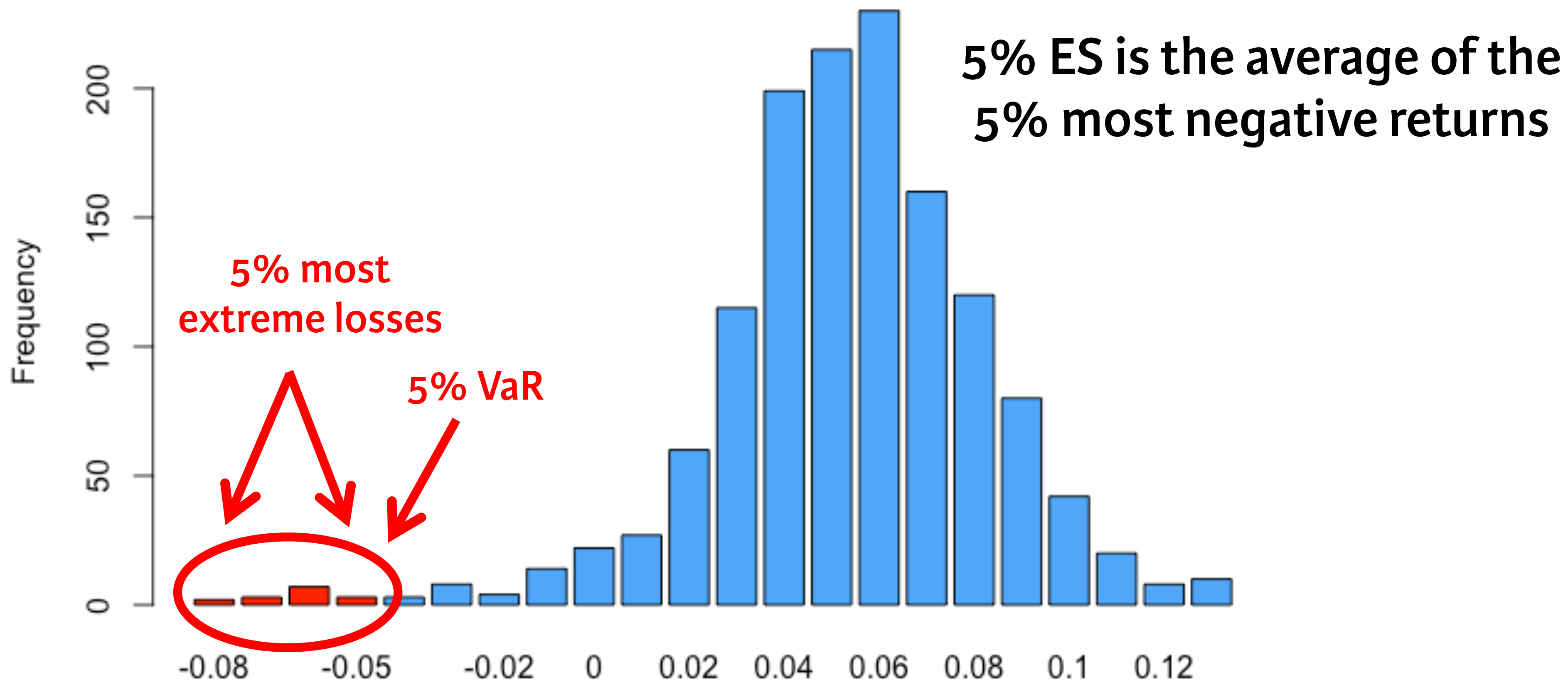
$$SD = \sqrt{\frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + \dots + (R_T - \mu)^2}{T - 1}}$$

- Semi-Deviation of Portfolio Returns:
 - Take the *subset* of returns below the mean

$$SemiDev = \sqrt{\frac{(Z_1 - \mu)^2 + (Z_2 - \mu)^2 + \dots + (Z_n - \mu)^2}{n}}$$

Value-at-Risk & Expected Shortfall

NASDAQ Daily Returns

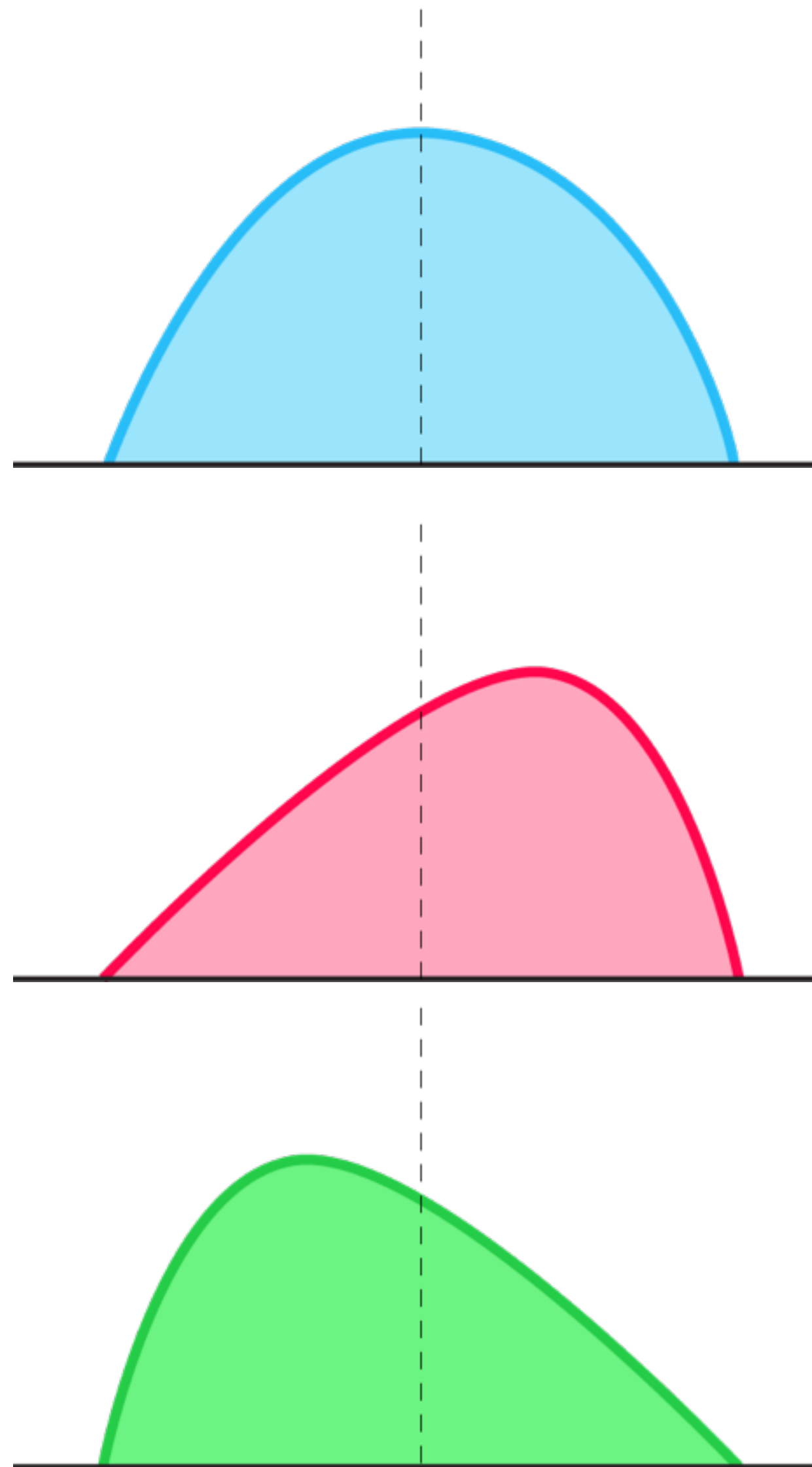


Shape of the Distribution

- Is it symmetric?
 - Check the skewness
- Are the tails fatter than those of the normal distribution?
 - Check the excess kurtosis

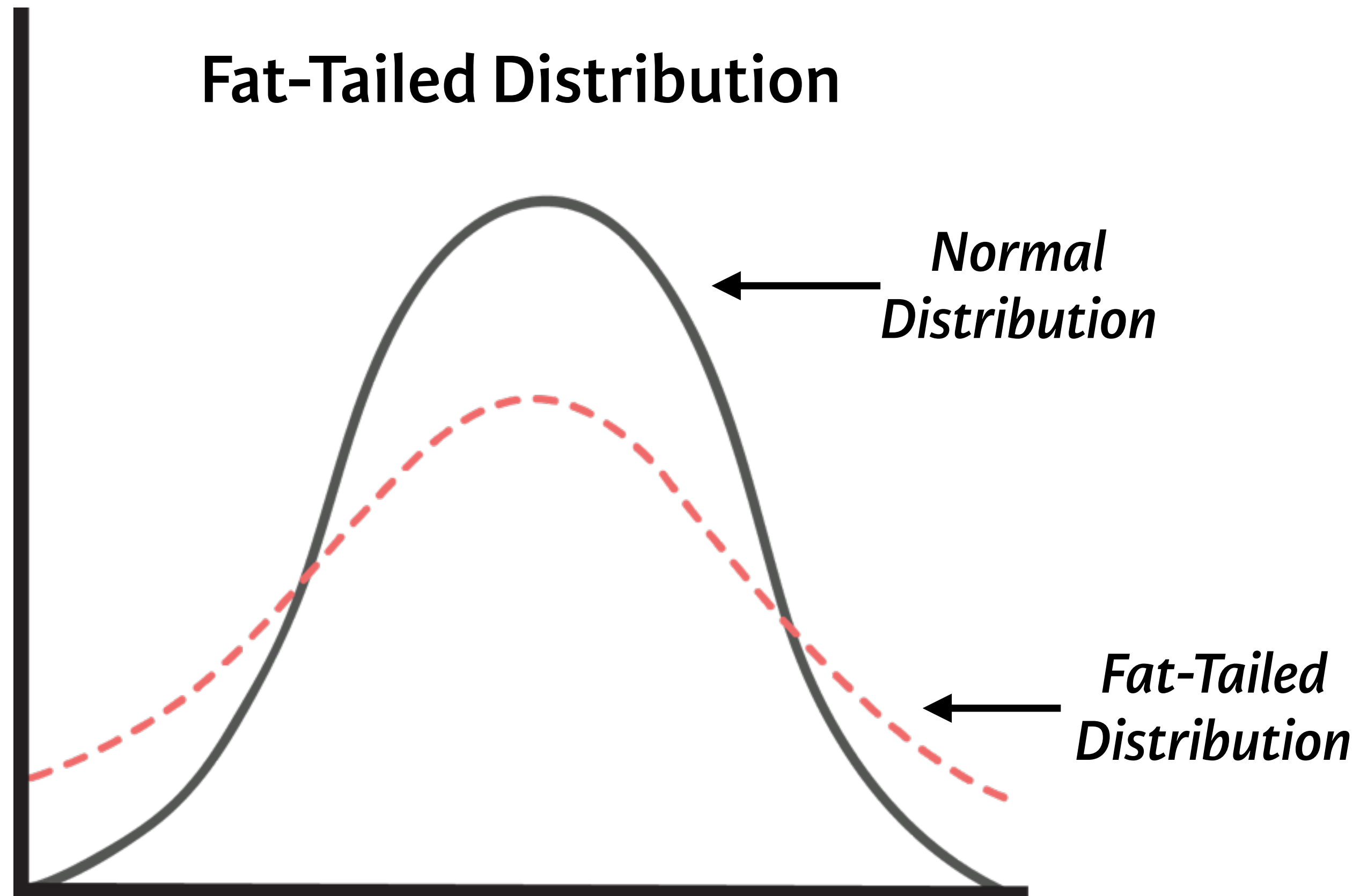
Skewness

- Zero Skewness
 - Distribution is symmetric
- Negative Skewness
 - Large negative returns occur more often than large positive returns
- Positive Skewness
 - Large positive returns occur more often than large negative returns



Kurtosis

- The distribution is fat-tailed when the excess kurtosis > 0





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Let's practice!