



## Introduction to Moments



# Optimization Inputs

Portfolio optimization problem inputs:

- Assets
- Constraints
- Objectives
- Moments of asset returns



#### Asset Return Moments

- First Moment: expected returns vector
- Second Moment: variance-covariance matrix
- Third Moment: coskewness matrix
- Fourth Moment: cokurtosis matrix



### Asset Return Moments

Moments to estimate are determined by objectives and constraints:

- Mean Variance
  - Expected returns vector
  - Covariance matrix
- Minimum Variance
  - Covariance matrix



### Asset Return Moment Estimates

Ledoit and Wolf (2003): "The central message of this paper is that nobody should be using the sample covariance matrix for the purpose of portfolio optimization."

#### Methods:

- Sample
- Shrinkage Estimators
- Factor Model
- Expressing Views
- Robust Statistics

20 Asset Portfolio		
Method	Sample	k = 3 factors
# of parameters	210	86



### Calculating Moments in PortfolioAnalytics

```
set.portfolio.moments(R,
portfolio,
method = c("sample", "boudt", "black_litterman", "meucci"),
...)
```

#### set.portfolio.moments() supports several methods:

- Sample
- Boudt
- Black-Litterman
- Meucci



### Example: Moments in PortfolioAnalytics



### Example: Moments in PortfolioAnalytics

```
> round(sample_moments$sigma, 6)
                               [,3]
                    [,2]
                                         [,4]
          [,1]
     0.000402 - 0.000034
                         0.000262
                                     0.000429
     -0.000034
                0.000632 - 0.000037 - 0.000010
[3,]
     0.000262 - 0.000037
                          0.000337
                                     0.000568
                         0.000568
[4,]
     0.000429 - 0.000010
                                     0.001488
> round(boudt_moments$sigma, 6)
                    [,2]
          [,1]
                               [,3]
                                         [,4]
      0.000403 - 0.000016
                          0.000224
                                     0.000523
     -0.000016
                0.000636 - 0.000019 - 0.000044
[3,]
     0.000224 - 0.000019 0.000337
                                     0.000614
[4,]
     0.000523 - 0.000044 0.000614
                                     0.001488
```





# Let's practice!





## Custom Moment Functions



### **Custom Moment Functions**

A custom moment function is a user defined function.

- Arguments
  - R for asset returns
  - portfolio for the portfolio specification object
- Return a named list where the elements represent the moments
  - mu: Expected returns vector
  - sigma: Variance-covariance matrix
  - m3: Coskewness matrix
  - m4: Cokurtosis matrix





## Example: Custom Moment Function

```
> library(MASS)
> custom_fun <- function(R, portfolio, rob_method = "mcd"){</pre>
    out <- list()
    out$sigma <- cov.rob(R, method = rob_method)</pre>
    return(out)
# Passing the rob_method argument to custom_fun
> optimize.portfolio(R, portfolio, momentFUN = custom_fun,
                      rob_method = "mcd")
> optimize.portfolio(R, portfolio, momentFUN = custom_fun,
                      rob_method = "mve")
```





# Let's practice!





# Objective Functions



# Objective Functions

Objective functions compute the objective value. In PortfolioAnalytics, objective functions can be any valid R function.

- Common portfolio risk measures
  - standard deviation, expected shortfall, value at risk, component contribution to risk, maximum drawdown, Sharpe Ratio
- Common benchmark relative performance measures
  - information ratio, tracking error, excess return, maximum relative drawdown



# Custom Objective Functions

User defined functions as objective functions.

- Argument naming
  - R for asset returns
  - weights for the portfolio weights
  - mu, sigma, m3, m4 for the moments
- Returns a single value





## Example: Custom Objective Function

```
> # Annualized sharpe ratio
> sr_annualized <- function(R, weights, sigma, scale, rfr){
    # Geometric annualized return
    r <- Return.annualized(Return.portfolio(R, weights),</pre>
                            scale = scale)
    # Annual excess return
    re <- r - rfr
    # Annualized portfolio standard deviation
    pasd <- sqrt(as.numeric(t(weights) %*%</pre>
                 sigma %*% weights)) * sqrt(scale)
    return(re /
```



## Example: Custom Objective Function

```
> data(edhec)
> asset_returns <- edhec[,1:4]</pre>
> # Setup spec and add constraints
> port_spec <- portfolio.spec(assets = colnames(asset_returns))</pre>
> port_spec <- add.constraint(portfolio = port_spec,</pre>
                               type = "full_investment")
> port_spec <- add.constraint(portfolio = port_spec,
                               type = "long_only")
> # Add custom objective function
> port_spec <- add.objective(portfolio = port_spec,
                     type = "return", name = "sr_annualized",
                     arguments = list(scale = 12,
                                       rfr = 0.02)
```





# Let's practice!