Bond Price Volatility and Price Value of a Basis Point

Bond Price Volatility

- Bond price volatility depends on many factors
- Some examples:
  - Size of yield change
  - Coupon rate
  - Time to maturity

Small Change, Symmetric Effect

- Small changes in yield: % change for most bonds are similar **whether yield goes up or down**
- Example:
  - 100 USD par value, 10% coupon rate, 20 years, 10% yield

```
> bondprc(100, 0.10, 20, 0.101) / bondprc(100, 0.10, 20, 0.10) - 1
[1] -0.008455776

> bondprc(100, 0.10, 20, 0.099) / bondprc(100, 0.10, 20, 0.10) - 1
[1] 0.008571998
```

Large Change, Asymmetric Effect

- For large changes in yield, the percentage change is higher when the yield decreases
- Example:
  - 100 USD par value, 10% coupon rate, 20 years, 10% yield

```
> bondprc(100, 0.10, 20, 0.14) / bondprc(100, 0.10, 20, 0.10) - 1
[1] -0.2649252

> bondprc(100, 0.10, 20, 0.06) / bondprc(100, 0.10, 20, 0.10) - 1
[1] 0.4587968
```

Lower Coupon, More Volatile

- Fixing the time to maturity and yield, bond price volatility is higher if the coupon rate is lower
- Example:
  - 100 USD par value, 20 years,
    10% initial yield, 8% new yield

```
> bondprc(100, 0.10, 20, 0.08) / bondprc(100, 0.10, 20, 0.10) - 1
[1] 0.1963629

> bondprc(100, 0.05, 20, 0.08) / bondprc(100, 0.05, 20, 0.10) - 1
[1] 0.228328

> bondprc(100, 0.00, 20, 0.08) / bondprc(100, 0.00, 20, 0.10) - 1
[1] 0.4433731
```

Shorter Maturity, More Volatile

- Fixing the coupon rate and yield, bond price volatility is higher if the **time to maturity** is longer
- Example:
    - 100 USD par value, 10% coupon rate, 10% initial yield, 8% new yield

```
> bondprc(100, 0.10, 20, 0.08) / bondprc(100, 0.10, 20, 0.10) - 1
[1] 0.1963629
```

```
> bondprc(100, 0.10, 10, 0.08) / bondprc(100, 0.10, 10, 0.10) - 1
[1] 0.1342016
```

```
> bondprc(100, 0.10, 5, 0.08) / bondprc(100, 0.10, 5, 0.10) - 1
[1] 0.0798542
```

Price Value of a Basis Point

- Or “dollar value of an 01” = measure of bond price volatility
- = price of the bond **if the required yield changes by 0.01%**
- Example:

```
> bondprc(100, 0.05, 20, 0.05)
[1] 100

> bondprc(100, 0.05, 20, 0.0501)
[1] 99.87548

> abs(bondprc(100, 0.05, 20, 0.0501) - bondprc(100, 0.05, 20, 0.05))
[1] 0.1245165
```

To make sure difference is positive

Let’s practice!

Duration

What is Duration?

- Estimated price change for a 100 basis point change in yield
    - Two bonds with the same duration will have same estimated price change
- A way to manage the risk of interest rate sensitive liabilities

Calculating Duration

Price When Yield Goes Down

Price When Yield Goes Up

Duration

$$D = \frac{P(\text{down}) - P(\text{up})}{2 * P * \Delta y}$$

Current Price

Change In Yield

Estimating Price Change

Percentage Change

$$\frac{\Delta P}{P} = -D * \Delta y$$

$$\Delta P = -D * \Delta y * P$$

Dollar Change

Duration

Change In Yield

Price

How Do You Use These Formulas?

- Example: $100 par value, 5% coupon rate, 10 years to maturity, initial yield = 4%, expected increase in yield = 1%

```
> (p <- bondprc(100, .05, 10, .04))
[1] 108.1109
> (p_down <- bondprc(100, .05, 10, .03))
[1] 117.0604
> (p_up <- bondprc(100, .05, 10, .05))
[1] 100
```

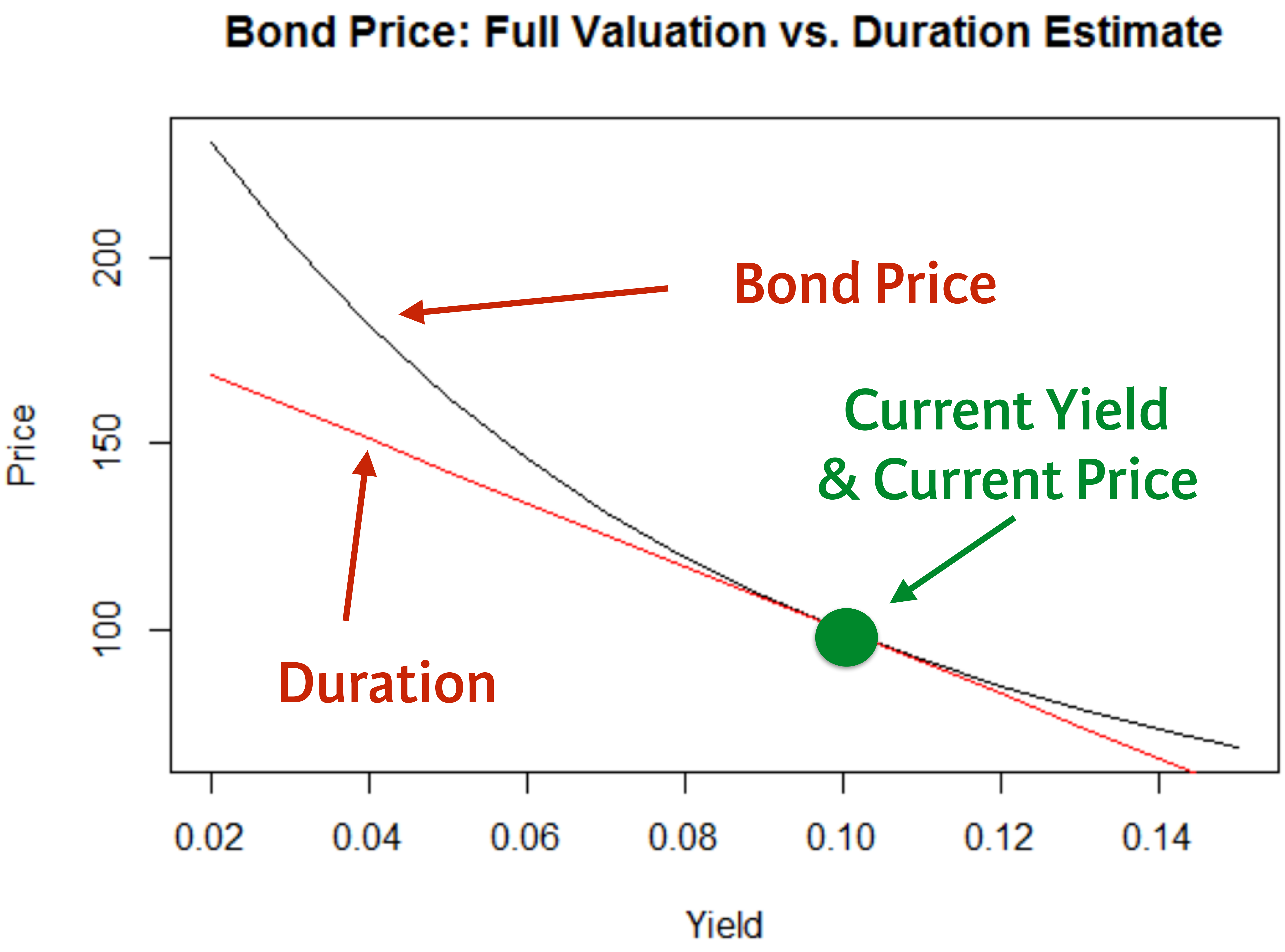
Inputs

```
> (duration <- (p_down - p_up) / (2 * p * 0.01))
[1] 7.890234

> (duration_pct_change <- - duration * 0.01)
[1] -0.07890234

> (duration_dollar_change <- duration_pct_change * p)
[1] -8.530203
```

Duration in a Chart



Let's practice!

Convexity

Convexity Measure

- Duration does a poor job when yield changes are large
- Convexity measure is used to adjust the duration estimate

Calculating the Convexity Measure

Convexity Measure

Price When Yield Goes Down

Price When Yield Goes Up

$$C = \frac{P(\text{down}) + P(\text{up}) - 2 * P}{P * (\Delta y)^2}$$

2x Current Price

Current Price

Change In Yield Squared

Estimating Effect on Price

Percentage Change

$$\frac{\Delta P}{P} = 0.5 * C * (\Delta y)^2$$

$$\Delta P = 0.5 * C * (\Delta y)^2 * P$$

Dollar Change

Convexity Measure

Change In Yield Squared

Current Price

How Do You Use These Formulas?

- Example (same as duration)
    - $100 par value, 5% coupon rate, 10 years to maturity, initial yield = 4%, expected increase in yield = 1%

Current price

```
> p
[1] 108.1109

> (convexity <- (p_down + p_up - 2 * p) / (p * (0.01^2)))
[1] 77.56981

> (convexity_pct_change <- 0.5 * convexity * 0.01^2)
[1] 0.00387849

> (convexity_dollar_change <- 0.5 * convexity * 0.01^2 * p)
[1] 0.4193071
```

Effect of Duration + Convexity

- Estimated Change in Price

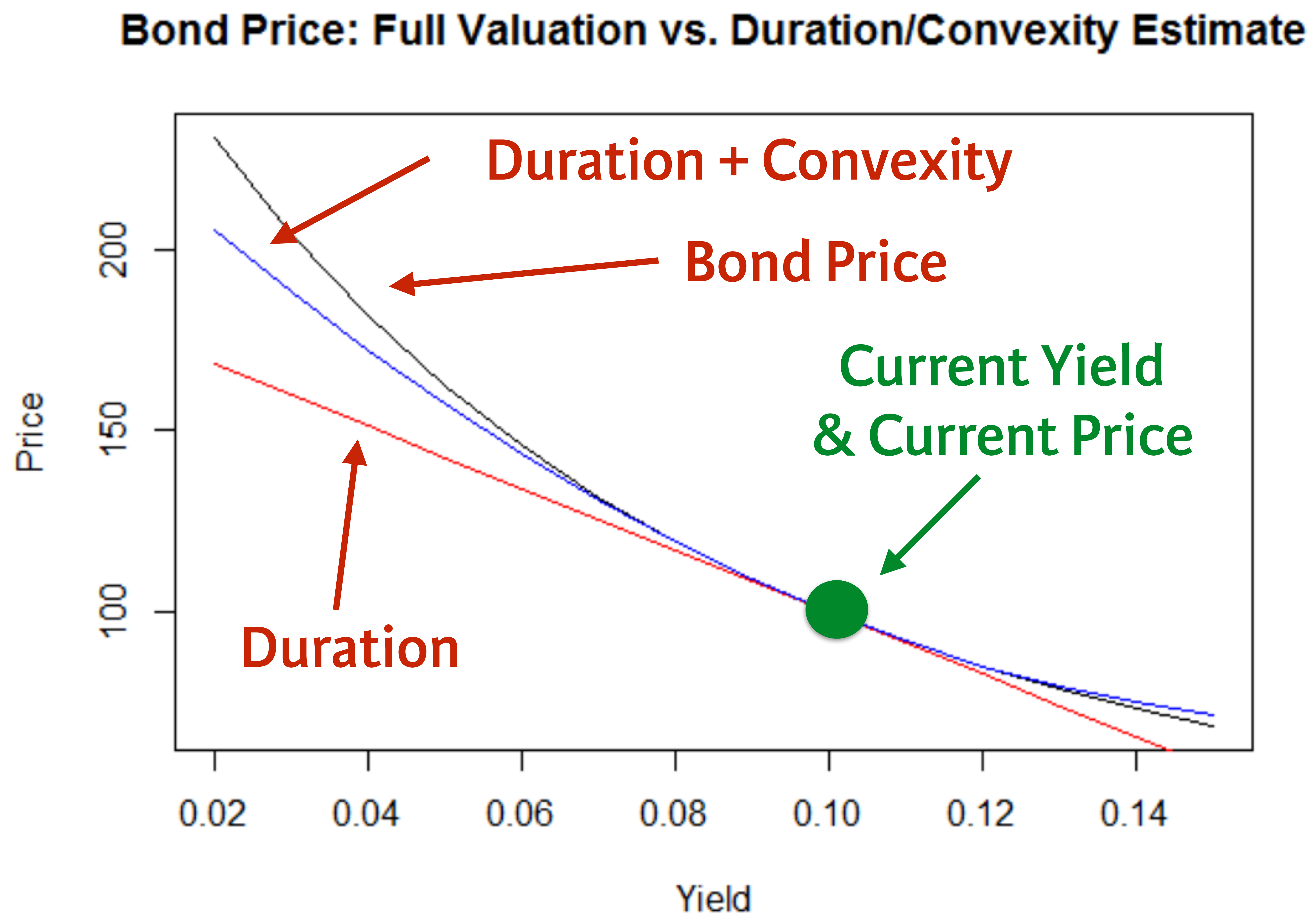
```
> duration_dollar_change
[1] -8.530203
> convexity_dollar_change
[1] 0.4193071
> duration_dollar_change + convexity_dollar_change
[1] -8.110896
```

- Estimated Price

Current Price

```
> p
[1] 108.1109
> duration_dollar_change + convexity_dollar_change + p
[1] 100
```

Convexity in a Chart



Let’s practice!