1.(a) A 90% C.I. is $\hat{\mu} \pm c\frac{\sigma}{\sqrt{n}}$, $c \sim N(0,1)$

$$= 20.59 \pm 1.645\frac{1.2}{\sqrt{15}}$$

$$= [20.08, 21.10]$$

(b) We want a 90% C.I. with width 0.1

$$1.645\frac{1.2}{\sqrt{n}} \leq 0.1$$

$$\sqrt{n} \geq 1.645\frac{1.2}{0.1}$$

$$\geq 19.74$$

$$n \geq 389.66$$

So n has to be at least 390

2. Let $\bar{y}_A$ be the mean in diet A and $\bar{y}_B$ be the mean in diet B. Let $\bar{y}_d = \bar{y}_A - \bar{y}_B$

$$\bar{y}_d \pm c\frac{s_d}{\sqrt{n_d}}, \quad c \sim t_8$$

$$= 14.889 - 12.444 \pm 2.306\frac{8.308}{\sqrt{9}}$$

$$= [-3.941, 8.831]$$

Since we are 95% confident that the true value of the difference lies in this interval. We conclude that the two diets do not have much difference

3. $H_0: \mu_b > \mu_c \quad H_a: \mu_b \leq \mu_c$

$$\hat{\mu}_b - \hat{\mu}_c = \frac{-1+5}{2} = 2$$

$$c\frac{s_d}{\sqrt{n_d}} = 3, \quad c \sim t_{16}$$

$$c = 2.12$$

$$\sqrt{\frac{s_b^2}{n_b} + \frac{s_c^2}{n_c}} = \frac{3}{2.12}$$

$$d = \frac{2}{\frac{3}{2.12}} = 1.413$$

$P(D \geq d) = P(D \geq 1.413)$ where $D \sim t_{16}$

$$= 1 - P(D < 1.413)$$

$= 0.088$

There is evidence reject H_0

4.(a) Let $w = \sum_{ij} r_{ij}^2 + \lambda(\tau_1 + \tau_2)$

$$= \sum_{ij}(y_{ij} - \mu - \tau_i)^2 + \lambda(\tau_1 + \tau_2)$$

$$= \sum_j (y_{1j} - \mu - \tau_1)^2 + \sum_j (y_{2j} - \mu - \tau_2)^2 + \lambda(\tau_1 + \tau_2)$$

$$\frac{\partial w}{\partial \mu} = -2\sum_{ij}(y_{ij} - \mu - \tau_i)$$

$$\frac{\partial w}{\partial \tau_1} = -2\sum_j (y_{1j} - \mu - \tau_1) + \lambda$$

$$\frac{\partial w}{\partial \tau_2} = -2\sum_j (y_{2j} - \mu - \tau_2) + \lambda$$

$$\frac{\partial w}{\partial \lambda} = \tau_1 + \tau_2$$

(b) $E(\tilde{\tau}_1) = E(\bar{Y}_{1+} - \bar{Y}_{++})$

$$= E\left(\sum_{j=1}^{5} \frac{Y_{1j}}{5} - \sum_{ij} \frac{Y_{ij}}{10}\right)$$

$$= \frac{1}{5} E\left(\sum_{j=1}^{5} Y_{1j}\right) - \frac{1}{10} E\left(\sum_{ij} Y_{ij}\right)$$

$$= \frac{1}{5} \sum_{j=1}^{5} E(\mu + \tau_1 + R_{1j}) - \frac{1}{10} \sum_{ij} E(\mu + \tau_i + R_{ij})$$

$$= \frac{1}{5}\left(\sum_{j=1}^{5} \mu + \tau_1\right) - \frac{1}{10}\left(\sum_{j=1}^{5} \mu + \tau_1 + \sum_{j=1}^{5} \mu + \tau_2\right) \qquad \text{Since } E(R_{ij}) = 0 \ \forall\ i,j$$

$$= \frac{1}{5}(5\mu + 5\tau_1) - \frac{1}{10} 10\mu \qquad \text{since } \tau_1 + \tau_2 = 0$$

$$= \tau_1$$

Since $E(\tilde{\tau}_1) = \tau_1$, $\tilde{\tau}_1$ is unbiased.

(c) $Var(\tilde{\tau}_1 - \tilde{\tau}_2)$

$$= Var(\bar{Y}_{1+} - \bar{Y}_{++} - \bar{Y}_{2+} + \bar{Y}_{++})$$

$$= Var(\bar{Y}_{1+} - \bar{Y}_{2+})$$

$$= Var(\bar{Y}_{1+}) + Var(\bar{Y}_{2+}) \qquad \text{Since independent}$$

$$= \frac{1}{25} Var\left(\sum_{j=1}^{5} \mu + \tau_1 + R_{1j}\right) + \frac{1}{25} Var\left(\sum_{j=1}^{5} \mu + \tau_2 + R_{2j}\right)$$

$$= \frac{1}{25} Var\left(\sum_{j=1}^{5} R_{1j}\right) + \frac{1}{25} Var\left(\sum_{j=1}^{5} R_{2j}\right) \qquad R_{ij} \sim N(0, \sigma^2)$$

$$= \frac{1}{25} \sum_{j=1}^{5} Var(R_{1j}) + \frac{1}{25} \sum_{j=1}^{5} Var(R_{2j})$$

$$= \frac{\sigma^2}{5} + \frac{\sigma^2}{5}$$

$$= \frac{2}{5}\sigma^2$$