a9

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2.(a)

```
income = c(60,55,78,19,21,85,42,48,58,67,110,95,94,63,54)
mean(income)
```

```
## [1] 63.26667
```

```
sqrt(var(income))
```

```
## [1] 25.99853
```

$\hat{\mu} = 63.26667$, $\hat{\sigma} = 25.99853$.
So a 95% C.I. is

$$
\begin{aligned}
&\hat{\mu} \pm c\sqrt{1-\frac{n}{N}}\frac{\hat{\sigma}}{\sqrt{n}} \quad \text{where } c \sim N(0,1) \\
=&63.26667 \pm 1.96\sqrt{1-\frac{15}{157000}}\frac{25.99853}{\sqrt{15}} \\
=&(50.11023, 76.42311)
\end{aligned}
$$

2.(b) $\hat{\pi} = \frac{6}{15} = 0.4$.
A 95% C.I. is

$$
\begin{aligned}
&\hat{\pi} \pm c\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}(1-\frac{n}{N})} \quad \text{where } c \sim N(0,1) \\
=&0.4 \pm 1.96\sqrt{\frac{0.4 \times 0.6}{15}(1-\frac{15}{157000})} \\
=&(0.1520893, 0.6479107)
\end{aligned}
$$

2.(c) Since $\frac{19}{20} = 0.95$.

$$
\begin{aligned}
c\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}(1-\frac{n}{N})} &\leq 0.05 \quad \text{where } c \sim N(0,1) \\
n &\geq (\frac{0.05^2}{0.24^2 \times 1.96^2} + \frac{1}{157000})^{-1} \\
n &\geq 88.46059
\end{aligned}
$$

Hence, we need at least 89 people.

2.(d)

```
income_lib = c(55,21,42,48,94,63)
income_con = c(60,78,19,85,58,67,110,95,54)
mean(income_lib)
```

```
## [1] 53.83333
```

```
sqrt(var(income_lib))
```

```
## [1] 24.29335
```

$\hat{\mu}_1 = 53.83333$, $\hat{\sigma}_1 = 24.29335$.

```
mean(income_con)
```

```
## [1] 69.55556
```

```
sqrt(var(income_con))
```

```
## [1] 26.50996
```

$\hat{\mu}_2 = 69.55556$, $\hat{\sigma}_2 = 26.50996$.

```
3 / 5 * mean(income_con) + 2 / 5 * mean(income_lib)
```

```
## [1] 63.26667
```

$\hat{\mu} = 63.26667$

```
4 / 25 * var(income_lib) / 6 * (1 - 5 * 6 / (157000 * 2))
```

```
## [1] 15.73627
```

```
9 / 25 * var(income_con) / 9 * (1 - 5 * 9 / (157000 * 3))
```

```
## [1] 28.10843
```

$w_1^2 \frac{\sigma_1^2}{n_1}(1 - \frac{n_1}{N_1}) = 15.73627$.
$w_2^2 \frac{\sigma_2^2}{n_2}(1 - \frac{n_2}{N_2}) = 28.10843$.
So a 95% C.I. is

$$\hat{\mu} \pm c\sqrt{\sum_{i=1}^{2} w_i^2 \frac{\sigma_i^2}{n_i}(1 - \frac{n_i}{N_i})} \quad \text{where } c \sim N(0,1)$$
$$=(50.28847, 76.24487)$$

2.(e)

```r
mean(income_con)
```

```
## [1] 69.55556
```

```r
sqrt(var(income_con))
```

```
## [1] 26.50996
```

$\hat{\mu} = 69.55556$, $\hat{\sigma} = 26.50996$. a 95% C.I. is

$$
\begin{aligned}
&\hat{\mu} \pm c\sqrt{1 - \frac{n}{N}}\frac{\hat{\sigma}}{\sqrt{n}} \quad \text{where } c \sim N(0,1) \\
=&69.55556 \pm 1.96\sqrt{(1 - \frac{9 \times 5}{157000 \times 3})}\frac{26.50996}{\sqrt{9}} \\
=&(52.23654, 86.87457)
\end{aligned}
$$

2.(f)

```r
mean(income_lib)
```

```
## [1] 53.83333
```

```r
sqrt(var(income_lib))
```

```
## [1] 24.29335
```

$\hat{\mu} = 53.83333$, $\hat{\sigma} = 24.29335$. a 95% C.I. is

$$\begin{aligned}
&\hat{\mu} \pm c\sqrt{1 - \frac{n}{N}}\frac{\hat{\sigma}}{\sqrt{n}} \quad \text{where } c \sim N(0, 1) \\
=&53.83333 \pm 1.96\sqrt{(1 - \frac{6 \times 5}{157000 \times 2})}\frac{24.29335}{\sqrt{6}} \\
=&(34.39554, 73.27113)
\end{aligned}$$