a2

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1.(a) If $y < y_{(k-1)}$, $a(y_1, \ldots, y_{N-1}, y) = y_{(k-1)}$.\ If $y_{(k-1)} \leq y \leq y_{(k)}$, $a(y_1, \ldots, y_{N-1}, y) = y$.\ If $y_{(k)} < y$, $a(y_1, \ldots, y_{N-1}, y) = y_{(k)}$\ Hence,

$$SC(y) = \begin{cases} N(y_{(k-1)} - y_{(k)}) & y < y_{(k-1)} \\ N(y - y_{(k)}) & y_{(k-1)} \leq y \leq y_{(k)} \\ 0 & y_{(k)} < y \end{cases}$$

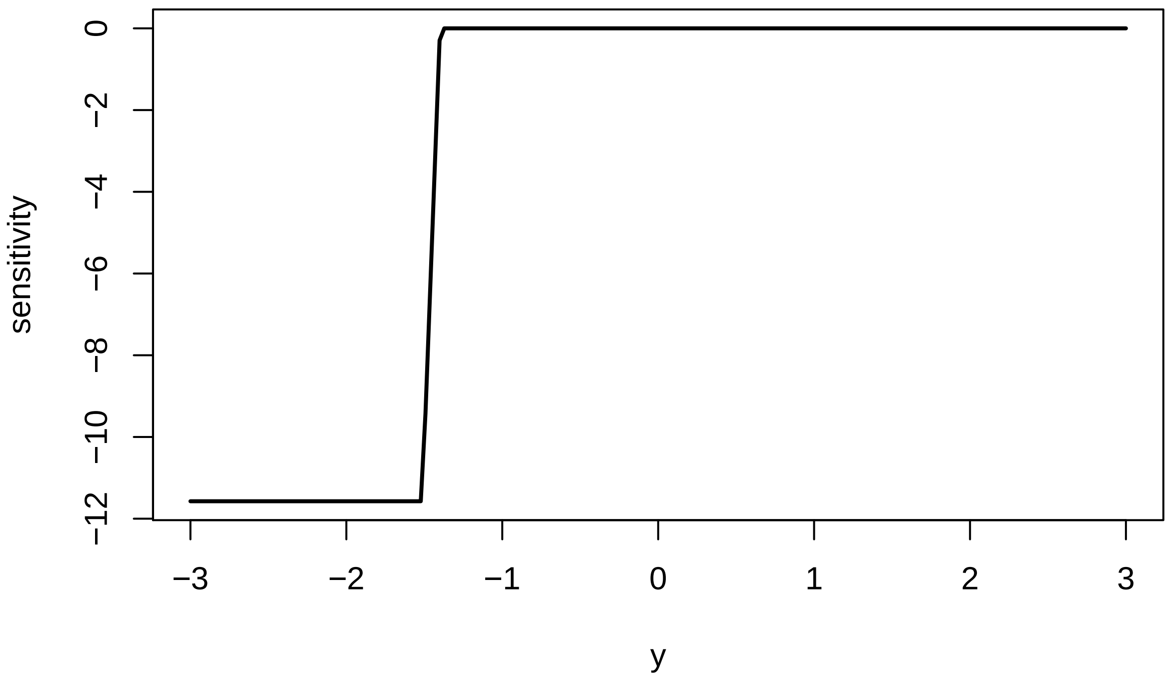
1.(b)

```
set.seed(444)
ys = rnorm(100)
N = length(ys) + 1
k = 5
y_ordered = sort(ys)
y_k = y_ordered[k]
y_kminusOne = y_ordered[k - 1]
y = seq(-3 , 3, length.out=200)

sc = function(y, y_kminusOne, y_k) {
  if(y < y_kminusOne) {
    return(N * (y_kminusOne - y_k))
  } else if(y_k < y) {
    return(0)
  } else {
    return(N * (y - y_k))
  }
}

sensitivity = vector("numeric", 200)
for(i in 1:200) {
  sensitivity[i] = sc(y[i], y_kminusOne, y_k)
}

plot(y, sensitivity, type="l", lwd = 2)
```



The sensitivity curve is bounded and has a linear increase between $y_{(k-1)}$ and y_k.

1.(c) The break down point is $min\{\frac{k}{N}, 1 - \frac{k}{N}\}$