Let $X_{aug} = \sqrt{\lambda} I_{p\times p}$ and $Y_{aug} = \vec{0}_{p\times p}$.
So

$$X^* = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ X_{aug} \end{bmatrix}$$

where $x_i = [x_{i1}, \cdots, x_{ip}]$
and

$$Y^* = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{aug} \end{bmatrix}$$

Let $n' = n + dim(X_{aug})$
Hence,

$$\begin{aligned} Rss(\lambda) &= \sum_{i=1}^{n'} (Y_i^* - X^*\beta)^2 \\ &= \sum_{i=1}^{n'} (Y_i - \beta_1 x_{i1} - \cdots - \beta_n x_{ip})^2 \\ &= \sum_{i=1}^{n} (Y_i - X\beta)^2 + (0 - \sqrt{\lambda}\beta_1)^2 + \cdots + (0 - \sqrt{\lambda}\beta_p)^2 \\ &= \sum_{i=1}^{n} (Y_i - X\beta)^2 + \lambda \sum_{j=1}^{p} \beta_j^2 \end{aligned}$$

Since, $Rss(\lambda) = Rss_{Ridge}(\lambda)$,
$\hat{\beta}_{Ridge} = (X^{*T}X^*)^{-1}X^{*T}Y^* = (X^TX + \lambda I)^{-1}X^TY$