1. (a) $\sum_{i=1}^{n}(y_i-\hat{\mu})^2$

$\frac{\partial}{\partial\hat{\mu}}\sum_{i=1}^{n}(y_i-\hat{\mu})^2=\sum_{i=1}^{n}-2(y_i-\hat{\mu})$

$=0$

$\Rightarrow \hat{\mu}=\frac{\sum_{i=1}^{n}y_i}{n}=\bar{y}$

$\tilde{\mu}=\bar{Y}$

(b) $\hat{\sigma}^2=\frac{\sum_{i=1}^{n}(y_i-\hat{\mu})^2}{n-1}$

(c) $E(\tilde{\mu})=E(\frac{\sum_{i=1}^{n}Y_i}{n})$

$=\frac{1}{n}\sum_{i=1}^{n}E(Y_i)$ since $R_i\perp R_j\ \forall i\neq j \Rightarrow Y_i\perp Y_j\ \forall i\neq j$

$=\frac{1}{n}\sum_{i=1}^{n}E(\mu+R_i)$

$=\frac{1}{n}\sum_{i=1}^{n}\mu+0$ $E(R_i)=0\ \forall i\in\{1,\cdots,n\}$

$=\frac{n}{n}\mu$

$=\mu$

2. $E(\tilde{\mu})=E(\bar{Y})$

$=\frac{1}{n}E(\sum_{i=1}^{n}Y_i)$

$=\frac{1}{n}\sum_{i=1}^{n}E(Y_i)$ $Y_i\perp Y_j\ \forall i\neq j$

$=\frac{1}{n}\sum_{i=1}^{n}\mu+\delta+E(R_j)$

$=\frac{1}{n}n(\mu+\delta)$ $E(R_j)=0$

$=\mu+\delta\neq\mu$

The bias is δ

3. A 95% C.I. is $\hat{\mu}\pm c\frac{s}{\sqrt{n}}$, $c\sim t_{n-1}=t_{19}$

$\hat{\mu}=85$, $s=5$

$85\pm\frac{5}{\sqrt{20}}\cdot 2.093$

$=[82.66, 87.34]$