

a) $Y_{ijk} = \mu + \tau_{ij} + \beta_k + R_{ijk}$ $R_{ijk} \sim N(0, \sigma^2)$ where i is filter type
 $i = 1, 2$ j is Ground cluster type
 $j = 1, 2, 3$ k is operators
 $k = 1, 2, 3, 4$

$$\sum_{i=1}^2 \sum_{j=1}^3 \tau_{ij} = 0, \quad \sum_{k=1}^4 \beta_k = 0$$

$$W = \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau}_{ij} - \hat{\beta}_k)^2 + \lambda_1 \sum_i \sum_j \hat{\tau}_{ij} + \lambda_2 \sum_k \hat{\beta}_k$$

$$\frac{\partial W}{\partial \hat{\mu}} = -2 \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau}_{ij} - \hat{\beta}_k)$$

$$\frac{\partial W}{\partial \hat{\tau}_{ij}} = -2 \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau}_{ij} - \hat{\beta}_k) + \lambda_1$$

$$\frac{\partial W}{\partial \hat{\beta}_k} = -2 \sum_i \sum_j (y_{ijk} - \hat{\mu} - \hat{\tau}_{ij} - \hat{\beta}_k) + \lambda_2$$

$$\frac{\partial W}{\partial \lambda_1} = \sum_i \sum_j \hat{\tau}_{ij}$$

$$\frac{\partial W}{\partial \lambda_2} = \sum_k \hat{\beta}_k$$

$$(b) \hat{\sigma}^2 = \frac{W}{24 - 2 - 3 - 4 + 2 + 2} = \frac{\sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau}_{ij} - \hat{\beta}_k)}{15}$$