a7

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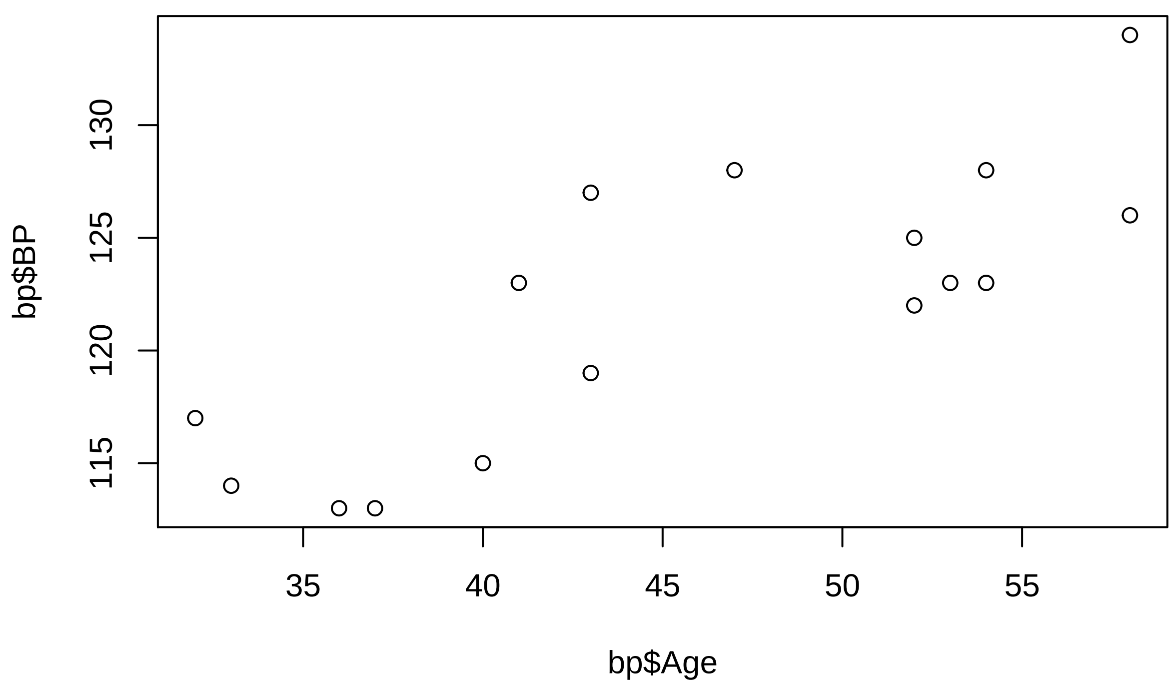
q1

```
bp = read.csv("BP.csv")
```

(a)

```
plot(bp$Age, bp$BP)
```



Since the age and blood pressure has a reasonably linear relationship, we use regression.

```
sample_age = bp$Age - mean(bp$Age)
bp_age = lm(bp$BP ~ sample_age)
summary(bp_age)
```

```
##
## Call:
## lm(formula = bp$BP ~ sample_age)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9123 -3.3517 -0.9754  3.1364  6.7089
##
```

```
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 121.8750     0.9662 126.143  < 2e-16 ***
## sample_age    0.5631     0.1134   4.968 0.000206 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.865 on 14 degrees of freedom
## Multiple R-squared:  0.6381, Adjusted R-squared:  0.6122
## F-statistic: 24.68 on 1 and 14 DF,  p-value: 0.0002064
```

since $\mu_x = 50$, and $\bar{x}$ is

```r
mean(bp$Age)
```

```
## [1] 45.8125
```

$\hat{\mu}_{reg} = 121.8750 + 0.5631(50 - 45.8125) = 124.233$

$\hat{\sigma}^2 = 3.865^2 \frac{14}{15} = 13.94234$

Therefore $\hat{\mu}_{reg} \pm c\sqrt{1 - \frac{16}{201}} \frac{\hat{\sigma}}{\sqrt{16}}$ where $C \sim N(0, 1)$ Hence a 95% CI is $[122.478, 125.988]$

(b)

```
var(bp$BP)
```

```
## [1] 38.51667
```

```
mean(bp$BP)
```

```
## [1] 121.875
```

So $\hat{\sigma^2} = 38.51667$ and $\hat{\mu} = 121.875$

$\hat{\mu} \pm c\sqrt{1 - \frac{16}{201}}\frac{\hat{\sigma}}{\sqrt{16}}$ where $C \sim N(0,1)$ Hence a 95% CI is $[118.958, 124.792]$

q2 (a) $Y_{ijk} = \mu + \tau_{ij} + \beta_k + R_{ijk}$ where $R_{ijk} \sim N(0, \sigma^2)$ and
$i = 1, 2,$
$j = 1, 2, 3,$
$k = 1, 2, 3, 4,$
$\sum_{i=1}^{2} \sum_{j=1}^{3} \tau_{ij} = 0$, $\sum_{k=1}^{4} \beta_k = 0$.
$W = \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau_{ij}} - \hat{\beta_k})^2 + \lambda_1 \sum_i \sum_j \hat{\tau_{ij}} + \lambda_2 \sum_k \hat{\beta_k}$

So the partial derivatives are:
$\frac{\partial W}{\partial \hat{\mu}} = -2 \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau_{ij}} - \hat{\beta_k})$
$\frac{\partial W}{\partial \hat{\tau_{ij}}} = -2 \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau_{ij}} - \hat{\beta_k}) + \lambda_1$
$\frac{\partial W}{\partial \hat{\beta_k}} = -2 \sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau_{ij}} - \hat{\beta_k}) + \lambda_2$
$\frac{\partial W}{\partial \lambda_1} = \sum_i \sum_j \hat{\tau_{ij}}$
$\frac{\partial W}{\partial \lambda_2} = \sum_k \hat{\beta_k}$

(b)

```
options(contrasts = c('contr.sum', 'contr.poly'))
intensity = c(90, 86, 96, 84, 100, 92, 92, 81, 102, 87, 106, 90, 105, 97, 96, 80, 114, 93, 112, 91, 108
optr = as.factor(c(rep(c(1, 1, 2, 2, 3, 3, 4, 4), 3)))
eqp = as.factor(c(rep(c(1, 2), 4), rep(c(3, 4), 4), rep(c(5, 6), 4)))
radar = lm(intensity ~ optr + eqp)
summary(radar)
```

```
##
## Call:
## lm(formula = intensity ~ optr + eqp)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.9167 -1.8542 -0.0833  1.8958  5.5833
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  94.9167     0.6797 139.638  < 2e-16 ***
## optr1         0.4167     1.1773   0.354 0.728334
## optr2         1.5833     1.1773   1.345 0.198657
## optr3         4.5833     1.1773   3.893 0.001442 **
## eqp1         -0.4167     1.5199  -0.274 0.787720
## eqp2         -9.1667     1.5199  -6.031 2.30e-05 ***
## eqp3          7.3333     1.5199   4.825 0.000223 ***
## eqp4         -6.4167     1.5199  -4.222 0.000740 ***
## eqp5         13.0833     1.5199   8.608 3.45e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.33 on 15 degrees of freedom
## Multiple R-squared:  0.9188, Adjusted R-squared:  0.8755
## F-statistic: 21.21 on 8 and 15 DF,  p-value: 7.517e-07
```

$\hat{\sigma}^2 = \frac{w}{24-2-3-4-2+2} = \frac{\sum_i \sum_j \sum_k (y_{ijk} - \hat{\mu} - \hat{\tau_{ij}} - \hat{\beta}_k)}{15} = 3.33$

(c)

```r
anova(radar)
```

```
## Analysis of Variance Table
##
## Response: intensity
##           Df  Sum Sq Mean Sq F value    Pr(>F)
## optr       3  402.17 134.056  12.089 0.0002771 ***
## eqp        5 1479.33 295.867  26.681 5.793e-07 ***
## Residuals 15  166.33  11.089
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$H_0 : \tau_{11} = \cdots = \tau_{23} = 0$ vs H_a : at least one of them is not 0
$f = \frac{ms_{eqp}}{ms_{res}} = 26.681$, $f \sim F_{5,15}$
From anova table, the p-value is 5.793×10^{-07}
So we have tons of evidence against H_0
Hence ground clutter and filter affect the operators ability to detect the target.

H_0 : no interaction vs H_a : interaction

```r
filter = as.factor(c(rep(c(1, 2), 12)))
clutter = as.factor(c(rep(1, 8), rep(2, 8), rep(8, 8)))
radar2 = lm(intensity ~ optr + filter + clutter + filter * clutter)
anova(radar2)
```

```
## Analysis of Variance Table
##
## Response: intensity
##                Df  Sum Sq Mean Sq F value    Pr(>F)
## optr            3  402.17  134.06 12.0892 0.0002771 ***
## filter          1 1066.67 1066.67 96.1924 6.447e-08 ***
## clutter         2  335.58  167.79 15.1315 0.0002527 ***
## filter:clutter  2   77.08   38.54  3.4757 0.0575066 .
## Residuals      15  166.33   11.09
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$f = \frac{ms_{int}}{ms_{res}} = 3.4757$, $f \sim F_{2,15}$
From anova table, the p-value is 0.0575066
So there is evidence against H_0 Hence there may exist interaction between ground clutter and filter.

$H_0 : \beta_1 = \cdots = \beta_4 = 0$ vs H_a : at least one of them is not 0
$f = \frac{ms_{optr}}{ms_{res}} = 12.0892$, $f \sim F_{3,15}$
From anova table, the p-value is 0.0002771
So there is tons of evidence against H_0
Hence blocking by operator is useful.