

$$(a) E(\tilde{\tau}_1 - \tilde{\tau}_2) = E(\tilde{\tau}_1) - E(\tilde{\tau}_2) = \tau_1 - \tau_2 \text{ (unbiased)}$$

$$\text{Var}(\tilde{\tau}_1 - \tilde{\tau}_2) = \text{Var}(\bar{Y}_{1+} - \bar{Y}_{2+} - \bar{Y}_{2+} + \bar{Y}_{++})$$

$$= \text{Var}(\bar{Y}_{1+} - \bar{Y}_{2+})$$

$$= \text{Var}(\bar{Y}_{1+}) + \text{Var}(\bar{Y}_{2+})$$

$$= \frac{1}{5} \sigma^2 + \frac{1}{5} \sigma^2$$

$$= \frac{2}{5} \sigma^2$$

$$H_0: \tau_1 - \tau_2 = 0 \quad H_a: \tau_1 - \tau_2 \neq 0$$

$$d = \frac{\bar{y}_1 - \bar{y}_2 - 0}{\sigma \sqrt{\frac{2}{5}}} = \frac{35.2 - 35.7}{1.029 \sqrt{\frac{2}{5}}} = -1.206 \quad D \sim t_{10-5-2-1+2} = t_4$$

$$p\text{-value} = P(D \geq |d|) = 2(1 - P(D < d)) = 0.294$$

We have no evidence reject  $H_0$ . Hence there is no difference between the effects of treatments.

$$(b) a = \# \text{ of blocks} - 1 = 4$$

$$b = \# \text{ of trts} - 1 = 1$$

$$c = n - q + 1 = 10 - 5 - 1 + 1 = 4$$

$$d = a + b + c = 4 + 1 + 4 = 9$$

$$e = SS_{\text{tot}} - SS_{\text{blk}} - SS_{\text{trt}} = 76 - 70 - 1.6 = 4.4$$

$$f = \frac{SS_{\text{blk}}}{df_{\text{blk}}} = \frac{70}{4} = 17.5$$

$$g = \frac{MS_{\text{trt}}}{MS_{\text{res}}} = \frac{1.6}{1.1} = 1.4545$$

$$h = P(F > 15.9091) = 1 - P(F \leq 15.9091) = 0.01008$$

(c) Since the p-value of  $\tau_1 - \tau_2 = 0$  is large, we have no evidence against  $H_0$  which is there is no difference between the effects of the treatment. Hence

$$(d) H_0: \tau_1 = \tau_2 = \tau_3 = \beta_1 = \dots = \beta_6 = 0 \quad H_a: \text{at least one of them are not } 0.$$

$$f = \frac{\frac{SS_{\text{blk}} + SS_{\text{trt}}}{2}}{MS_{\text{res}}} = \frac{\frac{80.4 + 11.6}{2}}{\frac{21.2}{11}} = 6.819, \quad F \sim F_{2,11}$$

$$p\text{-value} = P(F > f) = 1 - P(F < f) = 0.00273 \text{ from } 1 - pt(6.819, 2, 11)$$

There is tons of evidence reject  $H_0$ . Not all averages across the rows and columns are the same.