2.1(a) $Y_{fi} = \pi_f + R_{fi}$, $R_{fi} \sim N(0, \sigma_f^2)$ where π_H and π_f are the proportions of getting concusion among

$Y_{Hi} = \pi_H + R_{Hi}$, $R_{Hi} \sim N(0, \sigma_H^2)$ Hockey player and football player

(b) $Var(\tilde{\pi}_f - \tilde{\pi}_H) = Var(\tilde{\pi}_f) + Var(\tilde{\pi}_H)$

$$= Var\left(\frac{\sum_{i=1}^{n} Y_{fi}}{n_f}\right) + Var\left(\frac{\sum_{i=1}^{n} Y_{Hi}}{n_H}\right)$$

$$= \frac{1}{n_f^2} \sum_{i=1}^{n_f} Var(Y_{fi}) + \frac{1}{n_H^2} \sum_{i=1}^{n_f} Var(Y_{Hi})$$

$$= \frac{\pi_f(1-\pi_f)}{n_f} + \frac{\pi_H(1-\pi_H)}{n_f}$$ since Y_{Hi} and Y_{fi} are indicator r.v.s

95% C.I. : $\hat{\pi}_f - \hat{\pi}_H \pm c\sqrt{\hat{\sigma}_H^2 + \hat{\sigma}_f^2}$, $c \sim N(0,1)$

where $\sigma_H^2 = \dfrac{\hat{\pi}_H(1-\hat{\pi}_H)}{n_H} = \dfrac{\frac{85}{142}(1-\frac{85}{142})}{142} = 0.00169$

$\sigma_f^2 = \dfrac{\hat{\pi}_f(1-\hat{\pi}_f)}{n_f} = \dfrac{\frac{43}{98}(1-\frac{43}{98})}{98} = 0.00259$

95% C.I. : $\frac{43}{98} - \frac{85}{142} \pm 1.96\sqrt{0.0169 + 0.00259}$

$= (-0.287, -0.0327)$

3. Let $Y_L = \mu_L + R_{Li}$, $R_{Li} \sim N(0, \sigma_L^2)$

$Y_R = \mu_R + R_{Ri}$, $R_{Ri} \sim N(0, \sigma_R^2)$

where μ_L is the average grades of left handed students

where μ_R is the average grades of right handed students

$H_0: \mu_L = \mu_R$ $\quad$ $H_a: \mu_L \neq \mu_R$

$$d = \frac{\hat{\mu}_L - \hat{\mu}_R - 0}{\sqrt{\frac{s_L^2}{n_L} + \frac{s_R^2}{n_R}}}$$

$$= \frac{82 - 78}{\sqrt{\frac{6^2}{12} + \frac{4^2}{28}}}$$

$$= 2.117$$

$P\text{-value} = 2P(D \geq d) = 0.0409$, $D \sim t_{38}$

There is some evidence reject H_0, there is no difference between the grades of right and left handed students.