Q1.1 $R_1 \setminus \{B, D\}$ $R_2 \setminus \{A, B, C, E\}$

$R_1 \setminus \{B, D\}$ $R_2 \setminus \{A, E\}$ $R_3 \setminus \{A, B, C\}$

For R_1, compute$X^+(B, F_1) = \{B, D\} = R_1$, by theorem B is a superkey.

Since compute$X^+(D, F_1) = \{D\}$, so $D \to B \notin F^+$. So R_1 is BCNF

For R_2, compute$X^+(E, F_2) = \{A, E\} = R_2$, by theorem E is a superkey.

Since compute$X^+(A, F_2) = \{A\}$, so $A \to E \notin F^+$. So R_2 is BCNF

For R_3, compute $X^+(A, F_3) = \{A, B, C\}$, by theorem A is a superkey

since compute $X^+(B, F_3) = \{B\}$, and compute $X^+(C, F_3) = \{C\}$, $\{B \to A, B \to C, C \to A, C \to B\}$ are not in F^+,

so R_3 is BCNF

Q1.2 (a) Replace $A \to BCD$ with $A \to B$, $A \to C$, $A \to D$, replace $BC \to DE$ with $BC \to D$, $BC \to E$

$\{A \to B, A \to C, A \to D, BC \to D, BC \to E, B \to D, D \to A\}$

Remove $A \to D$, $BC \to D$

$\{A \to B, A \to C, BC \to E, B \to D, D \to A\}$

Remove $BC \to E$

$\{A \to B, A \to C, B \to D, D \to A\}$

(b) candidate keys are AF, BF, DF

There is no candidate key in result = {AB, AC, BD, DA}

we add AF to result

The decomposition is $R_1 \setminus \{AB\}$, $R_2 \setminus \{A, C\}$, $R_3 \setminus \{B, D\}$, $R_4 \setminus \{DA\}$, $R_5 \setminus \{AF\}$

Clearly AF is in decomposition, and its a candidate key.

Q2.1 $X \to Y \Rightarrow XZ \to YZ$ (augmentation)

$X \to Z \Rightarrow XX \to XZ$ (augmentation)

$XX = X \subseteq X \Rightarrow X \to XX$ (reflexivity)

$X \to XX, XX \to XZ \Rightarrow X \to XZ$ (transitivity)

$X \to XZ, XZ \to YZ \Rightarrow X \to YZ$ (transitivity)

$Y \subseteq YZ \Rightarrow YZ \to Y$ (reflexivity)

$X \to YZ, YZ \to Y \Rightarrow X \to Y$ (transitivity)

Q3.1. elim $\pi_{\#2}(\sigma_{\#1=\#4}(\text{book} \times \text{publication}))$

2. elim $\pi_{\#2}(\sigma_{\#2=\#4}(\text{wrote} \times \text{wrote})) - \pi_{\#2}(\sigma_{\#1=\#3}(\text{wrote} \times \text{wrote}))$

3. $\text{wrote} - \pi_{\#1,\#2}(\sigma_{\#2=\#3}(\text{wrote} \times \text{book})) - \pi_{\#1,\#2}(\sigma_{\#2=\#3}(\text{wrote} \times \text{journal}))$