MM553/837 - Computational Physics
Tutorial Exercise - Week 39

This tutorial concerns the testing and comparison of different integration schemes for the simple pendulum. The **simple pendulum** consists of a ball of mass m attached to a massless rigid rod of length L and allowed to swing in a plane under the influence of gravity. It has Hamiltonian

$$H(\theta, \ell) = \frac{\ell^2}{2I} - mgL\cos\theta$$

where θ is the angle the rod makes with respect to the vertical, ℓ is the angular momentum associated with θ, $I = mL^2$ is the moment of inertia and g is the acceleration due to a uniform gravitational field. Note that $\ell = I\dot{\theta}$. For illustration consider $m = L = 1$ and $g = 9.8$.

- Starting with $\ell(0) = 0$ and $\theta(0) = \pi/4$, integrate the simple pendulum motion for some time interval t. Make sure that t is large enough so that many cycles of the pendulum motion have passed. Choose a fixed number of iterations N, resulting in a fixed stepsize ϵ, and calculate (θ_n, ℓ_n) for all the steps using the Euler, Euler-Cromer, and Leapfrog integration schemes.

- Plot the phase space trajectory for each of these three schemes and compare them for different step sizes ϵ. Do they behave as expected?

- Calculate the total energy at each step and plot it to illustrate how well each of the integration schemes conserve the Hamiltonian. Then plot the energy after the final step for different N, but adjusting ϵ so that t is the same. How can you tell that Leapfrog is a second-order integrator?

- For one of the integrators, use the python animation script to create a movie of the motion. Can you identify energy violations in the motion?

- Try adapting the code to handle the forced-damped pendulum described in Sec. 4.6 of Anagnostopolous. You should us the RK4 integrator for this. Why doesn't it make sense to use a symplectic integrator for this problem? Can you see chaotic behavior?