# Jacobs University Bremen

# General Electrical Engineering 2 Lab
# Electrical and Computer Engineering

## Spring Semester 2021

# Lab Experiment 5 – Filter

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Place of execution :R1 EE Lab
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# 1 Introduction

## 1.1 Objective:

Through this week's experimentations, we learned about the behavior of simple passive RLC Networks with sinusoidal signal of different frequencies as an input signal. We learned how to measure the properties of these circuits. Then, we learned about the analysis of these circuits, and representation of the measurements.

Our objectives for the experiment are the follows:

- Understanding the concept of passive filters.
- Measuring the impedances of RLC circuits and the input and output voltages
- Analyzing the properties of different RLC circuits

## 1.2 Theory:

### 1.2.1 Filter

When we need to extract a desirable frequency or a range of frequencies from an input signal, we use filters. There are several methods for the construction of filters – using active components like transistors and Op-amps or with networks of resistors, capacitors and inductors, or using digital signal processors with analog to digital signal converters or vice versa. Simplest way would be through usage of a passive network of resistors, capacitors or inductors.

### 1.2.2 Frequency response:

This is the measure of the filter's response at the output to a signal of varying frequency but constant amplitude at the input. It is usually characterized by the magnitude of the system's response, measured in dB, and the phase shift relative to the input signal, measured in radians, versus frequency.

### 1.2.3 Cutoff frequency:

As one of the properties of the filer, the cut-off frequency – also known as corner frequency – is the frequency either above which or below which the power output of the filter is half the power of the passband. Since voltage is proportional to the power P, we know that $Vout$ is $\frac{1}{\sqrt{2}}$ of the $Vout$ in the passband. Considering this is close to -3 decibels, and the cutoff frequency is referred to as the -3 dB point. A bandpass circuit and a notch filter have two cutoff frequencies, whose geometric mean is known as the center frequency:

$$f_{bw} = \sqrt{f_1 \cdot f_2}$$

### 1.2.4 Center frequency

This is the geometric mean of the cutoff frequencies of a bandpass circuit and a notch filter.

$$f_{bw} = \sqrt{f1 * f2}$$

### 1.2.5 Bandwidth:

For a bandpass or notch filter, the bandwidth is the difference between the upper and lower cutoff frequencies in a continuous band of frequencies.

### 1.2.6 High Pass:

This is a circuit which allows to transfer signals with high frequencies in original form. The signal is attenuated and the phase shift is positive with low signals. The High Pass filter using LR-circuit and RC-circuit is shown below:

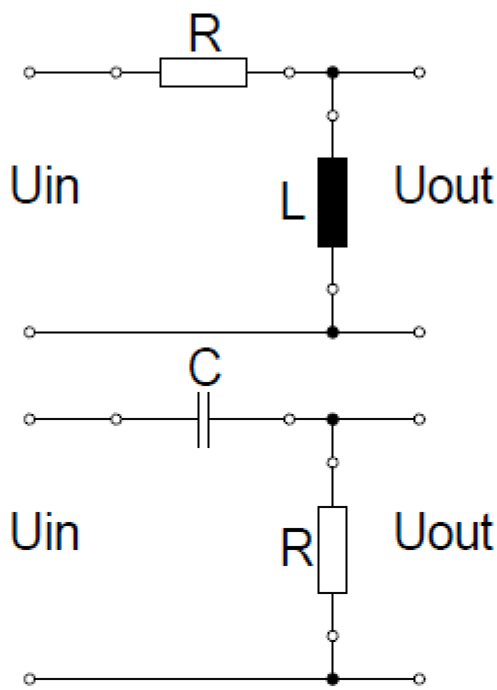



Fig 1: High Pass filter with R-L and R-C Circuit

For a high pass filter, we know the following:

$$|A| = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}} \text{ and } \varphi = \arctan\left(\frac{1}{\omega RC}\right)$$

We determine the cutoff frequency as follows:

$$f_{-3dB} = \frac{1}{2\pi RC}$$

### 1.2.7 Low Pass filter:

A Low Pass filter allows to transfer signals with low frequencies to pass in their original form. With high frequencies, the signal is attenuated and the phase shift is negative. The Low Pass filter using LR- circuit and RC- circuit is shown below:

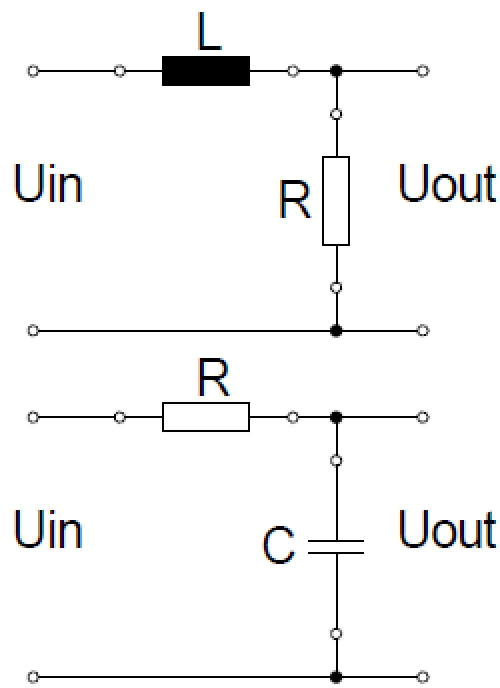



Fig 2: Low pass filter with R-L and R-C Circuits

We have the following for a low pass filter:

$$|A| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \; and \; \varphi = -\arctan(\omega RC)$$

We obtain the cutoff frequency as follows:

$$f_{-3dB} = \frac{1}{2\pi RC}$$

### 1.2.8 Bandpass filter:

This is a filter that only allows frequencies within a certain range to pass, and attenuates frequencies outside the range. The Band Pass filter is a combination of high and low pass filters using an RLC combination, as shown below:

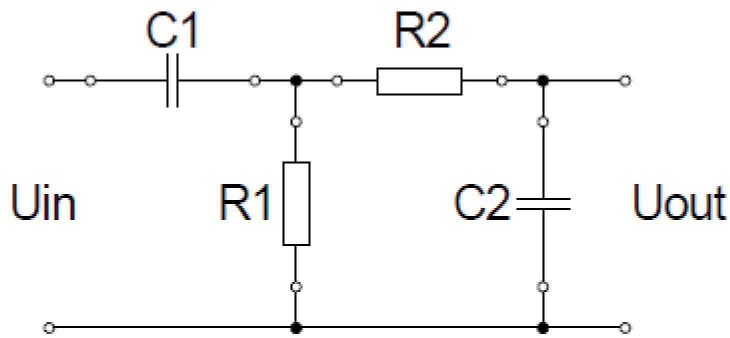



Fig 3: Bandpass filter circuit

We can obtain the formula for magnitude and frequency response of bandpass filter by combining the formulae for high and low pass filters, as shown below:

$$\underline{A}_{hi}(j\omega) = \frac{\underline{U}_{out_{hi}}}{\underline{U}_{in}} \; and \; \underline{A}_{lo}(j\omega) = \frac{\underline{U}_{out_{lo}}}{\underline{U}_{out_{hi}}}$$

Multiplying these equations:

$$\underline{A}_{hi}(j\omega) \cdot \underline{A}_{lo}(j\omega) = \frac{\underline{U}_{out_{lo}}}{\underline{U}_{in}}$$

$$\underline{A}_{hi} \cdot \underline{A}_{lo} \cdot e^{(\omega_{hi} + \omega_{lo})} = \frac{\underline{U}_{out_{lo}}}{\underline{U}_{in}}$$

### 1.2.9 Notch Filter:

This is a filter that passes most of the frequencies in original form, but attenuates those in a specific range to very low levels. It is the opposite of the band-pass filter. A notch filter is made using an RLC combination, as shown below:

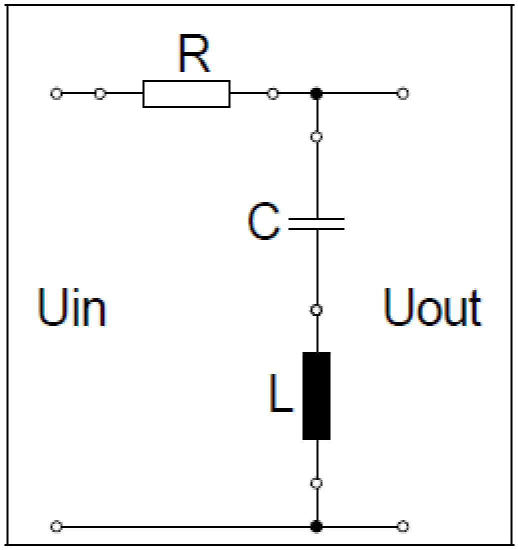



Fig 4: Notch Filter circuit

### 1.2.10 Phase Shift:

A phase shift is defined as the difference in phase of the input signal with respect to the output signal. It is generally represented by the symbol ($\varphi$). If the input signal lags the output signal, it is called a positive shift, and if the output signal lags input signal then it is called a negative shift. The diagram for phase- shift is provided below:

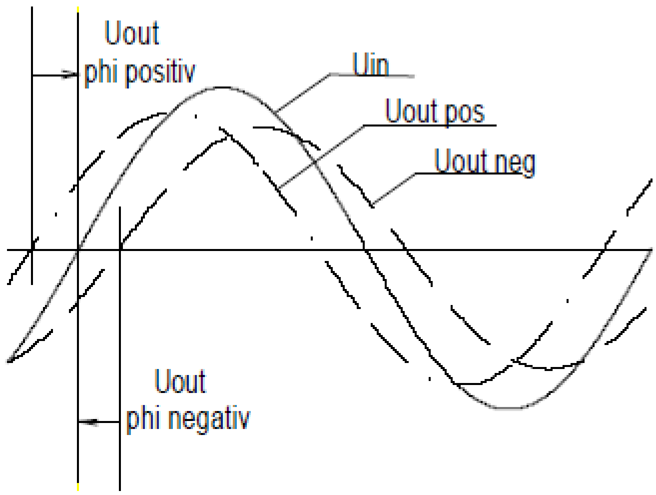



Fig 5: Diagram for Phase shift

### 1.2.11 Bode Plot:

A Bode plot consists of two types of plots: a magnitude plot and a phase plot. A Bode magnitude plot is a graph of magnitude in dB against frequency on a logarithmic scale. Likewise, a bode phase plot is a graph of phase against frequency on a logarithmic scale.

### 1.2.12 Nyquist Plot:

A Nyquist plot is a parametric plot of a frequency response. In Cartesian coordinates, the real part of the transfer function is plotted on the X axis and the imaginary part is plotted on the Y axis. The frequency is swept as a parameter, resulting in a plot per frequency.

# 2 Execution

## 2.1 Experimental set-up part 1:

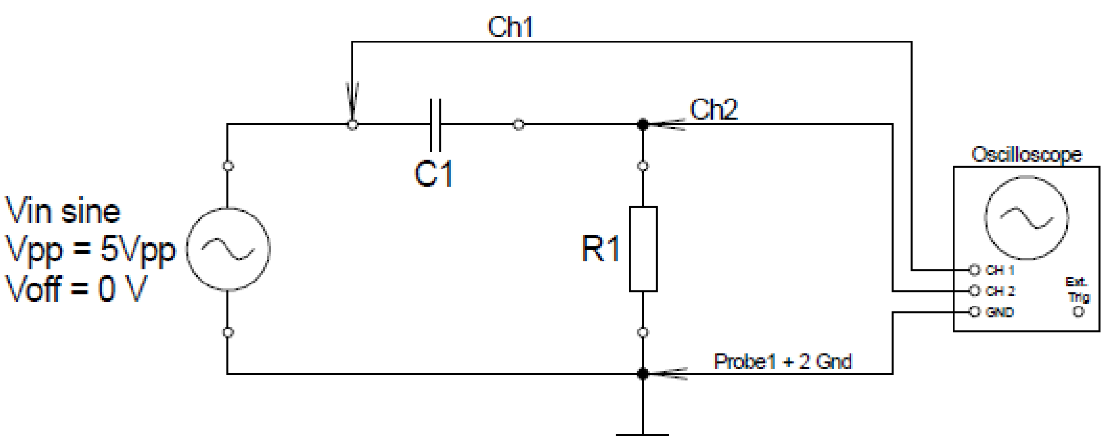
Used tools and equipment:

- Agilent Signal Generator
- TEKTRONIX Oscilloscope
- Resistors, Inductors and capacitors
- BNC Cable, BNC T connector, BNC Banana Connector

### 2.1.1 Part 1: Experimental Set-up and Results:

In this experiment, we determine the properties of a High-Pass filter and its characteristics over the frequency range.

We assembled the high pass filter according to the following schematic:

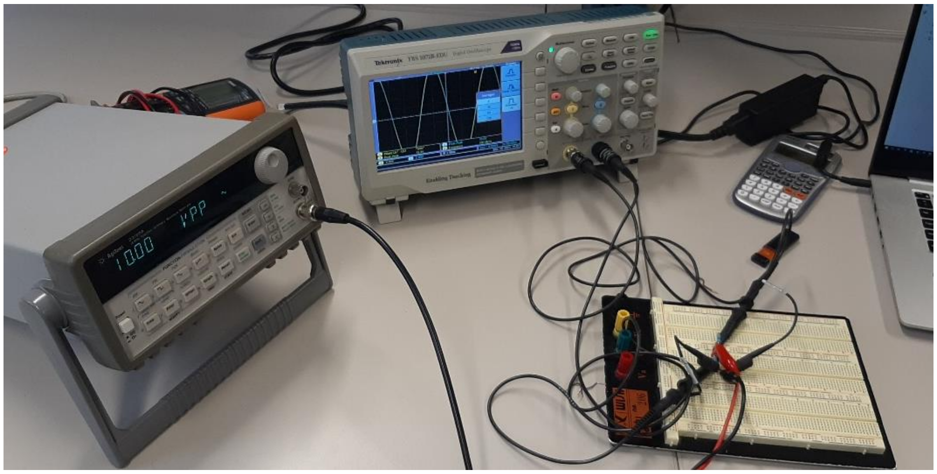


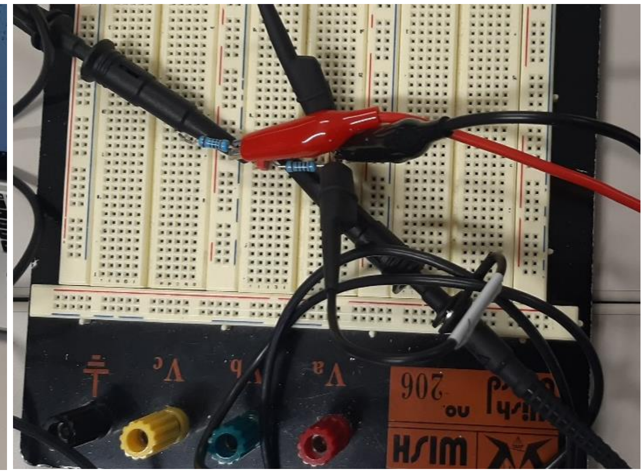

$$C_1 = 100n\,\mathrm{F} \text{ and } R_1 = 1K\Omega$$

Figure 1: Resistor and Capacitor Network for Circuit 1

After we have our set-up, we vary the frequency of the generator from 50Hz to 100KHz in steps of 1, 2 and 5 (50Hz, 100Hz, 200Hz, 500Hz…). Then we measure and record the input and output amplitude and the phase shift between the signals. Finally, we take the recorded data and put it in a table.

The lab set-up looks as follows:





Pictures: Lab set-up for High Pass Filter

The results of our experimentation are provided as follows:

| f[Hz] | Vin[V] | Vout[V] | Phase [degrees] | 20log (Vo/Vi) |
|---|---|---|---|---|
| **50** | 10.2 | 0.338 | 88.5 | -29.5937 |
| **100** | 10.2 | 0.672 | 85.6 | -23.6246 |
| **200** | 10.2 | 1.34 | 82 | -17.6299 |
| **500** | 10.1 | 3.2 | 70.9 | -9.98343 |
| **1000** | 10 | 5.48 | 56.8 | -5.22439 |
| **2000** | 9.92 | 7.84 | 38.1 | -2.04391 |
| **5000** | 9.76 | 9.36 | 15.8 | -0.36348 |
| **10000** | 9.76 | 9.52 | 7.56 | -0.21626 |
| **20000** | 9.68 | 9.68 | 3.17 | 0 |
| **50000** | 9.68 | 9.68 | 2.88 | 0 |
| **100000** | 9.68 | 9.68 | 1.44 | 0 |

Table 1: Measured values for High pass filter

## 2.2 Part 2: Notch Filter:

### 2.2.1 Part 2: Experimental Set-up and Results:

The objective of this experiment is to determine the properties of a Notch filter, and understand its characteristics over a frequency range.

To do this, build a notch filter using a 2.7kΩ resistor, a 2.2nF capacitor and a 10mH inductor. Then, we connect a signal generator via the BNC-to-Kleps cable to the input, and set up a sine signal with a 5V amplitude and no offset. Then, we use the oscilloscope to measure the input signal (Ch1), the output signal (Ch2), and the phase shift. We use Ch1 as the reference signal.

We assembled the notch filter with the following component values:

$$R = 2.7\ K\Omega, C = 2.2\ nF, L = 10\ mH$$

We can make the following deductions about the properties of the notch filter:

$$Center\ frequency\ f_{cf} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}} = 33931.95 Hz$$

For cutoff frequencies,

$$f_{cut-low} = \frac{1}{2\pi} \cdot \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$f_{cut-low} = \frac{1}{2\pi} \cdot \frac{-2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9} + \sqrt{(2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9})^2 + 4 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}}{2 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}$$

$$f_{cut-low} = 18676.52\ Hz$$

$$f_{cut-high} = \frac{1}{2\pi} \cdot \frac{RC + \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$f_{cut-high} = \frac{1}{2\pi} \cdot \frac{2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9} + \sqrt{(2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9})^2 + 4 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}}{2 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}$$

$$f_{cut-high} = 61648.36\ Hz$$

By experiment, $f_{cf} = 35400\ Hz$, which is close to the calculated value.

Next, we make a table for f, Vin, Vout and $\varphi$. We then vary the frequency from 10KHz to 100KHz. In this table, we include cutoff values, and calculated and measured center frequency, and then inserted five extra values between each of the cutoffs (calculated) and center frequency (experimentally).

We then measure and record the input voltage, the output voltage and the phase shift between the input and the output. The results are tabulated below:

| f[Hz] | Vin[V] | Vout[V] | Phase [degrees] |
|---|---|---|---|
| **10** | 10.3 | 9.68 | -20.8 |
| **13** | 10.3 | 9.2 | -25.7 |
| **15** | 10.3 | 8.72 | -30.8 |
| **18.7** | 10.2 | 7.76 | -40.9 |
| **21** | 10.1 | 6.96 | -47.4 |
| **24** | 10.1 | 5.76 | -56.2 |
| **27** | 10.1 | 4.48 | -60.9 |
| **30** | 10 | 2.96 | -70.3 |
| **33** | 10.1 | 1.42 | -75.2 |
| **35.4** | 10 | 0.204 | 0 |
| **36** | 10.1 | 0.352 | 54.9 |
| **40** | 10.1 | 2.1 | 74.5 |
| **44** | 10.1 | 3.56 | 65 |
| **48** | 10.1 | 4.8 | 59.4 |
| **52** | 10.1 | 5.72 | 53.8 |
| **56** | 10.2 | 6.44 | 47.9 |
| **60** | 10.1 | 7.08 | 43.6 |
| **61.6** | 10.2 | 7.24 | 43.4 |
| **62.5** | 10.1 | 7.36 | 40.5 |
| **70** | 10.2 | 8.24 | 33.6 |
| **80** | 10.3 | 8.88 | 28.4 |
| **100** | 10.2 | 9.52 | 20.8 |

Table 2: Measured values for Notch filter

# 3 Evaluation:

## 3.1 Part 1: High Pass

### 3.1.1 Drawing the Bode magnitude and phase plot from the values measured:

The magnitude of transfer function is $|A\ | = \frac{V_{out}}{V_{in}}$

For the frequency of $50\ Hz, V_{in} = 10.2\ V\ and\ V_{out} = \ 0.338\ V$

$$|A| = \frac{0.338\ V}{10.2\ V} = 0.03314$$

Scaling the magnitude to logarithmic scale, we do $20\log(|A|)$ to obtain it in decibels.

$20\log(|A|) = 20\log(0.03314) = -29.59$ dB

Similarly, by calculating the gain for each frequency, we get:

| f[Hz] | Vin[V] | Vout[V] | Phase [degrees] | 20log (Vo/Vi) |
|---|---|---|---|---|
| 50 | 10.2 | 0.338 | 88.5 | -29.5937 |
| 100 | 10.2 | 0.672 | 85.6 | -23.6246 |
| 200 | 10.2 | 1.34 | 82 | -17.6299 |
| 500 | 10.1 | 3.2 | 70.9 | -9.98343 |
| 1000 | 10 | 5.48 | 56.8 | -5.22439 |
| 2000 | 9.92 | 7.84 | 38.1 | -2.04391 |
| 5000 | 9.76 | 9.36 | 15.8 | -0.36348 |
| 10000 | 9.76 | 9.52 | 7.56 | -0.21626 |
| 20000 | 9.68 | 9.68 | 3.17 | 0 |
| 50000 | 9.68 | 9.68 | 2.88 | 0 |
| 100000 | 9.68 | 9.68 | 1.44 | 0 |

Table 3: Magnitude calculator for measured values

We can use this data to construct the Bode plots for magnitude and phase. These have been provided below:

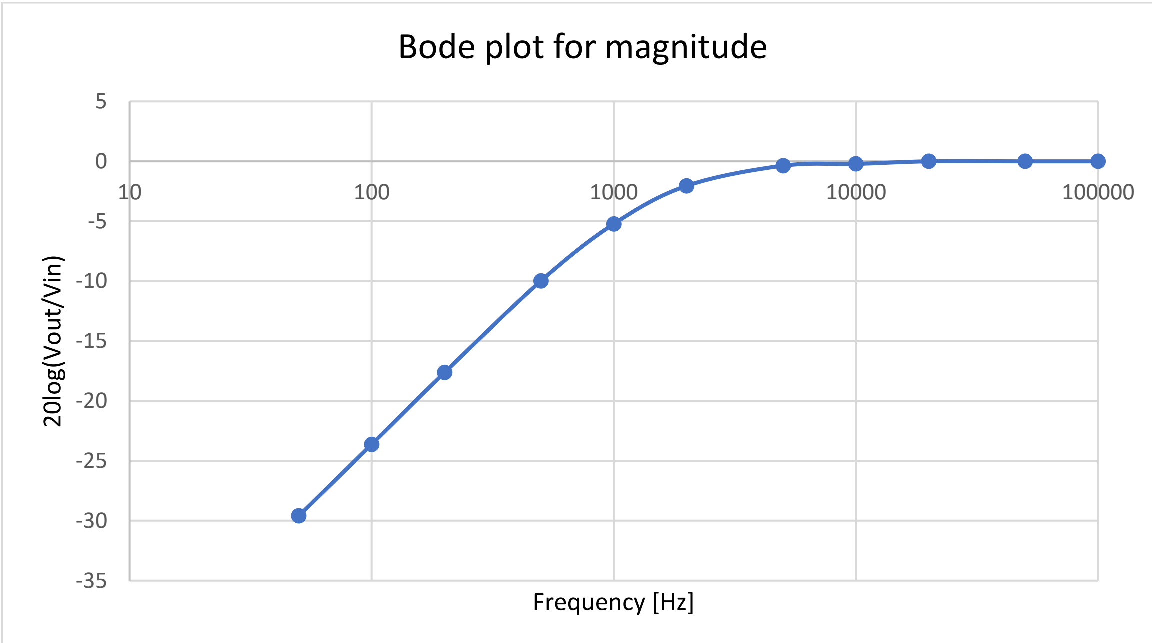



Fig 2: Bode magnitude plot for low pass filter

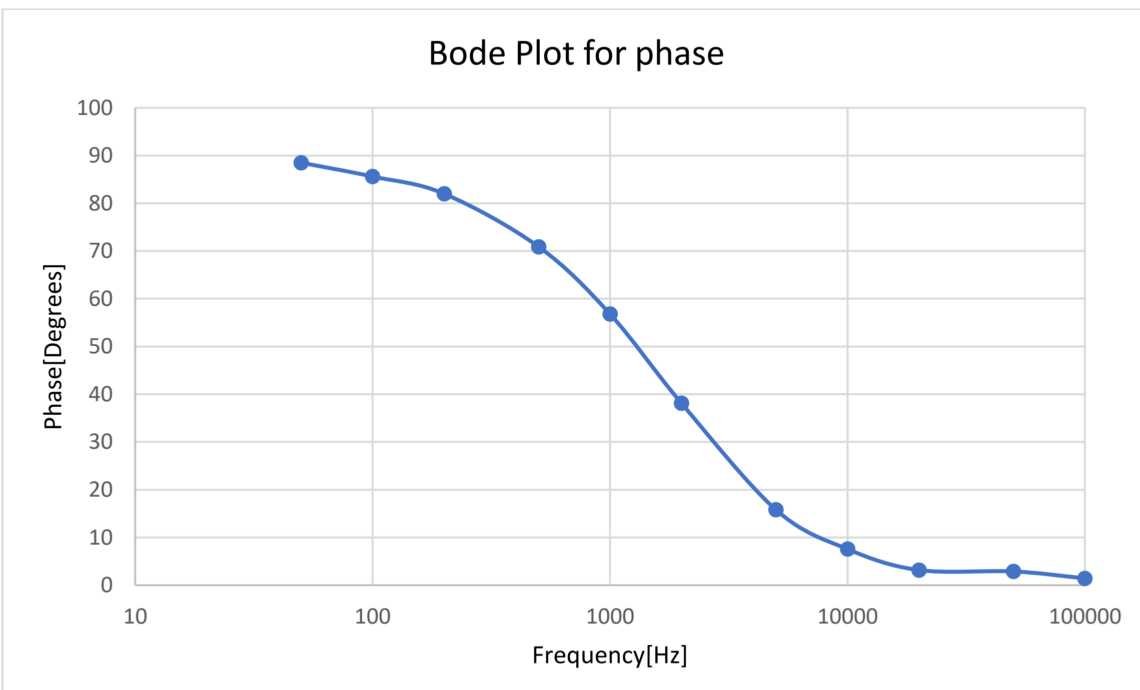



Fig 3: Bode phase plot for low pass filter

### 3.1.2 Calculating theoretical Bode plots from the formula given:

We know,

$$|A| = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}} \; and \; \varphi = \arctan\left(\frac{1}{\omega RC}\right)$$

For the frequency of 50Hz, we have,

$$\omega = 2\pi f = 314.159 \frac{rad}{s}, C = 100\, nF, R = 1\, K\Omega$$

$$|A| = \frac{1}{\sqrt{1 + 1/(314.159 \cdot 1 \cdot 10^3 \cdot 100 \cdot 10^{-9})^2}} = 0.0314$$

We convert it to decibel as follows, $20\log(|A|) = -30.061$

For phase

$$\varphi = \arctan\left(\frac{1}{\omega RC}\right)$$

$$\varphi = \arctan\left(\frac{1}{\omega RC}\right) = \arctan\left(1/(314.159 \cdot 1 \cdot 10^3 \cdot 100 \cdot 10^{-9})\right)$$

$$\varphi = 88.20^o$$

Similarly, we calculate the magnitude and phase for all frequencies as follows:

| f[Hz] | $\omega[rad/s]$ | Phase [degrees] | 20log (Vo/Vi) |
|---|---|---|---|
| **50** | 314.1593 | 88.20059 | -30.0613 |
| **100** | 628.3185 | 86.40473 | -24.0535 |
| **200** | 1256.637 | 82.83754 | -18.0838 |
| **500** | 3141.593 | 72.55941 | -10.4658 |
| **1000** | 6283.185 | 57.85809 | -5.48147 |
| **2000** | 12566.37 | 38.51189 | -2.13055 |
| **5000** | 31415.93 | 17.65679 | -0.41914 |
| **10000** | 62831.85 | 9.043061 | -0.10864 |
| **20000** | 125663.7 | 4.549865 | -0.02742 |
| **50000** | 314159.3 | 1.823166 | -0.0044 |
| **100000** | 628318.5 | 0.911814 | -0.0011 |

Table 4: Theoretical values of phase and |A| for high pass

Now, we have enough data to plot the following:

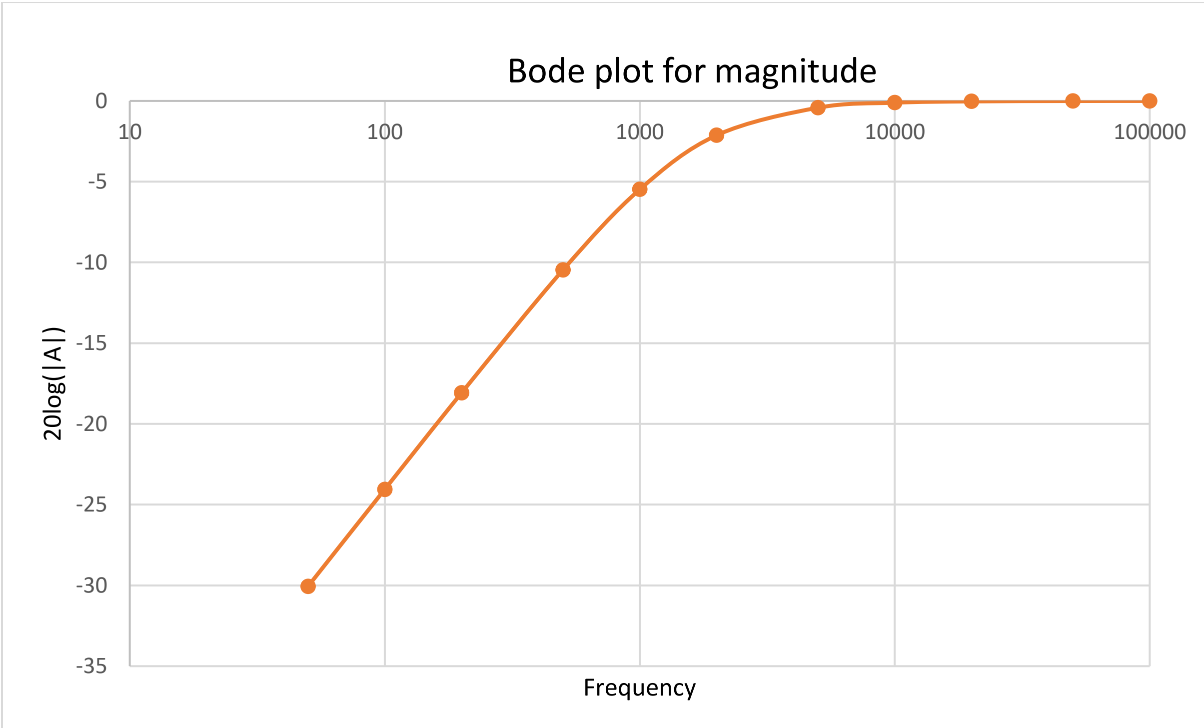



Fig 4: Theoretical Bode plot for magnitude


Fig 5: Theoretical Bode plot for phase

We can make comparisons between the Bode plots as follows:

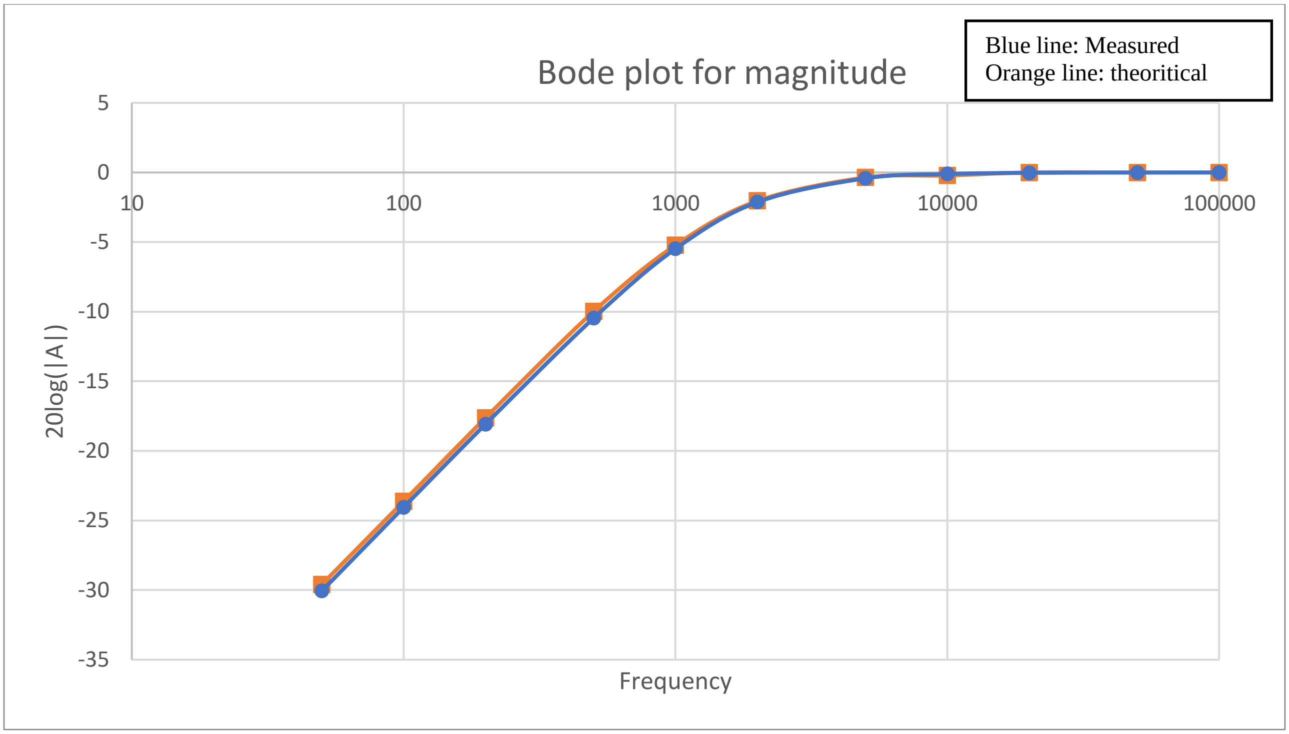



Fig 6: Comparison of theoretical and measured Bode magnitude plot

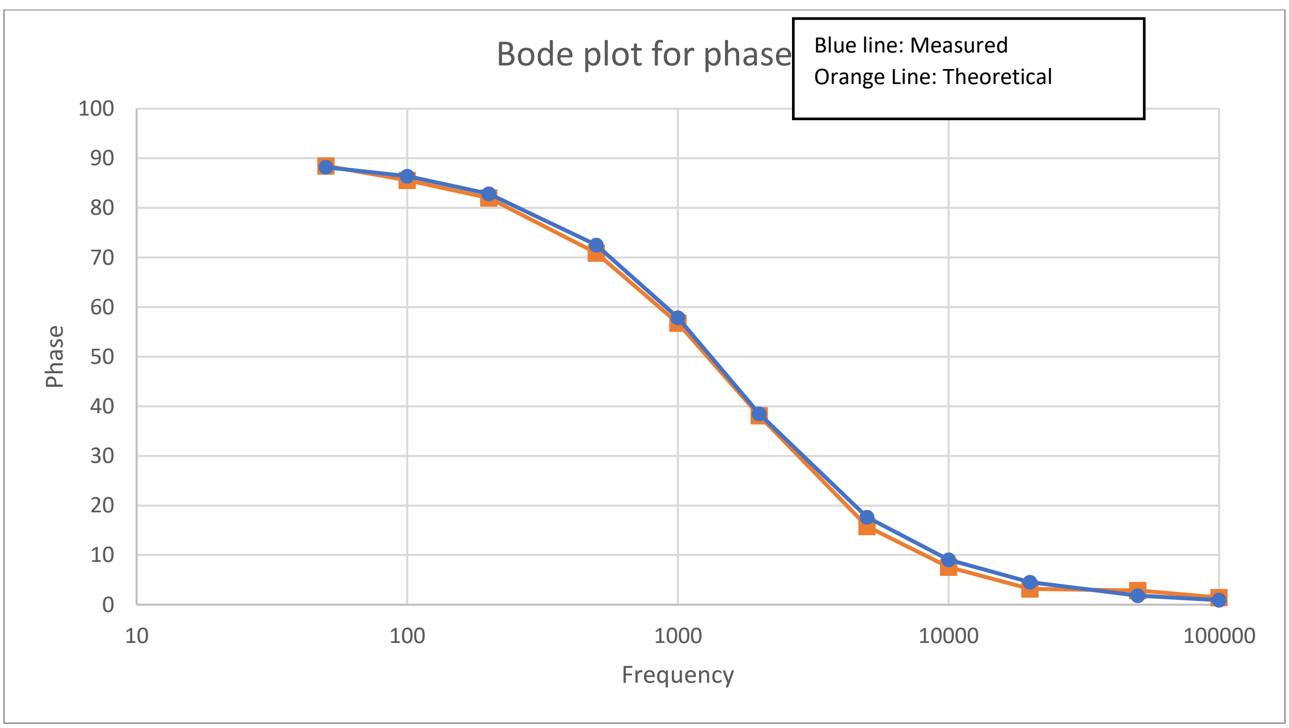



Fig 7: Comparison of theoretical and measured Bode phase plot

As we can see, the graphs made from theoretical and measured values for these Bode plots almost coincide with each other. However, some deviation in the phase plot for higher frequencies.

### 3.1.3 Calculating the frequency for -3dB:

For theoretical value of -3 dB, we have the following characteristic values:

$$20\log(A) = -3$$

$$A = 10^{-\frac{3}{20}} = 0.707946$$

$$A = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

$$0.707946 = \frac{1}{\sqrt{1 + \frac{1}{(\omega \cdot 1 \cdot 10^3 \cdot 100 \cdot 10^{-9})^2}}}$$

$$\omega_{-3dB} = 10023.779 \frac{rad}{s}$$

We can calculate the frequency as follows:

$$f = \frac{2\pi}{\omega_{-3dB}} = \frac{2\pi}{10023.779} = 1595.33\ Hz$$

Using the graph, we can calculate the corresponding value of A for the frequency 1595.33 Hz, as shown below:

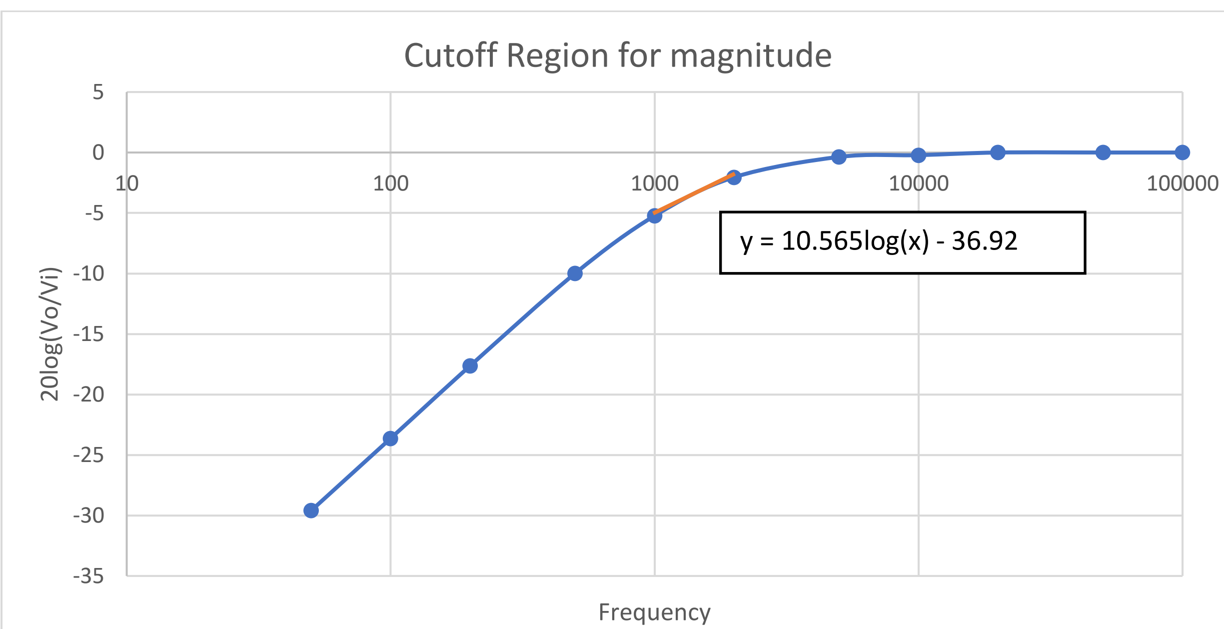



Fig 8: Approximation of cutoff region for magnitude

The approximate equation for the portion of the graph where the frequency of 1595.33 Hz lies is $y = 10.565\log(x) - 36.92$

When y = -3 dB,

$$-3 = 10.565\log(x) - 36.92$$

$$x = 1624.05$$

Which means the corresponding frequency for -3dB is 1624.05 Hz

From the above calculated values, it can be seen that there is a difference in the calculated value and the value we got from the graph and the main reason behind it is the linear approximation of the portion of the curve, which is not completely linear.

### 3.1.4 Calculating the phase shift for -3dB:

$$\omega_{-3dB} = 10023.779\frac{rad}{s}, f_{-3db} = 1595.33\ Hz$$

$$\varphi_{-3dB} = \arctan\left(\frac{1}{\omega RC}\right) = \arctan\left(\frac{1}{10023.7791 \cdot 10^3 \cdot 100 \cdot 10^{-9}}\right)$$

$$\varphi_{-3dB} = 45.07^o$$

The comparison of the value with the graph is given below:

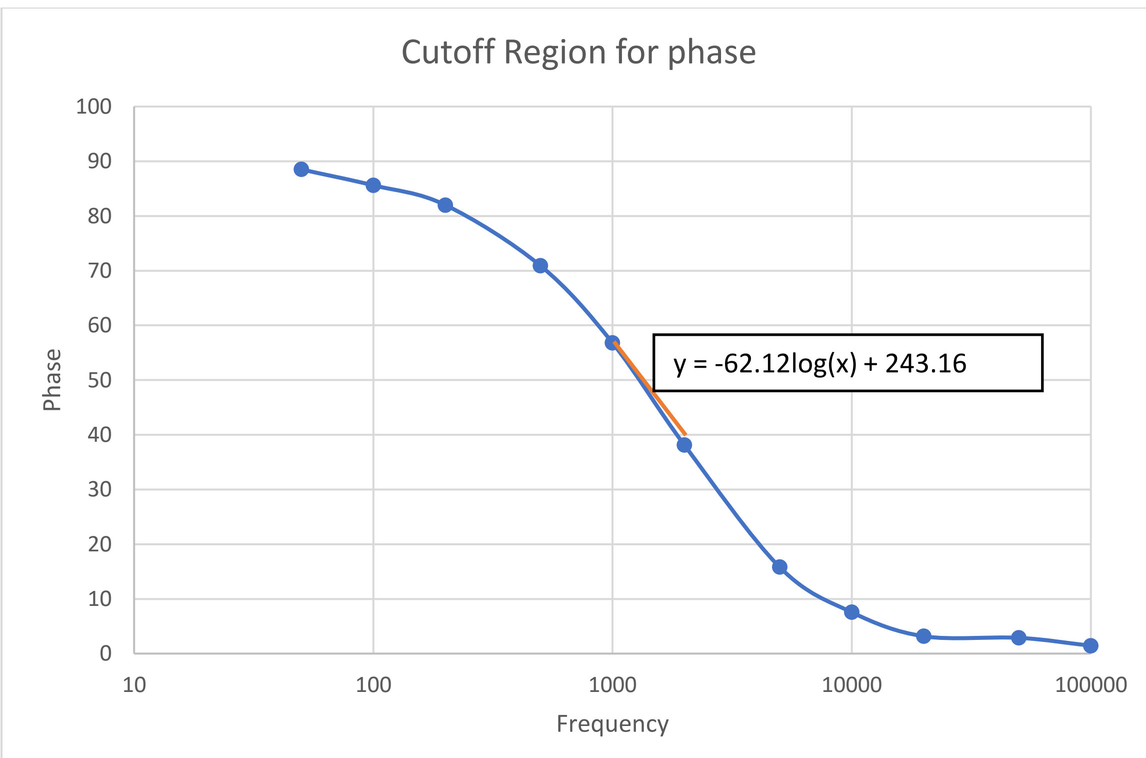



Fig 9: Approximation of cutoff region for phase

The region within the 1595.33 Hz frequency has the following equation:

$$y = -62.12\log(x) + 243.16$$

$$y = -62.12\log(1595.33) + 243.16 = 44.198^o$$

From this, we can infer that for frequency = 1595.33 Hz, the phase is $44.198^o$. This is close to the calculated phase of $45.07^o$. The slight deviation can be attributed to the linear approximation of the portion of the curve we used, which is not completely linear.

### 3.1.5 Gradient of |A| per decade:

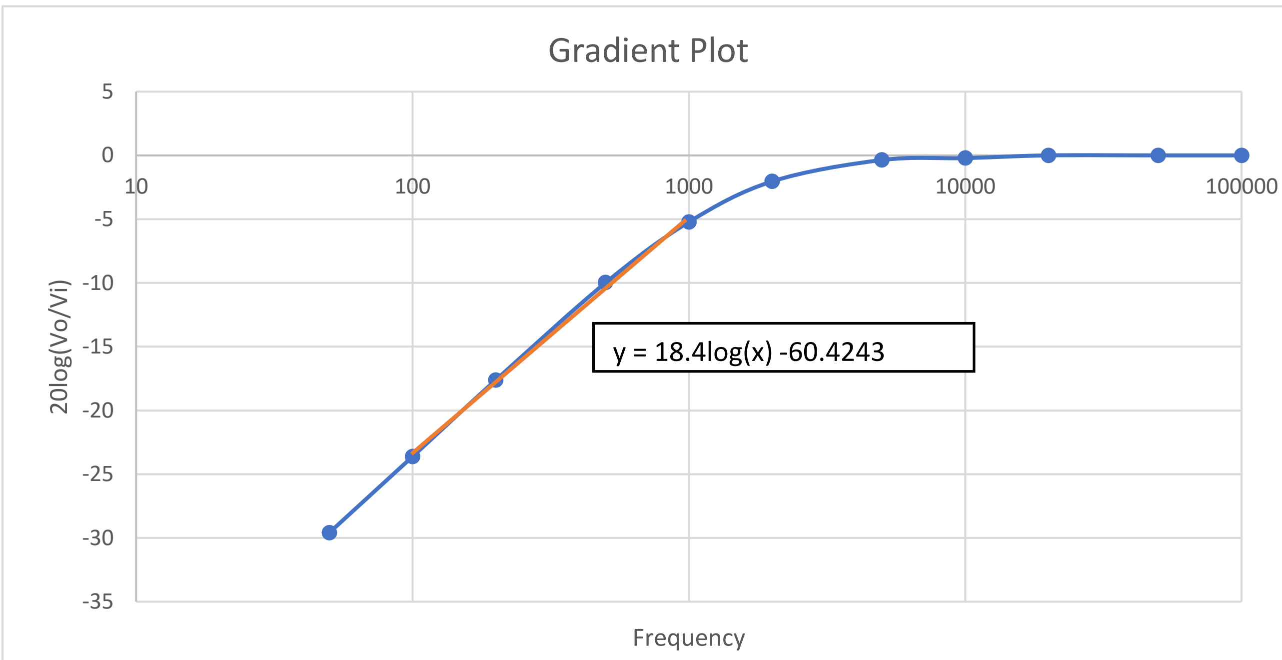



Fig 10: Linear approximation for gradient

The equation of the extrapolated line is as follows:

$$y = 18.4\log(x) - 60.424$$

Therefore, the gradient of the line is 18.4, which means the gradient of |A| per decade is 18.4.

### 3.1.6 Limits of amplitude:

We can determine the limit of amplitude as follows:

$$A = \frac{1}{\sqrt{1+\frac{1}{(\omega RC)^2}}}$$

$$20\log|A| = -10\log\left(1+\frac{1}{(\omega RC)^2}\right)$$

a. When $f \ll f_{-3dB}$

Magnitude:

$$\lim_{f\to 0,\omega\to 0} 20logA = \lim_{f\to 0,\omega\to 0} -10\log\left(1+\frac{1}{(\omega RC)^2}\right) = 0$$

Phase:

$$\lim_{f\to 0,\omega\to 0} \arctan(\varphi) = \lim_{f\to 0,\omega\to 0} \arctan\left(\frac{1}{\omega RC}\right) = 90^o$$

b. When $f \gg f_{-3dB}$

Magnitude:

$$\lim_{f\to \infty,\omega\to \infty} 20logA = \lim_{f\to \infty,\omega\to \infty} -10\log\left(1+\frac{1}{(\omega RC)^2}\right) = 20\log(\omega \mathrm{RC})$$

Phase:

$$\lim_{f\to \infty,\omega\to \infty} \arctan(\varphi) = \lim_{f\to \infty,\omega\to \infty} \arctan\left(\frac{1}{\omega RC}\right) = 0^o$$

c. When $f = f_{-3dB}$

$$f = f_{-3dB}$$
$$\frac{\omega}{2\pi} = \frac{1}{2\pi RC}$$
$$\omega = \frac{1}{RC}$$

Magnitude:

$20\log|\mathrm{A}| = -10\log\left(1+\frac{1}{(\omega RC)^2}\right) = -3.01\ dB$

Phase:

$$\varphi = \arctan\left(\frac{1}{\omega RC}\right) = 45^o$$

## 3.2 Part 2: Notch Filter

### 3.2.1 Drawing the Bode magnitude and phase plot from the values measured:

The magnitude of transfer function is the following:

$$|A| = \frac{V_{out}}{V_{in}}$$

For a frequency of $10\ Hz, V_{in} = 10.3\ V\ and\ V_{out} = 9.68\ V$

$$|A| = \frac{9.68\ V}{10.3\ V} = 0.9398$$

Scaling the magnitude to a logarithmic scale, we do $20\log(|A|)$ to obtain it in decibels.

$20\log(|A|) = 20\log(0.9398) = -0.53924\text{ dB}$

Similarly, calculating the gain for each frequency, we get the following:

| f[Hz] | Vin[V] | Vout[V] | Phase [degrees] | 20log (Vo/Vi) |
|---|---|---|---|---|
| **10** | 10.3 | 9.68 | -20.8 | -0.53924 |
| **13** | 10.3 | 9.2 | -25.7 | -0.98099 |
| **15** | 10.3 | 8.72 | -30.8 | -1.44641 |
| **18.7** | 10.2 | 7.76 | -40.9 | -2.37477 |
| **21** | 10.1 | 6.96 | -47.4 | -3.23424 |
| **24** | 10.1 | 5.76 | -56.2 | -4.87798 |
| **27** | 10.1 | 4.48 | -60.9 | -7.06087 |
| **30** | 10 | 2.96 | -70.3 | -10.5742 |
| **33** | 10.1 | 1.42 | -75.2 | -17.0407 |
| **35.4** | 10 | 0.204 | 0 | -33.8074 |
| **36** | 10.1 | 0.352 | 54.9 | -29.1556 |
| **40** | 10.1 | 2.1 | 74.5 | -13.642 |
| **44** | 10.1 | 3.56 | 65 | -9.05743 |
| **48** | 10.1 | 4.8 | 59.4 | -6.4616 |
| **52** | 10.1 | 5.72 | 53.8 | -4.93851 |
| **56** | 10.2 | 6.44 | 47.9 | -3.99429 |
| **60** | 10.1 | 7.08 | 43.6 | -3.08576 |
| **61.6** | 10.2 | 7.24 | 43.4 | -2.97723 |
| **62.5** | 10.1 | 7.36 | 40.5 | -2.74887 |
| **70** | 10.2 | 8.24 | 33.6 | -1.85346 |
| **80** | 10.3 | 8.88 | 28.4 | -1.28849 |
| **100** | 10.2 | 9.52 | 20.8 | -0.59926 |

Table 5: Magnitude calculator for measured values

Using these values, we can construct the Bode plots of magnitude and phase.

The Bode plots are as follows:

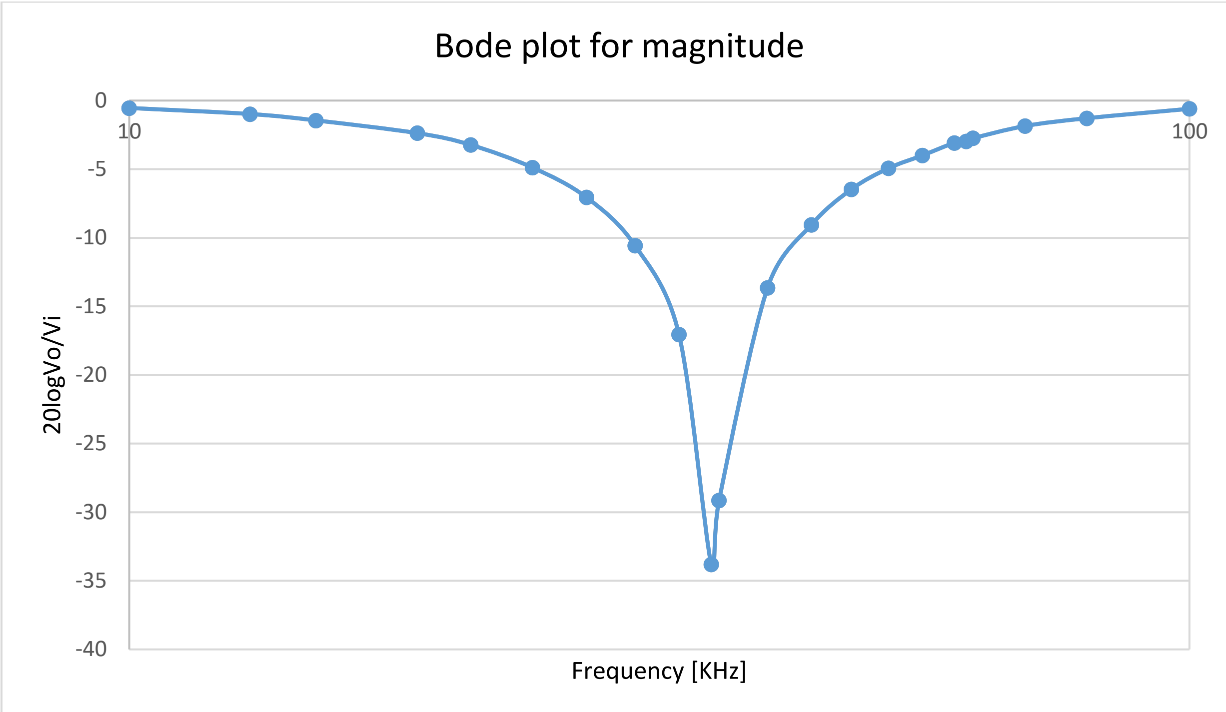



Fig 11: Bode magnitude plot for notch filter

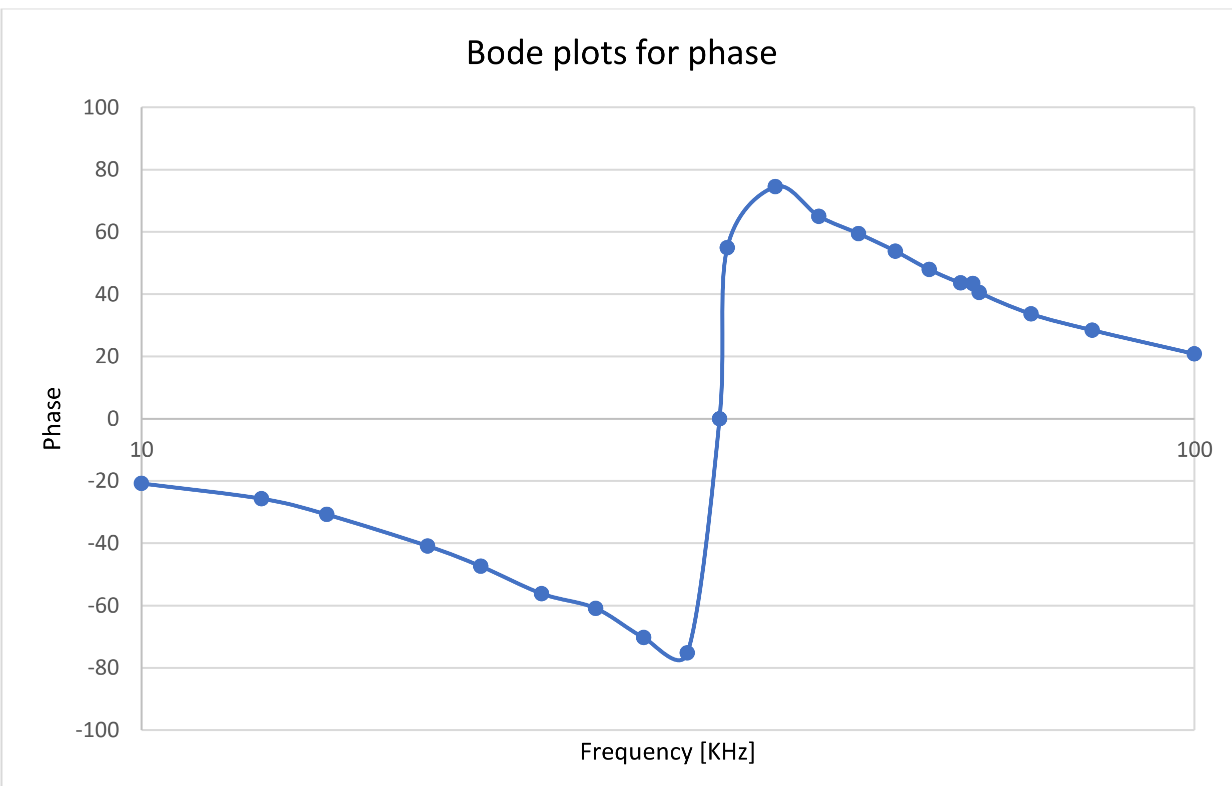



Fig 12: Bode phase plot for notch filter

### 3.2.2 Calculating theoretical Bode plots from the formula given

We will use the following known equations:

$$|A| = \frac{1}{\sqrt{1+\left(\frac{R}{\omega L-\frac{1}{\omega C}}\right)^2}} \quad and\ \varphi = \arctan\left(\frac{R}{\omega L-\frac{1}{\omega C}}\right)$$

We shall use the following parameters:

$f$ = 10KHz, $\omega = 2\pi f = 62831.85\ rad/s, C = 2.2\ nF, R = 2.7\ K\Omega,$ L $= 10$mH

Using the equations we have, we can make the following calculations:

$$|A| = \frac{1}{\sqrt{1+\left(\frac{2.7\cdot 10^3}{62831.85\cdot 10\cdot 10^{-3}-\frac{1}{62831.85\cdot 2.2\cdot 10^{-9}}}\right)^2}} = 0.925667$$

Converting it to decibel, we have $20\log(|A|) = -0.67090$

We can obtain the phase as follows:

$$\varphi = \arctan\left(\frac{R}{\omega L-\frac{1}{\omega C}}\right) = \arctan\left(\frac{R}{\omega L-\frac{1}{\omega C}}\right)$$

$$\varphi = \arctan\left(\frac{2.7\cdot 10^3}{62831.85\cdot 10\cdot 10^{-3}-\frac{1}{62831.85\cdot 2.2\cdot 10^{-9}}}\right)$$

$$\varphi = -22.23^o$$

We can now calculate the magnitude and phase for all frequencies and plot them as follows:

| f[KHz] | Phase [degrees] | 20log (Vo/Vi) |
|---|---|---|
| **10** | -22.231 | -0.6709008 |
| **13** | -29.625 | -1.2168087 |
| **15** | -34.830 | -1.7147609 |
| **18.7** | -45.067 | -3.0205181 |
| **21** | -51.790 | -4.1725925 |
| **24** | -60.843 | -6.2457195 |
| **27** | -69.996 | -9.3174330 |
| **30** | -78.966 | -14.3616571 |
| **33** | -87.481 | -27.1418499 |
| **35.4** | 86.172 | -23.5096451 |
| **36** | 84.659 | -20.6227774 |
| **40** | 75.372 | -11.9534495 |
| **44** | 67.463 | -8.3295519 |
| **48** | 60.803 | -6.2349753 |
| **52** | 55.207 | -4.8732190 |
| **56** | 50.487 | -3.9273892 |
| **60** | 46.478 | -3.2402133 |
| **61.6** | 45.042 | -3.0166785 |
| **62.5** | 44.272 | -2.9013259 |
| **70** | 38.745 | -2.1587615 |
| **80** | 33.224 | -1.5503215 |
| **100** | 25.903 | -0.9196205 |

Table 6: Theoretical values for Notch filter

We can use this data to construct our Bode plots, which are shown in the next page.

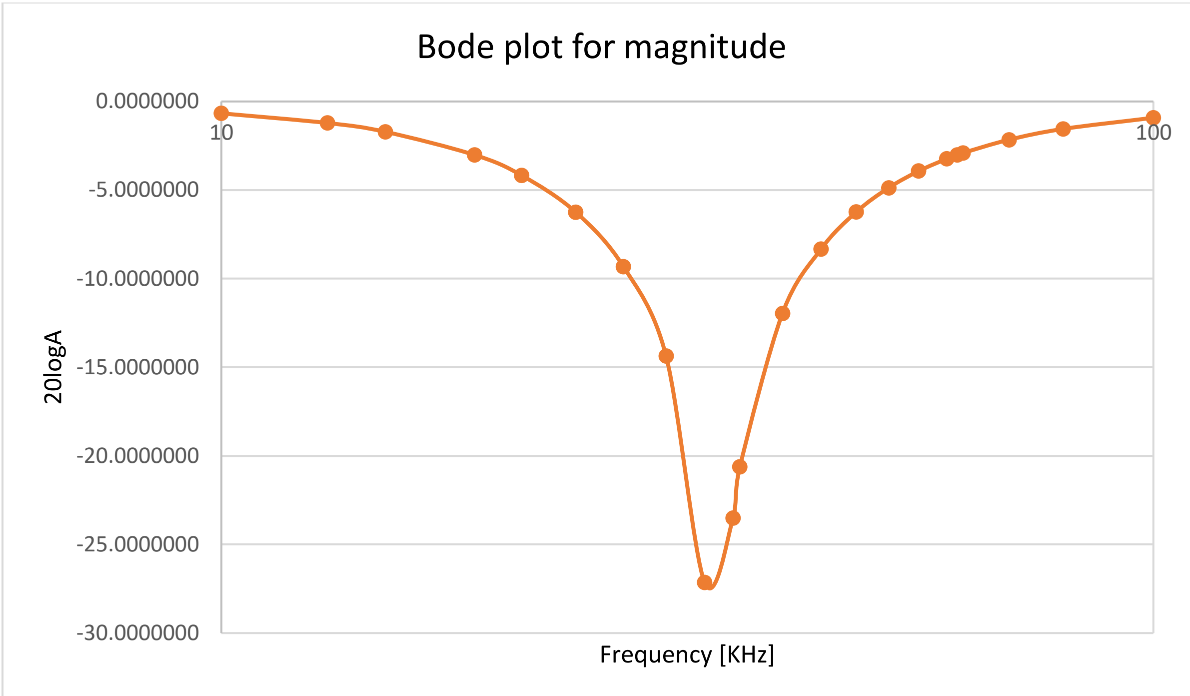



Fig 13: Theoretical Bode plot for magnitude

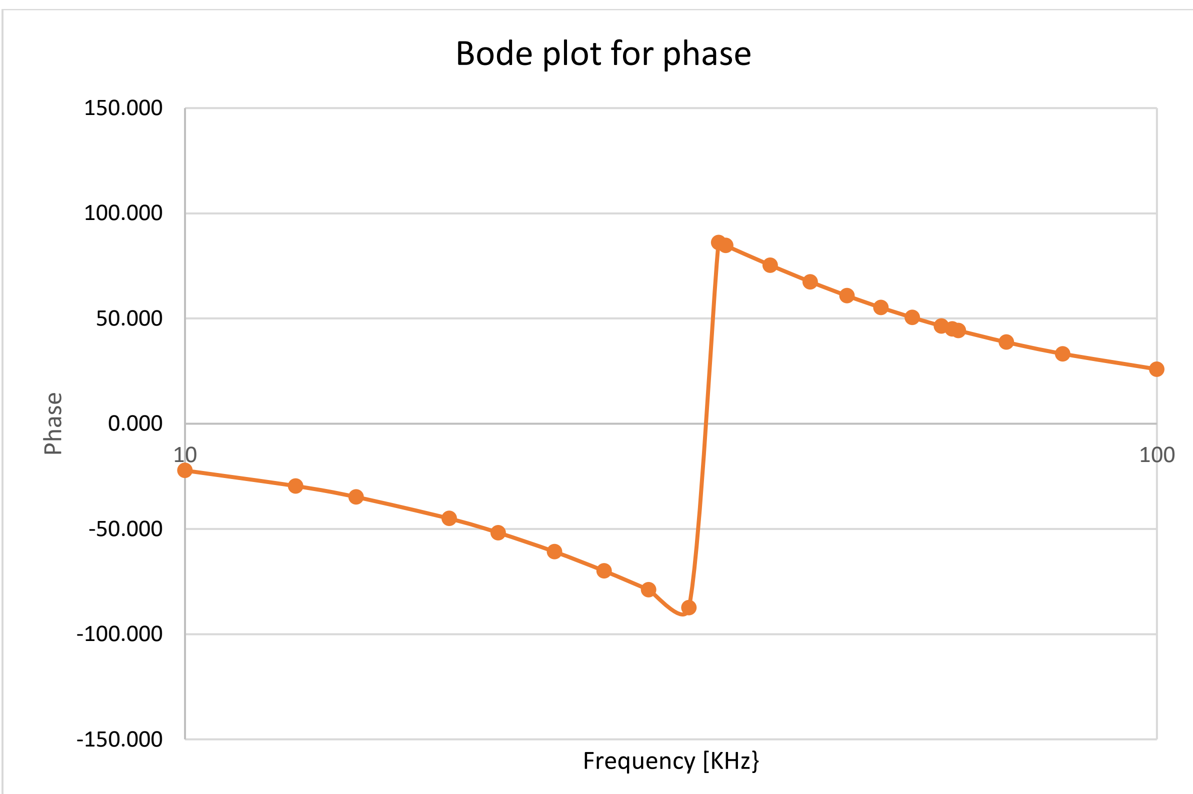



Fig 14: Theoretical Bode plot for phase

### 3.2.3 Calculating the center frequency, cutoff frequency and bandwidth

$$Center\ frequency\ f_{cf} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{\sqrt{10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}} = 33931.95Hz$$

We can calculate the cutoff frequencies as follows:

$$f_{cut-low} = \frac{1}{2\pi} \cdot \frac{-RC + \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$f_{cut-low} = \frac{1}{2\pi} \cdot \frac{-2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9} + \sqrt{(2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9})^2 + 4 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}}{2 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}$$

$$f_{cut-low} = 18676.52\ Hz$$

$$f_{cut-high} = \frac{1}{2\pi} \cdot \frac{RC + \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$f_{cut-high} = \frac{1}{2\pi} \cdot \frac{2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9} + \sqrt{(2.7 \cdot 10^3 \cdot 2.2 \cdot 10^{-9})^2 + 4 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}}{2 \cdot 10 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-9}}$$

$$f_{cut-high} = 61648.36\ Hz$$

The Bandwidth is given by the following:

$$\beta = f_{cut-high} - f_{cut-low} = 42971.84\ Hz$$

### 3.2.4 Comparing measured and theoretical Bode plots

We start by putting our Bode plots for both situations in the same graph, as shown below:

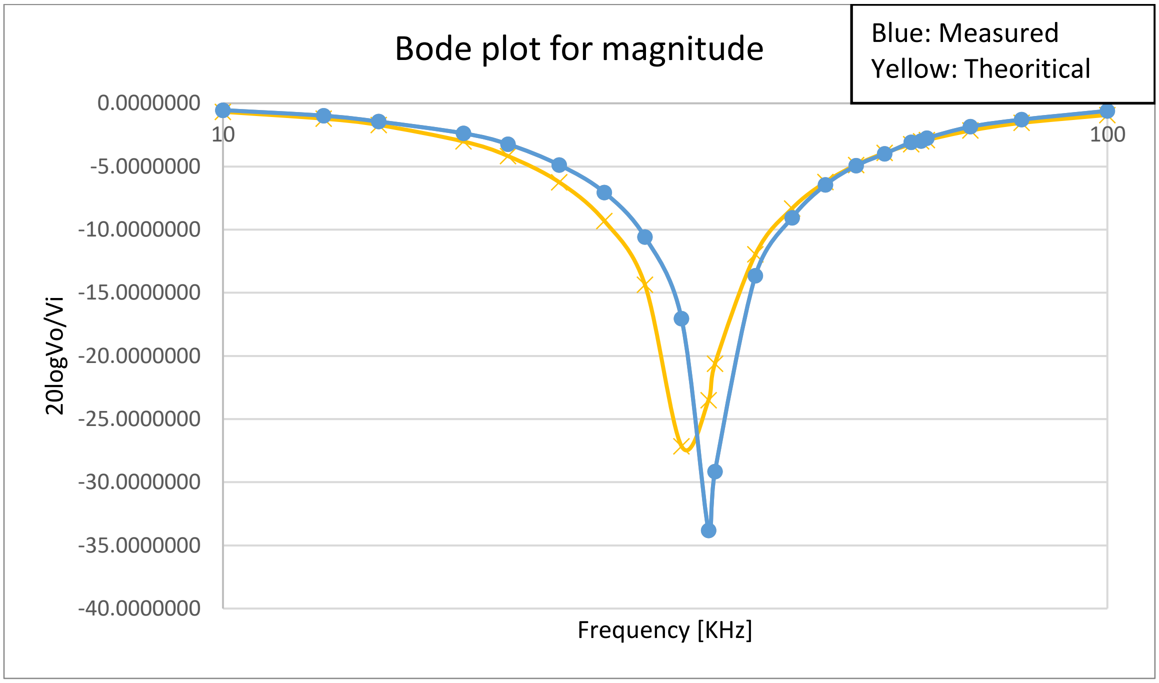



Fig 15: Comparison of theoretical and measured bode magnitude plot

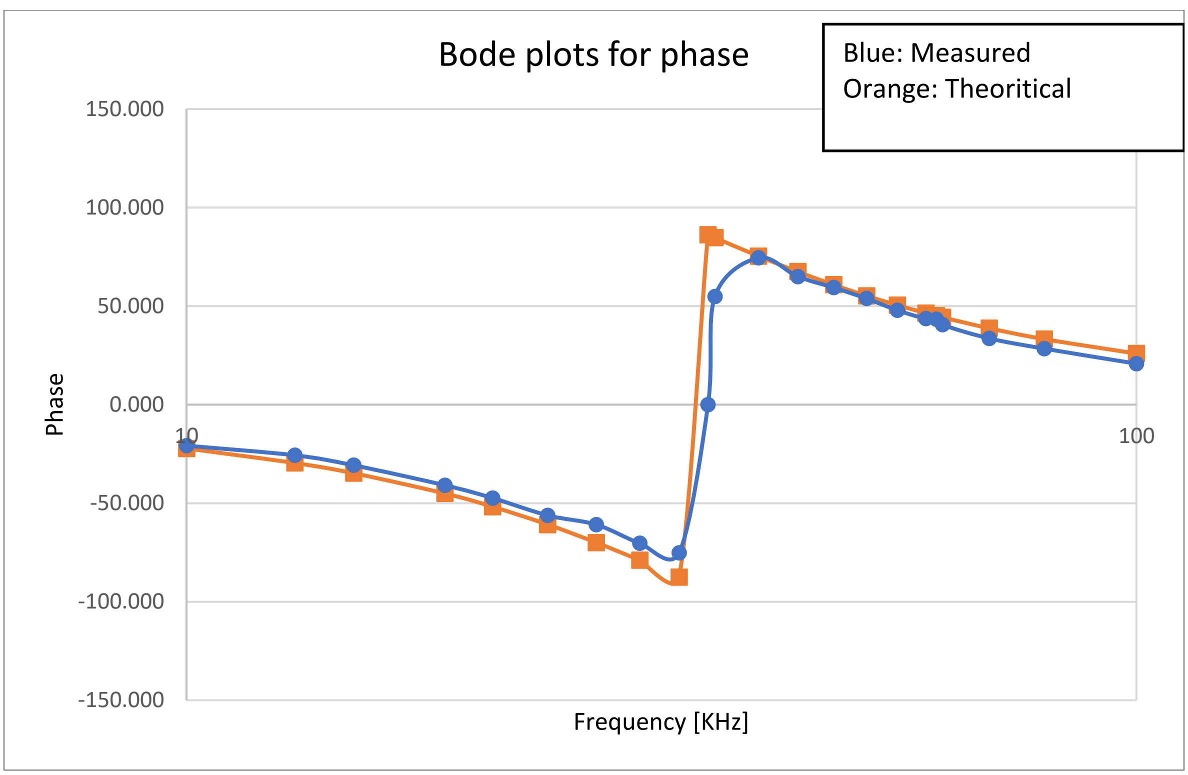



Fig 16: Comparison of theoretical and measured bode phase plot

From the graph, we can see that the Bode Plots for magnitude are similar in shape. However, there is a deviation in the values of A for lower frequencies.

For phase also the Bode plots are similar. Yet, here also, there is a deviation at the cutoff frequencies.

We see this disparity because we make calculations using theoretical values of R, L and C, which are exact. However, the same cannot be said in practice. The values of R, L and C for the resistors, capacitors and inductors that we used are not exact. Hence, these small variations in components results in large deviations in analysis or our data.

Some of the error contribution also comes from measuring devices. For example, the oscilloscope we used has an error range of 5-10%, which is a large source of error.

### 3.2.5 For Nyquist Plot:

At f=10KHz, we have the following characteristic values:

$$V_{in} = 10.3V, V_{out} = 9.68V \ and \ \varphi = -20.8^{o} or - 0.363 \ rad$$

$$A = \frac{V_{out}}{V_{in}} = \frac{9.68V}{10.3V} = 0.939805$$

$$A = 0.939805 \cdot (\cos(-0.363\,) + jsin(-0.363\,))$$

$$A = 0.878554 - \mathrm{j}0.33373$$

$$Re[A] = 0.878554, \mathrm{Im}[A] = -0.33373$$

We can tabulate the data we obtained for multiple frequencies as follows:

| f[KHz] | Vin[V] | vout[V] | Phase[º] | Phase[rad] | \|A\| |
|---|---|---|---|---|---|
| **10** | 10.3 | 9.68 | -20.8 | -0.363028484 | 0.939805825 |
| **13** | 10.3 | 9.2 | -25.7 | -0.448549618 | 0.893203883 |
| **15** | 10.3 | 8.72 | -30.8 | -0.53756141 | 0.846601942 |
| **18.7** | 10.2 | 7.76 | -40.9 | -0.713839664 | 0.760784314 |
| **21** | 10.1 | 6.96 | -47.4 | -0.827286065 | 0.689108911 |
| **24** | 10.1 | 5.76 | -56.2 | -0.98087504 | 0.57029703 |
| **27** | 10.1 | 4.48 | -60.9 | -1.062905514 | 0.443564356 |
| **30** | 10 | 2.96 | -70.3 | -1.226966464 | 0.296 |
| **33** | 10.1 | 1.42 | -75.2 | -1.312487597 | 0.140594059 |
| **35.4** | 10 | 0.204 | 0 | 0 | 0.0204 |
| **36** | 10.1 | 0.352 | 54.9 | 0.958185759 | 0.034851485 |
| **40** | 10.1 | 2.1 | 74.5 | 1.300270293 | 0.207920792 |
| **44** | 10.1 | 3.56 | 65 | 1.134464014 | 0.352475248 |
| **48** | 10.1 | 4.8 | 59.4 | 1.036725576 | 0.475247525 |
| **52** | 10.1 | 5.72 | 53.8 | 0.938987138 | 0.566336634 |
| **56** | 10.2 | 6.44 | 47.9 | 0.836012712 | 0.631372549 |
| **60** | 10.1 | 7.08 | 43.6 | 0.760963554 | 0.700990099 |
| **61.6** | 10.2 | 7.24 | 43.4 | 0.757472895 | 0.709803922 |
| **62.5** | 10.1 | 7.36 | 40.5 | 0.706858347 | 0.728712871 |
| **70** | 10.2 | 8.24 | 33.6 | 0.586430629 | 0.807843137 |
| **80** | 10.3 | 8.88 | 28.4 | 0.495673508 | 0.862135922 |
| **100** | 10.2 | 9.52 | 20.8 | 0.363028484 | 0.933333333 |

Tables 7: Calculation of parameters for Nyquist plot

We prepare the following table for our Nyquist plot:

| f[KHz] | A[a+jb] | Re[A] | Im[A] |
|---|---|---|---|
| **10** | 0.878554616263439-0.333731591855414j | 0.87855462 | -0.333732 |
| **13** | 0.80484549477313-0.387345978466544j | 0.80484549 | -0.387346 |
| **15** | 0.727197116624796-0.433496483657416j | 0.72719712 | -0.433496 |
| **18.7** | 0.575041462817734-0.498116540632015j | 0.57504146 | -0.498117 |
| **21** | 0.466441262276368-0.507251062015183j | 0.46644126 | -0.507251 |
| **24** | 0.317253737156609-0.473907974556449j | 0.31725374 | -0.473908 |
| **27** | 0.215721040029429-0.387574213782068j | 0.21572104 | -0.387574 |
| **30** | 0.0997801964932323-0.278675281264363j | 0.09978020 | -0.278675 |
| **33** | 0.03591415606622-0.135929624932508j | 0.03591416 | -0.135930 |
| **35.4** | 0.0204 | 0.02040000 | 0.000000 |
| **36** | 0.0200397870019044+0.0285137327260998j | 0.02003979 | 0.028514 |
| **40** | 0.0555644148281524+0.200358807102783j | 0.05556441 | 0.200359 |
| **44** | 0.148962476415534+0.319451061569354j | 0.14896248 | 0.319451 |
| **48** | 0.24192067283186+0.409065517784052j | 0.24192067 | 0.409066 |
| **52** | 0.334481625622374+0.45701118667946j | 0.33448163 | 0.457011 |
| **56** | 0.423288963342615+0.468463178027742j | 0.42328896 | 0.468463 |
| **60** | 0.507637304849235+0.48341647224243j | 0.50763730 | 0.483416 |
| **61.6** | 0.515725550767634+0.487697409629806j | 0.51572555 | 0.487697 |
| **62.5** | 0.55411761453626+0.47326115205051j | 0.55411761 | 0.473261 |
| **70** | 0.672869708181492+0.447053565271094j | 0.67286971 | 0.447054 |
| **80** | 0.758376633694714+0.410052716169325j | 0.75837663 | 0.410053 |
| **100** | 0.87250396463628+0.331433164914261j | 0.87250396 | 0.331433 |

Tables 8: Calculation of parameters for Nyquist plot

We can make the following plot using the data we have obtained:

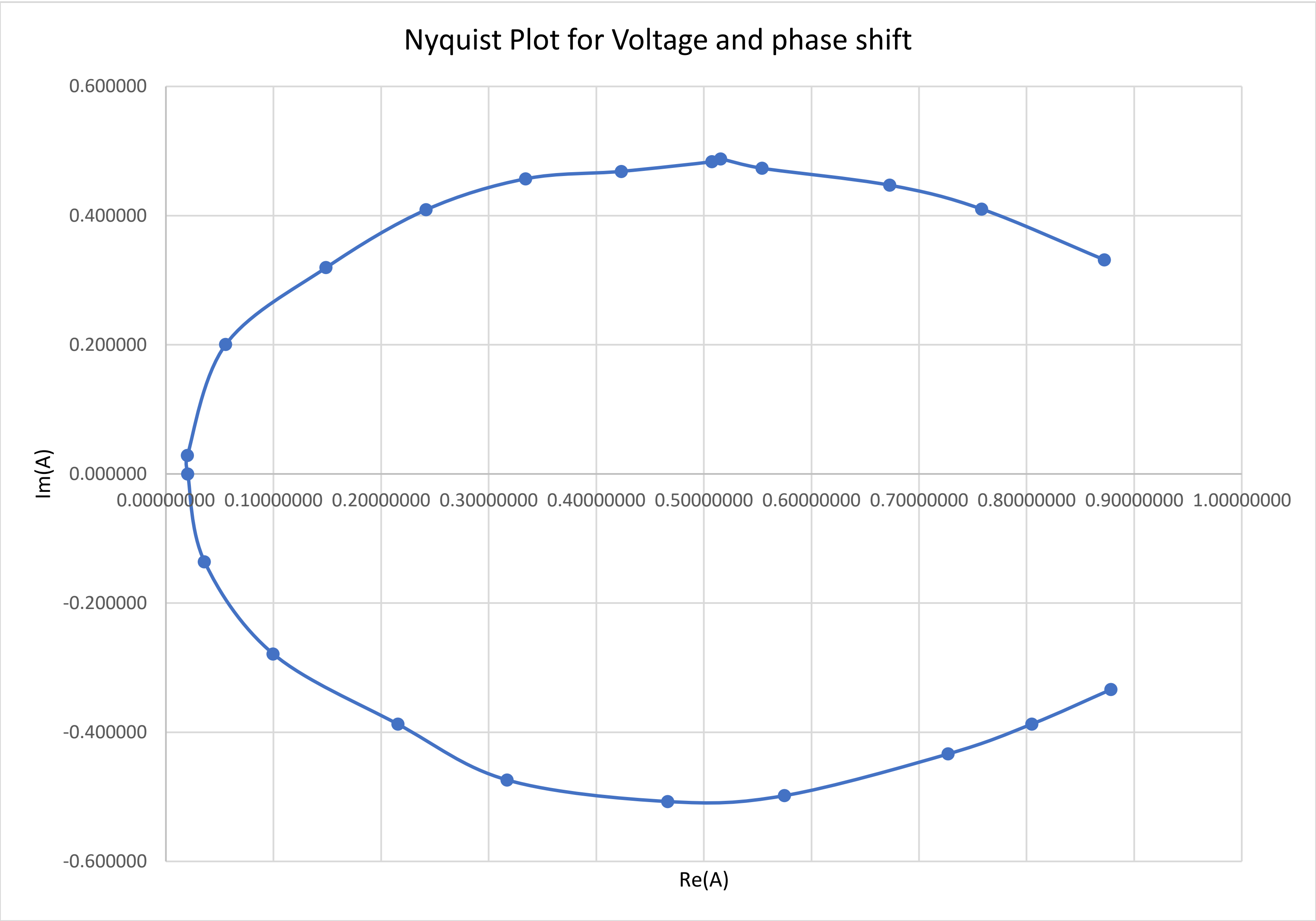



Fig 17: Nyquist plot for voltage and phase shift

# 4 Conclusion:

In this lab we learned about filters. We used capacitors, resistors, inductors, and other components to build filters and run some experiments on them. Then, we determined some characteristic data about the filters, and using this information, constructed the Bode Magnitude and Phase plots, and in one event also a Nyquist plot. We then made another set of Bode Magnitude and Phase plots using theoretical values, and compared our results.

In the first experiment we built a High Pass filter, and executed for it the steps stated above. Then, we built a notch filter and repeated the steps. Comparing the results for these filters helped us understand what differences we can expect for these two filters in practice, compared to our theoretical models. For example, for high pass filters, we observed that the Bode magnitude and phase plots differ at high frequencies, while they stay close in lower frequencies. However, in the Bode plots for the notch filter, we saw that a large amount of inconsistency is distributed throughout the whole of the plot.

During analysis of these filters, we were able to investigate and determine sources that contribute to these errors, which could help us perform better experiments using filters in the future. We learned how error in measured values can propagate during calculations and deviate our results from true values. We observed this not only in our Bode plots, but also in our calculations for the -3dB frequency, where the value calculated from measured data was somewhat different from that calculated from theoretical data.

Overall, we obtained a wholesome and practical understanding with regards to the build and characteristics of a high pass filter and a notch filter. We were introduced to analysis and testing techniques for these filters, which gave us an overview of the factors that need to be kept in mind when using these filters to solve real-world problems.

# 5 Appendix:

Measured data from Experiment 6: The Operational Amplifier:

## 5.1 Part 1: Inverting Amplifier:

First case: $R_1 = R_2 = 10K0$, $R_3 = 50K0$

| f[kHz] | Vin[v] | Vout[V] | Phase shift |
|---|---|---|---|
| 1 | 0.504 | 0.504 | 180 |
| 2 | 0.512 | 0.504 | 179 |
| 3 | 0.508 | 0.504 | 179 |
| 5 | 0.512 | 0.508 | 179 |
| 10 | 0.512 | 0.512 | 179 |
| 20 | 0.512 | 0.512 | 176 |
| 50 | 0.516 | 0.508 | 171 |
| 100 | 0.516 | 0.5 | 164 |
| 200 | 0.516 | 0.48 | 148 |
| 500 | 0.52 | 0.324 | 104 |
| 1000 | 0.524 | 0.16 | 63.5 |
| 2000 | 0.52 | 0.064 | 4.31 |
| 4000 | 0.532 | 0.0224 | -27 |
| 5000 | 0.544 | 0.016 | -57.9 |

Second Case: $R_1 = 22K0$, $R_2 = 10K0$, $R_3 = 50K0$

| f[kHz] | Vin[v] | Vout[V] | Phase shift |
|---|---|---|---|
| 1 | 0.528 | 0.248 | 180 |
| 2 | 0.536 | 0.248 | 177 |
| 3 | 0.544 | 0.256 | 177 |
| 5 | 0.536 | 0.252 | 177 |
| 10 | 0.536 | 0.256 | 178 |
| 20 | 0.544 | 0.256 | 176 |
| 50 | 0.544 | 0.256 | 176 |
| 100 | 0.544 | 0.248 | 165 |
| 200 | 0.552 | 0.252 | 160 |
| 500 | 0.528 | 0.208 | 120 |
| 1000 | 0.528 | 0.124 | 68.1 |
| 2000 | 0.52 | 0.0456 | 1.44 |
| 4000 | 0.544 | 0.0128 | -42 |
| 5000 | 0.544 | 0.0104 | -67.3 |

Third Case: $R_1$ = 1K00, $R_2$ = 10K0, $R_3$ = 50K

| f[kHz] | Vin[v] | Vout[V] | Phase shift |
|---|---|---|---|
| **1** | 0.496 | 4.92 | 179 |
| **2** | 0.496 | 4.92 | 178 |
| **3** | 0.5 | 5 | 177 |
| **5** | 0.5 | 5 | 176 |
| **10** | 0.492 | 4.92 | 171 |
| **20** | 0.504 | 4.84 | 161 |
| **50** | 0.508 | 3.68 | 133 |
| **100** | 0.516 | 2.08 | 108 |
| **200** | 0.516 | 1.08 | 95.7 |
| **500** | 0.516 | 0.412 | 75.1 |
| **1000** | 0.52 | 0.192 | 55.9 |
| **2000** | 0.524 | 0.0824 | 13.6 |
| **4000** | 0.536 | 0.0336 | 17.2 |
| **5000** | 0.544 | 0.0265 | -12.9 |

## 5.2 Part 2: Inverting Integrator

When frequency is set to 1KHz for square input wave, the signal is as expected, and the values for variable frequencies are:

| f[Hz] | Vin[v] | Vout[V] | Phase shift |
|---|---|---|---|
| **200** | 1.02 | 3.7 | 92.1 |
| **500** | 1.02 | 1.5 | 92.8 |
| **1000** | 1.02 | 0.74 | 90.4 |
| **2000** | 1.02 | 0.38 | 91.8 |
| **5000** | 1.02 | 0.156 | 90.1 |
| **10000** | 1.02 | 0.08 | 89.2 |
| **15000** | 1.02 | 0.0526 | 90.2 |
| **20000** | 1.02 | 0.04 | 92.8 |

### 5.3 Part 3: Differential Amplifier:

While measuring tenma, voltage is -1.36V

Current in the ammeter = 98.82 mA

Voltage drop across ammeter = 0.1229V

$U^-$ = 5.098V

$U^+$= 4.965 V

Vout = -1.3011 V

# 6 References:


- Electrical Engineering-II Lab Manual (Uwe Pagel)
- http://www.faculty.jacobs-university.de/upagel/
- https://physics.stackexchange.com