Jacobs University Bremen

CO-520-B

Signals and Systems Lab

RLC-Circuit – Transient Response

Priontu Chowdhury

Introduction

Objective

On this lab, we studied the transient response of second order systems. We changed different parameters of the circuit and analyzed the response of the circuit to them. The oscilloscope was used to record changes across the capacitor of the RLC circuit, so we made changes to the resistance and observed the response across the capacitor to those changes, namely the overdamped, underdamped and critically damped responses.

Theory

RLC circuits are systems containing two energy storage elements, the capacitor and the inductor. They are second order electrical systems, so they can be modeled as second order differential equations.

A typical RLC circuit looks as follows:

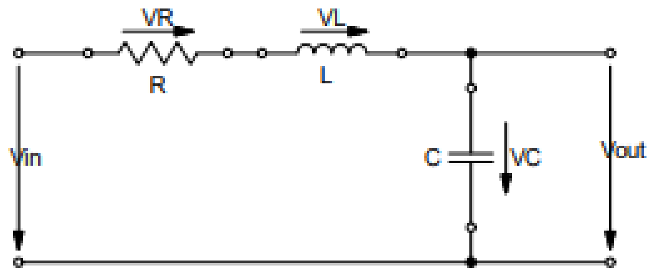


Figure 1: A Second-Order Series RLC circuit

From the circuit above, we can deduce that $V_{in} = V_R + V_L + V_c$. We know, $V_c = V_{out}$.

Therefore,

$$i = i_c = C\frac{dV_{out}}{dt}$$

$$V_R = iR = RC\frac{dV_{out}}{dt}$$

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left(C\frac{dV_{out}}{dt}\right) = LC\frac{d^2V_{out}}{dt^2}$$

Substituting these equations into the first equation, this circuit can be modeled as follows:

$$LC\frac{d^2V_{out}}{dt^2} + RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

Which can be formatted as follows:

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

From the above models, we can deduce the undamped natural frequency, the damping ratio and the gain of the circuit as follows:

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$K = 1$$

The complete solution to this system can be modeled as follows:

1) The transient response:

   The transient response of a system is the response of the system to a change in its equilibrium. In an electric circuit, when an input is provided, it takes a certain amount of time for the system to reach steady state. That is the transient response of the circuit.

   The transient response can be categorized into the following:

   a. Over-damped response ($\zeta > 1$):
      When the damping ratio is greater than one, the general solution to the homogeneous equation is as follows:
      $y(t) = C_1 \exp\left((-\zeta + \sqrt{\zeta^2 - 1})\omega_n t\right) + C_2 \exp\left((-\zeta - \sqrt{\zeta^2 - 1})\omega_n t\right)$
   b. Critically damped response ($\zeta = 1$):
      For this case, the general homogenous solution is the following:
      $y(t) = C_1 \exp(-\zeta\omega_n t) + C_2 t \exp(-\zeta\omega_n t)$
   c. Under-damped response ($0 < \zeta < 1$):
      In this case, the general solution looks as follows:
      $y(t) = \exp(-\zeta\omega_n t)(C_1 \cos(\omega_n\sqrt{(1-\zeta^2)}\,t) + C_2 \sin(\omega_n\sqrt{(1-\zeta^2)}\,t))$
      where,
      $\omega_d = \omega_n\sqrt{1-\zeta^2}$
      And therefore, the equation can be rewritten as follows:
      $y(t) = \exp(-\zeta\omega_n t)(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$

Visually, the responses look as follows:

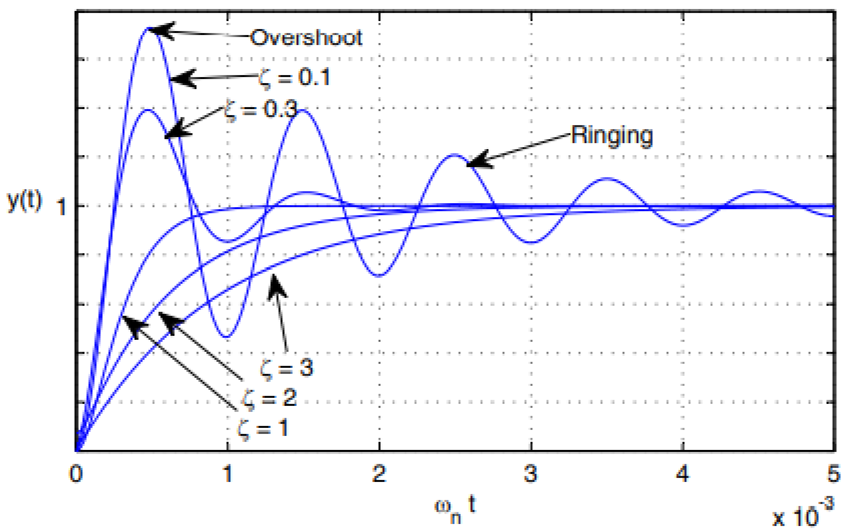


Figure 2: Overdampred, underdamped and critically damped systems

2) The steady state response
   This is the constant DC input signal to the circuit to the circuit.

The solution to the differential equation can be found by first finding the homogenous solution and then the particular solution. We find the homogenous solution by setting x(t) = 0 and solving the equation. x(t) is the forcing function, which is a result of having an external source which causes a force on the circuit. The homogenous part of the solution is the natural response.

The response to the forcing function is usually the same form as the forcing function, which we find by solving the non-homogenous differential equation for the particular solution. In case of DC-input, the solution is usually the steady state response of the component under examination.

Steady-State Value

The voltage or current magnitude of the system after it reaches stability is called the steady-state response.

Ringing

When a system is underdamped, it results in an oscillation phenomenon which is called ringing.

Overshoot

Overshoot occurs when the transient signal exceeds the final steady state value. Percentage overshoot is described as follows:

$$\text{Percentage overshoot} = \frac{V_{max} - V_{SteadyState}}{V_{SteadyState}} * 100\%$$

The Complete Response

We can determine the complete response of the second order system as follows:

For transient response

1) We use Ohm's law, KVL and KCL to obtain a second order nonhomogeneous differential equation.
2) We can solve the homogenous counterpart to the non-homogeneous equation to obtain the transient solution.

For DC steady state response we replace all the capacitances with open circuits and replace all the inductances with short circuits, and solve the circuit for the required steady state response.

We sum all the solutions we obtain to find the complete solution, which contains unknown coefficients C1 and C2. We can solve for these coefficients using the initial conditions.

Experimental Set-up and Results

The following tools were used for the experiment:

- Signal Generator
- Breadboard
- Oscilloscope
- BNC Cable
- 1n5F Capacitor
- 10mH
- Resistor Decade

We used the above tools to implement the following set-up:

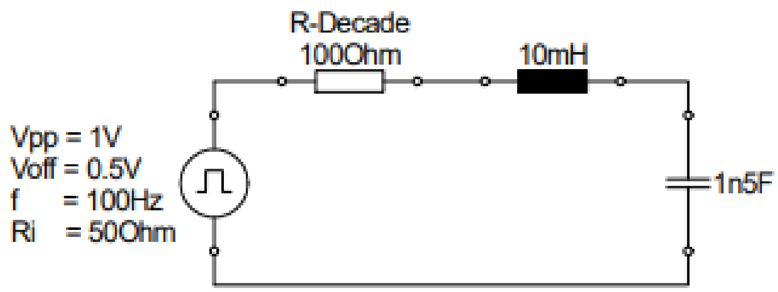


Figure 3: The series RLC circuit used in our experiment.

We used the function generator to produce a 100 Hz square wave with an amplitude of 0.5V and an offset of 0.5V. The result was a signal that modulated between 0V and 1V. Then, we set the R-Decade to 100 Ohms, and connected the capacitor in parallel to the oscilloscope.

The result was the following response that could be seen on the oscilloscope:

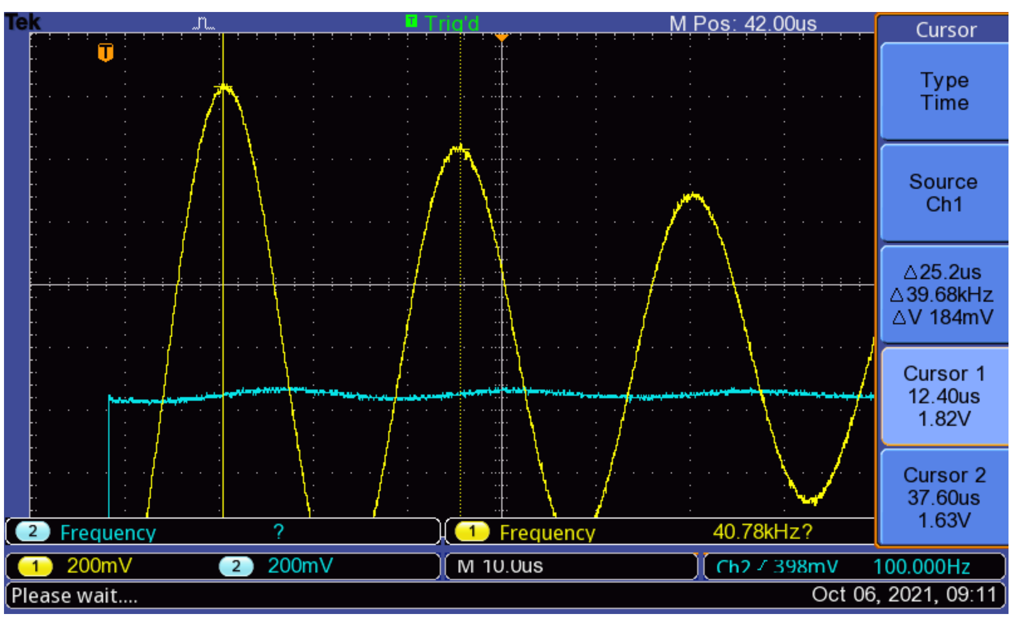


Figure 4: Ringing Phenomenon

What we see above is the ringing phenomenon of the under-damped response of the RLC circuit. Now that we had the response, we were able to measure the damped frequency f_d using the cursors of the oscilloscope. We did this by zooming on the picture obtained and setting the cursors at the two peaks.

The results looked as follows:

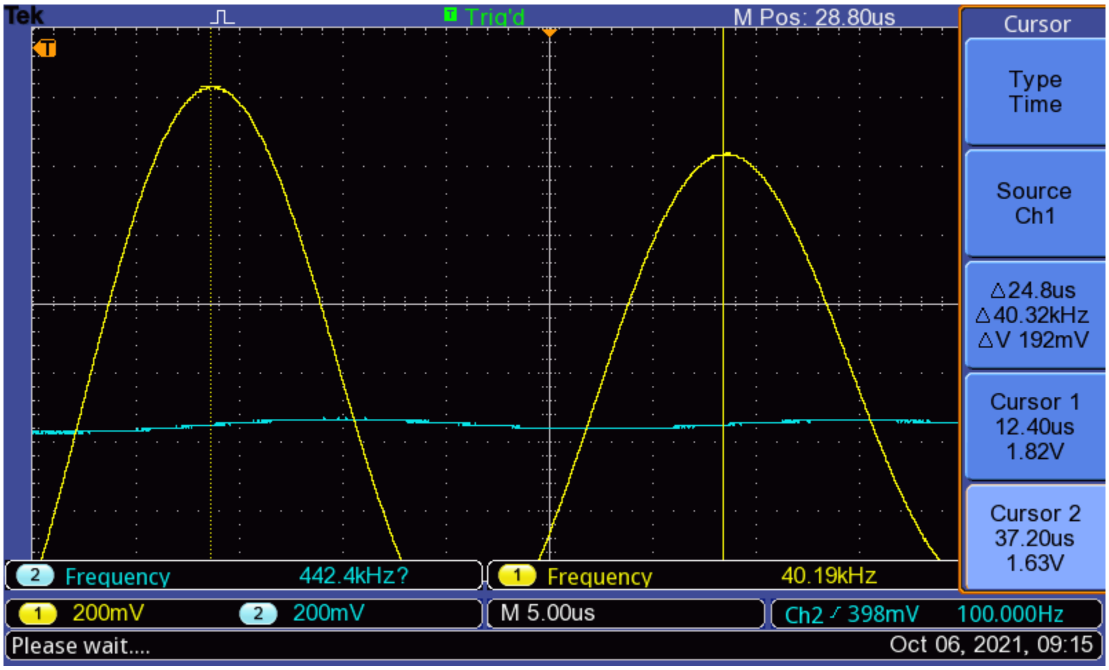


Figure 5: One signal period

We can see the results on the right. According to the measurements of the oscilloscope, the damping frequency $f_d = 40.32\ kHz$.

We then calculated the theoretical damped radian frequency ω_d as follows:

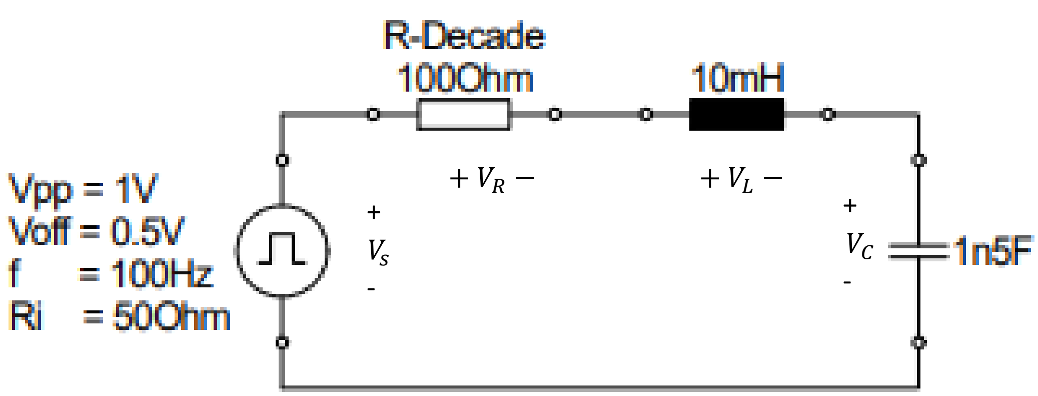


Figure 6: The RLC Circuit with voltage polarities

Supporting information:

$$v_s = potential\ difference\ across\ source$$

$$v_R = potential\ difference\ across\ resistance$$

$$v_L = potential\ difference\ across\ inductor$$

$$v_C = potential\ difference\ across\ capacitor$$

$$i = current\ in\ the\ circuit$$

$$R = Resistance$$

$$L = Inductance$$

$$C = Capacitance$$

$$v_R = iR$$

$$v_L = L\frac{di}{dt}$$

$$i = C\frac{dv_c}{dt}$$

$$R_{Decade} = 100\ \Omega$$

$$R_{internal} = 50\ \Omega$$

$$R = R_{Decade} + R_{internal} = 150\ \Omega$$

$$L = 10^{-2}\ H$$

$$C = 1.5 \times 10^{-9}\ F$$

$$v_s = \frac{V_{pp}}{2} + V_{offset} = 1V$$

$$-v_s + v_R + v_L + v_c = 0$$

$$v_R + v_L + v_C = v_s$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = v_s$$

$$i = C\frac{dv_c}{dt}$$

$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = v_s$$

$$\frac{LC}{LC}\frac{d^2v_c}{dt^2} + \frac{RC}{LC}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{v_s}{LC}$$

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

Homogenous part of the solution:

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

This can be rewritten as:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\alpha = \frac{R}{2L} = 7500$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 258198.89$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 258089.94\, rad.s^{-1}$$

Therefore, the theoretical value for damped radian frequency is:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 258089.94\, rad.s^{-1}$$

The experimental value of ω_d can be calculated as follows:

$$\omega_{d_{experimental}} = 2\pi f_d = 253338.03\, rad.s^{-1}$$

As we can see, the calculated and the experimental values for the damped radian frequency are very close. As a result, we can say that they are consistent with each other.

Next, we calculated the resistance so that the circuit would be critically damped. This can be calculated as follows:

For critical damping on the above circuit, we require the following from its system definition:

$$\alpha = \omega_0$$

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

$$R = 2\sqrt{\frac{L}{C}}$$

$$R = 5163.978\ \Omega$$

$$R_{Decade} = R - R_{internal} = 5113.978\ \Omega$$

Then, we configured the circuit according to the above setting by setting the resistance on the R-Decade to 5114 ohms, which was the closest value possible according to our findings. The results on the oscilloscope were as follows:

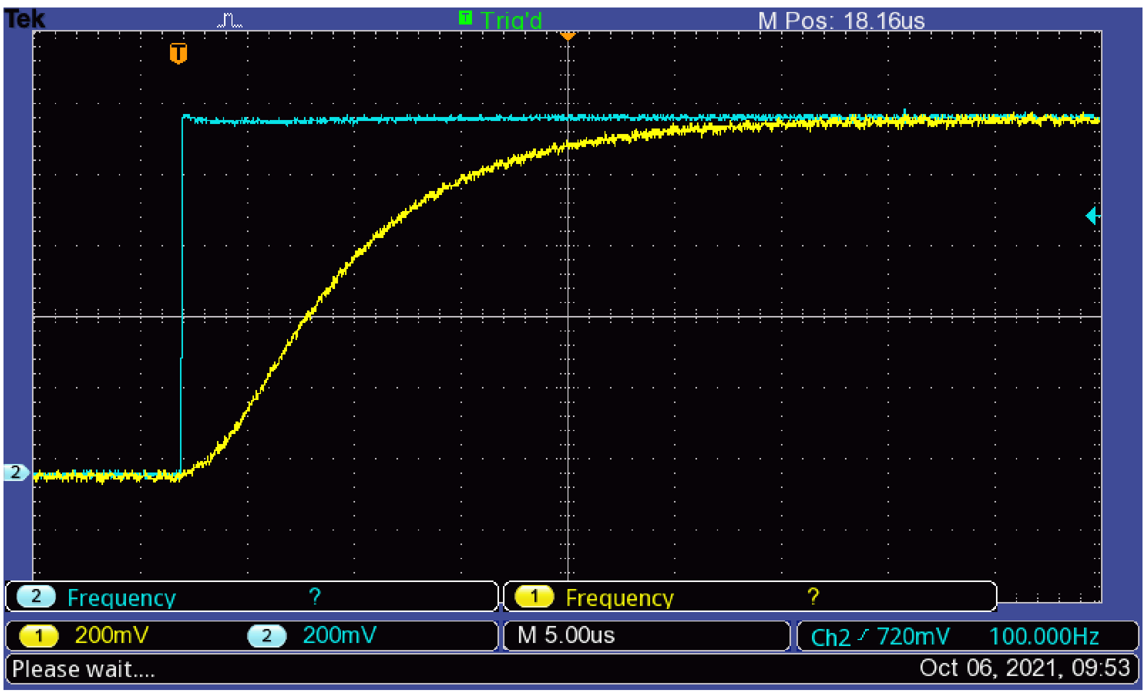


Figure 7: Voltage variation across Capacitor at R = 5164 Ohms

On varying the value on the R-Decade as instructed, we saw that at the value theoretically obtained, the circuit was still not critically damped. Reconfiguring the circuit brought us to the conclusion that the actual critical damping was obtained at R-decade = 3830 Ohms.

At this final value, the oscilloscope showed the following display:

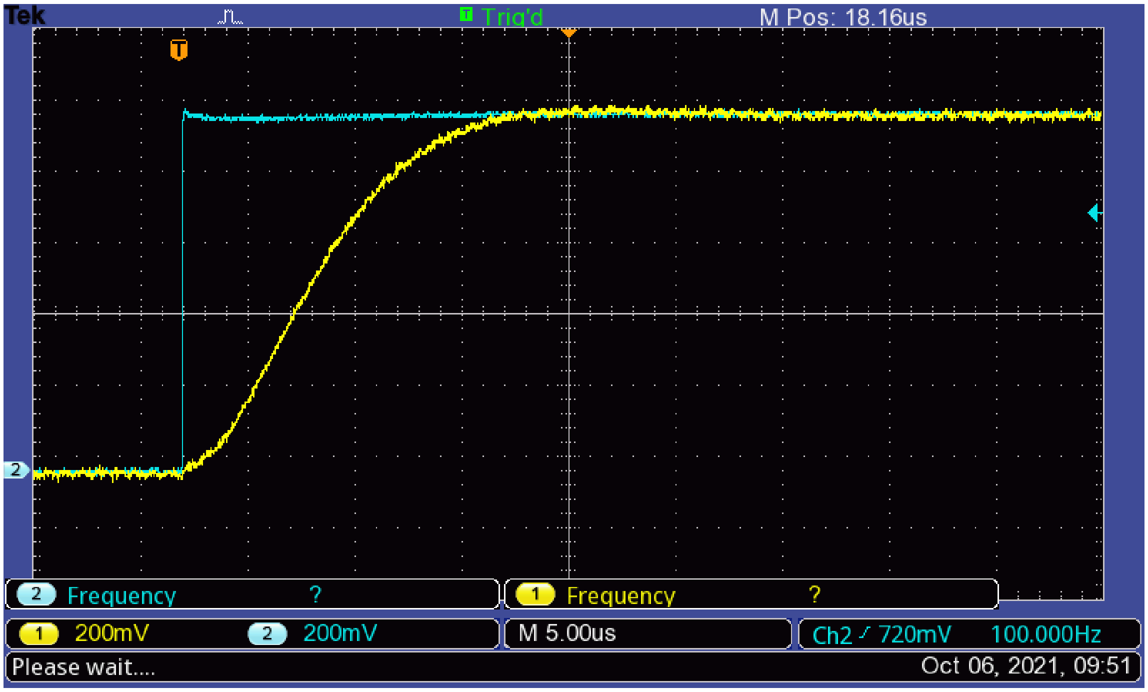


Figure 8: Display on the oscilloscope at R = 3880Ω

As we can see, the differences are very defined for the two resistance values.

After this, we're instructed to set the R-decade value to $30k\Omega$, so that the circuit is overdamped. On doing this, we could see the transient voltage across the capacitor as follows:

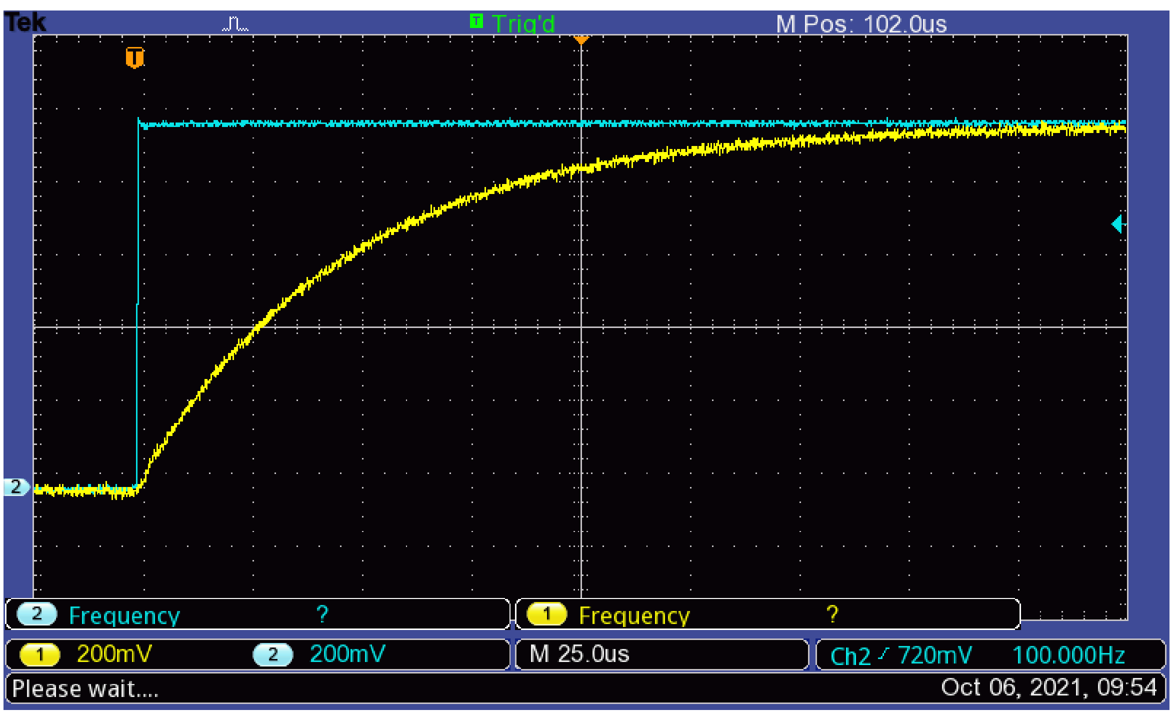


Figure 9: Display on the oscilloscope at R = $30k\Omega$

Evaluation

Task 1

On the first task, we are required to obtain the differential equation for the voltage $v_c(t)$ across the capacitor for the circuit we used in our experiment.

$\underline{Supporting\ information}$:

$$v_s = potential\ difference\ across\ source$$

$$v_R = potential\ difference\ across\ resistance$$

$$v_L = potential\ difference\ across\ inductor$$

$$v_C = potential\ difference\ across\ capacitor$$

$$i = current\ in\ the\ circuit$$

$$R = Resistance$$

$$L = Inductance$$

$$C = Capacitance$$

$$v_R = iR$$

$$v_L = L\frac{di}{dt}$$

$$i = C\frac{dv_c}{dt}$$

$$R_{Decade} = 100\ \Omega$$

$$R_{internal} = 50\ \Omega$$

$$R = R_{Decade} + R_{internal} = 150\ \Omega$$

$$L = 10^{-2}\ H$$

$$C = 1.5 \times 10^{-9}\ F$$

$$v_s = \frac{V_{pp}}{2} + V_{offset} = 1V$$

$$-v_s + v_R + v_L + v_c = 0$$

$$v_R + v_L + v_C = v_s$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = v_s$$

$$i = C\frac{dv_c}{dt}$$

$$LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = v_s$$

$$\frac{LC}{LC}\frac{d^2v_c}{dt^2} + \frac{RC}{LC}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{v_s}{LC}$$

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$v_s = \frac{V_{pp}}{2} + V_{offset} = 1V$$

$$\frac{R}{L} = \frac{150}{10^{-2}} = 15000$$

$$\frac{1}{LC} = \frac{1}{10^{-2} \times 1.5 \times 10^{-9}} = 6.667 \times 10^{10}$$

$$\frac{v_s}{LC} = \frac{1}{10^{-2} \times 1.5 \times 10^{-9}} = 6.667 \times 10^{10}$$

Finally,

$$\frac{d^2v_c}{dt^2} + 15000\frac{dv_c}{dt} + (6.667 \times 10^{10})v_c = 6.667 \times 10^{10}$$

As we can see from above, the final differential equation we obtain for the system is as follows:

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

$$\frac{d^2v_c}{dt^2} + 15000\frac{dv_c}{dt} + (6.667 \times 10^{10})v_c = 6.667 \times 10^{10}$$

Particular solution to the system is as follows:

$$v_{c_p} = v_{ss} = v_s = 1V$$

The homogeneous form of the equation is as follows:

$$\frac{d^2v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = 0$$

This can be formulated as follows:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Which can be rewritten as:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

We observe the following:

$$\alpha = \frac{R}{2L} = 7500 \ and \ \omega_0 = \frac{1}{\sqrt{LC}} = 258198.894$$

$$\alpha < \omega_0$$

Therefore, the system is under-damped.

We can calculate the damping frequency as:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 258089.94 \ rad.s^{-1}$$

The final equation can be formulated as follows:

$$v_c(t) = v_{ss} + e^{-\alpha t}(C_1\cos(\omega_d t) + C_2\sin(\omega_d t))$$

In the above equation, C_1 and C_2 are unknown. In order to find them, we require the initial conditions across v_c.

We know that the voltage drop across a capacitor cannot change instantaneously. Therefore,

$$v_c(0^-) = v_c(0^+) = 0$$

The second initial condition can be found as follows:

$$i_c(0^-) = i(0^-) = i_c(0^+) = 0 = C\frac{dv_c(0^+)}{dt}$$

Therefore,

$$\frac{dv_c(0^+)}{dt} = 0$$

Substituting $v_c(0) = 0$ into the equation, we get:

$$v_c(0) = 0 = C_1 + v_{ss}$$

$$C_1 = -v_{ss} = -1$$

Differentiating v_c, we obtain the following:

$$\frac{dv_c}{dt} = e^{-\alpha t}(-C_1\omega_d \sin(\omega_d t) + C_2\omega_d \cos(\omega_d t)) + (-\alpha)e^{-\alpha t}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

$$\frac{dv_c}{dt}(0) = 0 = C_2\omega_d - \alpha C_1$$

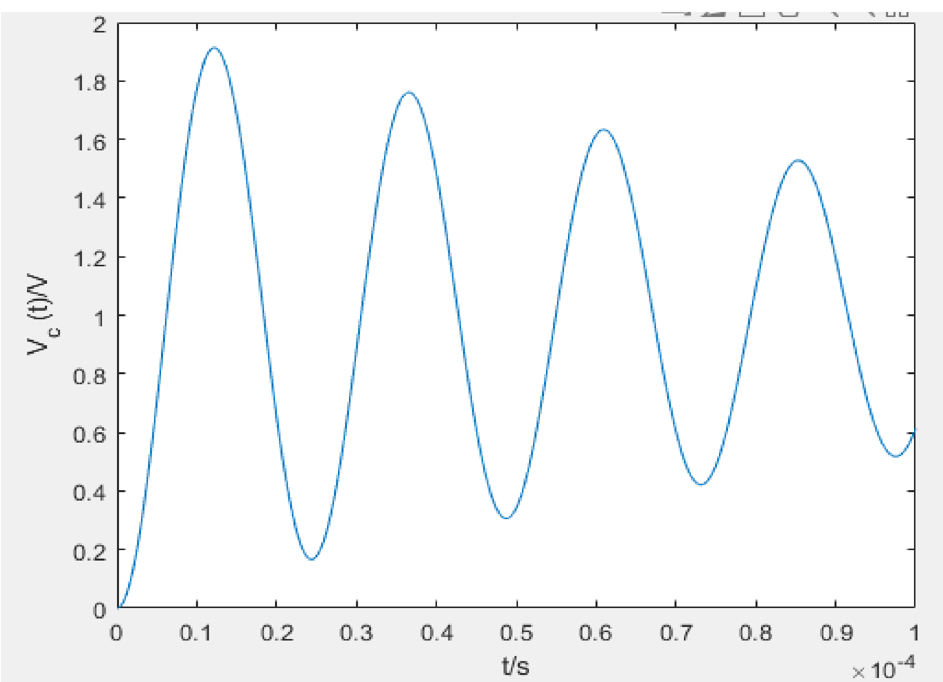
$$C_2 = \frac{\alpha C_1}{\omega_d} = \frac{7500 \times -1}{258089.94} = -0.029$$

The final equation becomes:

$$v_c(t) = 1 - e^{-7500t}(\cos(258089.94t) + 0.029 \sin(258089.94t))$$

Task 2

We can now plot the curve for the function v_c in MATLAB. The plot is provided below:



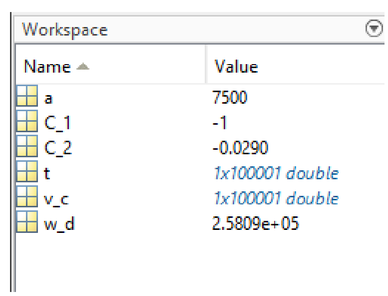


Figure 10: MATLAB Plot of v_c

Editor - C:\Users\hp\Documents\MATLAB\lab_1_ev_q2.m

lab_1_ev_q2.m | lab_1_ev_q3.m

```
a = 7500;
w_d = 258089.94;
C_1 = -1;
C_2 = -0.029;

t = 0:0.0000001:0.01;
v_c = 1 + exp((-1*a).*t).*(C_1.*cos(w_d.*t))+ (C_2.*sin(w_d.*t));
plot(t, v_c);
ylim([0 2]);
xlim([0 10^-4]);
xlabel('t/s');
ylabel('V_c (t)/V');
```

Figure 11: MATLAB script for plot of v_c

Task 3

From our previous calculations, we know that the circuit can be defined by the following system:

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{v_s}{LC}$$

This has the homogeneous counterpart:

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = 0$$

This can be formulated as follows:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Which can be rewritten as:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

Critical damping is achieved when $\alpha = \omega_0$ for the above system, which gives us the following:

$$\alpha = \omega_0 = \frac{R}{2L} = \frac{1}{\sqrt{LC}} = 258198.89$$

$$R = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{10^{-2}}{1.5 \times 10^{-2}}} = 5164\ \Omega$$

$$v_c(t) = v_{ss} + (C_1 + C_2 t)e^{-\alpha t}$$

The steady state voltage value is the DC-input voltage, which means we have:

$$v_{ss} = 1V$$

Again, we know that the voltage drop across a capacitor cannot change instantaneously. Therefore,

$$v_c(0) = 0V$$

$$\frac{dv_c}{dt}(0) = 0$$

Using these values, we can find the unknowns as follows:

$$v_c(0) = 0 = v_{ss} + C_1$$

$$C_1 = -v_{ss} = -1V$$

$$\frac{dv_c}{dt} = (C_1 + C_2 t)(-\alpha)e^{-\alpha t} + e^{-\alpha t}C_2$$

$$\frac{dv_c}{dt}(0) = 0 = -\alpha C_1 + C_2$$

$$C_2 = \alpha C_1 = -258198.89$$

The final equation is as follows:

$$v_c(t) = 1 - (1 + 258198.89t)e^{-258198.89t}$$

The plot for this function looks as follows:

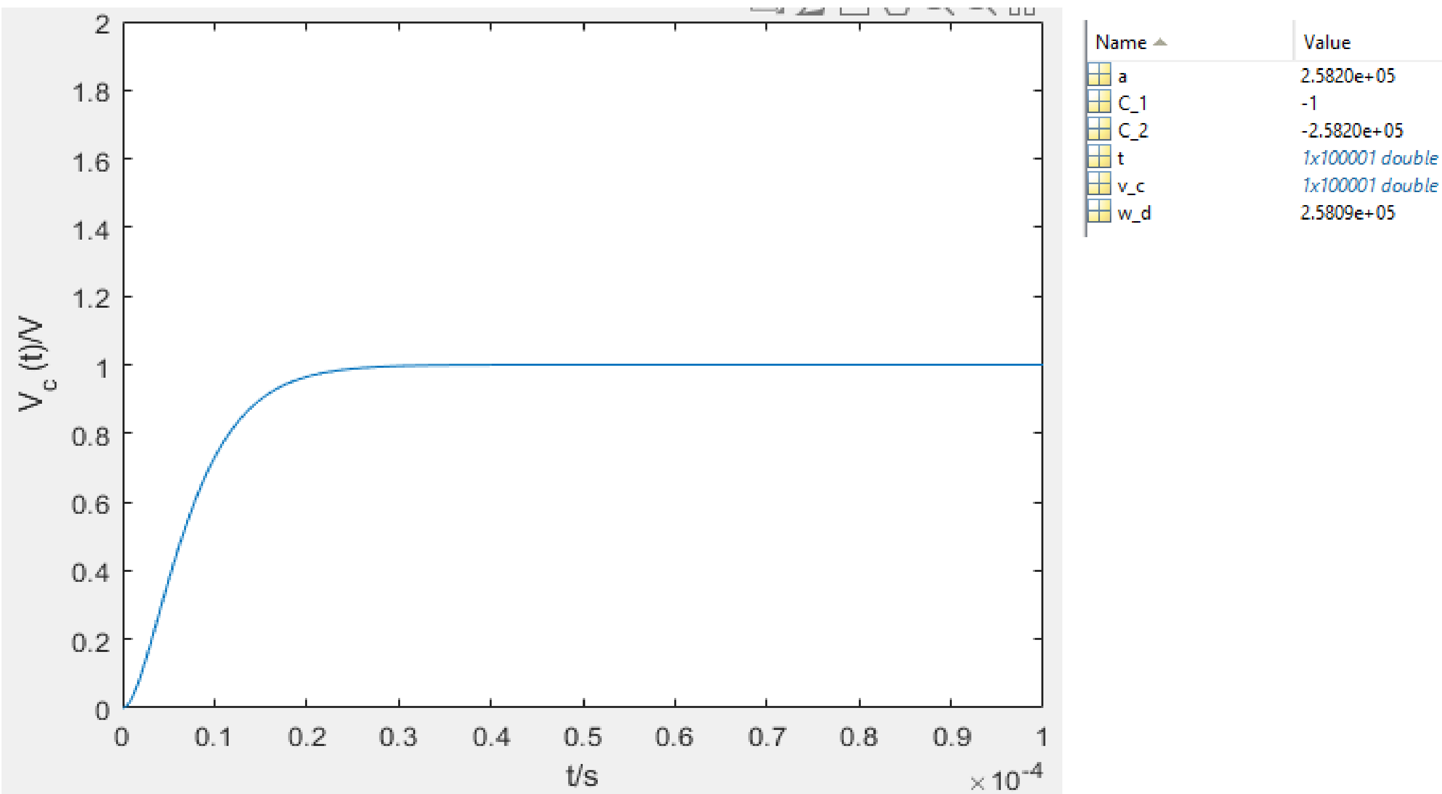


Figure 12: MATLAB Plot of v_c

The script is provided below:

Editor - C:\Users\hp\Documents\MATLAB\lab_1_ev_q3.m

lab_1_ev_q2.m | lab_1_ev_q3.m

```
a = 258198.89;
C_1 = -1;
C_2 = -a;

t = 0:0.0000001:0.01;
v_c = 1 + exp((-1*a).*t).*(C_1 + C_2.*t);
plot(t, v_c);
ylim([0 2]);
xlim([0 10^-4]);
xlabel('t/s');
ylabel('V_c (t)/V');
```

Figure 13: MATLAB script of v_c

Task 4

Our theoretical and experimental values have the following differences:

$$R_{theoretical} = 5164\ \Omega$$
$$\mathrm{R}_{\mathrm{experimental}} = 3880\ \Omega$$

The values have high relative difference between each other. This could be the result of multiple causes. For example, the actual values for the capacitance of capacitor and inductance of the inductor might not be as accurate as we consider them to be. These values might be different from the values we used in our theoretical calculations, and the differences might be sufficient enough to cause deviations in our experimental data.

The same could be said about the R-decade, which contains a certain error bound, which means the actual value of the R-decade is within a range rather than the actual value we are setting on the equipment.

The coil of the inductor could also be the source of error, as we don't consider the internal resistance of the wire used in the coil of the inductor in our calculations.

The error propagation from these components creates a large enough variation in the experimental results so that they could be differentiated from the theoretical calculations.

Task 5

(a) We are provided the following circuit:

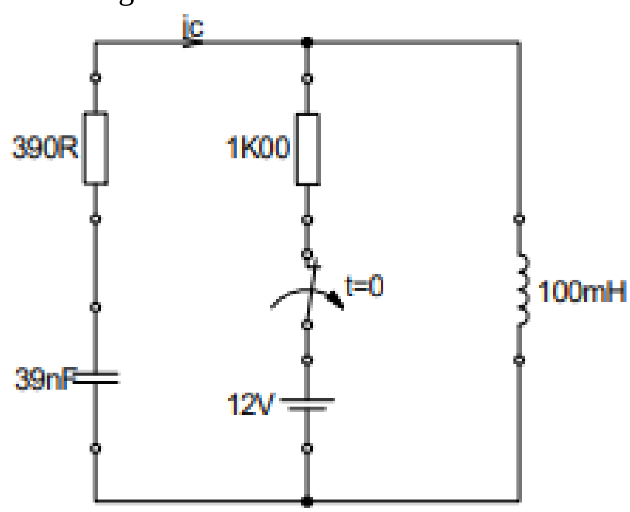


Figure 14: RLC Circuit for Task 5

We are required to obtain the differential equation for the current $i_c(t)$ through the capacitor, identify the damping nature of the circuit and determine the values for the coefficients C_1 and C_2.

To achieve this, we first figure out the initial conditions by considering that the circuit above had the shown settings for infinite time. This provides us with the following values:

$$v_c(0) = -12V, V_{R_1}(0) = 0V, V_{R_2}(0) = 12V, V_L(0) = 0$$

$$i_c(0) = 0, i_{R_2}(0) = i_L(0) = \frac{12}{1000} = 0.012$$

At t > 0, the circuit changes to the following:

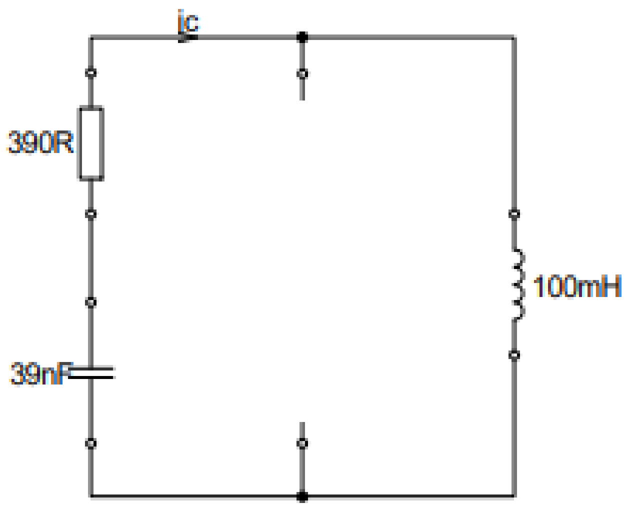


Figure 15: RLC Circuit at t > 0

From here, we follow the following steps:

$\underline{Supporting\ information}$:

$$v_R = potential\ difference\ across\ resistor$$

$$v_L = potential\ difference\ across\ inductor$$

$$v_C = potential\ difference\ across\ capacitor$$

$$i_c = current\ through\ capacitor$$

$$R = Resistance$$

$$L = Inductance$$

$$C = Capacitance$$

$$v_R = iR, v_L = L\frac{di}{dt}, i = C\frac{dv_c}{dt}$$

$$L = 10^{-1}\ H, C = 39 \times 10^{-9}\ F$$

$$R_1 = 390\Omega, R_2 = 1000\Omega$$

$$V_{R_1} + V_L + V_c = 0$$

$$V_{R_1} + V_L + V_C = 0$$

$$i_c R + L\frac{di_c}{dt} + \frac{1}{C}\int i_c\, dt = 0$$

$$L\frac{d^2 i_c}{dt^2} + R_1\frac{di_c}{dt} + i_c = 0$$

$$\frac{d^2 i_c}{dt^2} + \frac{R_1}{L}\frac{di_c}{dt} + \frac{1}{LC} i_c = 0$$

$$\frac{R_1}{L} = \frac{390}{10^{-1}} = 3900$$

$$\frac{1}{LC} = \frac{1}{10^{-1} \times 39 \times 10^{-9}} = 2.564 \times 10^8$$

$$\frac{d^2 i_c}{dt^2} + (3900)\frac{di_c}{dt} + (2.564 \times 10^8) i_c = 0$$

The system for this circuit is defined as follows:

$$\frac{d^2 i_c}{dt^2} + \frac{R_1}{L}\frac{di_c}{dt} + \frac{1}{LC} i_c = 0$$

$$\frac{d^2 i_c}{dt^2} + (3900)\frac{di_c}{dt} + (2.564 \times 10^8) i_c = 0$$

This can be formulated as follows:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Which can be rewritten as:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

From this, we find that

$$\alpha = \frac{R}{2L} = 1950$$

$$\omega_0 = 16012.815$$

We see that $\alpha < \omega_0$, therefore, the circuit is under-damped.

The equation defining the current through the capacitor is defined as follows:

$$i_c(t) = i_{c_{ss}} + e^{-\alpha t}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

Now, we solve for the unknown parameters:

$$i_{c_{ss}} = i_c(t \to \infty) = 0$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 15893.63$$

$$i_c(0^+) = i_{R_2}(0^-) = i_L(0^-) = 0.012\ A$$

$$i_c(0) = C_1 = 0.012$$

$$\frac{di_c}{dt}(0) = \frac{v_L(0)}{L} = 0$$

$$\frac{di_c}{dt} = e^{-\alpha t}(-C_1\omega_d \sin(\omega_d t) + C_2\omega_d \cos(\omega_d t)) + (-\alpha)e^{-\alpha t}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

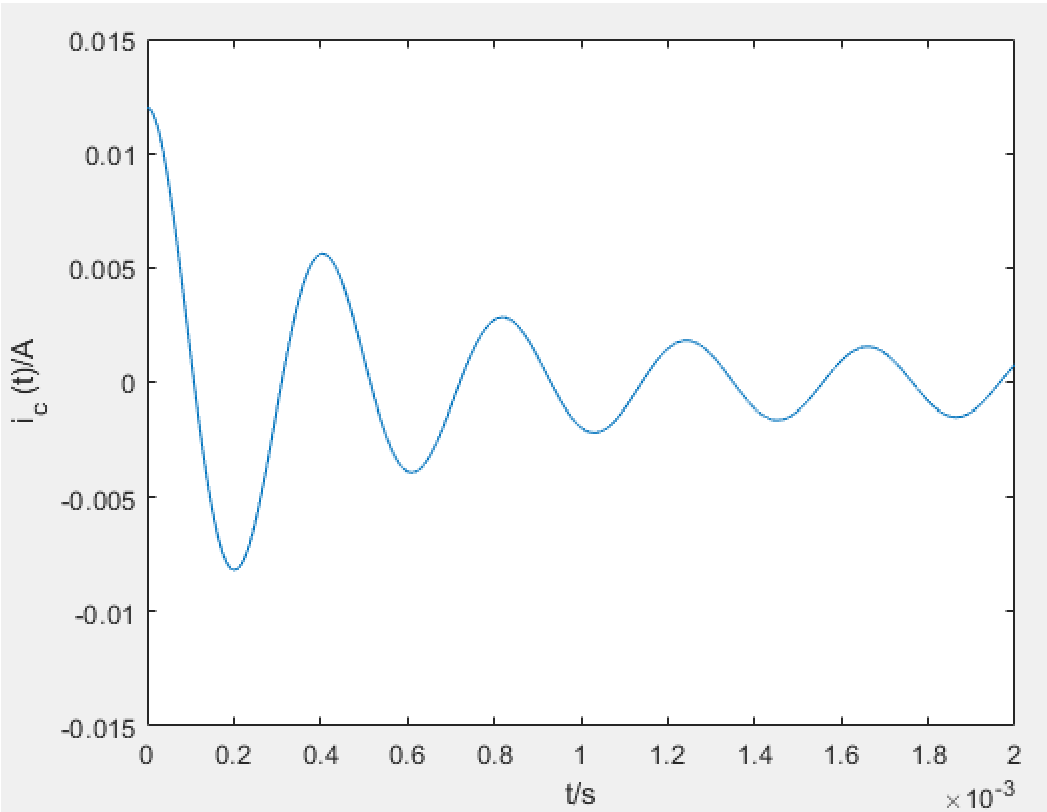
$$\frac{di_c}{dt}(0) = 0 = C_2\omega_d - \alpha C_1$$

$$C_2 = \frac{\alpha C_1}{\omega_d} = \ 1.472 \times 10^{-3}$$

The final equation is the following:

$$i_c(t) = e^{-1950t}(0.012\cos(15893.63t) + 0.001472\sin(15893.63t))$$

The plot for the equation is provided below:



Workspace

Name	Value
a	1950
C_1	0.0120
C_2	0.0015
i_c	1x100001 double
t	1x100001 double
v_c	1x100001 double
w_d	1.5894e+04

Figure 16: MATLAB plot of i_c

The script has been provided below:

Editor - C:\Users\hp\Documents\MATLAB\lab_1_ev_q5b.m

lab_1_ev_q2.m | lab_1_ev_q3.m | lab_1_ev_q5b.m | +

```
a = 1950;
w_d = 15893.63;
C_1 = 0.012;
C_2 = 0.001472;

t = 0:0.0000001:0.01;
i_c = exp((-1*a).*t).*(C_1.*cos(w_d.*t))+ (C_2.*sin(w_d.*t));
plot(t, i_c);
ylim([-15*10^-3 15*10^-3]);
xlim([0 2*10^-3]);
xlabel('t/s');
ylabel('i_c (t)/A');
```

Figure : MATLAB script of i_c

Conclusion

The purpose of this experiment was to study the transient response of second order systems: second order RLC circuits. We did this by observing the circuit response to different changes in resistance. We began with the set-up provided to us which allowed us to observe an under-damped RLC system. We constructed the set-up as instructed and observed the output across the capacitor using the oscilloscope, which allowed us to study the ringing phenomenon. We were also able to use the oscilloscope functions to obtain a value for the damped frequency and compare it with our own calculations to confirm the validity of our work.

Then, we studied the effects of changing the resistance on the system. We calculated the theoretical value of R to obtain a critically damped RLC circuit, and observed the response across the capacitor when the resistance was changed to this value. However, here we were able to study another important occurrence. On varying the resistance, we noticed the actual value of resistance which results in critical damping experimentally is lower than the value we calculated. We were able to deduce that this was a result of deviation in circuit component characteristics from the expected ones.

We then observed an overdamped system that was the result of setting the R-Decade to $30\ k\Omega$, and with this our experimentation concluded.

In evaluation section, we took the time to theoretically rework and study the different characteristics of our circuit again, and solve the circuit for its voltage across the capacitor in each case. We also studied another circuit with a different structure, which has different initial conditions, and solved the circuit for its current through the resistor as a function of time. In each case we are able to study the characteristics of the circuit by plotting the graph for the output function in MATLAB, which gives us a visual representation of the variation of the output with time. A practical outcome of these experimentations is that it allowed us to understand how comparable our theoretical results are to our experimental data, as our MATLAB plots were similar to the output we found on the oscilloscope.

References

- Signals and Systems Lab Manual (Uwe Pagel)
- http://www.faculty.jacobs-university.de/upagel/
- https://de.mathworks.com/help/control/ref/tf.html
- https://de.mathworks.com/help/ident/ref/lti.bode.html

Appendix

Prelab: RLC Circuits – Frequency Response

Task 1

```
%% Initialization
R = 390;
C = 270*10^-9;
L = 10*10^-3;
V = 5;
s = tf('s');
Z_C = 1/(s*C);
Z_L = s*L;
Z_R = R;
Z_tot = Z_C + Z_L + Z_R;

%% Bode plots
%Ratio across Capacitor
H_C = Z_C/Z_tot;
bode(H_C, 'b');
grid on;
hold on;

%Ratio across Resistor
H_R = Z_R/Z_tot;
bode(H_R, 'r');
grid on;
hold on;

%Ratio across Inductor
H_L = Z_L/Z_tot;
bode(H_L, 'g');
grid on;
hold on;

%Ratio across capacitor and Inductor
H_LC = (Z_C + Z_L)/Z_tot;
bode(H_LC, 'm');
grid on;
hold on;
```

The magnitude and phase plots are provided below:

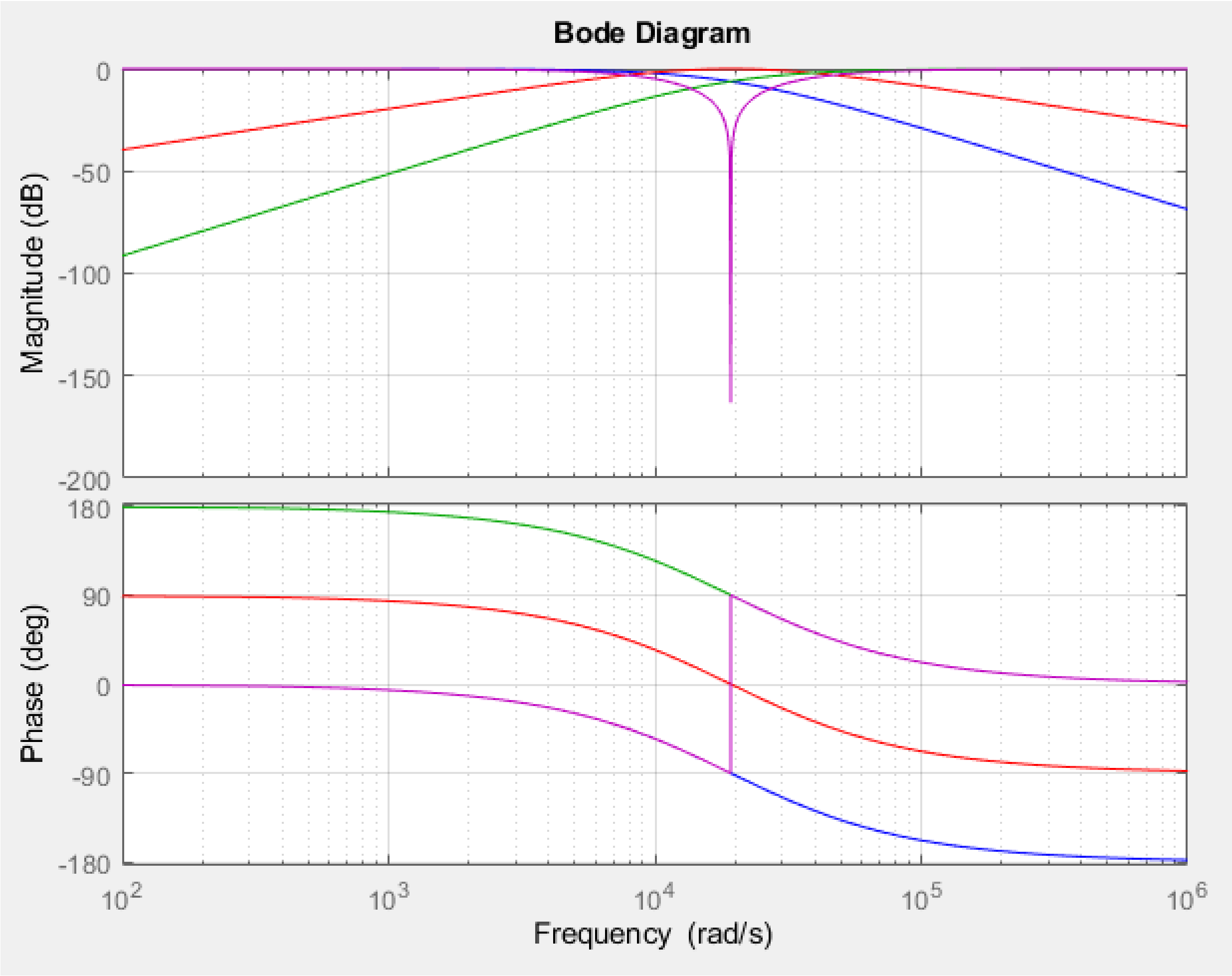


Figure 17: Bode Magnitude and Phase plots

Task 2

In order to find the bandwidth, we plotted the Bode plot separately and extracted the intersection frequencies of the Bode plots with the -3dB line and the zero line.

The plot is provided in the next page.

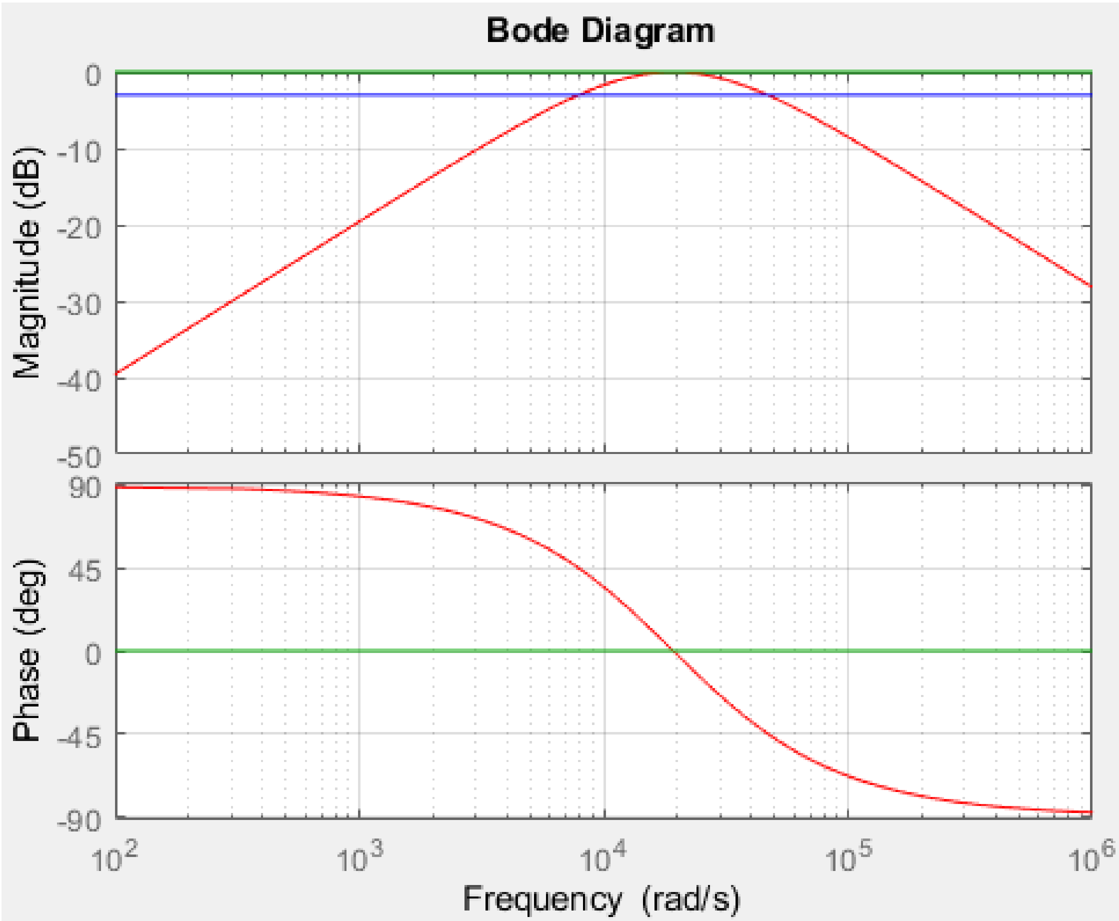


Figure 18: Bode magnitude plot for transfer function of R

The script is provided below:

```
%% Initialization
R = 390;
C = 270*10^-9;
L = 10*10^-3;
V = 5;
s = tf('s');
Z_C = 1/(s*C);
Z_L = s*L;
Z_R = R;
Z_tot = Z_C + Z_L + Z_R;

%% Plot across Resistor
H_R = Z_R/Z_tot;
bode(H_R, 'r');
grid on;
hold on;

H_ref = (10^(-3/20))*s/s;
bode(H_ref, 'b');
grid on;
hold on;

H_cut = 1*s/s;
bode(H_cut, 'g');
grid on;
hold on;
```

On zooming in on the points of intersection, we see that the curves intersect at the following radian frequencies:

$$\omega_1 = 7925\ rad/s$$
$$\omega_2 = 4.674 \times 10^4\ rad/s$$

From the information above, we can calculate the -3dB frequencies and bandwidth:

$$f_1 = \frac{7925}{2\pi} = 1261.3\ Hz$$
$$f_2 = \frac{4.674 \times 10^4}{2\pi} = 7438.9\ Hz$$
$$B = f_2 - f_1 = 6177.6\ Hz$$

The cutoff frequency was also extracted from the plot using the same method:

$$\omega_0 = 1.925 \times 10^4\ rad/s$$

$$f_0 = 3063.733 Hz$$

$$Q = \frac{f_0}{B} = 0.496$$

Task 3

The filter characteristics when measured over different components, component combinations is as follows:

Resistor: Band Pass Filter

Capacitor: Low Pass Filter

Inductor: High Pass Filter

Inductor and Capacitor: Notch Filter