Jacobs University Bremen

General Electrical Engineering 2 Lab

Electrical and Computer Engineering

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Lab Experiment 2 – Two-Port Networks

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Place of execution : Teaching Lab EE
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1 Introduction

1.1 Objective:

This week, we worked with two-port networks. We constructed two-port networks on the breadboard, analysed them, and constructed their characteristic Z and Y matrix models. Then we formed interconnected two-port networks using the individual networks, and analysed their characteristics as well.

The objectives of this experiment are:

- Understanding the concept of a two-port network.
- Learning to build Y and Z models of two-port networks using theoretical and measured values.
- Measuring currents and voltages of a two-port network and learning to use these measurements
- Learning the use of the table for converting from one set of two-port parameters to another set.

1.2 Theory:

1.2.1 Linear Two-Port Network

A port consists of a pair of terminals; current enters through one of the terminals and the same current leaves through the other terminal. A resistor is a one-port network. A general two-port network is as shown in Figure 1.

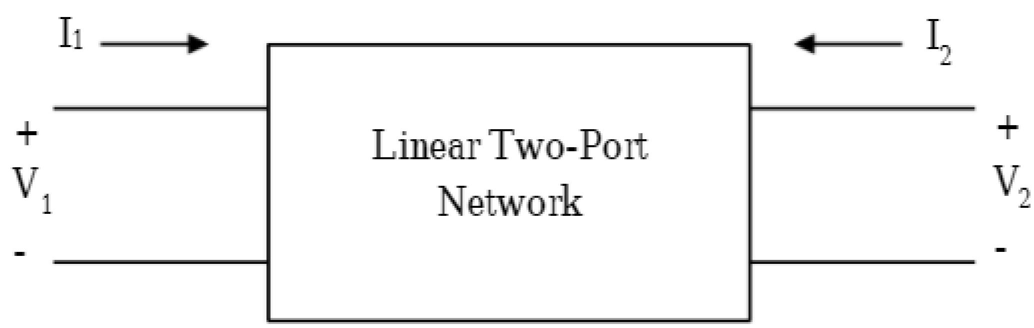


Fig 1: General Two-port Network

I_1 and V_1 are input current and input voltage respectively while I_2 and V_2 are the output current and output voltage respectively.

1.2.2 Z-Parameter Representation (Impedance):

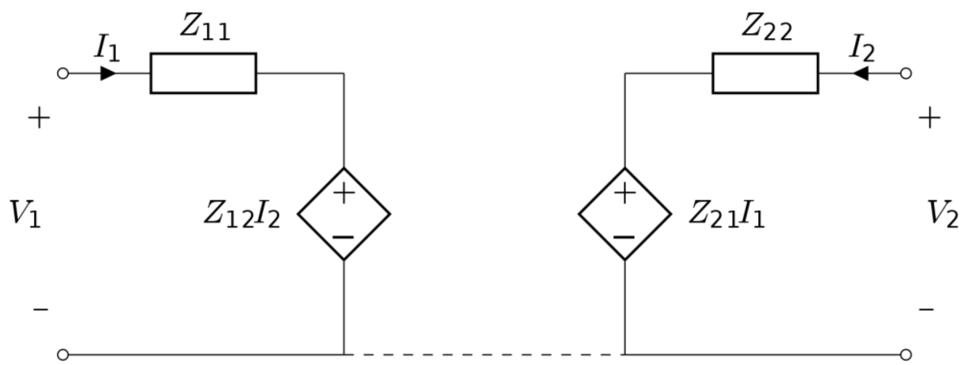


Fig 2: General Form of Two port Impedance network

The value of z-parameter can be determined by applying the open-circuit test. If we open circuit the output terminals ($I_2 = 0$) and solve for z_{11} and z_{21}, the result is as follows:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

The value of z_{12} and z_{22} can be determined by applying the similar test to the output with the input open-circuited.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

This can be written as:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

1.2.3 Y-Parameter Representation (Admittance):

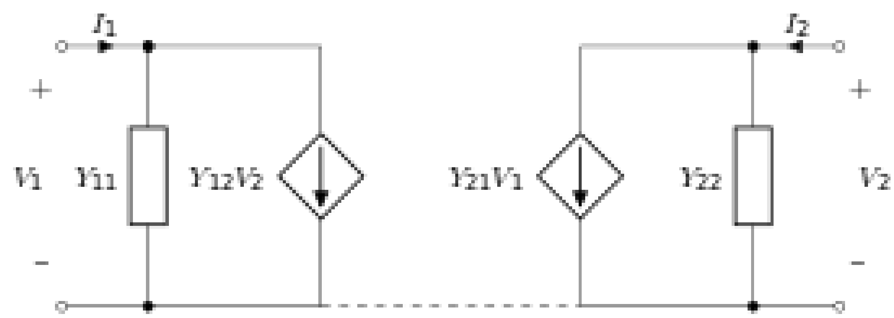


Fig 3: General form of Two port Admittance Network

The value of y-parameter can be determined by applying the short-circuit test. If we short circuit the output terminals ($V_2 = 0$) and solve for y_{11} and y_{21}, the result is as follows:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

The value of y_{12} and y_{22} can be determined by applying the similar test to the output with the input short circuited.

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad z_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

This can be written as:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

1.2.4 Two port Transmission ABCD-parameter network:

The value of ABCD-parameter can be determined by applying the open-circuit and short-circuit tests. If we open the output terminals (I_2 = 0) and solve for A and C and, the result is as follows:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

The value of B and D can be determined by applying the similar test to the output with the output being short-circuited (V_2 =0).

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

This can be written as:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

2 Execution

2.1 Experimental set-up part 1:

Used tools and equipment:

- Agilent Signal Generator
- TEKTRONIX Oscilloscope
- TENMA Multimeter
- ELABO Multimeter/Power Supply
- BNC Cable, BNC T connector, BNC Banana Connector

2.1.1 Part 1: Setup:

In this experiment, we explored the experimental determination of Z and Y parameters.

We started by building the following circuits on the breadboard:

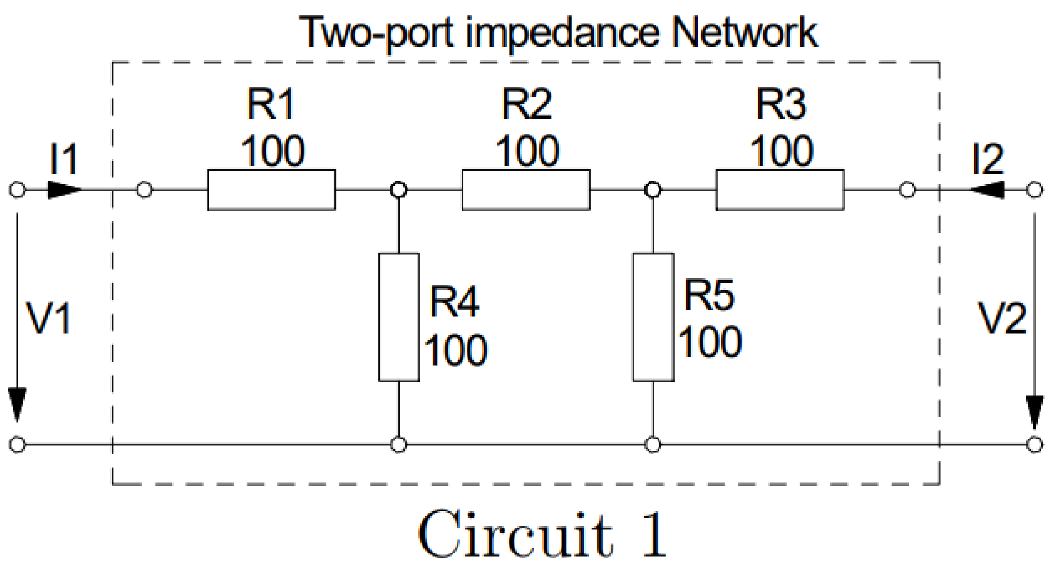


Figure 1: Resistor Network for Circuit 1

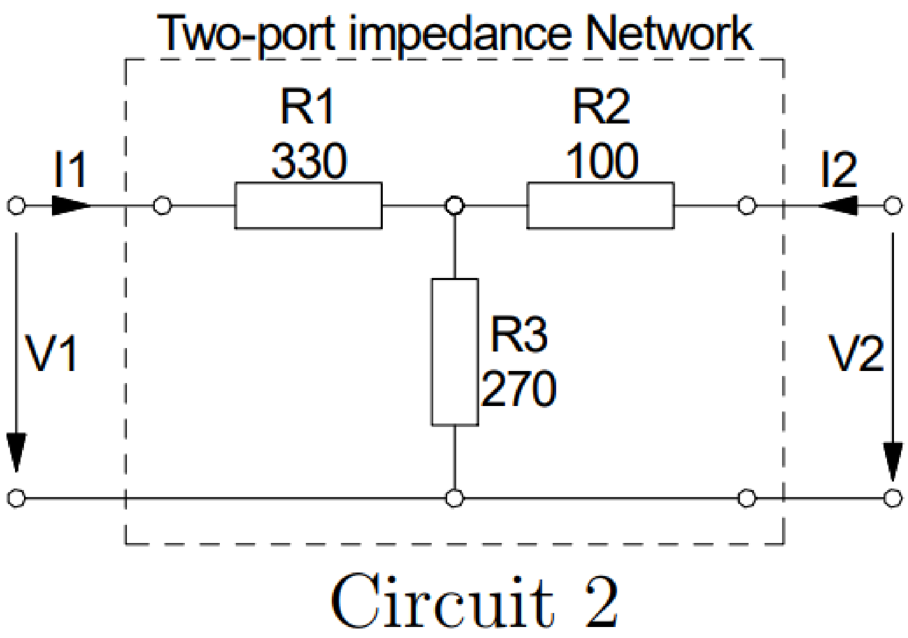


Figure 2: Resistor Network for Circuit 2

2.1.2 Part 1-Execution and Results:

Then, we found the Z parameters for Circuit 1 and the Y parameters for Circuit 2.

We found Z_{11} and Z_{22} for the first circuit by directly measuring resistance of the circuit across the left and right ends of the circuit using a TENMA multimeter while the other end remained open-circuited. To obtain Z_{12}, we first connected a 5V power supply to the right end of the circuit while the left side was open-ended, and measured voltage V_{R3} across R_3 using the multimeter. We found I_2 using $I_2 = V_{R3}/R_3$. Then we found V_1 by measuring potential difference across the ends on the left side while the 5V power supply was connected at V_2. We calculated Z_{12} as $Z_{12} = V_1/I_2$.

The values are provided below:

$$V_{R_3} = -3.0430V, R_3 = 100\,\Omega, V_2 = 5V$$

$$I_2 = \frac{V_{R_3}}{R_3} = -\frac{3.0430\,V}{100\,\Omega} = -0.3043A$$

$$V_1 = -1.0188\,V$$

$$Z_{12} = \frac{V_1}{I_2} = -\frac{1.0188V}{-0.3043A} = 33.48\,\Omega$$

For Z_{21}, we connected a 5V power supply to V_1 end while the V_2 end was open-circuited and measured the voltage V_{R1} across R_1, and obtained I_1 using $I_1 = V_{R1}/R_1$.

Then we measured the potential difference across the open-circuit on the right to obtain V_2. We found Z_{21} using $Z_{21} = V_2/I_1$. The measurements are provided below:

$$V_{R_1} = -3.0444 \text{ V}, \mathrm{R}_1 = 100\ \Omega, V_1 = 5V$$

$$\mathrm{I}_1 = \frac{\mathrm{V}_{\mathrm{R}_1}}{\mathrm{R}_1} = -\frac{3.0444\mathrm{V}}{100\Omega} = -0.030444 \text{ A}$$

$$\mathrm{V}_2 = -1.0170\mathrm{V}$$

$$\mathrm{Z}_{21} = \frac{\mathrm{V}_2}{\mathrm{I}_1} = \frac{-1.0170\mathrm{V}}{-0.030444\mathrm{A}} = 33.41\Omega$$

After obtaining all the impedances, we compared the diagonal values of our Z matrix. The diagonal and anti-diagonal values were almost same, which verified our results. Considering we had a symmetric Two-Port network, the diagonal Impedance values should be the same. However, a small deviation is possible in calculated and measured values due to error as a result of numerous factors involved in experimentation.

Finally, we obtain the following Z parameters for Circuit 1:

$$Z_{11} = 166.72\ \Omega,\ Z_{12} = 33.48\ \Omega,$$

$$Z_{21} = 33.41\ \Omega,\ \mathrm{Z}_{22} = 166.72\ \Omega$$

$$Z = \begin{bmatrix} 166.72 & 33.48 \\ 33.41 & 166.72 \end{bmatrix}$$

To obtain the Y parameters for Circuit 2, we used a different method. We shorted V_2 and measured resistance across ends on the left-hand side of the network using the TENMA multimeter, which provided us Z_{11}. Then we shorted V_1 and measured the same on the right-hand side, which provided us Z_{22}. Then we calculated Y_{11} and Y_{22} as follows:

$$Z_{11} = 404.8\ \Omega$$

$$\mathrm{Z}_{22} = 248.71\Omega$$

$$Y_{11} = \frac{1}{Z_{11}} = \frac{1}{404.8\ \Omega} = 2.47 \times 10^{-3}\ \Omega^{-1}$$

$$Y_{22} = \frac{1}{Z_{22}} = \frac{1}{248.71\Omega} = 4.021 \times 10^{-3}\ \Omega^{-1}$$

To obtain Y_{12}, we shorted V_1 and connected the right side of Circuit 2 to a 5V power supply, which meant V_2 = 5V. Then we measured voltage V_{R1} across R_1 using the TENMA multimeter. After that, we found Y_{12} as follows:

$$V_2 = 5V, V_{R_1} = -3.0169V, R_1 = 330\Omega,$$

$$I_{R_1} = \frac{V_{R1}}{R_1} = -\frac{3.0169V}{330\Omega} = -9.142 \times 10^{-3}\ A$$

$$Y_{12} = \frac{I_{R_1}}{V_2} = \frac{-9.142 \times 10^{-3}A}{5.0V} = -1.828 \times 10^{-3}\Omega^{-1}$$

We obtained Y_{21} by first shorting V_2 and connecting V_1 to a 5V power supply. Then we measured voltage V_{R2} across R_2 and measured the voltage V_2 using the TEMNA multimeter. We made the following calculations to obtain a value:

$$V_1 = 5.0V, V_{R_2} = -0.9112V, R_2 = 100\Omega$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{-9.112V}{100\Omega} = -9.112 \times 10^{-3}A$$

$$Y_{21} = \frac{I_{R_2}}{V_1} = \frac{-9.112 \times 10^{-3}A}{5V} = -1.8224 \times 10^{-3}\ \Omega^{-1}$$

For Circuit 2, we obtained the following Y parameters:

$Y_{11} = 2.47 \times 10^{-3}\ \Omega^{-1},\ Y_{12} = -1.828 \times 10^{-3}\ \Omega^{-1},$

$Y_{21} = -1.8224 \times 10^{-3}\ \Omega^{-1},\ Y_{22} = 4.021 \times 10^{-3}\ \Omega^{-1}$

$$Y = \begin{bmatrix} 2.47 \times 10^{-3} & -1.828 \times 10^{-3} \\ -1.8224 \times 10^{-3} & 4.021 \times 10^{-3} \end{bmatrix}$$

Next, we connected Circuit 1 and 2 to the voltage supply with V_1 = 5V. Then, we used a load, R_L= 1 kΩ, at the outputs, and measured V_1, V_2, I_1, I_2 for both circuits. We obtained the following results:

For Circuit 1,

$$V_1 = 5.055V, R_2 = 1000\Omega, V_2 = 0.8702V, V_{R_3} = -0.0876V\ \ R_3 = 100\Omega,$$

$$I_2 = I_{R_3} = \frac{V_{R_3}}{R_3} = -8.76 \times 10^{-4}A$$

$$V_{R_1} = 3.0423V, R_1 = 100\Omega,$$

$$I_1 = I_{R_1} = \frac{V_{R_1}}{R_1} = 0.030423A$$

For Circuit 2,

$$V_1 = 5.055V, R_L = 1000\Omega, V_2 = 1.8094V, V_{R_2} = -0.1826V, R_2 = 100\Omega$$

$$I_2 = I_{R_2} = \frac{V_{R_2}}{R_2} = -1.826 \times 10^{-3}A$$

$$V_{R_1} = 3.0571V, R_1 = 330\Omega,$$

$$I_1 = I_{R_1} = \frac{V_{R_1}}{R_1} = 9.264 \times 10^{-3}A$$

2.2 Part 2: Interconnection of Two port Networks:

2.2.1 Part 2- Setup:

In this experiment, we explored the parallel connection of Two-port Networks. We did this by first connecting the circuits from the previous experiment in parallel, according to the design below:

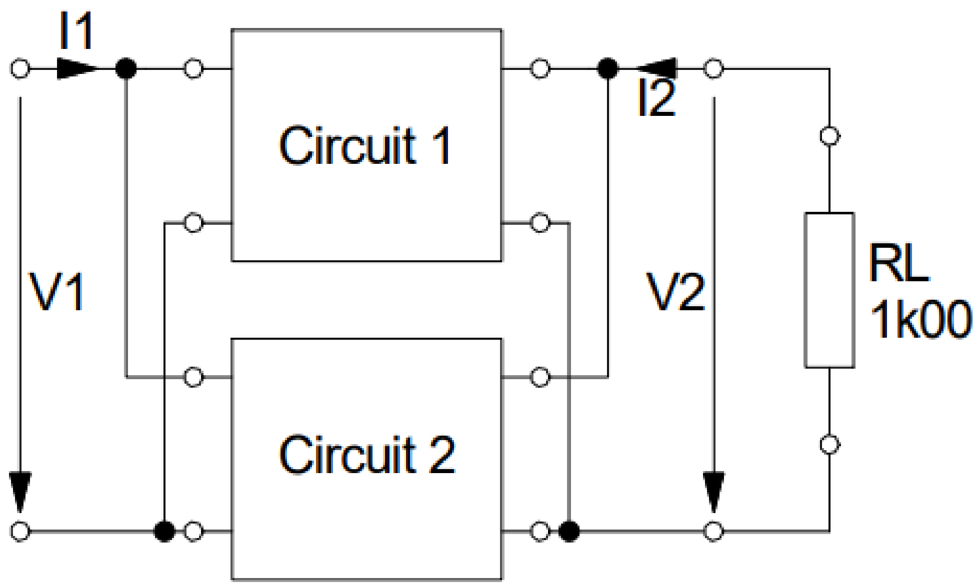


Figure 3: Two-Port Networks connected in parallel

2.2.2 Part 2- Execution

The first objective of this experiment was to calculate the Z parameters for this network. The first parameter we focused on was Z_{11}. So, we connected V_1 to a 5V power supply such that V_1 = 5V, and kept the right end open-circuited such that I_2 = 0. Then we measured the voltage across the 100 Ω resistor and the 330 Ω resistor using the TENMA multimeter. The data we obtained and our calculations are provided below:

$$V_1 = 5.055V, V_{R_{100}} = 2.9584V, R_{100} = 100\Omega,$$

$$I_{R_{100}} = \frac{V_{R_{100}}}{R_{100}} = 0.029584A$$

$$V_{R_{330}} = 3.2359V, R_{330} = 330\Omega,$$

$$I_{R_{330}} = 9.8058 \times 10^{-3}A$$

$$I_1 = I_{R_{100}} + I_{R_{330}}$$

$$I_1 = 0.0393898A$$

$$Z_{11} = \frac{V_1}{I_1} = 128.33\Omega$$

Using the same set-up, we were able to find Z_{21} as well. In order to accomplish this, we measured V_2 using the TENMA multimeter. We already had the rest of the values from our measurements before:

$$V_2 = 1.4978V, I_1 = 0.0393898\ A,$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{1.4978V}{0.0393898A} = 38.025\ \Omega$$

After obtaining Z_{11} and Z_{21}, we moved our focus into finding Z_{12} and Z_{22}. To do this, we modified the circuit by making V_1 open-circuited and connected V_2 to a 5V power supply. Then we measured V_1, V_{1R100} (Voltage across 100Ω resistor in Circuit 1), V_{2R100} (Voltage across 100Ω resistor in Circuit 2).

$$V_1 = 1.7674V$$

Data from Circuit 1:

$$V_{1R_{100}} = 2.9293V$$

$$I_{1R_{100}} = \frac{V_{1R_{100}}}{R_{100}} = 0.029293A$$

Data from Circuit 2:

$$V_{2R_{100}} = 1.7126V$$

$$I_{2R_{100}} = \frac{V_{2R_{100}}}{R_{100}} = 0.017126A$$

Considering that we now had currents that separated from I_2 at the junction where the circuits were connected, we were ready to make the following further calculations:

$$I_2 = I_{1R_{100}} + I_{2R_{100}} = 0.046419\ A$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{1.7674V}{0.046419A} = 38.075\ \Omega$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{5.054V}{0.046419A} = 108.88\ \Omega$$

We now had all the values we required to model our two-port network.

$$Z = \begin{bmatrix} 128.33 & 38.075 \\ 38.025 & 108.88 \end{bmatrix}$$

Next, we connected the load R_L = 1kΩ to V_2 and connected a 5V supply to V_1. Then we obtained V_1, V_2, I_1 and I_2 using the TENMA multimeter. To obtain I_1 and I_2 we first measured the voltages across the 100Ω, 330Ω and 1000Ω resistors, then calculated the currents across these resistors. The data is provided below:

$$V_1 = 5.055V, V_2 = 1.3645V, V_{R_L} = -1.3645V, R_L = 1000\Omega,$$

$$V_{R_{100}} = 2.9782V, V_{R_{330}} = 3.3146V, R_{100} = 100\Omega, R_{330} = 330\Omega,$$

$$I_{R_{100}} = \frac{V_{R_{100}}}{R_{100}} = 0.029782A,$$

$$I_{R_{330}} = \frac{V_{R_{330}}}{R_{330}} = 0.010044A,$$

$$I_1 = I_{R_{100}} + I_{R_{330}} = 0.03983A$$

$$I_2 = \frac{V_{R_L}}{R_L} = -1.3645 \times 10^{-3}A$$

2.3 Part 3: Complex Two port Networks/Cascading

2.3.1 Setup:

To start with this experiment, we first built the following circuits on the breadboard:

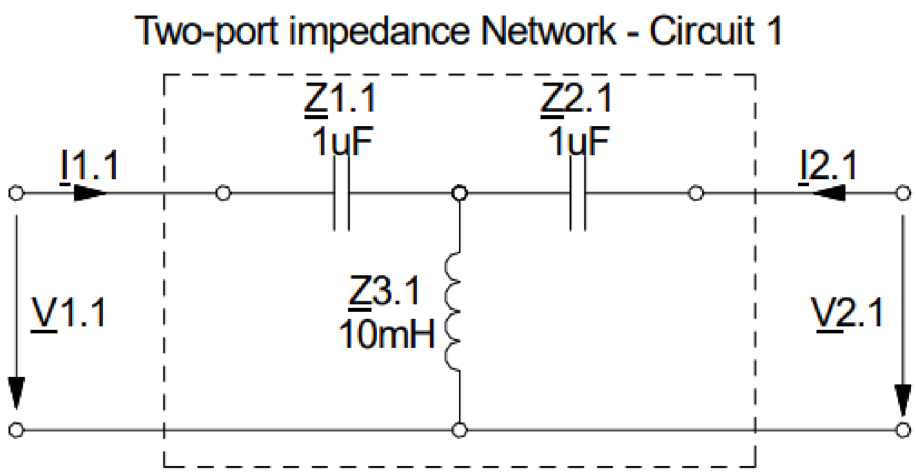


Figure 4: Two-Port Impedance Network – Circuit 1

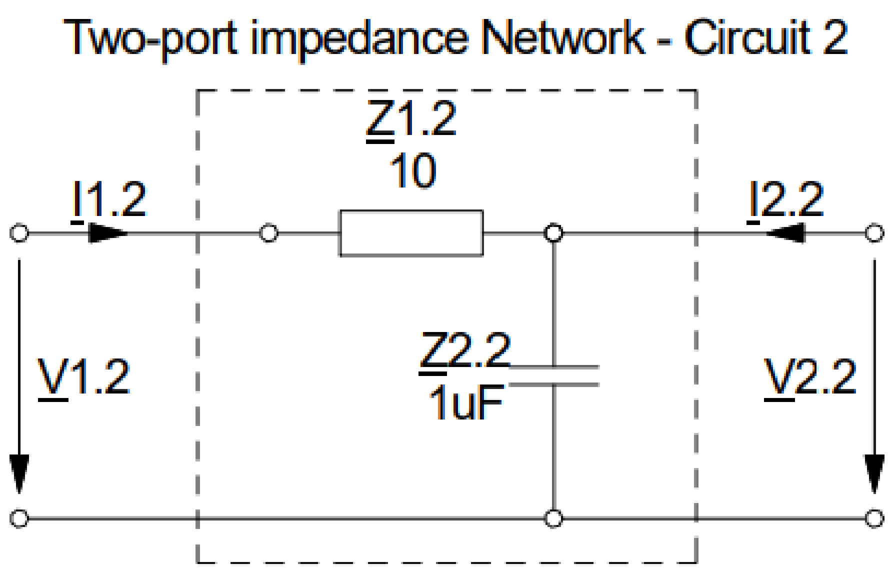


Figure 6: Two-Port Impedance Network – Circuit 2

After we had these networks, we formed a Cascading Two-Port Network by connecting Circuit 1 and Circuit 2 in cascade. Then we connected the signal generator to the left of the resulting circuit, in series with a resistor Rs = 100Ω. We also connected a load to the right R_L = 1000Ω.

The resulting circuit was based on the following schematic:

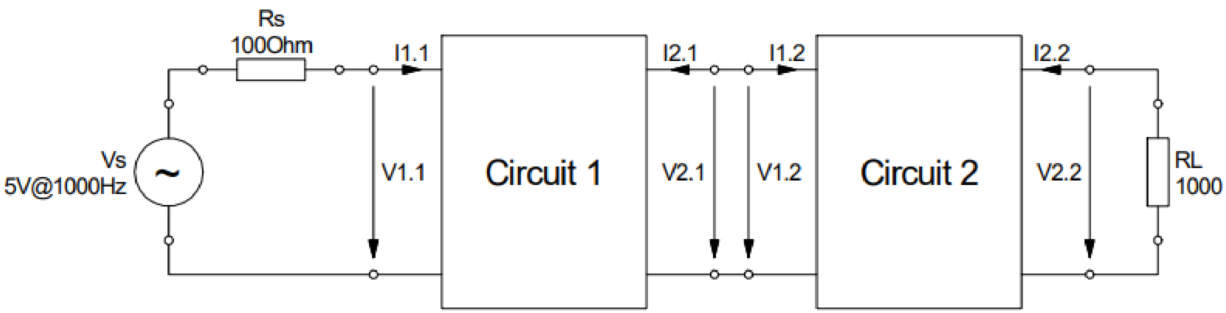
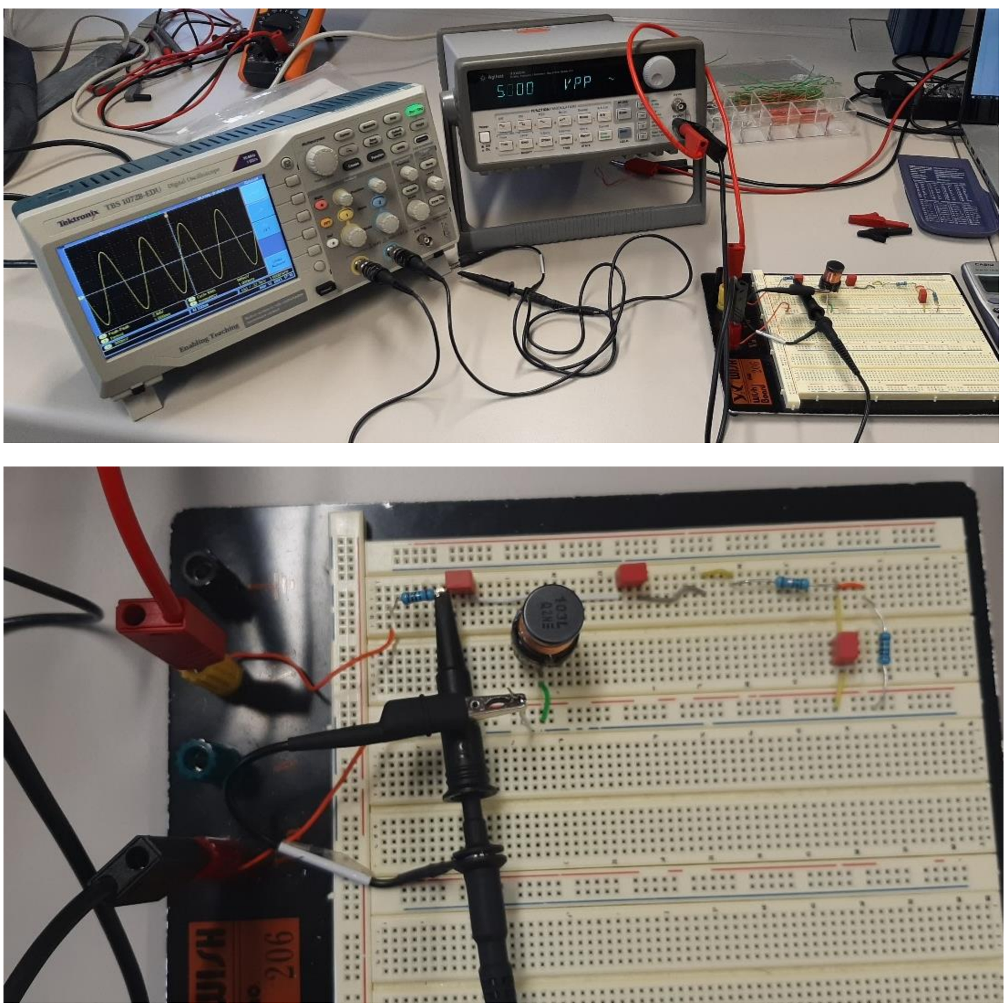


Figure 7: Cascading Two-Port Network

Pictures of the resulting set-up can be found below:



Pictures: Experimental Set-up

2.3.2 Execution and Results:

Considering that we now had a complete circuit, we were ready to take measurements. We first measured $\underline{\widehat{v_1}}, \underline{\widehat{\imath_1}}$ at Circuit 1 and $\underline{\widehat{v_2}}$, $\underline{\widehat{\imath_2}}$ at Circuit 2 using the oscilloscope. We use $\underline{\widehat{v_1}}$ as reference for our measurements. The data we obtain is provided below:

$$\underline{\widehat{v_1}} = 2.0423\angle 0°\, V$$

$$\underline{\widehat{\imath_1}} = \frac{4\angle 51.1° - 2.0423\angle 0°}{100} = 0.0315\angle 81.4°\, A$$

$$\underline{\widehat{v_2}} = V_{R_L} = 0.7826\angle 172°\, V$$

$$\underline{\widehat{\imath_2}} = \frac{\underline{\widehat{v_2}}}{1000} = (7.826 \times 10^{-4})\angle 172°\, A = -(7.826 \times 10^{-4})\angle -8^o A$$

After obtaining these measurements, we used the RLC meter to obtain the impedance (R, jX) parameter for every component. Pictures were taken while taking measurements with the RLC meter for reference. A sample has been provided below:

Picture: Taking measurements with the RLC meter

We obtained the following data:

$$Z_{1.1} = 148.83\angle -89.77°\, \Omega = 0.597 - j148.83\, \Omega$$

$$Z_{2.1} = 171.05\angle -89.77°\, \Omega = 0.687 - j171.05\, \Omega$$

$$Z_{2.2} = 152.70\angle -89.76°\, \Omega = 0.6397 - j152.699\Omega$$

$$Z_{3.1} = 65.033\angle 86.17°\, \Omega = 4.344 + j64.89\, \Omega$$

$$Z_{1.2} = 10 + j0\, \Omega$$

3 Evaluation:

3.1 Part 1: Two port Z/Y Network

<u>Calculating Z parameters for Circuit 1 and 2</u>

Circuit 1:

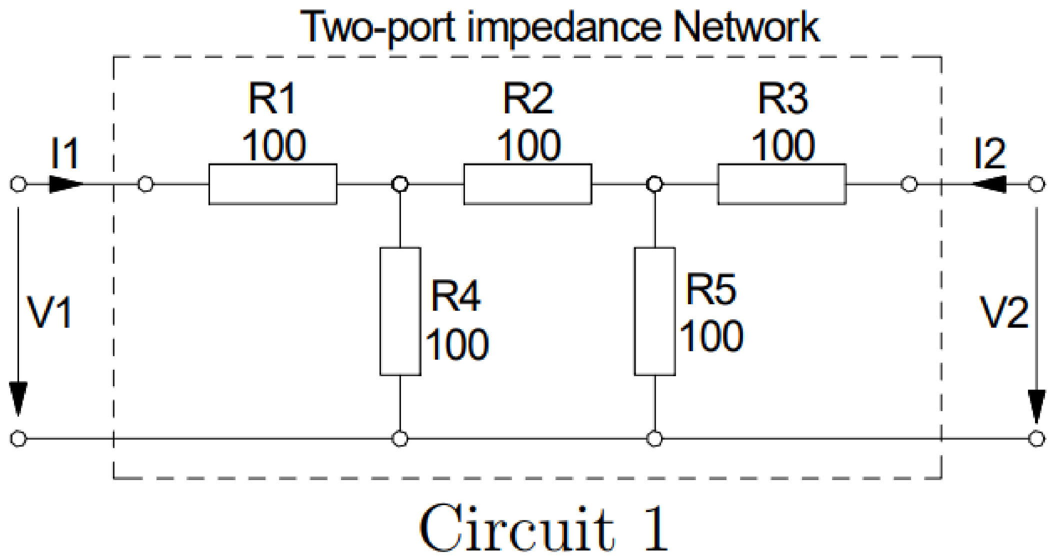


Figure 8: Two-port Network for Circuit 1

Theoretical calculation of Z_{11} and Z_{22}:

$$Z_{11} = R1 + R4||(R2 + R5)$$

$$Z_{11} = 100 + 100||(100 + 100) = 100 + 100||(200)$$

$$Z_{11} = 100 + \frac{100 \times 200}{100 + 200} = 100 + \frac{200}{3} = \frac{500}{3} = 166.667\Omega$$

$$Z_{22} = \ R3 + R5||(R2 + R4)$$

$$Z_{22} = 100 + 100\,||(100 + 100) = 166.667\Omega$$

Theoretical calculation of Z_{21}:

$$V_2 = V_{R5} = R5 \times I_1 \times \frac{R4}{R2 + R4 + R5}$$

$$Z_{21} = \frac{V_2}{I_1} = \ R5 \times \frac{R4}{R2 + R4 + R5} = 33.33\Omega$$

Theoretical calculation of Z_{12}:

$$V_1 = R4 \times I_2 \times \frac{R5}{R2 + R4 + R5}$$

$$Z_{12} = \frac{V_1}{I_2} = R4 \times \frac{R5}{R2 + R4 + R5} = 33.33\Omega$$

The final impedance matrix for Circuit 1 is the following:

$$Z = \begin{bmatrix} 166.667 & 33.33 \\ 33.33 & 166.667 \end{bmatrix} \Omega$$

Admittance matrix for Circuit 1 can be found using the following construct:

$$Y = Z^{-1}$$

$$Y = \begin{bmatrix} 0.00625 & -0.00125 \\ -0.00125 & 0.00625 \end{bmatrix} \Omega^{-1}$$

The Z matrix we constructed using the measured values for Circuit 1 is the following:

$$Z = \begin{bmatrix} 166.72 & 33.48 \\ 33.41 & 166.72 \end{bmatrix} \Omega$$

We can calculate the Y parameter using the same method as before:

$$Y = Z^{-1} = \begin{bmatrix} 0.00624958 & -0.00125501 \\ -0.00125239 & 0.00624958 \end{bmatrix} \Omega^{-1}$$

As we can see, even though the matrices constructed from measured values are not exactly the same as matrices constructed from theoretical values, they are very similar. Our theoretical values are very close to our calculated values. Therefore, this meets our expectations, considering we expected to obtain Z and Y matrices that had values close to the theoretical values. We did not expect substantial differences, and therefore our expectations have been met in this case.

For third section of our execution, we set V_1 = 5.055V and measured V_2 = 0.8702V, I_1 = 0.030423A and I_2 = -0.000876A.

We can verify the voltage values by using the following construct:

$$V = ZI$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 166.72 & 33.48 \\ 33.41 & 166.72 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 166.72 & 33.48 \\ 33.41 & 166.72 \end{bmatrix} \begin{bmatrix} 0.030423 \\ -0.000876 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 166.72 \times 0.030423 + 33.48 \times -0.000876 \\ 33.41 \times 0.030423 + 166.72 \times -0.000876 \end{bmatrix} = \begin{bmatrix} 5.042794 \\ 0.87039 \end{bmatrix}$$

We can verify the current values using the following construct:

$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.00625 & -0.00125 \\ -0.00125 & 0.00625 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.00625 & -0.00125 \\ -0.00125 & 0.00625 \end{bmatrix} \begin{bmatrix} 5.055 \\ 0.8702 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.00625 \times 5.055 \pm 0.00125 \times 0.8702 \\ -0.00125 \times 5.055 + 0.00625 \times 0.8702 \end{bmatrix} = \begin{bmatrix} 0.03049951 \\ -0.00089245 \end{bmatrix}$$

As we can see, except for the small deviation due to error of the measuring equipment, the values we calculated for V_1, V_2, I_1 and I_2 are very close to the values we measured.

Circuit 2:

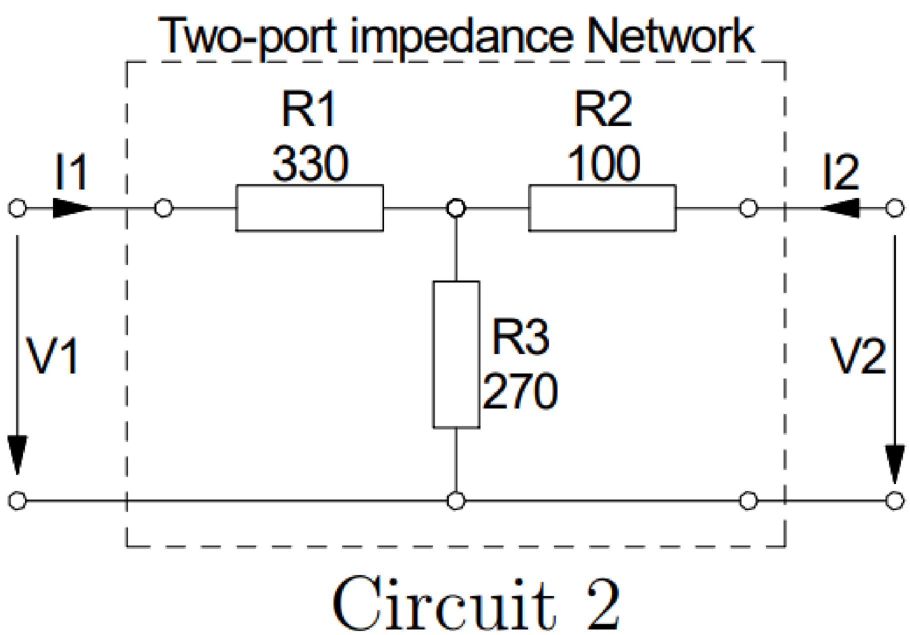


Figure 9: Two-Port Network for Circuit 2

Theoretical calculation of Z_{11} and Z_{22}:

$$Z_{11} = R1 + R3 = 330 + 270 = 600\Omega$$

$$Z_{22} = R2 + R3 = 100 + 270 = 370\Omega$$

Theoretical calculation of Z_{21}:

$$V_2 = I_1 \times R3$$

$$Z_{21} = \frac{V_2}{I_1} = R3 = 270\Omega$$

Theoretical calculation of Z_{12}:

$$V_1 = \ \ I_2 \times R3$$

$$Z_{12} = \frac{V_1}{I_2} = R_3 = 270\Omega$$

Now that we have all the parameters, we can construct the following matrix:

$$Z = \begin{bmatrix} 600 & 270 \\ 270 & 370 \end{bmatrix} \Omega$$

We can obtain the admittance parameter as follows:

$$Y = Z^{-1}$$

$$Y = \begin{bmatrix} 0.00248156 & -0.00181087 \\ -0.00181087 & 0.00402414 \end{bmatrix} \Omega^{-1}$$

The Y matrix we constructed using the measured values from Circuit 2 is provided below:

$$Y = \begin{bmatrix} 2.47 \times 10^{-3} & -1.828 \times 10^{-3} \\ -1.8224 \times 10^{-3} & 4.021 \times 10^{-3} \end{bmatrix}$$

The Z matrix is the inverse of this Y matrix:

$$Z = Y^{-1} = \begin{bmatrix} 609.1942 & 276.9478 \\ 276.0993 & 374.2128 \end{bmatrix}$$

As we can see from above, the Y parameters for the matrix constructed from measured values are close to the ones from the theoretical model. However, this cannot be said for the Z matrix. The Z parameters of the matrix constructed from measured values are close to those of the theoretical model. However, we can still see a deviation from the theoretical model in the Z parameters that is somewhat significant.

We had expected to see Y and Z parameters that were close to the theoretical values. Therefore, while the Y parameters meet our expectations, the Z parameters do not. However, we did not expect such substantial differences.

For the third section of our execution, we set V_1 = 5.055V, and measured V_2 = 1.8094V, I_1 = 0.009264A and I_2 = -0.001826A.

We can verify the voltage values using the following construct:

$$V = ZI$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 609.1942 & 276.9478 \\ 276.0993 & 374.2128 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 609.1942 & 276.9478 \\ 276.0993 & 374.2128 \end{bmatrix} \begin{bmatrix} 0.00926 \\ -0.001826 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 609.1942 \times 0.000926 + 276.9478 \times -0.001826 \\ 276.0993 \times 0.00926 + 374.2128 \times -0.001826 \end{bmatrix} = \begin{bmatrix} 5.13543 \\ 1.87336 \end{bmatrix}$$

We can verify the current values using the following construct:

$$I = YV$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2.47 \times 10^{-3} & -1.828 \times 10^{-3} \\ -1.8224 \times 10^{-3} & 4.021 \times 10^{-3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2.47 \times 10^{-3} & -1.828 \times 10^{-3} \\ -1.8224 \times 10^{-3} & 4.021 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 5.055 \\ 1.8094 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.009178 \\ -0.001937 \end{bmatrix}$$

As we can see, except for the small deviation due to error of the measuring equipment, the values we calculated for $V_{1,}$ V_2, I_1 and I_2 are very close to the values we measured. We had expected to obtain values for $V_{1,}$ V_2, I_1 and I_2 that were close to our calculated values. Therefore, in this case our expectations have been met.

We have clearly noticed that all of the impedance and admittance matrices we constructed using measured values have deviations from the theoretical models. There are very good reasons for these deviations. To start with, we construct the theoretical model assuming ideal conditions, which are not true in practice. While we use exact values for resistance in our theoretical calculations, in our experiment none of the resistances have values that exactly match the theoretical values. All of them have values that are only close to the theoretical values. Secondly, we have to consider that our measuring equipment have significant error. The TENMA multimeter has an error range of 1-3%. We also disregard the contact resistance of the breadboard, which also contributes to the overall error. All these factors result in the accumulation of the error in our measured values to become something significant.

3.2 Part 2: Interconnection of two port networks:

In order to determine the Y and Z parameters for the parallel interconnection of two-port networks constructed using Circuit 1 and Circuit 2, we simply need to add up the admittance parameters. Essentially, this boils down to the following:

$$Y_{Total} = Y_1 + Y_2$$

$$Y_{Total} = \begin{bmatrix} 0.00624958 & -0.00125501 \\ -0.00125239 & 0.00624958 \end{bmatrix} + \begin{bmatrix} 2.47 \times 10^{-3} & -1.828 \times 10^{-3} \\ -1.8224 \times 10^{-3} & 4.021 \times 10^{-3} \end{bmatrix}$$

$$Y_{Total} = \begin{bmatrix} 8.71958 \cdot 10^{-3} & -3.08301 \cdot 10^{-3} \\ -3.0748 \cdot 10^{-3} & 0.010270 \end{bmatrix}$$

$$\text{Now, } Z_{Total} = (Y_{Total})^{-1}$$

$$Z_{Total} = \begin{bmatrix} 8.71958 \cdot 10^{-3} & -3.08301 \cdot 10^{-3} \\ -3.0748 \cdot 10^{-3} & 0.010270 \end{bmatrix}^{-1}$$

$$Z_{Total} = \begin{bmatrix} 128.262 & 38.503 \\ 38.401 & 108.898 \end{bmatrix}$$

We found the following Z parameters from direct measurements:

$$Z = \begin{bmatrix} 128.33 & 38.075 \\ 38.025 & 108.88 \end{bmatrix}$$

When we compare the measured values of the Z parameters with those of the calculated values, we see that they are very close to each other. There are minor differences in the Z parameters but that is expected considering our measurements with the TENMA multimeter has 1-2% error and all measurements with the ELABO multimeter have about 3-4% error.

We also do not account for the errors due to contact of components with the breadboard. However, significant error contribution can be attributed to the resistors we use, which are not exactly equal to their theoretical values. All these factors contribute to the overall error of the measured values, and the errors are propagated when we make calculations using these values.

We did not expect substantial differences between our measured values and calculated values. In this case, our expectations have been met.

Now that we have our Z and Y matrices, we can use them for the verification of V_1, V_2, I_1, and I_2:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 128.33 & 38.075 \\ 38.025 & 108.88 \end{bmatrix} \begin{bmatrix} 0.03983 \\ -1.3645 \times 10^{-3} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5.0594 \\ 1.3659 \end{bmatrix}$$

The measured value of V_1 is 5.055V, and that of V_2 is 1.3645V. As expected, the calculated values are very close to the measured values – even though they are not exact. The small difference can be attributed to the error factors due to measuring tools and circuit equipment as discussed before.

Again, for the verification of I_1 and I_2 using Y parameters:

$$Y = Z^{-1}$$

$$Y = \begin{bmatrix} 128.33 & 38.075 \\ 38.025 & 108.88 \end{bmatrix}^{-1}$$

$$Y = \begin{bmatrix} 0.0086932 & -0.0030399 \\ -0.0030359 & 0.010246 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.0086932 & -0.0030399 \\ -0.0030359 & 0.010246 \end{bmatrix} \begin{bmatrix} 5.055 \\ 1.3645 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.039796 \\ -0.001366 \end{bmatrix}$$

The measured value of I_1 is 0.03983A, and that I_2 is -0.0013645A. These values are, again, very close to the calculated values. The small difference can be attributed to the error factors due to measuring tools and circuit equipment as discussed before.

When two-port networks are connected in series, we have the following set-up:

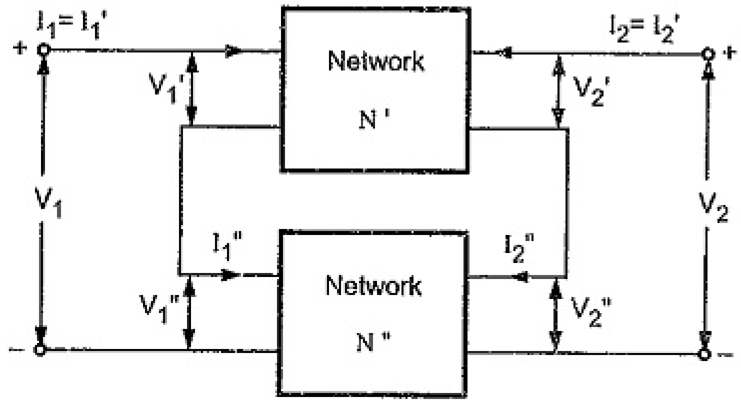


Figure 10: Series Interconnection of Two-Port Networks

We can obtain the following relations from this set-up:

$$\begin{aligned} V_1 &= V_1' + V_1'' \\ V_2 &= V_2' + V_2'' \\ I_1 &= I_1' = I_1'' \\ I_2 &= I_2' = I_2'' \end{aligned}$$

For network N′ , z-parameter equations are,

$$\begin{aligned} V_1' &= z_{11}'I_1' + z_{12}'I_2' \\ V_2' &= z_{21}'I_1' + z_{22}'I_2' \end{aligned}$$

For network N″, z-parameter equations are,

$$\begin{aligned} V_1'' &= z_{11}''I_1'' + z_{12}''I_2'' \\ V_2'' &= z_{21}''I_1'' + z_{22}''I_2'' \end{aligned}$$

From the above relations, we can construct the following resulting relation:

$$\begin{aligned} V_1 &= \left(z_{11}' + z_{11}''\right)I_1 + \left(z_{12}' + z_{12}''\right) I_2 \\ V_2 &= \left(z_{21}' + z_{21}''\right)I_1 + \left(z_{22}' + z_{22}''\right) I_2 \end{aligned}$$

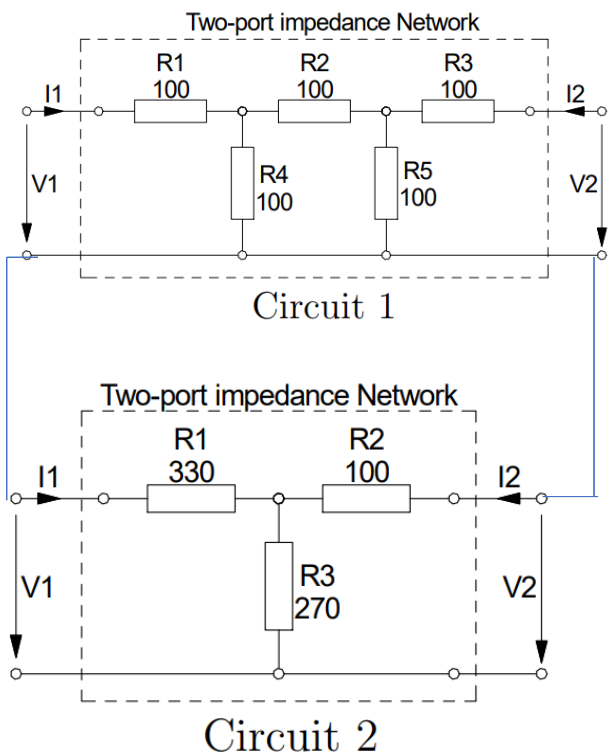
The matrix form of the above equation is:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (z'_{11} + z''_{11}) & (z'_{12} + z''_{12}) \\ (z'_{21} + z''_{21}) & (z'_{22} + z''_{22}) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Therefore, we obtain the following relation for two-port networks interconnected in series:

$$[z] = \begin{bmatrix} z'_{11} + z''_{11} & z'_{12} + z''_{12} \\ z'_{21} + z''_{21} & z'_{22} + z''_{22} \end{bmatrix}$$

However, this is not possible in our situation. In our situation, the two interconnected circuits have the following setup:



As we can see, in this set-up, the R1 and R2 resistors of circuit 2 end up becoming part of circuit 1, and therefore the overall two-port network doesn't exist anymore. Hence, the two-port network structure is broken in this case. Hence, in this case, series interconnection is not allowed. As a result, even if we constructed a series connection of these circuits to determine z by combining the z parameters, it would not be possible in practice.

3.3 Part 3: Complex Two port Networks/Cascading:

In order to calculate Z parameters, we refer to the measured values of impedance from the RLC meter.

For Circuit 1:

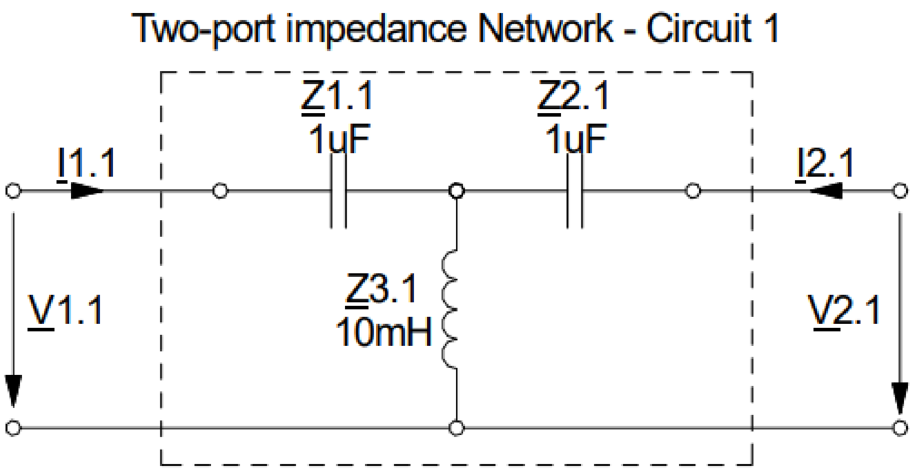


Figure 11: Two-Port Impedance Network – Circuit 1

$$Z_{12} = Z3.1 = 4.344 + j64.89\ \Omega$$
$$Z_{11} = Z1.1 + Z_{12} = 4.941 - j83.94\ \Omega$$

$$Z_{22} = Z2.1 + Z_{12} = 5.031 - j106.16\ \Omega$$

$$Z_{21} = Z_{12} = 4.344 + j64.89\ \Omega$$

Therefore, for Circuit 1, the Z-parameters are:

$$Z_1 = \begin{bmatrix} 4.94 - j83.94 & 4.34 + j64.89 \\ 4.34 + j64.89 & 5.03 - j106.16 \end{bmatrix}$$

We convert from Z parameters to ABCD parameters using the following formula:

$$T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Converting the Z parameters to ABCD parameters or Transmission parameters:

$$T_1 = \begin{bmatrix} -1.28 - j0.162 & -28.0 + j70.5 \\ 0.00103 - j0.0153 & -1.62 - j0.186 \end{bmatrix}$$

For Circuit 2:

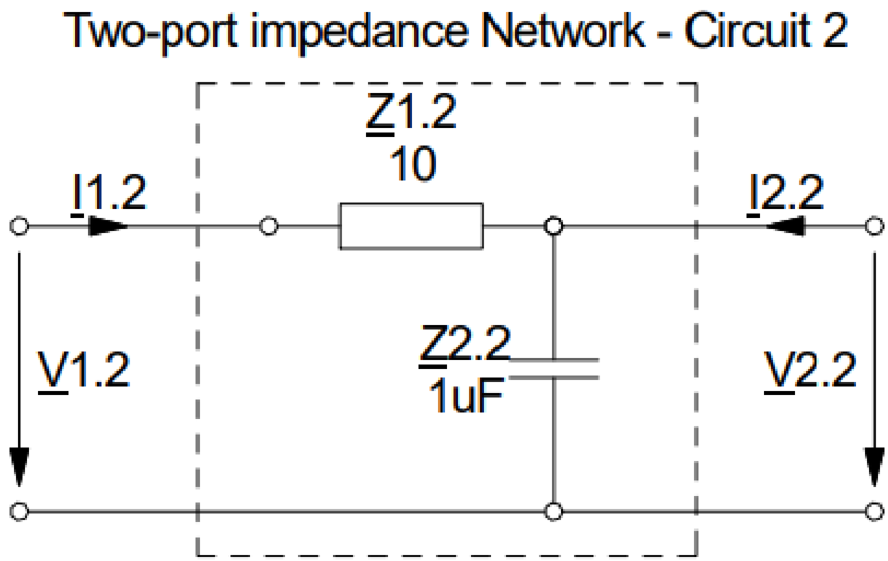


Figure 12: Two-Port Impedance Network – Circuit 2

$$Z_{12} = Z2.2 = 0.6397 - j152.699\ \Omega$$
$$Z_{11} = Z1.2 + Z_{12} = 10.6397 - j152.699\ \Omega$$

$$Z_{22} = Z_{12} = 0.6397 - j152.699\ \Omega$$

$$Z_{21} = Z_{12} = 0.6397 - j152.699\ \Omega$$

Therefore, for Circuit 2, the Z-parameters are:

$$Z_2 = \begin{bmatrix} 10.64 - j152.70 & 0.640 - j152.70 \\ 0.640 - j152.70 & 0.640 - j152.70 \end{bmatrix}$$

We convert from Z parameters to ABCD parameters using the following formula:

$$T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Converting the Z parameters to ABCD parameters or Transmission parameters:

$$T_2 = \begin{bmatrix} 1.0003 + j0.0655 & 10 + j9.5156 \times 10^{-15} \\ 2.745 \times 10^{-5} + j0.00655 & 1 \end{bmatrix}$$

To calculate the resulting cascaded ABCD parameters or T-parameters, we make the following operations:

$$T = T_1 T_2$$

$$T = \begin{bmatrix} -1.28 - j0.162 & -28.0 + j70.5 \\ 0.00103 - j0.0153 & -1.62 - j0.186 \end{bmatrix} \begin{bmatrix} 1.0003 + j0.0655 & 10 + j9.5156 \times 10^{-15} \\ 2.745 \times 10^{-5} + j0.00655 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -1.7347 - j0.4273 & -40.8093 + j68.85242 \\ 3.2053 \cdot 10^{-3} - j0.02592 & -1.61329 - j0.339524 \end{bmatrix}$$

Calculation of abcd/t parameters:

$$t = \begin{bmatrix} \frac{D}{\Delta T} & \frac{B}{\Delta T} \\ \frac{C}{\Delta T} & \frac{\mathrm{A}}{\Delta \mathrm{T}} \end{bmatrix}$$

$$t = \begin{bmatrix} -1.61329 - j0.339524 & -40.08093 + j68.85242 \\ 3.20563 \cdot 10^{-3} - j0.025492 & -1.73476 - \mathrm{j}0.427279 \end{bmatrix}$$

Now, we need to verify if the measured values of input/output voltage and current correspond with the calculated values of ABCD and abcd parameters:

From ABCD Parameters, we have the following:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} -1.7347 - j0.4273 & -40.08093 + j68.85242 \\ 3.2053 \cdot 10^{-3} - j0.02592 & -1.61329 - j0.339524 \end{bmatrix} \begin{bmatrix} 0.7826\angle 172^{o} \\ 7.826 \cdot 10^{-4}\angle 172^{o} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1.4151 + j0.084385 \\ 0.001626 + j0.02052 \end{bmatrix} = \begin{bmatrix} 1.4176\angle 3.413° \\ 0.020586\angle 85.470° \end{bmatrix}$$

From abcd parameters, we have the following:

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} -1.61329 - j0.339524 & -40.08093 + j68.85242 \\ 3.20563 \cdot 10^{-3} - j0.025492 & -1.73476 - \mathrm{j}0.427279 \end{bmatrix} \begin{bmatrix} 2.0423 \angle 0^{o} \\ 0.0315 \angle 81.4^{o} \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} -0.95926 + j0.253072 \\ 0.001412 + j0.003096 \end{bmatrix} = \begin{bmatrix} 0.99208 \angle 165.221° \\ 0.0034027 \angle 65.648° \end{bmatrix}$$

The phases we calculated from the measured V_1, V_2, I_1, and I_2 are close to the values we measured through the oscilloscope. The magnitude of the calculated value for I_1 and V_2 are also close to the measured values. However, though somewhat close, we can see that the calculated values for magnitude of V_1 and I_2 have some noticeable difference when compared with the measured values.

Our first estimation is some error or mistake was made in the measurement of V1 or Vs. The error due to this was significant, which propagated every time we made an operation with the faulty value. That is why our results show some disparity.

In the event that no mistakes were made when taking measurements, two big reasons for this difference could be the 5-10% error contributed to all measurements taken through the oscilloscope and 2% error contributed when taking measurements with the RLC meter. When we made calculations using these erroneous values, this error was propagated, and therefore the difference between measured and calculated values was enlarged. Therefore, even though we expected to obtain calculated values that were close to the measured values, we obtained results with substantial differences.

4 Conclusion:

In this lab, we mainly worked with Two-Port Networks. Firstly, we set up two two-port network circuits using resistors, and made measurements appropriately in order to obtain their Z and Y matrices. Also, we calculated the theoretical models for the Z and Y matrices using the theoretical values of circuit components and compared them with the measured Z and Y matrices.

We delved into analysis of two-port networks, where we interconnected the two two-port networks in parallel, and obtained their Z and Y matrices using measurements with tools such as the TENMA multimeter. Then, we constructed the Y model of the interconnected parallel two-port network by adding the Y matrices of the individual two-port network components (since this is a parallel interconnection of two-port networks), then calculated its Z parameter by inverting the Y parameter. We also measured V_1, V_2, I_1 and I_2 values for the interconnected two-port network – after connecting a load and a voltage supply to the network – and used them to verify our Z and Y models for our circuits.

Then, we built two more two-port networks which now included reactive components like capacitors and inductors as well. We made measurements and constructed their Z model, and using this we obtained their T parameters. We then put the two networks in cascade, and made some analysis on the resulting cascaded network. Through matrix multiplication of the T-models of the individual networks, we obtained a T-model for the resulting cascaded network. We obtained the t-model by inverting the T-model. Then, we connect the cascaded network to a signal generator on the left and a load on the right, and took measurements for V_1, V_2, I_1 and I_2, which we used to verify the T-model we constructed before. However, we saw that our resulting values were far different from our measured values, and the model did not fit.

Overall, we learned how to work with two-port networks, model their Z, Y and ABCD matrices, and verify these models using the tools at our disposal

5 Appendix:

Measured data from Experiment 3: The Wheatstone Bridge:

5.1 Part 1: Balanced DC Wheatstone Bridge:

Resistor	Resistance [KΩ], measured in 20KΩ range
R_1	21.93
R_3	2.192
R_4	8.212

When $U_{AB} = 0$ Resistance of decade box = 81.110 KΩ

Resistance of decade box using Elabo Multimeter = 82.25 KΩ

5.2 Part 2: Unbalanced DC Wheatstone Bridge

V_S	V_{Out}
1.0091 V	-21.38 mV
10.0072 V	-236.28 mV

5.3 Part 3: Balanced AC Wheatstone Bridge:

Values of Resistors using Elabo Multimeter

Resistor	Resistance [Ω]
R_1	993.2
R_3	992.7
R_4	999.0

Component Values of Y_1:

R_p = 38.691 KΩ

C_P = 1.069 µF

Theoritical values of R_{padj} = 1015.52 Ω, and C_{padj} = 23.49 µF

6 References:

1. Electrical Engineering-II Lab Manual (Uwe Pagel)
2. http://www.faculty.jacobs-university.de/upagel/
3. https://physics.stackexchange.com
4. https://www.eeeguide.com/interconnection-of-two-port-network/