MATHEMATICS-VIII
MODULE-1

INDEX

RATIONAL NUMBERS & RADICALS

DEFINITION

A number $\frac{a}{b}$ is a rational number if 'a' and 'b' are integers and 'b' is not equal to zero. 'b' cannot be equal to zero because division by zero is not allowed. Further, a rational number is said to be in the standard form or simplest form when the numerator and denominator have no common factor other than 1.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS CLOSURE PROPERTY

When two rational numbers are added, the result is always a rational number, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ is always a rational number. For example, $\frac{2}{5} + \frac{3}{6} = \frac{12+15}{30} = \frac{27}{30}$, which is also a rational number.

COMMUTATIVE PROPERTY

When two rational numbers are added, the order of addition does not matter, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

For example, $\frac{3}{4} + \frac{4}{5} = \frac{15+16}{20} = \frac{31}{20}$ and $\frac{4}{5} + \frac{3}{4} = \frac{16+15}{20} = \frac{31}{20}$. Both results are equal.

ASSOCIATIVE PROPERTY

If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ three rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d}+\frac{e}{f}\right)$. Consider the fractions

$\frac{2}{5}$, $\frac{1}{4}$, and $\frac{2}{3}$.

$\left(\frac{2}{5}+\frac{1}{4}\right)+\frac{2}{3}$, $\frac{2}{5}+\left(\frac{1}{4}+\frac{2}{3}\right)$

$= \left(\frac{8+5}{20}\right)+\frac{2}{3} = \frac{2}{5}+\left(\frac{3+8}{12}\right)$

$= \frac{13}{20}+\frac{2}{3} = \frac{2}{5}+\frac{11}{12} = \frac{39+40}{60}$

$= \frac{24+55}{60} \qquad = \frac{79}{60} = \frac{79}{60}$

Additive identity If $\frac{a}{b}$ is a rational number, then there exists a rational number zero such that $\frac{a}{b} + 0 = \frac{a}{b}$. Zero is called the identity element of addition. Addition of zero does not change the value of the rational number.

Additive identity

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\left(\frac{-a}{b}\right)$, called the additive inverse, such that $\frac{a}{b}+\left(\frac{-a}{b}\right)=0$

The additive inverse is also referred to as 'negative' of the given number..

SUBTRACTION OF RATIONAL NUMBERS

When we have to subtract a rational number, say $\frac{5}{9}$ from $\frac{8}{9}$, we add the additive inverse of $\frac{5}{9}$, i.e., $\frac{-5}{9}$ to $\frac{8}{9}$. Thus, $\frac{8}{9}-\frac{5}{9}=\frac{8}{9}+\left(\frac{-5}{9}\right)=\frac{8-5}{9}=\frac{3}{9}=\frac{1}{3}$

MULTIPLICATION OF RATIONAL NUMBERS

Multiplication is the process of successive addition.

Like 6 × 8 = 8 + 8 + 8 + 8 + 8 + 8 = 48.

Similarly, $6\times\frac{1}{3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{6}{3}=2$

Alternatively, $6\times\frac{1}{3}=\frac{6}{1}\times\frac{1}{3}=\frac{6\times1}{1\times3}=\frac{6}{3}=\frac{2}{1}2$

So, when we multiply two rational numbers, we multiply the numerator with the numerator and the denominator with the denominator.

Thus, $-5\times(-7)=\frac{-5}{1}\times\left(\frac{-7}{1}\right)=\frac{(-5)(-7)}{1\times1}=35$

and $\frac{-2}{11}\times\frac{3}{5}=\frac{-2\times3}{11\times5}=\frac{-6}{55}$

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBER CLOSURE PROPERTY

The rational number are closed under multiplication. It means that the product of two rational numbers is always a rational number, i.e., if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ is always a rational number.

For example, $\frac{-3}{7}\times\frac{5}{8}=-\frac{15}{56}$ which is rational number.

COMMUTATIVE PROPERTY

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$, i.e., $\frac{ac}{bd}=\frac{ca}{db}$

ASSOCIATIVE PROPERTY

If $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$ are three rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$

i.e. $\frac{ac}{bd}\times\frac{e}{f}=\frac{a}{b}\times\frac{ce}{df}$ or $\frac{ace}{bdf}=\frac{ace}{bdf}$

Thus, rational numbers can be multiplied in any order.

Multiplicative identity :

When any rational number, say $\frac{a}{b}$, is multiplied by the rational number 1, the product is always $\frac{a}{b}$. $\frac{a}{b} \times 1 = \frac{a \times 1}{b} = \frac{a}{b}$

or $1 \times \frac{a}{b} = \frac{1 \times a}{b} = \frac{a}{b}$

Multiplicative inverse, or reciprocal :

For every non-zero rational number $\frac{a}{b}$, there exists a rational number $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.

This is so, because $\frac{a}{b} \times \frac{b}{a}$

$= \frac{a \times b}{b \times a} = \frac{ab}{ba} = 1$

Distributive property :

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

Multiplication of a Rational Number by Zero

When any rational number $\frac{a}{b}$ is multiplied by 0, the product is always zero.

$\frac{a}{b} \times 0 = \frac{a \times 0}{b} = \frac{0}{b} = 0$

DIVISION OF RATIONAL NUMBERS

Division is the inverse process of multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

PROPERTIES OF DIVISION OF RATIONAL NUMBERS CLOSURE PROPERTY

When a rational number is divided by another rational number, the quotient is always a rational number.

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$, which is again a rational number since b, c, d are non-zero integers.

Division is not commutative :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers in which b, c and d $\neq$ 0, then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$ because,

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ and $\frac{c}{d} \div \frac{a}{b} = \frac{c}{d} \times \frac{b}{a} = \frac{cb}{da}$

So $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

Ordering of Rational Numbers Law of Trichotomy :

Given two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, then either $\frac{a}{b} > \frac{c}{d}$, $\frac{a}{b} = \frac{c}{d}$, or $\frac{a}{b} < \frac{c}{d}$.

LAW OF TRANSITIVITY

If $\frac{a}{b}, \frac{c}{d}$, and $\frac{e}{f}$ are three rational numbers

(i) If $\frac{a}{b} > \frac{c}{d}$ and $\frac{c}{d} > \frac{e}{f}$, then $\frac{a}{b} > \frac{e}{f}$.

(ii) If $\frac{a}{b} < \frac{c}{d}$ and $\frac{c}{d} < \frac{e}{f}$, then $\frac{a}{b} < \frac{e}{f}$.

(iii) If $\frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$, then $\frac{a}{b} = \frac{e}{f}$. (all are equivalent).

LAW OF ADDITION

Given $\frac{a}{b}, \frac{c}{d}$, and $\frac{e}{f}$ are three rational numbers.

(i) If $\frac{a}{b} > \frac{c}{d}$ then $\frac{a}{b} + \frac{e}{f} > \frac{c}{d} + \frac{e}{f}$.

(ii) If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} + \frac{e}{f} = \frac{c}{d} + \frac{e}{f}$.

(iii) If $\frac{a}{b} < \frac{c}{d}$ then $\frac{a}{b} + \frac{e}{f} < \frac{c}{d} + \frac{e}{f}$.

PROPERTY OF MULTIPLICATION

Let $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ be three rational numbers.

If $\frac{e}{f}$ is a positive number, then

(i) If $\frac{a}{b} > \frac{c}{d}$, then $\frac{a}{b} \times \frac{e}{f} > \frac{c}{d} \times \frac{e}{f}$.

(ii) If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} \times \frac{e}{f} < \frac{c}{d} \times \frac{e}{f}$.

(iii) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} \times \frac{e}{f} = \frac{c}{d} \times \frac{e}{f}$.

POWERS EXPONENTIAL NOTATION AND RATIONAL NUMBERS

Exponential notation can be extended to rational numbers. For example: $\left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right)$ can be written as

$\left(\frac{4}{5}\right)^3$ which is read as $\frac{4}{5}$ raised to the power 3.

(i) $\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) = \frac{3^3}{4^3} = \frac{27}{64}$

(ii) $\left(\frac{-5}{6}\right)^2 = \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \frac{(-5)^2}{6^2} = \frac{25}{36}$

(iii) $\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) = \frac{(-2)^3}{3^3} = \frac{-8}{27}$

In general, if $\frac{x}{y}$ is a rational number and a is a positive integer, then

$$\boxed{\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}}$$

Reciprocals with Positive Integral Exponents:

The reciprocal of 2 is $\frac{1}{2}$, reciprocal of 2^3 is $\frac{1}{2^3}$.

Reciprocal of $\left(\frac{2}{3}\right)^4 = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\frac{2^4}{3^4}} = \frac{3^4}{2^4} = \left(\frac{3}{2}\right)^4$

Reciprocal of $\left(\frac{-4}{5}\right)^4 = \left(\frac{-5}{4}\right)^6$ and, Reciprocal of $\left(\frac{1}{3}\right)^5 = \left(\frac{3}{1}\right)^5 = 3^5$

Reciprocals with Negative Integral Exponents

Reciprocal of 2 $= \frac{1}{2} = \frac{1}{2^1}$. Therefore, the reciprocal of 2 is 2^{-1}. The reciprocal of $3^2 = \frac{1}{3^2} = 3^{-2}$.

Reciprocal of $\left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)^{-2}$, Reciprocal of $\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)^{-3}$, etc.

In general, if x is any rational number other than zero and a is any positive integer, then:

$$\boxed{x^{-a} = \frac{1}{x^a}}$$

LAWS OF EXPONENTS

1. Consider the following.

(i) $3^3 \times 3^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7 = 3^{3+4}$

(ii) $\left(\frac{5}{2}\right)^2 \times \left(\frac{5}{2}\right)^3 = \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \left(\frac{5}{2}\right)^5 = \left(\frac{5}{2}\right)^{2+3}$

$\therefore$ $\boxed{x^a \times x^b = x^{a+b}}$

2. (i) $2^5 \div 2^2 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2 \times 2 \times 2 = 2^3 = 2^{5-2}$

(ii) $\left(\frac{2}{3}\right)^6 \div \left(\frac{2}{3}\right)^2 = \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{6-2}$

$\therefore$ $\boxed{x^a \div x^b = x^{a-b}}$

3. (i) $(2^3)^2 = (2 \times 2 \times 2)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6 = 2^3 \times 2^3$

(ii) $\left\{\left(\frac{2}{3}\right)^3\right\}^2 = \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right)^2 = \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) \times \left(\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\right) = \left(\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^{3\times 2}$

$\therefore$ $\boxed{(x^a)^b = x^{ab}}$

4. (i) $2^4 \times 3^4 = (2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3)$
$= (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) = (2 \times 3)^4$

(ii) $\left(\frac{3}{5}\right)^4 \times \left(\frac{1}{2}\right)^4 = \left(\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\right) \times \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right) = \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) \times \left(\frac{3}{5}\times\frac{1}{2}\right) = \left(\frac{3}{5}\times\frac{1}{2}\right)^4$

$\therefore$ $\boxed{x^a \times y^a = (x \times y)^a}$

5. (i) $2^4 \div 3^4 = \frac{2\times 2\times 2\times 2}{3\times 3\times 3\times 3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4$

(ii) $\left(\frac{3}{5}\right)^4 \div \left(\frac{1}{2}\right)^4 = \frac{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}}{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}} = \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) \times \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right) = \left(\frac{\frac{3}{5}}{\frac{1}{2}}\right)^4$

$\therefore$ $\boxed{x^a \div y^a = \left(\frac{x}{y}\right)^a}$

If x is any rational number different from zero and a, b are any integers, then,

Law I: $\boxed{x^a \times x^b = x^{a+b}}$

Law II: $\boxed{x^a \div x^b = x^{a-b}}$

Law III: $\boxed{(x^a)^b = x^{ab}}$

Law IV: $\boxed{x^a \times y^a = (x \times y)^a}$ (where y is also a non zero rational number)

Law I: $\boxed{x^a \div y^a = \left(\frac{x}{y}\right)^a}$ (where y is also a non- zero rational number)

SOLVED EXAMPLES

Ex.1 Add $\frac{3}{5}$ and $\frac{13}{5}$.

Sol. We have,

$\frac{3}{5}+\frac{13}{5}=\frac{3+13}{5}=\frac{16}{5}$ [$\because$ 3 + 13 = 16]

Ex.2 Add $\frac{7}{9}$ and $\frac{-12}{9}$.

Sol. We have,

$\frac{7}{9}+\frac{-12}{9}=\frac{7+(-12)}{9}=\frac{-5}{9}$

[$\because$ 7 + (–12) = – 5]

Ex.3 Add $\frac{-5}{9}$ and $\frac{-17}{9}$.

Sol. We have,

$\frac{-5}{9} + \frac{-17}{9} = \frac{(-5)-(17)}{9} = \frac{-22}{9}$

[$\because$ (–5) + (–17) = – 22]

Ex.4 Add $\frac{4}{-11}$ and $\frac{7}{11}$.

Sol. We first express $\frac{4}{-11}$ as a rational with positive denominator.

We have, $\frac{4}{-11} = \frac{4\times(-1)}{(-11)\times(-1)} = \frac{-4}{11}$

$\therefore \frac{4}{-11} + \frac{7}{11} = \frac{-4}{11} + \frac{7}{11} = \frac{(-4)+7}{11}$

$= \frac{3}{11}$

[$\because$ (–4) + 7 = 3]

Ex.5 Add $\frac{5}{12}$ and $\frac{3}{8}$.

Sol. Clearly, denominator of the given numbers are positive.

The LCM of denominators 12 and 8 is 24.

Now, we express $\frac{5}{12}$ and $\frac{3}{8}$ into forms in which both of them have the same denominator 24.

We have,

$\frac{5}{12} = \frac{5\times2}{12\times2} = \frac{10}{24}$ and, $\frac{3}{8} = \frac{3\times3}{8\times3} = \frac{9}{24}$

$\therefore \frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{10+9}{24} = \frac{19}{24}$

Ex.6 Add $\frac{7}{9}$ and 4.

Sol. We have, 4 = $\frac{4}{1}$.

Clearly, denominators of the two rational numbers are positive. We now rewrite them so that they ahve a common denominator 3equal to the LCM of the denominators.

LCM of 9 and 1 is 9.

We have, $\frac{4}{1} = \frac{4\times9}{1\times9} = \frac{36}{9}$

$\therefore \frac{7}{9} + 4 = \frac{7}{9} + \frac{4}{1} = \frac{7}{9} + \frac{36}{9}$

$= \frac{7+36}{9} = \frac{43}{9}$

Ex.7 Add $\frac{3}{8}$ and $\frac{-5}{12}$.

Sol. The denominators of the given rational numbers are 8 and 12 respectively. The LCM of 8 and 12 is 24. Now we re-write the given ratinoal numbers into forms in which both of them have the same dnominator.

$\frac{3}{8}=\frac{3\times3}{8\times3}=\frac{9}{24}$ and, $\frac{-5}{12}=\frac{-5\times2}{12\times2}=\frac{-10}{24}$

$\therefore \quad \frac{3}{8} + \frac{-5}{12} = \frac{9}{24} + \frac{(-10)}{24} = \frac{9-10}{24} = \frac{-1}{24}$

Ex.8 Simplify : $\frac{8}{-15} + \frac{4}{-3}$.

Sol. We have,

$\frac{8}{-15} + \frac{4}{-3} = \frac{-8}{15} + \frac{-4}{3}$

$\left[\therefore \frac{8}{-15}=\frac{8\times-1}{(-15)\times(-1)}=\frac{-8}{15} \text{ and } \frac{4}{-3}=\frac{4\times-1}{(-3)\times(-1)}=\frac{-4}{3}\right]$

LCM of 15 and 3 is 15.

Re-writing $\frac{-4}{3}$ in the form in which is has denominator 15, we get

$\frac{-4}{3} = \frac{-4\times5}{3+5} = \frac{-20}{15}$

$\therefore \quad \frac{8}{-15} + \frac{4}{-3} = \frac{-8}{15} + \frac{-4}{3}$

$= \frac{-8}{15} + \frac{-20}{15} \qquad \left[\because \frac{-4}{3}=\frac{-20}{15}\right]$

$= \frac{-(-8)+(-20)}{15}=\frac{-28}{15}$

Ex.9 Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) 5^{-3} (ii) $(-2)^{-5}$ (iii) $\left(\frac{4}{3}\right)^{-3}$ (iv) $\left(\frac{-2}{5}\right)^{-4}$ (v) $\frac{1}{2^{-3}}$

Sol. We know that, if a is a non-zero rational number and n is a positive integer, then $a^{-n} = \frac{1}{a^n}$

(i) $5^{-3} = \frac{1}{5^3} = \frac{1}{125} \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$

(ii) $(-2)^{-5} = \frac{1}{(-2)^5}=\frac{1}{-32} = -\frac{1}{32} \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$

(iii) $\left(\frac{4}{3}\right)^{-3} = \frac{1}{\left(\frac{4}{3}\right)^3} = \frac{1}{\frac{4^3}{3^3}} = \frac{1}{\frac{64}{27}} = \frac{27}{64}$

$\left[\because \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \text{ when n is a whole number}\right]$

(iv) $\left(\frac{-2}{5}\right)^{-4} = \frac{1}{\left(\frac{-2}{5}\right)^4} = \frac{1}{\frac{(-2)^4}{5^4}} = \frac{1}{\frac{16}{625}} = \frac{625}{16} \qquad \left[\because \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \text{ for } n>0\right]$

(v) $\frac{1}{2^{-3}} = \frac{1}{\frac{1}{2^3}} = \frac{2^3}{1} = \frac{8}{1} = 8 \qquad \left[\because a^{-n}=\frac{1}{a^n}\right]$

Ex.10 Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3}$ (ii) $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-7}{5}\right)^{2}$

Sol. (i) We have,

$$\left(\frac{3}{8}\right)^{-2} \times \left(\frac{4}{5}\right)^{-3} = \frac{1}{\left(\frac{3}{8}\right)^{2}} \times \frac{1}{\left(\frac{4}{5}\right)^{3}}$$

$$\left[\because\ a^{-n} = \frac{1}{a^n}\right]$$

$$= \frac{1}{\frac{3^2}{8^2}} \times \frac{1}{\frac{4^3}{5^3}} \quad \left[\because \left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right)\right]$$

$$= \frac{1}{\frac{9}{64}} \times \frac{1}{\frac{64}{125}} = \frac{64}{9} \times \frac{125}{64} = \frac{125}{9}$$

(ii) We have, $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-7}{5}\right)^{2}$

$$= \frac{1}{\left(\frac{-2}{7}\right)^{4}} \times \left(\frac{-7}{5}\right)^{2} = \frac{1}{\frac{(-2)^4}{7^4}} \times \frac{(-7)^2}{5^2}$$

$$= \frac{7^4}{(-2)^4} \times \frac{(-7)^2}{5^2} = \frac{7\times7\times7\times7}{16} \times \frac{-7\times-7}{25}$$

$$= \frac{7^6}{16\times25} = \frac{7^6}{16\times25} = \frac{7^6}{400} = \frac{117649}{400}$$

Ex.11 Express each of the following as power of a rational number with positive exponent :

(i) $\left(\frac{1}{4}\right)^{-3}$ (ii) $5^{-3} \times 5^{-6}$ (iii) $\left(\frac{-1}{4}\right)^{-5} \times \left(\frac{-1}{4}\right)^{-7}$

Sol. (i) We have,

$$\left(\frac{1}{4}\right)^{-3} = \frac{1}{\left(\frac{1}{4}\right)^{3}} = \frac{1}{\frac{1^3}{4^3}} = \frac{4^3}{1^3} = 4^3$$

(ii) We have,
$5^{-3} \times 5^{-6}$

$$= \frac{1}{5^3} \times \frac{1}{5^6} = \frac{1\times1}{5^3\times5^6} = \frac{1}{5^{3+6}} = \frac{1^9}{5^9} = \left(\frac{1}{5}\right)^9$$

(iii) We have, $\left(\frac{-1}{4}\right)^{-5} \times \left(\frac{-1}{4}\right)^{-7}$

$$= \frac{1}{\left(\frac{-1}{4}\right)^{5}} \times \frac{1}{\left(\frac{-1}{4}\right)^{7}} = \frac{1}{\frac{(-1)^5}{4^5}} \times \frac{1}{\frac{(-1)^7}{4^7}} \times \frac{1}{\frac{-1}{4^5}} \times \frac{1}{\frac{-1}{4^7}}$$

$$= \frac{4^5}{-1} \times \frac{4^7}{-1} = \frac{4^5\times4^7}{(-1)\times(-1)} = \frac{4^{5+7}}{1} = 4^{12}$$

Ex.12 Simplify:

(i) $\left(2^{-1}\div 5^{-1}\right)^{2}\times\left(\frac{-5}{8}\right)^{-1}$ (ii) $\left(6^{-1}-8^{-1}\right)^{-1}+(2^{-1}-3^{-1})^{-1}$

(iii) $\left(5^{-1}\times 3^{-1}\right)^{-1}\div 6^{-1}$ (iv) $\left(4^{-1}+8^{-1}\right)\div\left(\frac{2}{3}\right)^{-1}$

Sol. (i) We have,

$(2^{-1}\div 5^{-1})^{2}\times\left(\frac{-5}{8}\right)^{-1}$

$= \left(\frac{1}{2}\div\frac{1}{5}\right)^{2}\times\frac{1}{\left(\frac{-5}{8}\right)} = \left(\frac{1}{2}\times\frac{5}{1}\right)^{2}\times\left(\frac{8}{-5}\right)$

$\left[\because a^{-1}=\frac{1}{a}\right]$

$= \left(\frac{5}{2}\right)^{2}\times\left(\frac{8}{-5}\right)=\frac{5^{2}}{2^{2}}\times\frac{8}{-5}=\frac{5}{4}\times\frac{8}{-1}$

$= \frac{5}{1}\times\frac{2}{-1}=-10$

(ii) We have, $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$

$= \left(\frac{1}{6}-\frac{1}{8}\right)^{-1}+\left(\frac{1}{2}-\frac{1}{3}\right)^{-1}$

$= \left(\frac{4-3}{24}\right)^{-1}+\left(\frac{3-2}{6}\right)^{-1}$

$= \left(\frac{1}{24}\right)^{-1}+\left(\frac{1}{6}\right)^{-1}=\frac{1}{\frac{1}{24}}+\frac{1}{\frac{1}{6}}-\frac{24}{1}+\frac{6}{1}=30$

$\left[\because a^{-1}=\frac{1}{a}\right]$

(iii) We have, $(5^{-1}\times 3^{-1})^{-1} \div 6^{-1}$

$= \left(\frac{1}{5}\times\frac{1}{3}\right)^{-1}\div\frac{1}{6}$

$= \left(\frac{1}{15}\right)^{-1}\div\frac{1}{6}=\frac{1}{\frac{1}{15}}\div\frac{1}{6}= 15\div\frac{1}{6}=15\times\frac{6}{1}=90$

(iv) We have, $(4^{-1}+8^{-1})\div\left(\frac{2}{3}\right)^{-1}$

$= \left(\frac{1}{4}+\frac{1}{8}\right)\div\frac{1}{\frac{2}{3}}=\left(\frac{2+1}{8}\right)\div\left(\frac{3}{2}\right)$

$= \frac{3}{8}\div\frac{3}{2}=\frac{3}{8}\times\frac{2}{3}=\frac{1}{4}$

Ex.13 Simplify

(i) $\left(\frac{1}{4}\right)^{-2}+\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}$ (ii) $\left\{6^{-1}\left(\frac{3}{2}\right)^{-1}\right\}^{-1}$

Sol. (i) We have, $\left(\frac{1}{4}\right)^{-2}+\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}$

$$= \frac{1}{\left(\frac{1}{4}\right)^2}+\frac{1}{\left(\frac{1}{2}\right)^2}+\frac{1}{\left(\frac{1}{3}\right)^2}$$

$$= \frac{1}{\frac{1^2}{4^2}}+\frac{1}{\frac{1^2}{2^2}}+\frac{1}{\frac{1^2}{3^2}} = \frac{4^2}{1^2}+\frac{2^2}{1^2}+\frac{3^2}{1^2}$$

$$= \frac{4^2}{1}+\frac{2^2}{1}+\frac{3^2}{1}$$

$= 4^2 + 2^2 + 3^2 = 16 + 4 + 9 = 29$

(ii) We have, $\left\{6^{-1}+\left(\frac{3}{2}\right)^{-1}\right\}^{-1}$

$$= \left\{6^{-1}+\frac{1}{\frac{3}{2}}\right\}^{-1} = \left(\frac{1}{6}+\frac{2}{3}\right)^{-1} = \left(\frac{1+2\times 2}{6}\right)^{-1}$$

$$= \left(\frac{5}{6}\right)^{-1} = \frac{1}{\frac{5}{6}} = \frac{6}{5}$$

Ex.14 Express each of the following as a rational number of the form $\frac{p}{q}$

(i) $(2^{-1} + 3^{-1})^2$ (ii) $(2^{-1} - 4^{-1})^2$ (iii) $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Sol. We know that for any positive integer n and any rational number a, $a^{-n}=\frac{1}{a^n}$. Thus, we have

(i) $(2^{-1}+3^{-1})^2 = \left(\frac{1}{2}+\frac{1}{3}\right)^2 = \left(\frac{3+2}{6}\right)^2$

$$= \left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$$

(ii) $(2^{-1}-4^{-1})^2 = \left(\frac{1}{2}-\frac{1}{4}\right)^2 = \left(\frac{2-1}{4}\right)^2 = \left(\frac{1}{4}\right)^2$

$$= \frac{1^2}{4^2} = \frac{1}{16}$$

(iii) $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

$$= \left(\frac{1}{\frac{3}{4}}-\frac{1}{\frac{1}{4}}\right)^{-1} = \left(\frac{4}{3}-\frac{4}{1}\right)^{-1}$$

$$= \left(\frac{4-3\times 4}{3}\right)^{-1}\left(\frac{-8}{3}\right)^{-1} = \frac{1}{\frac{-8}{3}} = \frac{3}{-8} = -\frac{3}{8}$$

Ex.15 By what number should $(-8)^{-1}$ be multiplied so that the product may be equal to 10^{-1} ?

Sol. Let $(-8)^{-1}$ be multiplied by x to get 10^{-1}. Then, $x \times (-8)^{-1} = 10^{-1}$

$\Rightarrow \quad x = 10^{-1} \div (-8)^{-1}$

$\Rightarrow \quad x = \frac{1}{10} \div \frac{1}{-8} \qquad \left[\because a^{-1} = \frac{1}{a}\right]$

$\Rightarrow \quad x = \frac{1}{10} \times \frac{-8}{1} = \frac{-8}{10} = \frac{-4}{5}$

Hence, the required number is $\frac{-4}{5}$

Ex.16 Using the laws of exponents, simplify each of the following and express in exponential form:

(A) $3^7 \times 3^{-2}$ (B) $2^{-7} \div 2^{-3}$ (C) $(5^2)^{-3}$ (D) $2^{-3} \times (-7)^{-3}$ (E) $\frac{3^{-5}}{4^{-5}}$

Sol. Using laws of exponents, we have:

(i) $3^7 \times 3^{-2} = 3^{7+(-2)} = 3^5 \qquad [\because a^m \times a^n = a^{m+n}]$

(ii) $2^{-7} \div 2^{-3} = \frac{2^{-7}}{2^{-3}} = 2^{-7-(-3)} = 2^{-7+3} = 2^{-4} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

(iii) $(5^2)^{-3} = 5^{2 \times -3} = 5^{-6} \qquad [\because (a^m)^n = a^{mn}]$

(iv) $2^{-3} \times (-7)^{-3} = (2 \times (-7))^{-3} = (-14)^{-3} \qquad [\because a^n \times b^n = (ab)^n]$

(v) $\frac{3^{-5}}{4^{-5}} = \left(\frac{3}{4}\right)^{-5} \qquad \left[\because \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n\right]$

Ex.17 Using the laws of exponents simplify and express each of the following in exponential form with positive exponent:

(i) $(-4)^4 \times (-4)^{-10}$ (ii) $2^{-5} \div 2^2$ (iii) $3^{-4} \times 2^{-4}$

(iv) $\left(\frac{1}{2^3}\right)^2$ (v) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (vi) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Sol. (i) We have, $(-4)^4 \times (-4)^{-10} = (-4)^{4+(-10)} \qquad [\because a^m \times a^n = a^{m+n}]$

$= (-4)^{-6}$

$= \frac{1}{(-4)^6} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1^6}{(-4)^6} \qquad [\because 1^6 = 1]$

$= \left(\frac{1}{-4}\right)^6 \qquad \left[\because \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n\right]$

$= \left(\frac{-1}{4}\right)^6 \qquad \left[\because \frac{1}{-4} = \frac{-1}{4}\right]$

(ii) We have,

$2^{-5} \div 2^2 = \frac{2^{-5}}{2^2} = 2^{-5-2} \qquad \left[\because \frac{a^m}{a^n} = a^{m-n}\right] \qquad = 2^{-7} = \frac{1}{2^7} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right] \qquad = \frac{1^7}{2^7} = \left(\frac{1}{2}\right)^7$

(iii) We have,

$3^{-4} \times 2^{-4} = (3 \times 2)^{-4}$ $\quad [\because a^n \times b^n = (ab)^n]$

$= 6^{-4} = \frac{1}{6^4}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$ $\quad = \frac{1^4}{6^4}$ $[\because 1^4 = 1]$

$= \left(\frac{1}{6}\right)^4$ $\quad \left[\because \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n\right]$

(iv) We have,

$$\left(\frac{1}{2^3}\right)^2 = \left(\frac{1^3}{2^3}\right)^2 \left\{\left(\frac{1}{2}\right)^3\right\}^2 = \left(\frac{1}{2}\right)^{3\times 2} = \left(\frac{1}{2}\right)^6$$

(v) We have, $(3^{-7} \div 3^{-10}) \times 3^{-5}$

$= \left(\frac{3^{-7}}{3^{-10}}\right) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5}$

$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)} = 3^{-2}$

$= \frac{1}{3^2} = \frac{1^2}{3^2} = \left(\frac{1}{3}\right)^2$

(vi) We have,

$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1\times 3)^4 \times \left(\frac{5}{3}\right)^4$ $\quad [\because -3 = -1 \times 3]$

$= \{(-1)^4 \times 3^4\} \times \frac{5^4}{3^4}$

$\left[\because (ab)^n = a^n b^n \text{ and} \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

$= (1 \times 3^4) \times \frac{5^4}{3^4}$ $\quad [\because (-1)^4 = 1]$

$= 3^4 \times \frac{5^4}{3^4} = 3^{4-4} \times 5^4 \quad = 3^0 \times 5^4 = 1 \times 5^4 = 5^4$

Ex.18 Simplify and write the answer in the exponetial form:

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$ (ii) $(-4)^3 \times (5)^{-3} \times (-5)^{-3}$ (iii) $\frac{1}{8} \times 3^{-3}$

Sol. (i) We have, $(2^5 \div 2^8)^5 \times 2^{-5}$

$= \left(\frac{2^5}{2^8}\right)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5}$

$= (2^{-3})^5 \times 2^{-5} = 2^{-3\times 5} \times 2^{-5}$

$= 2^{-15} \times 2^{-5} = 2^{-15-5} = 2^{-20}$

(ii) We have, $(-4)^{-3} \times 5^{-3} \times (-5)^{-3}$

$= \{-4 \times 5 \times (-5)\}^{-3}$ $\quad [\because a^n \times b^n \times c^n = (abc)^n]$

$= (100)^{-3} = (10^2)^{-3} = 10^{2 \times -3} = 10^{-6}$

(iii) We have,

$$\frac{1}{8}\times 3^{-3} = \frac{1}{2^3}\times 3^{-3} = 2^{-3}\times 3^{-3} = (2\times 3)^{-3} = 6^{-3}$$

Ex.19 Simplify each of the following :

(i) $\left[\left\{\left(\frac{-1}{5}\right)^{-2}\right\}^{2}\right]^{-1}$ (ii) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

Sol. We have,

(i) $\left[\left\{\left(\frac{-1}{5}\right)^{-2}\right\}^{2}\right]^{-1} = \left\{\left(\frac{-1}{5}\right)^{-2}\right\}^{2\times -1}$ $\left[\because (a^m)^n = a^{mn}\right]$

$= \left\{\left(\frac{-1}{5}\right)^{-2}\right\}^{-2} = \left(\frac{-1}{5}\right)^{(-2)\times(-2)}$

$= \left(\frac{-1}{5}\right)^{4} = \frac{(-1)^4}{5^4} = \frac{1}{625}$

(ii) We have,

$\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$ $\left\{\left(\frac{3}{1}\right)^{2} - \left(\frac{2}{1}\right)^{3}\right\} \div \left(\frac{4}{1}\right)^{2}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = (9 - 8) \div 16$

$= 1 \div 16 = \frac{1}{16}$

Ex.20 Simplify :

(i) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5}$ (ii) $\left(\frac{-2}{3}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3}$ (iii) $\left(\frac{3}{4}\right)^{-4}\div\left(\frac{3}{2}\right)^{-3}$ (iv) $\left(\frac{3}{7}\right)^{-2}\times\left(\frac{7}{6}\right)^{-3}$

Sol. (i) We have,

$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}}\times\frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}}\times\frac{8^{-5}}{8^{-7}}$

$= 5^{-7-(-5)} \times 8^{-5-(-7)}$

$= 5^{-7+5} \times 8^{-5+7} = 5^{-2} \times 8^{2} = \frac{8^2}{5^2} = \frac{64}{25}$

(ii) We have, $\left(\frac{-2}{3}\right)^{-2}\times\left(\frac{4}{5}\right)^{-3} = \frac{(-2)^{-2}}{3^{-2}}\times\frac{4^{-3}}{5^{-3}}$

$= \frac{3^2}{(-2)^2}\times\frac{5^3}{4^3} = \frac{9}{4}\times\frac{125}{64} = \frac{9\times 125}{4\times 64} = \frac{1125}{256}$

(iii) We have,

$$\left(\frac{3}{4}\right)^{-4} \div \left(\frac{3}{2}\right)^{-3} = \left(\frac{3}{4}\right)^{-4} \times \frac{1}{\left(\frac{3}{2}\right)^{-3}} = \left(\frac{3}{4}\right)^{-4} \times \left(\frac{3}{2}\right)^{3}$$

$$= \frac{3^{-4}}{4^{-4}} \times \frac{3^3}{2^3}$$

$$= \frac{3^{-4} \times 3^3}{(2^2)^{-4} \times 2^3} = \frac{3^{-4} \times 3^3}{2^{-8} \times 2^3} = \frac{3^{-4+3}}{2^{-8+3}} = \frac{3^{-1}}{2^{-5}} = \frac{2^5}{3^1} = \frac{32}{3}$$

(iv) We have, $\left(\frac{3}{7}\right)^{-2} \times \left(\frac{7}{6}\right)^{-3} = \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{6^{-3}}$ $\left[\because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$

$$= \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{(2 \times 3)^{-3}}$$

$$= \frac{3^{-2}}{7^{-2}} \times \frac{7^{-3}}{2^{-3} \times 3^{-3}}$$

$$= \frac{3^{-2}}{3^{-3}} \times \frac{7^{-3}}{7^{-2}} \times \frac{1}{2^{-3}}$$

$$= 3^{-2+3} \times 7^{-3+2} \times 2^3$$

$$= 3 \times 7^{-1} \times 2^3 = 3 \times \frac{1}{7} \times 8 = \frac{24}{7}$$

Ex.21 Evaluate: $\frac{8^{-1} \times 5^3}{2^{-4}}$

Sol. We have,

$$\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{3\times -1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}}$$

$$= 2^{-3+4} \times 5^3 \, 2^1 \times 5^3 = 2 \times 125 = 250$$

Ex.22 Simplify:

(i) $\frac{25 \times a^{-4}}{5^{-3} \times 10 \times a^{-8}}$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Sol. (i) We have,

$$\frac{25 \times a^{-4}}{5^{-3} \times 10 \times a^{-8}} = \frac{5^2 \times a^{-4}}{5^{-3} \times (2 \times 5) \times a^{-8}} = \frac{5^2 \times a^{-4}}{5^{-3+1} \times 2 \times a^{-8}}$$

$$= \frac{5^2 \times a^{-4}}{5^{-2} \times 2 \times a^{-8}} = \frac{5^{2-(-2)} \times a^{-4+8}}{2} = \frac{5^4 \times a^4}{2} \quad = \frac{5^4}{2} \times a^4 = \frac{625}{2} a^4$$

(ii) We have,

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)}$$

$$= 3^0 \times 2^0 \times 5^{-5+3+7} = 1 \times 1 \times 5^5 = 5^5$$

Ex.23 By what number should $(-4)^{-2}$ be multiplied so that the product may be equal to 10^{-2} ?

Sol. Let $(-4)^{-2}$ = be multiplied by x to get 10^{-2}.

Then, $x \times (-4)^{-2} = 10^{-2}$

$\Rightarrow x = 10^{-2} \div (-4)^{-2}$

$\Rightarrow x = 10^{-2} \times \frac{1}{(-4)^{-2}}$

$\Rightarrow x = \frac{10^{-2}}{(-4)^{-2}}$

$\Rightarrow x = \frac{(-4)^2}{10^2} = \frac{16}{100} = \frac{4}{25}$

Hence, required number is $\frac{4}{25}$

Ex.24 By what number should $(-12)^{-1}$ be divided so that the quotient may be $\left(\frac{2}{3}\right)^{-1}$?

Sol. Let the required number be x. Then,

$(-12)^{-1} \div x = \left(\frac{2}{3}\right)^{-1}$

$\Rightarrow \frac{(-12)^{-1}}{x} = \left(\frac{2}{3}\right)^{-1}$ $\Rightarrow x = (-12)^{-1} \div \left(\frac{2}{3}\right)^{-1}$

$\Rightarrow x = \frac{1}{-12} \div \left(\frac{3}{2}\right)$ $\Rightarrow x = \frac{1}{-12} \times \frac{2}{3} = \frac{1}{-18} = \frac{-1}{18}$ $\left[\because a^{-1} = \frac{1}{a} \text{ and } \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}\right]$

Ex.25 By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{4}{27}\right)^{-2}$?

Sol. Let the required number be x. Then,

$\left(\frac{-3}{2}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$

$\Rightarrow \left(\frac{-3}{2}\right)^{-3} \times \frac{1}{x} = \left(\frac{4}{27}\right)^{-2}$ $\Rightarrow x = \left(\frac{-3}{2}\right)^{-3} \div \left(\frac{4}{27}\right)^{-2}$

$\Rightarrow x = \left(\frac{2}{-3}\right)^{3} \div \left(\frac{27}{4}\right)^{2}$ $\left[\because \left(\frac{a}{b}\right)^{n} = \left(\frac{b}{a}\right)^{n}\right]$ $\Rightarrow x = \left(\frac{2}{-3}\right)^{3} \times \frac{1}{\left(\frac{27}{4}\right)^{2}}$

$\Rightarrow x = \frac{2^3}{(-3)^3} \times \frac{1}{\frac{27^2}{4^2}}$ $\Rightarrow x = \frac{8}{-27} \times \frac{4^2}{27^2} = \frac{-8}{27} \times \frac{4^2}{27^2} = \frac{-2 \times 4^3}{27^3}$

$= -2 \times \left(\frac{4}{27}\right)^{3}$

EXERCISE - I UNSOLVED PROBLEMS

Q.1 What number should be added to $\frac{-5}{11}$ so as to get $\frac{26}{33}$?

Q.2 What number should be added to $\frac{-5}{7}$ to get $\frac{-2}{3}$?

Q.3 What number should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

Q.4 What number should be subtracted from $\frac{3}{7}$ to get $\frac{5}{4}$?

Q.5 What should be added to $\left(\frac{2}{3}+\frac{3}{5}\right)$ to get $\frac{-2}{15}$?

Q.6 What should be added to $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{5}\right)$ to get 3?

Q.7 What should be subtracted from $\left(\frac{3}{4}-\frac{2}{3}\right)$ to get $\frac{-1}{6}$?

Q.8 Simply each of the following and write as a rational number of the from $\frac{p}{q}$:

(i) $\frac{3}{4}+\frac{5}{6}+\frac{-7}{8}$ (ii) $\frac{2}{3}+\frac{-5}{6}+\frac{-7}{9}$

Q.9 Express each of the following as a rational number of the form $\frac{p}{q}$:

(i) $\frac{-8}{3}+\frac{-1}{4}+\frac{-11}{6}+\frac{3}{8}-3$

(ii) $\frac{6}{7}+1+\frac{-7}{9}+\frac{19}{21}+\frac{-12}{7}$

Q.10 Simplify:

(i) $\frac{-3}{2}+\frac{5}{4}-\frac{7}{4}$ (ii) $\frac{5}{3}-\frac{7}{6}+\frac{-2}{3}$

Q.11 Multiply:

(i) $\frac{7}{11}$ by $\frac{5}{4}$ (ii) $\frac{5}{7}$ by $\frac{-3}{4}$

Q.12 Multiply:

(i) $\frac{-5}{17}$ by $\frac{51}{-60}$ (ii) $\frac{-6}{11}$ by $\frac{-55}{36}$

Q.13 Simplify each of the following and express the result as a rational number in standard form:

(i) $\frac{-16}{21}\times\frac{14}{5}$ (ii) $\frac{7}{6}\times\frac{-3}{28}$

Q.14 Simplify:

(i) $\left(\frac{25}{8}\times\frac{2}{5}\right)-\left(\frac{3}{5}\times\frac{-10}{9}\right)$ (ii) $\left(\frac{1}{2}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times 6\right)$

Q.15 Simplify:

(i) $\left(\frac{3}{2}\times\frac{1}{6}\right)+\left(\frac{5}{3}\times\frac{7}{2}\right)-\left(\frac{13}{8}\times\frac{4}{3}\right)$

(ii) $\left(\frac{1}{4}\times\frac{2}{7}\right)-\left(\frac{5}{14}\times\frac{-2}{3}\right)+\left(\frac{3}{7}\times\frac{9}{2}\right)$

Q.16 Express each of the following as a rational number in the form $\frac{p}{q}$:

(i) 6^{-1} (ii) $(-7)^{-1}$

(iii) $\left(\frac{1}{4}\right)^{-1}$ (iv) $(-4)^{-1}\times\left(\frac{-3}{2}\right)^{-1}$

Q.17 Simplify

(i) $\{4^{-1}\times 3^{-1}\}^2$ (ii) $\{5^{-1}\div 6^{-1}\}^3$

Q.18 Express each of the following rational numbers with a negative exponent:

(i) $\left(\frac{1}{4}\right)^3$ (ii) 3^5

Q.19 Express each of the following rational numbers with a positive exponent:

(i) $\left(\frac{3}{4}\right)^{-2}$ (ii) $\left(\frac{5}{4}\right)^{-3}$

Q.20 Simplify

(i) $\left\{\left(\frac{3}{2}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-3}$

(ii) $(3^2-2^2)\times\left(\frac{2}{3}\right)^{-3}$

Q.21 By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Q.22 By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{-4}{7}^{-1}\right)$?

Q.23 By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Q.24 By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Q.25 Find x, if

(i) $\left(\frac{1}{4}\right)^{-4}\times\left(\frac{1}{4}\right)^{-8}=\left(\frac{1}{4}\right)^{-4x}$

(ii) $\left(\frac{-1}{2}\right)^{-19}\div\left(\frac{-1}{2}\right)^{8}=\left(\frac{-1}{2}\right)^{-2x+1}$

Q.26 If $x=\left(\frac{3}{2}\right)^{2}\times\left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}.

Q.27 Find the value of x for which $5^{2x}\div5^{-3}=5^5$.

Q.28 Express the following numbers in standard form:
(i) 6020000000000000
(ii) 0.00000000000942

Q.29 Write the following numbers in the usual form:
(i) 4.83×10^7 (ii) 3.02×10^{-6}
(iii) 4.5×10^4 (iv) 3×10^{-8}

ANSWER KEY

1. $\frac{41}{33}$ **2.** $\frac{1}{21}$ **3.** $\frac{-5}{2}$ **4.** $\frac{47}{28}$

5. $\frac{-7}{5}$ **6.** $\frac{59}{30}$ **7.** $\frac{1}{4}$

8. (i) $\frac{17}{24}$ (ii) $\frac{-17}{18}$

9. (i) $-7\frac{3}{8}$ (ii) $\frac{17}{63}$

10. (i) -2 (ii) $\frac{-1}{6}$

11. (i) $\frac{35}{44}$ (ii) $\frac{-15}{28}$

12. (i) $\frac{1}{4}$ (ii) $\frac{5}{6}$

13. (i) $-2\frac{2}{15}$ (ii) $\frac{-1}{8}$

14. (i) $1\frac{11}{12}$ (ii) $3\frac{1}{8}$

15. (i) $3\frac{11}{12}$ (ii) $2\frac{5}{21}$

16. (i) $\frac{1}{6}$ (ii) $\frac{-1}{7}$ (iii) 4 (iv) $\frac{1}{6}$

17. (i) $\frac{1}{144}$ (ii) $\frac{216}{125}$

18. (i) 4^{-3} (ii) $\left(\frac{1}{3}\right)^{-5}$

20 (i) $\frac{-13}{108}$ (ii) $\frac{135}{8}$

21. $\frac{-5}{7}$ **22.** $(-56)^{-1}$ **23.** $(3)^{-1}$

24. $\frac{25}{21}$

25. (i) 3 (ii) 14

26. $\left(\frac{2}{3}\right)^{12}$

27. 1

28 (i) 6.02×10^{15} (ii) 9.42×10^{-12}

29 (i) 48300000 (ii) 0.00000302
(iii) 45000 (iv) 0.00000003

EXERCISE – II | SCHOOL EXAM/BOARD

Q.1 Simplify : $\frac{4}{3}+\frac{3}{5}+\frac{-2}{3}+-\frac{11}{5}$

Q.2 Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

Q.3 Represent these numbers on the number line:

(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

Q.4 Represent $\frac{-2}{11},\frac{-5}{11},\frac{-9}{11}$ on the number line.

Q.5 Write five rational numbers which are smaller than 2.

Q.6 Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Q.7 Find five rational numbers between

(i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$

Q.8 Write five rational numbers greater than –2.

Q.9 Simplify : $\frac{-3}{10}+\frac{7}{15}+\frac{3}{-20}-\frac{9}{10}+\frac{13}{15}+\frac{13}{-20}$

Q.10 The sum of two rational numbers is – 6 and one of them is $\frac{-7}{2}$. Find the other.

Q.11 Subtract the sum of the two numbers is $\frac{-8}{5}$ and $\frac{-5}{3}$ from the sum of $\frac{3}{2}$ and $\frac{-31}{28}$.

Q.12 Rimmi bought $4\frac{3}{4}$ litres milk and used $3\frac{7}{8}$ litres to prepare a sweet dish. How much milk is left?

Q.13 A train goes 80 km in one hour. How much distance will it cover in 45 minutes?

Q.14 A man has Rs. 100 with him. He bought $3\frac{1}{2}$ litres of milk at Rs. $16\frac{1}{2}$ per litre. How much money is left with him.

Q.15 Praneeta bought $3\frac{1}{2}$ m ribbon at Rs. $5\frac{3}{7}$ per metre, $4\frac{3}{4}$ m cloth at Rs. $27\frac{1}{2}$ per metre. How much money did she spend?

Q.16 By taking $x=\frac{-3}{4}$, $y=\frac{2}{3}$ and $z=\frac{-5}{6}$, verify that :

(i) $x \times (y + z) = x \times y + x \times z$

(ii) $x(y - z) = x \times y - x \times z$

Q.17 The product of two numbers is $-17\frac{1}{2}$. If one of them is $1\frac{1}{6}$, find the other.

Q.18 Divide the sum of $\frac{-3}{4}$ and $\frac{-5}{12}$ by their product.

Q.19 A shirt needs $2\frac{1}{4}$ m cloth. How many shirts can be made from $31\frac{1}{2}$ m cloth?

Q.20 The length of 21 skipping ropes is $36\frac{3}{4}$ m. Find the length of 1 rope.

ANSWER KEY

1. $-\frac{14}{15}$

2. Yes, because the product is 1

5.	$1, -\frac{1}{2}, 0, -1, -\frac{1}{2},$	**9.**	$-\frac{2}{3}$
10.	$-\frac{5}{2}$	**11.**	$3\frac{17}{84}$
12.	$\frac{7}{8}$ Litre	**13.**	60 km
14.	$42\frac{1}{4}$	**15.**	Rs. $149\frac{5}{8}$
17.	– 15	**18.**	$-\frac{56}{15}$
19.	14 shirts	**20.**	$\frac{7}{4}$

EXERCISE - III MULTIPLE CHOICE QUESTIONS

Q.1 If x = $0.1\overline{6}$, then 3x is -

(A) $0.4\overline{8}$ (B) $0.4\overline{9}$

(C) $0.\overline{5}$ (D) 0.5

Q.2 Find the value of x when $\left(\frac{3}{5}\right)^{2x-3} = \left(\frac{5}{3}\right)^{x-3}$

(A) x = 2 (B) x = – 2

(C) x = 1 (D) x = – 1

Q.3 If $2^x - 2^{x-1} = 4$, then x^x is equal to

(A) 1 (B) 27

(C) 3 (D) None of these

Q.4 The value of $\dfrac{(0.6)^0 - (0.1)^{-1}}{(3/2^3)^{-1}(3/2)^3 + \left(-\frac{1}{3}\right)^{-1}}$ is

(A) 3/2 (B) – 3/2

(C) 2/3 (D) –1/2

Q.5 What must be added to the sum of $4x^2 + 3x - 7$ and $3x^2 + 6x + 5$ to get : 1?

(A) $7x^2 + 9x - 3$ (B) $3 - 9x - 7x^2$

(C) $7x^2 + 9x - 2$ (D) None of these

Q.6 $1.\overline{3}$ is equal to –

(A) 3/4 (B) 2/3

(C) 4/3 (D) 2/5

Q.7 $0.\overline{585}$ is equal to –

(A) $\frac{585}{99}$ (B) $\frac{585}{999}$

(C) $\frac{999}{585}$ (D) none of these

Q.8 $5.\overline{2}$ is equal to –

(A) 45/9 (B) 46/9

(C) 47/9 (D) None of these

Q.9 Which of the following numbers is different from others ?

(A) $\sqrt{2}$ (B) $\sqrt{3}$

(C) $\sqrt{4}$ (D) $\sqrt{5}$

Q.10 Which of the following numbers is different from others ?

(A) $\sqrt{7}$ (B) $\sqrt{8}$

(C) $\sqrt{13}$ (D) $\sqrt{16}$

Q.11 If x = $0.\overline{7}$, then 2x is -

(A) $1.\overline{4}$ (B) $1.\overline{5}$

(C) $1.\overline{54}$ (D) $1.\overline{45}$

Q.12 Evaluate $\sqrt[3]{\left(\frac{1}{64}\right)^{-2}}$

(A) 4 (B) 16

(C) 32 (D) 64

Q.13 The value of $(256)^{0.16} \times (256)^{0.09}$ is -

(A) 64 (B) 256.25

(C) 16 (D) 4

Q.14 If a = $2 + \sqrt{3}$ and b = $2 - \sqrt{3}$, then $\frac{1}{a} + \frac{1}{b}$ is equal to -

(A) $2\sqrt{3}$ (B) $-2\sqrt{3}$

(C) 4 (D) – 4

Q.15 If a = $2 + \sqrt{3}$ and b = $2 - \sqrt{3}$, then $\frac{1}{a^2} - \frac{1}{b^2}$ is equal to

(A) 14 (B) – 14

(C) $8\sqrt{3}$ (D) $-8\sqrt{3}$

Q.16 If a = $\frac{2+\sqrt{3}}{2-\sqrt{3}}$, b = $\frac{2-\sqrt{3}}{2+\sqrt{3}}$, then the value of a + b is -

(A) 14 (B) – 14

(C) $8\sqrt{3}$ (D) $-\sqrt{3}$

Q.17 If x = $3 + \sqrt{8}$ and y = $3 - \sqrt{8}$, then $\frac{1}{x^2} + \frac{1}{y^2}$ is equal to -

(A) 34 (B) – 34

(C) $12\sqrt{8}$ (D) $-12\sqrt{8}$

Q.18 $\frac{2^{n+3}-2(2^n)}{2(2^{n+2})}$ when simplified is -

(A) $1 - 2(2^n)$ (B) $2^{n+3} - \frac{1}{4}$

(C) $1 - \frac{1}{4}$ (D) $1 - \frac{1}{2}$

Q.19 $\left(\frac{1}{64}\right)^0 + 64^{-1/2} - (-32)^{4/5}$ is equal to

(A) $-15\frac{7}{8}$ (B) $16\frac{1}{8}$

(C) $-14\frac{7}{8}$ (D) $17\frac{1}{8}$

Q.20 A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{\sqrt{2}-\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\sqrt{3}}{2}$

(C) 1.4 (D) 1.5

Q.21 If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and y = 1, the value of $\frac{x-y}{x-3y}$ is -

(A) $\frac{\sqrt{6}+4}{5}$ (B) $\frac{5}{\sqrt{6}-4}$

(C) $\frac{5}{\sqrt{6}+4}$ (D) $\frac{\sqrt{6}-4}{5}$

Q.22 If $A = x - \frac{1}{x}$, then the value of $\left(A+\frac{1}{A}\right)$ is

(A) $\frac{x^4-x^2+1}{x(x^2-1)}$ (B) $\frac{x^4+x^2+1}{x(x^2-1)}$

(C) $\frac{x^4+1}{x^3-x^2}$ (D) 1

Q.23 The value of $\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{4x}{1+x^2}$ is –

(A) $\frac{8x}{1+x^4}$ (B) $\frac{8x}{1-x^4}$

(C) $\frac{8}{1-x^4}$ (D) $\frac{-8}{1-x^4}$

Q.24 The expression to be added to $(5x^2 - 7x + 2)$ to produce $(7x^2 - 1)$ is –
(A) $2x^2 + 7x + 3$ (B) $2x^2 + 7x - 3$
(C) $12x^2 - 7x + 1$ (D) $2x^2 - 3$

Q.25 What must be added to $1 - x + x^2 - 2x^3$ to obtain x^3 ?
(A) $x^3 - x^2 + x - 1$
(B) $-1 + x + x^2 - 3x^3$
(C) $3x^3 - x^2 + x - 1$
(D) None of these

Q.26 The product of $4\sqrt{6}$ and $3\sqrt{24}$ is –
(A) 124 (B) 134
(C) 144 (D) 154

Q.27 If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is –
(A) 34 (B) 35
(C) 36 (D) 37

Q.28 If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^3 + b^3$ is –
(A) 194 (B) 196
(C) 198 (D) 200

Q.29 If $x = (7 + 4\sqrt{3})$, then the value of $x^2 + \frac{1}{x^2}$ is –
(A) 193 (B) 194
(C) 195 (D) 196

Q.30 If $\sqrt{5}$ = 2.236 and $\sqrt{10}$ = 3.162, the value of $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$ on simplifying is –

(A) 0.455 (B) 0.855
(C) 0.655 (D) 0.755

Q.31 The value of $5\sqrt{3}-3\sqrt{12}+2\sqrt{75}$ on simplifying is –

(A) $5\sqrt{3}$ (B) $6\sqrt{3}$
(C) $\sqrt{3}$ (D) $9\sqrt{3}$

Q.32 If $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ = a + b$\sqrt{3}$, then the value of a and b is–

(A) a = 2, b = –1
(B) a = 2, b = 1
(C) a = –2, b = 1
(D) a = –2, b = –1

Q.33 The rational form of $2.74\overline{35}$ is –

(A) $\frac{27161}{9999}$ (B) $\frac{27}{99}$
(C) $\frac{27161}{9900}$ (D) $\frac{27161}{9000}$

Q.34 The sum of a number and its reciprocal is 125/22. The number is –

(A) 2/11 (B) 1/11
(C) 3/11 (D) None of these

Q.35 What must be added to x/y to make it y/x ?

(A) $\frac{y-x}{y^2x^2}$ (B) $\frac{y^2-x^2}{xy}$
(C) $\frac{xy}{x+y}$ (D) $\frac{x^2y^2}{x^2+y^2}$

ANSWER KEY

1.	D	**2.**	A	**3.**	B	**4.**	B
5.	B	**6.**	C	**7.**	B	**8.**	C
9.	C	**10.**	D	**11.**	B	**12.**	B
13.	D	**14.**	C	**15.**	D	**16.**	A
17.	A	**18.**	C	**19.**	C	**20.**	D
21.	A	**22.**	A	**23.**	B	**24.**	B
25.	C	**26.**	C	**27.**	A	**28.**	C
29.	B	**30.**	C	**31.**	D	**32.**	A
33.	C	**34.**	A	**35.**	B		

EXERCISE - IV OLYMPIAD / NTSE QUESTIONS

CHOOSE THE CORRECT ONE

1. Which of the following statement is true?
(A) Every whole number is a natural number
(B) Every natural number is a whole number
(C) '1' is the lest whole number
(D) None of these

2. A student was asked to multiply a number by $\frac{3}{2}$. Instead he divided the number by $\frac{3}{2}$ and obtained a number smaller by $\frac{3}{2}$; the number is :

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$
(C) $\frac{4}{5}$ (D) $\frac{1}{2}$

3. The two missing numbers shown with asterisk in the equation $5\frac{3}{*} \times *\frac{1}{2}$ = 19 are :-
(A) 6, 3 (B) 7, 3
(C) 8, 3 (D) 11, 3

4. Which of the following statements is true?
(A) $\frac{-2}{3} < \frac{4}{-9} < \frac{-5}{12} < \frac{7}{-18}$
(B) $\frac{7}{-18} < \frac{-5}{12} < \frac{4}{-9} < \frac{-2}{3}$
(C) $\frac{4}{-9} < \frac{7}{-18} < \frac{-5}{12} < \frac{-2}{3}$
(D) $\frac{-2}{3} < \frac{-5}{12} < \frac{4}{-9} < \frac{7}{-18}$

5. Which of the following rational numbers lie between $\frac{-3}{7}$ and $\frac{-9}{8}$?
(A) $\frac{-1}{2}$ (B) 0
(C) $\frac{12}{15}$ (D) None of these

6. $0.\overline{018}$ can be expressed in the rational form as :
(A) $\frac{18}{1000}$ (B) $\frac{18}{990}$
(C) $\frac{18}{9900}$ (D) $\frac{18}{999}$

7. The value of $4 - \cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2\frac{1}{4}}}}$ is :
(A) $\frac{40}{31}$ (B) $\frac{4}{9}$
(C) $\frac{1}{8}$ (D) $\frac{31}{40}$

8. Choose the rational number which does not lie between rational numbers $-\frac{2}{5}$ and $-\frac{1}{5}$:
(A) $-\frac{1}{4}$ (B) $-\frac{3}{20}$
(C) $-\frac{3}{10}$ (D) $-\frac{7}{20}$

9. Which of the following has fractions in ascending order?
(A) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$
(B) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$
(C) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$
(D) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$

10. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?
(A) $\frac{1}{4}$ (B) $\frac{23}{24}$
(C) $\frac{11}{12}$ (D) $\frac{17}{24}$

11. $5 - \left[\frac{3}{4} + \left\{2\frac{1}{2} - \left(0.5 + \overline{\frac{1}{6} - \frac{1}{7}}\right)\right\}\right]$:-
(A) $2\frac{23}{84}$ (B) $3\frac{1}{6}$
(C) $3\frac{3}{10}$ (D) $5\frac{1}{10}$

12. If 2805 ÷ 2.55 = 1100, then 280.5 ÷ 25.5 =
(A) 1.1 (B) 1.01
(C) 0.11 (D) 11

13. Evaluate : $\frac{8 - [5 - (-3 + 2)] \div 2}{|5 - 3| - |5 - 8| \div 3}$
(A) 2 (B) 3
(C) 4 (D) 5

14. The value of $0.\overline{4}$ is :- **[NTSE-2008]**
(A) $\frac{4}{10}$ (B) $\frac{4}{9}$

(C) $\frac{4}{100}$ (D) $\frac{9}{4}$

15. If a and b are natural numbers such that $\left(\frac{1}{a}\right)^{\frac{1}{b}} = 0.\overline{3}$, then the value of ab is :

(A) 81 (B) 24
(C) 192 (D) 375

16. If $x < -2$, then $|1 - |1+x||$ equals :

[NTSE-2008]

(A) $2 + x$ (B) x
(C) $-x$ (D) $-(2 + x)$

17. The value of the expresion $\frac{0.777... \times 0.33... \times 0.222}{0.777... + 0.333... + 0.222...}$ is equal to :

(A) $\frac{7}{162}$ (B) $\frac{162}{7}$
(C) $\frac{163}{7}$ (D) $\frac{7}{163}$

18. The expression $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + ... + \frac{1}{n(n+1)}$ for any natural number n, is :

(A) Always greater than 1

(B) Always less than 1

(C) Always equal to 1

(D) Not definite

19. When simplified the product $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right)....\left(1 - \frac{1}{n}\right)$ equals :

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$
(C) $\frac{2(n-1)}{n}$ (D) $\frac{2}{n(n+1)}$

20. The value of $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \times \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \times \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$ is :

(A) x (B) $\frac{1}{x}$
(C) -1 (D) 1

21. The value of the expression $\frac{1}{1 + x^{a-b} + x^{a-c}} + \frac{1}{1 + x^{b-c} + x^{b-a}} + \frac{1}{1 + x^{c-a} + x^{c-b}}$ is equal to :

(A) 0 (B) x^{a+b+c}
(C) $\frac{1}{x^{a+b+c}}$ (D) 1

22. $\left(\frac{a^{-1}b^{-1}}{a^{-1} + b^{-1}} - \frac{a^{-1}b^{-1}}{a^{-1} - b^{-1}}\right)$ equal to :

(A) $\frac{2b}{b^2 - a^2}$ (B) $\frac{2b}{a^2 - b^2}$
(C) $\frac{2a}{b^2 - a^2}$ (D) $\frac{2a}{a^2 - b^2}$

23. The decimal representation of $\frac{27}{400}$ is :

(A) Terminating
(B) Non-terminating recurring
(C) Non-terminating non recuring
(D) None of these

24. $2.2\overline{34} =$

(A) $\frac{1101}{495}$ (B) $\frac{1103}{495}$
(C) $\frac{1106}{495}$ (D) $\frac{1105}{495}$

25. Which of the following number are rational ?

(A) $\sqrt{19}$ (B) $\sqrt{16}$
(C) $\sqrt{17}$ (D) $\sqrt{18}$

26. The rational form of $2.74\overline{35}$ is :

(A) $\frac{27161}{9999}$ (B) $\frac{27}{99}$
(C) $\frac{27161}{9900}$ (D) $\frac{27161}{9000}$

27. The value of $0.4\overline{23}$ is :

(A) $\frac{423}{1000}$ (B) $\frac{419}{1000}$
(C) $\frac{423}{9000}$ (D) $\frac{419}{990}$

28. If $x = 3 + \sqrt{8}$ and $y = 3 - \sqrt{8}$ then $\frac{1}{x^2} + \frac{1}{y^2} =$

(A) -34 (B) 34
(C) $12\sqrt{8}$ (D) $-12\sqrt{8}$

29. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3}}}$ is equal to –

(A) $\frac{1}{3}$ (B) $\frac{11}{7}$
(C) 3 (D) $1\frac{1}{3}$

30. $\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a \times x^b \times x^c)^4} = ?$

(A) -1 (B) 0
(C) 1 (D) None of these

RATIONAL NUMBERS & RADICALS

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	B	A	A	D	C	B	C	D	A	D	D	B	A
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	A	C	B	D	D	D	A	C	B	C	D	B	B	C