Adaptive Filtering for Audio Denoising

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***Abstract*—A least mean squares (LMS) adaptive finite impulse response (FIR) filter was chosen for the purpose of denoising an audio signal. Two adaptive schemes were implemented with one using pure LMS and the other using normalized LMS for finding the filter coefficients. Analyzing the frequency content present in the distorted audio signal lead to an intuition of what the correct magnitude response of the adaptive filter would be. Ultimately, the pure adaptive scheme was unable to converge to the LMS solution which lead to a comparison of the adaptation step sizes chosen in both schemes. The filters found from using both designs were then critiqued to see if they adhered to the properties guaranteed by FIR filters such as linear phase, stability, and having a finite impulse response.**

I. INTRODUCTION

Many times, hardware is designed with a specific purpose in mind. Consider a bandpass filter that may be used in the receiver of an air to air radar. The goal here is to only pass a certain region of frequencies so that the aircraft is not confused by returns from other transmitting devices that may be in the airspace. A fixed filter with a specific bandpass would work well here since the radar knows what it transmitted and what to expect in return. However, fixed filter design may not be useful in applications where the region to be filtered is unknown –for these types of problems, an adaptive filter may be more fitting. In this project, an adaptive FIR filter was chosen for the purpose of filtering distorted audio signals. The audio signal was a two second clip of a female saying the phrase, “cottage cheese with chives is delicious”. Access to the clean audio signal, along with various distorted versions were applied to a pure LMS and normalized LMS adaptive scheme. Below the spectrograms of the clean, along with some of the distorted clips, are shown.

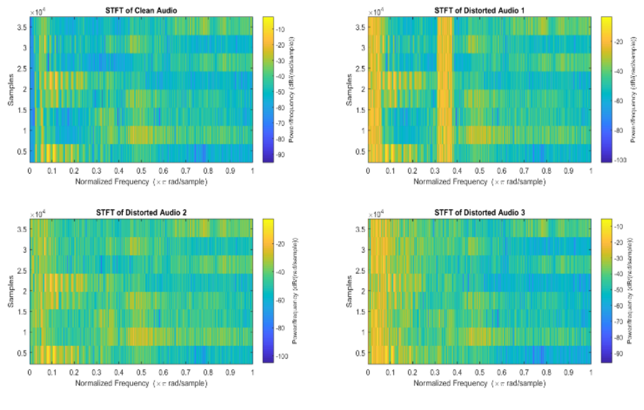

Figure 1: Short-time Fourier transforms (STFT) of the clean audio signal, and several distorted ones

II. METHODS

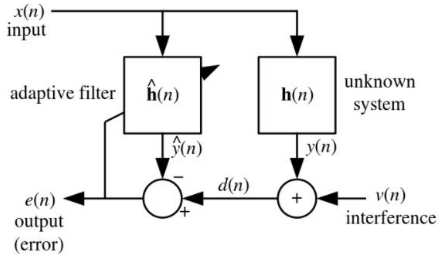

Figure 2: General scheme for an adaptive filter [1]

Figure 2 shows the schematic of a generic adaptive filter design. The signals that one has access to will therefore govern what application it is used for. Typically, when d(n) is equal to y(n), meaning the interference source is 0, it is known as filtering[2]. Here, y(n) was the clean audio signal while x(n) was the distorted one. The filter acted on each sample of x(n) whose output was compared with the clean signal at the same interval by taking the difference between the two –which produces e(n). At the next interval n, the goal is to reduce the error between the output of the filter and the desired signal and the only way to accomplish this is to change the filter coefficients in some way. One of the most common class of algorithms used for adaptive filtering is LMS –which gets its name from the fact that it minimizes the squared error and does so by using steepest gradient descent to find the new filter coefficients[3]. The update rule for finding the new filter coefficients is given by Equation 1.

$$w_{n+1} = w_n - \mu * x(n)e(n)$$

Equation 1

The algorithm requires a set of L weights, which was selected to be 16, where L is the number coefficients that defines the impulse response of the FIR filter. Since FIR filters can only act on current and previous samples of x(n), the input of the filter is defined by Equation 2 and the output is defined by Equation 3.

$$x(n) = [x(n)\, x(n-1) \ldots x(n-L+1)]^T$$

Equation 2

$$\hat{y}(n) = w(n)^T x(n)$$

Equation 3

Once the filtered value of the signal is obtained, the error with respect to the desired signal is computed.

$$e(n) = d(n) - w(n)^T x(n)$$

Equation 4

Now that nearly all the ingredients required for finding the updated weights have been identified, it is time to consider a value for mu –the adaptation step size. The proper selection of this term will ensure that the algorithm can reach the minimum of the convex error surface formed by the squared error criterion[3]. The proper value of mu should be bounded by what is defined in Equation 5.

$$0 < \mu < \frac{2}{\lambda_{max}}$$

$$where\ \lambda_{max} = \max(eig(x(n)x(n)^T))$$

Equation 5

However, the pure LMS algorithm may be sensitive to the scaling of the input x(n). For this reason, the value of mu may cause the filter to diverge regardless of how it is chosen. Since we have access to the desired signal with v(n) = 0, then we may select mu to have a value of 1[1]. In this case, x(n) will be scaled by the input power of x(n) to mitigate the sensitivity of the algorithm from the input. This approach is known as normalized least mean squares whose update rule is given by Equation 6.

$$w_{n+1} = w_n - \frac{x(n)e(n)}{x(n)x(n)^T}$$

Equation 6

Both methods were evaluated towards the application of filtering audio signals.

III. Results and Discussion

It was important to first visualize the frequency spectrum of both the clean and distorted audio signals. By taking the fast Fourier transform (FFT) of the audio clips, it is clearly seen that there are varying degrees of frequency content present between the two signals. Between the two images on the left, there is a large magnitude of frequency content centered around 250 and 2800 Hz. Furthermore, inspection of the spectrograms of the two audio signals reveals that this additional frequency content in the distorted audio is present throughout the entire clip. Spectrograms aid in the analysis of how the signal content may vary with time since it computes the FFT over segments of the signal which are then strung together. The spectrogram of the distorted audio signal reveals that the undesired content does not change with time, and therefore the entire signal could be filtered without any additional windowing applied.

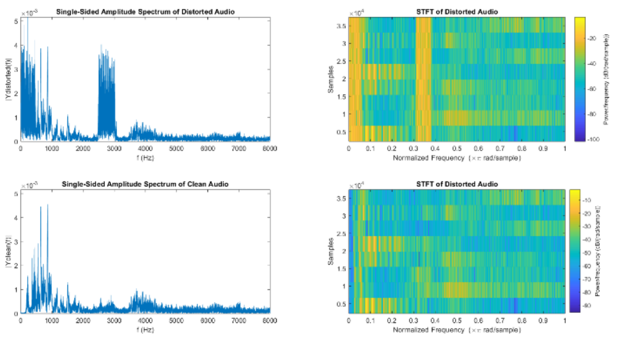

Figure 3: FFT and STFT of distorted (top) and clean (bottom) audio signals

A. *Pure LMS Adpative Scheme Results*

The first adaptive scheme used the update rule given by Equation 1, with the value of mu defined by Equation 5.

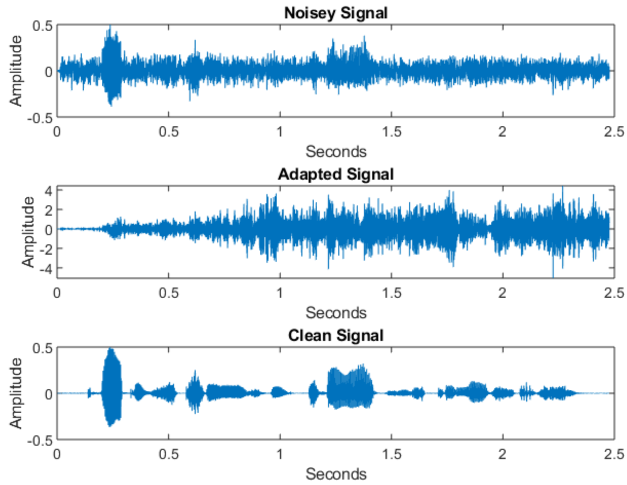

Figure 4: Time domain of noisy, adapted, and clean audio signals from using pure LMS

B. *Normalized LMS Adaptive Scheme Results*

The second adaptive scheme used the update rule given by Equation 6.

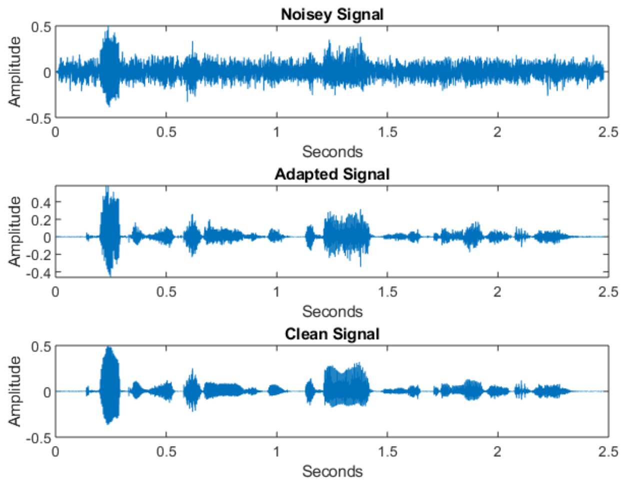

Figure 5: Time domain of noisy, adapted, and clean audio signals from using normalized LMS

C. *Comparison of Pure and Normalized LMS*

In order to compare the step sizes between the two adaptive schemes, a value for mu was extracted for the normalized LMS implementation as $\frac{1}{x(n)x(n)^T}$. As shown in Figure 6, using pure LMS resulted in much larger values for the step size.

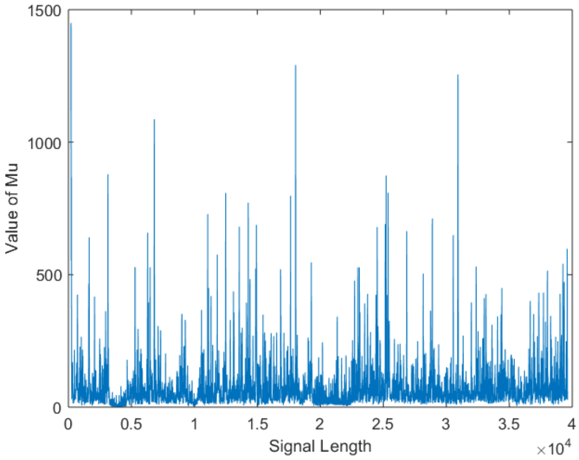

Figure 6: Values of mu used for each update with pure LMS

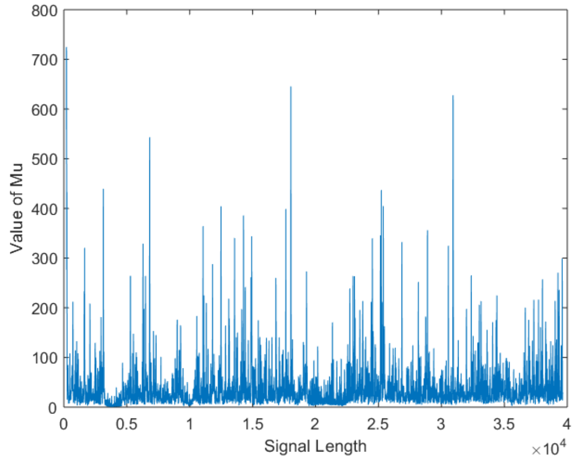

Figure 7: Values of mu used for each update with normalized LMS

Intuitively, by comparing the adapted signals from using the pure LMS versus normalized LMS schemes the step sizes played a key role in determining whether the filter would be able to converge to the minimum least squared error solution. The algorithms rely on mu to determine how far in the opposite direction of the gradient to move. The gradient, which in this case was the derivative of the mean squared error between our desired signal and the output from the filter at time n, points in the direction of the greatest increase of the squared error. An optimal mu would ensure that after the next iteration, the gradient is reduced. However, what resulted from using the pure LMS scheme reveals that mu continually overshot the local minimum of the objective function and did not move in the direction to reduce the gradient. In contrast, the normalized LMS scheme reached a solution quickly and supports the claim that a pure LMS implementation may be sensitive to the scaling of the input and unable to converge regardless of the mu chosen.

Now, the behavior of the FIR filter can be discussed in detail. In class, we learned that FIR filters possess the following features: guaranteed stability, linear phase behavior, and a finite impulse response. Stability is realized when a bounded input signal applied to the filter results in a bounded output –meaning that the output does not go towards infinity. In order to ensure this behavior all poles must lie within the unit circle and for this specific application, all poles were placed at the origin and the adaptive scheme was only used to choose the location of the zeros.

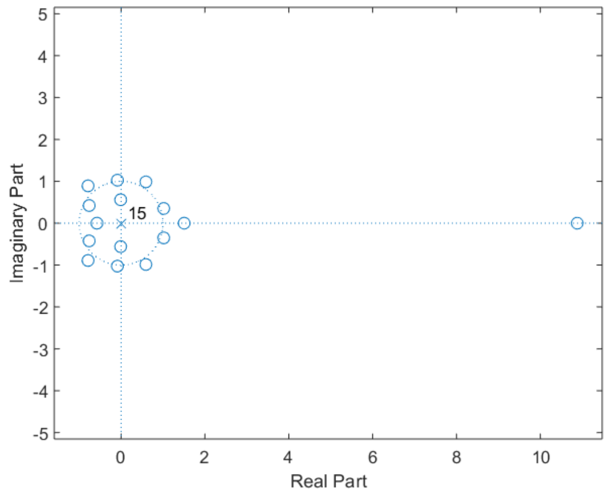

Figure 8: Pole-zero plot for filter found using pure LMS

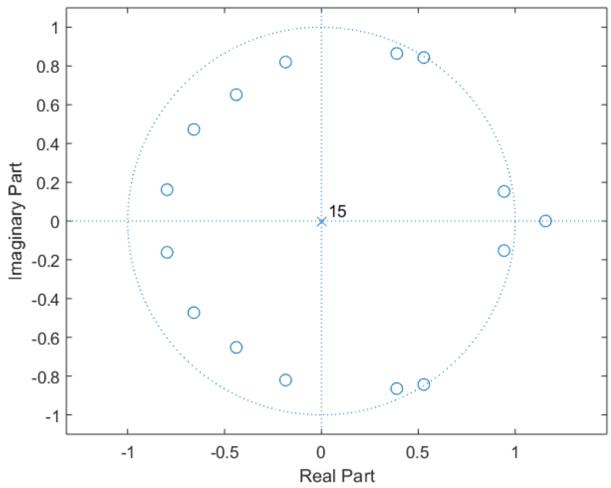

Figure 9: Pole-zero plot for filter found using normalized LMS

Through analyzing the pole-zero plots produced by the two schemes, it is confirmed that all poles lie within the unit circle and the system is stable. However, both systems contained zeros that lie outside of the unit circle which means that they are non-minimum phase systems. This non-minimum phase behavior may have resulted from feeding a sampled continuous time signal to the discrete model[4]. FIR filters guarantee that the system will be causal and stable, but with zeros outside of the unit circles the inverse $\frac{1}{H(z)}$ will be causal and unstable[5]. The pole-zero plot for the pure adaptive LMS scheme shows two zeros that lie outside of the unit circle, with the second falling almost ten times farther away on the real axis.

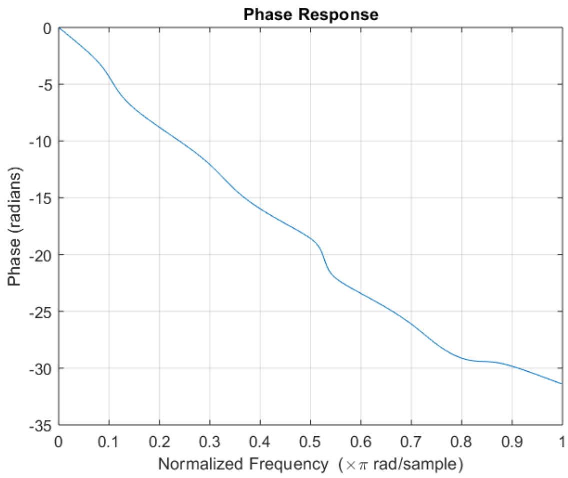

Figure 10: Phase response of filter found using pure LMS

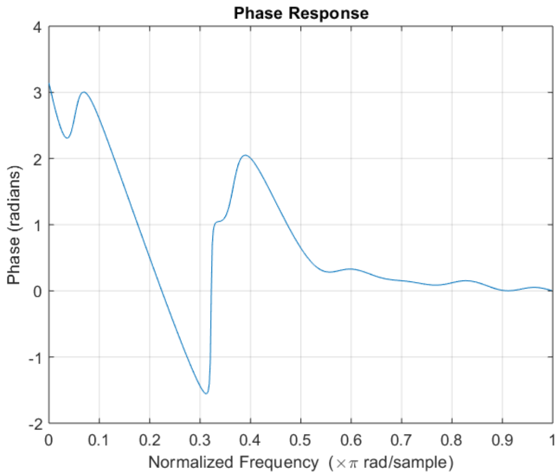

Figure 11: Phase response of filter found using normalized LMS

In Figures 10 and 11, the phase response of both filters using the different adaptive schemes are shown. Surprisingly, the phase response in Figure 10 seems to exhibit a linear phase response which means that the delay in the filter is applied equally across all frequencies. However, based on the pole-zero plot in Figure 8, this behavior was not expected. Similarly, inspection of Figure 11 reveals that the phase response of the FIR filter found from using normalized LMS resulted in non-linear phase behavior. There are different types of FIR filters, and only those whose filter coefficients are symmetric about the center coefficient are guaranteed to have linear phase behavior[6]. This leads to the inspection of the impulse responses of both filters.

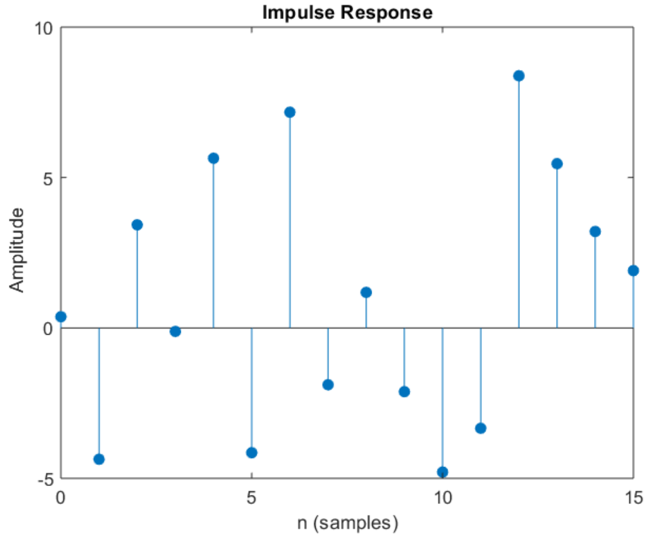

Figure 12: Coefficients of FIR filter found using pure LMS

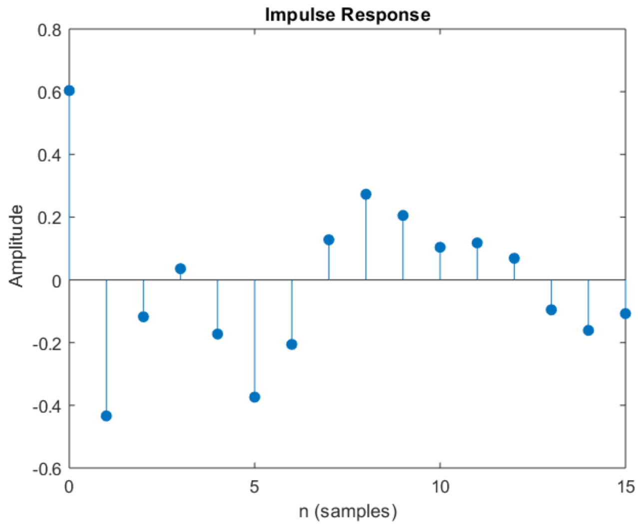

Figure 13: Coefficients of FIR filter found using normalized LMS

Both adaptive schemes resulted in asymmetric coefficients for the FIR filters. However, there is a little more symmetry in the coefficents shown in Figure 12 which may explain why the phase response had more linear behavior. Finally, since the impulse response of an FIR filter is just the set of filter coefficients then the final condition of a finite impulse response is met.

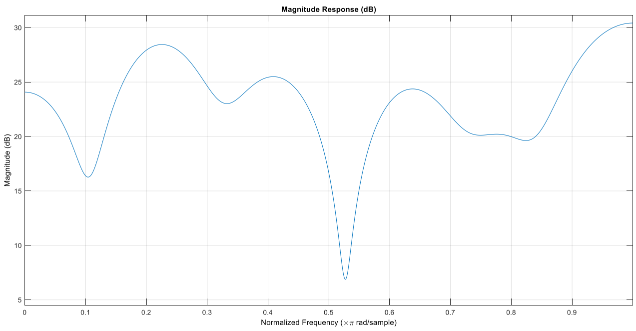

Figure 14: Magnitude response of filter found from pure LMS

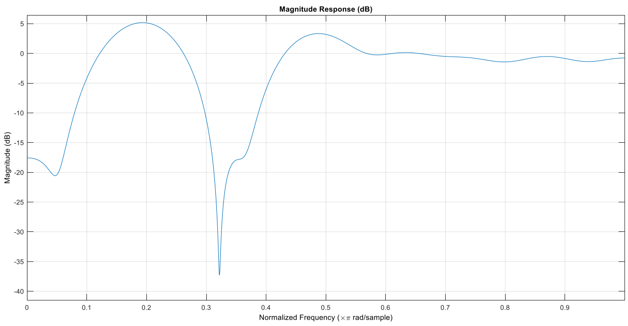

Figure 15: Magnitude response of filter found from normalized LMS

The magnitude responses of both FIR filters are given in Figures 14 and 15. Through analysis of the spectrogram of the distorted signal, one could expect what the proper magnitude response should be. Given that the undesired frequency content existed around 250 (0.03π) and 2800 (0.35π) Hz, the amplitude of the signal at those frequencies should be reduced. It is clearly seen in Figure 15 that the normalized LMS implementation found filter coefficients that focused on dampening the undesired spectral content at those frequencies and was successfully able to reduce it at 250 and 2800 Hz, by -18 and -37 dB, respectively.

IV. Conclusion

Implementing both adaptive schemes solidified the importance of selecting mu, the adaptation step size. When researching methods for this application, it was discovered that when using a pure adaptive scheme, it may be highly sensitive to the scaling of the input x(n) which could result in never converging to the minimum mean squared error solution regardless of the mu chosen. This was clearly demonstrated when seeing the adapted signal in Figure 4, which ended up with more distortion than it originally started with. In contrast, using a normalized LMS approach resulted with filter coefficients who produced a magnitude response that was expected based on simply seeing the FFT and STFT of the distorted audio signal. Finally, the properties that a FIR filter may guarantee when used in the design were examined and most were observed in the final design.

References

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