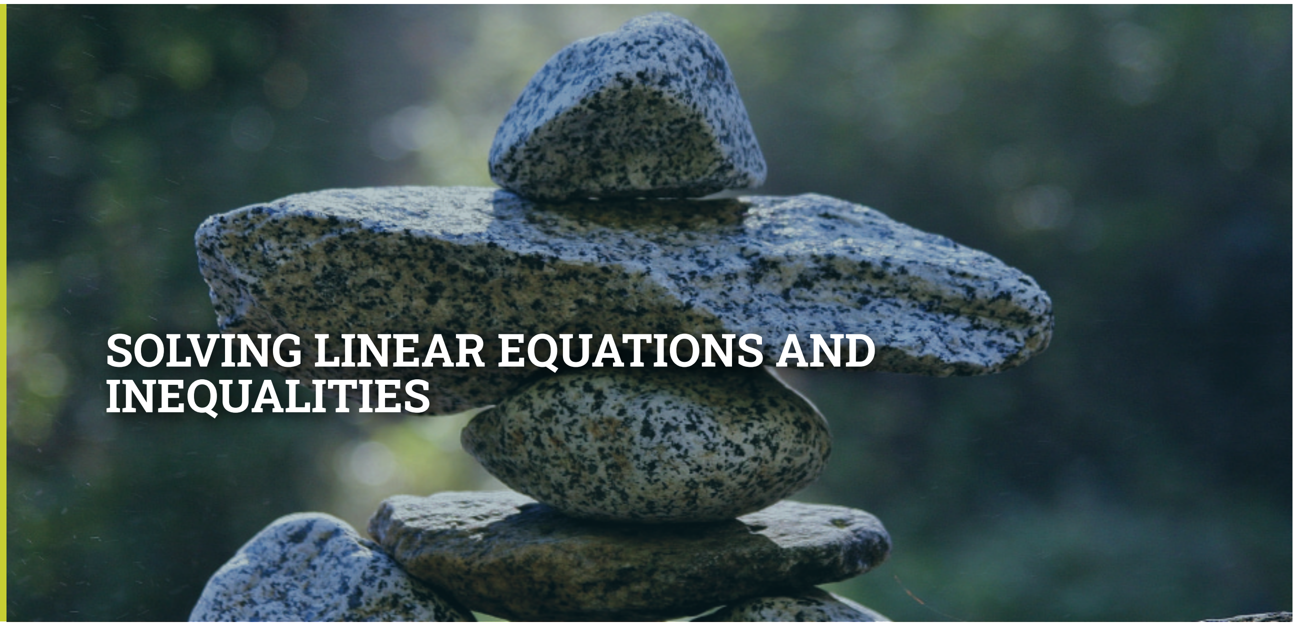


2 SOLVING LINEAR EQUATIONS AND INEQUALITIES

Figure 2.1 The rocks in this formation must remain perfectly balanced around the center for the formation to hold its shape.

Chapter Outline

Introduction

If we carefully placed more rocks of equal weight on both sides of this formation, it would still balance. Similarly, the expressions in an equation remain balanced when we add the same quantity to both sides of the equation. In this chapter, we will solve equations, remembering that what we do to one side of the equation, we must also do to the other side.

2.1 Solve Equations Using the Subtraction and Addition Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

BE PREPARED 2.1 Before you get started, take this readiness quiz.

Evaluate $x + 4$ when $x = -3$.
If you missed this problem, review Example 1.54.

BE PREPARED 2.2 Evaluate $15 - y$ when $y = -5$.
If you missed this problem, review Example 1.56.

BE PREPARED 2.3 Simplify $4(4n + 1) - 15n$.
If you missed this problem, review Example 1.138.

BE PREPARED 2.4 Translate into algebra "5 is less than x."
If you missed this problem, review Example 1.26.

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Solution of an equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

HOW TO

To determine whether a number is a solution to an equation.

Step 1. Substitute the number in for the variable in the equation.
Step 2. Simplify the expressions on both sides of the equation.
Step 3. Determine whether the resulting equation is true (the left side is equal to the right side)
- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

EXAMPLE 2.1

Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x - 2 = 2x + 1$
Substitute $\frac{3}{2}$ for x.	$4\left(\frac{3}{2}\right) - 2 \stackrel{?}{=} 2\left(\frac{3}{2}\right) + 1$
Multiply.	$6 - 2 \stackrel{?}{=} 3 + 1$
Subtract.	$4 = 4\checkmark$

Since $x = \frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x - 2 = 2x + 1$.

TRY IT 2.1 Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?

TRY IT 2.2 Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in Figure 2.2. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

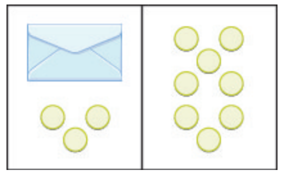


Figure 2.2 The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure 2.3 for an illustration of this process.

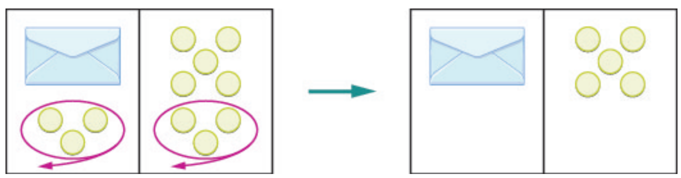


Figure 2.3 The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure 2.4 each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.

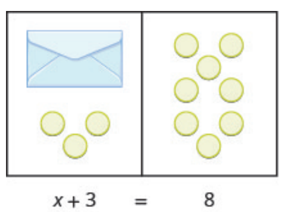


Figure 2.4 The illustration shows a model for the equation $x + 3 = 8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

$$5 + 3 = 8$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

Subtraction Property of Equality

For any numbers *a*, *b*, and *c*,

$$\begin{array}{lrcl} \text{If} & a & = & b, \\ \text{then} & a - c & = & b - c \end{array}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

MANIPULATIVE MATHEMATICS

Doing the Manipulative Mathematics activity “Subtraction Property of Equality” will help you develop a better understanding of how to solve equations by using the Subtraction Property of Equality.

Let’s see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

EXAMPLE 2.2

Solve: $y + 37 = -13$.

Solution

To get *y* by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

		$y + 37 = -13$
Subtract 37 from each side to ‘undo’ the addition.		$y + 37 - 37 = -13 - 37$
Simplify.		$y = -50$
Check:	$y + 37 = -13$	
Substitute $y = -50$	$-50 + 37 = -13$	
	$-13 \stackrel{?}{=} -13$ ✓	

Since $y = -50$ makes $y + 37 = -13$ a true statement, we have the solution to this equation.

TRY IT 2.3 Solve: $x + 19 = -27$.

TRY IT 2.4 Solve: $x + 16 = -34$.

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to ‘undo’ the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

Addition Property of Equality

For any numbers *a*, *b*, and *c*,

$$\begin{array}{lrcl} \text{If} & a & = & b, \\ \text{then} & a+c & = & b+c \end{array}$$

When you add the same quantity to both sides of an equation, you still have equality.

In Example 2.2, 37 was added to the *y* and so we subtracted 37 to 'undo' the addition. In Example 2.3, we will need to 'undo' subtraction by using the Addition Property of Equality.

EXAMPLE 2.3

Solve: $a - 28 = -37$.

Solution

		$a - 28 = -37$
Add 28 to each side to 'undo' the subtraction.		$a - 28 + 28 = -37 + 28$
Simplify.		$a = -9$
Check:	$a - 28 = -37$	
Substitute $a = -9$	$-9 - 28 = -37$	
	$-37 \stackrel{?}{=} -37$ ✓	
		The solution to $a - 28 = -37$ is $a = -9$.

TRY IT 2.5 Solve: $n - 61 = -75$.

TRY IT 2.6 Solve: $p - 41 = -73$.

EXAMPLE 2.4

Solve: $x - \frac{5}{8} = \frac{3}{4}$.

Solution

		$x - \frac{5}{8} = \frac{3}{4}$
Use the Addition Property of Equality.		$x - \frac{5}{8} + \frac{5}{8} = \frac{3}{4} + \frac{5}{8}$
Find the LCD to add the fractions on the right.		$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$
Simplify.		$x = \frac{11}{8}$
Check:	$x - \frac{5}{8} = \frac{3}{4}$	

Substitute $x = \frac{11}{8}$.	$\frac{11}{8} - \frac{5}{8} \stackrel{?}{=} \frac{3}{4}$	
Subtract.	$\frac{6}{8} \stackrel{?}{=} \frac{3}{4}$	
Simplify.	$\frac{3}{4} = \frac{3}{4}$ ✓	
		The solution to $x - \frac{5}{8} = \frac{3}{4}$ is $x = \frac{11}{8}$.

TRY IT 2.7 Solve: $p - \frac{2}{3} = \frac{5}{6}$.

TRY IT 2.8 Solve: $q - \frac{1}{2} = \frac{5}{6}$.

The next example will be an equation with decimals.

EXAMPLE 2.5

Solve: $n - 0.63 = -4.2$.

Solution

	$n - 0.63 = -4.2$
Use the Addition Property of Equality.	$n - 0.63 + 0.63 = -4.2 + 0.63$
Add.	$n = -3.57$

Check:	$n = -3.57$	
Let $n = -3.57$.	$-3.57 - 0.63 \stackrel{?}{=} -4.2$	
	$-4.2 = -4.2$ ✓	

TRY IT 2.9 Solve: $b - 0.47 = -2.1$.

TRY IT 2.10 Solve: $c - 0.93 = -4.6$.

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that simplification is different from the process used to solve an equation in which we apply an operation to both sides.

EXAMPLE 2.6

How to Solve Equations That Require Simplification

Solve: $9x - 5 - 8x - 6 = 7$.

Solution

Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x - 5 - 8x - 6 = 7$ $9x - 8x - 5 - 6 = 7$ $x - 11 = 7$
Step 2. Isolate the variable.	Now isolate *x*. Undo subtraction by adding 11 to both sides.	$x - 11 + 11 = 7 + 11$
Step 3. Simplify the expressions on both sides of the equation.		$x = 18$
Step 4. Check the solution.		**Check:** Substitute $x = 18$. $9x - 5 - 8x - 6 = 7$ $9(18) - 5 - 8(18) - 6 \stackrel{?}{=} 7$ $162 - 5 - 144 - 6 \stackrel{?}{=} 7$ $157 - 144 - 6 \stackrel{?}{=} 7$ $13 - 6 \stackrel{?}{=} 7$ $7 = 7\checkmark$ The solution to $9x - 5 - 8x - 6 = 7$ is $x = 18$.

TRY IT 2.11 Solve: $8y - 4 - 7y - 7 = 4$.

TRY IT 2.12 Solve: $6z + 5 - 5z - 4 = 3$.

EXAMPLE 2.7

Solve: $5(n - 4) - 4n = -8$.

Solution

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n - 4) - 4n = -8$
Distribute on the left.	$5n - 20 - 4n = -8$
Use the Commutative Property to rearrange terms.	$5n - 4n - 20 = -8$
Combine like terms.	$n - 20 = -8$

Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$
Check. Substitute $n=12$. $5(n-4)-4n=-8$ $5(12-4)-4(12)\overset{?}{=}-8z$ $5(8)-48\overset{?}{=}-8$ $40-48\overset{?}{=}-8$ $-8=-8$ ✓	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

TRY IT 2.13 Solve: $5(p-3)-4p=-10$.

TRY IT 2.14 Solve: $4(q+2)-3q=-8$.

EXAMPLE 2.8

Solve: $3(2y-1)-5y=2(y+1)-2(y+3)$.

Solution

We simplify both sides of the equation before we isolate the variable.

	$3(2y-1)-5y=2(y+1)-2(y+3)$
Distribute on both sides.	$6y-3-5y=2y+2-2y-6$
Use the Commutative Property of Addition.	$6y-5y-3=2y-2y+2-6$
Combine like terms.	$y-3=-4$
Each side is as simplified as possible. Next, isolate y.	
Undo subtraction by using the Addition Property of Equality.	$y-3+3=-4+3$
Add.	$y=-1$

Check. Let $y = -1$.

$$3(2y - 1) - 5y = 2(y + 1) - 2(y + 3)$$

$$3(2(-1) - 1) - 5(-1) \stackrel{?}{=} 2(-1 + 1) - 2(-1 + 3)$$

$$3(-2 - 1) + 5 \stackrel{?}{=} 2(0) - 2(2)$$

$$3(-3) + 5 \stackrel{?}{=} -4$$

$$-9 + 5 \stackrel{?}{=} -4$$

$$-4 = -4 \checkmark$$

	The solution to $3(2y - 1) - 5y = 2(y + 1) - 2(y + 3)$ is $y = -1$.

TRY IT 2.15 Solve: $4(2h - 3) - 7h = 6(h - 2) - 6(h - 1)$.

TRY IT 2.16 Solve: $2(5x + 2) - 9x = 3(x - 2) - 3(x - 4)$.

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the equals sign. Table 2.1 shows us some of the words that are commonly used.

Equals =
is is equal to is the same as the result is gives was will be

Table 2.1

The steps we use to translate a sentence into an equation are listed below.

HOW TO

Translate an English sentence to an algebraic equation.

Step 1. Locate the “equals” word(s). Translate to an equals sign (=).
Step 2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
Step 3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

EXAMPLE 2.9

Translate and solve: Eleven more than *x* is equal to 54.

Solution

Translate.	$\underbrace{\text{Eleven more than } x}_{x+11}$	$\underbrace{\text{is equal to}}_{=}$	$\underbrace{54}_{54}$
Subtract 11 from both sides.	$x + 11 - 11$	$=$	$54 - 11$
Simplify.	x	$=$	43
Check: Is 54 eleven more than 43? $43 + 11 \stackrel{?}{=} 54$ $54 = 54\checkmark$			

> **TRY IT** 2.17 Translate and solve: Ten more than x is equal to 41.

> **TRY IT** 2.18 Translate and solve: Twelve less than x is equal to 51.

EXAMPLE 2.10

Translate and solve: The difference of $12t$ and $11t$ is -14.

Solution

Translate.	$\underbrace{\text{The difference of } 12t \text{ and } 11t}_{12t - 11t}\ \underbrace{\text{is}}_{=}\ \underbrace{-14}_{-14}$
Simplify.	$t = -14$
Check: $12(-14) - 11(-14) \stackrel{?}{=} -14$ $-168 + 154 \stackrel{?}{=} -14$ $-14 = -14\checkmark$	

> **TRY IT** 2.19 Translate and solve: The difference of $4x$ and $3x$ is 14.

> **TRY IT** 2.20 Translate and solve: The difference of $7a$ and $6a$ is -8.

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

EXAMPLE 2.11

How to Solve Translate and Solve Applications

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 1st month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let $w =$ weight of the newspapers the 1st month
Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable w.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \stackrel{?}{=} 57$ $57 = 57 \checkmark$
Step 7. Answer the question with a complete sentence.	Write a sentence to answer "How much did the newspapers weigh the 1st month?"	The 1st month the newspapers weighed 29 pounds.

TRY IT 2.21 Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

TRY IT 2.22 Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

HOW TO

Solve an application.

Step 1. **Read** the problem. Make sure all the words and ideas are understood.
Step 2. **Identify** what we are looking for.
Step 3. **Name** what we are looking for. Choose a variable to represent that quantity.
Step 4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with the important information.
Step 5. **Solve** the equation using good algebra techniques.
Step 6. **Check** the answer in the problem and make sure it makes sense.
Step 7. **Answer** the question with a complete sentence.

EXAMPLE 2.12

Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	"What was the sticker price of the car?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let $s =$ the sticker price of the car.
Step 4. Translate into an equation. Restate the problem in one sentence.	$28,675 is $875 less than the sticker price
Step 5. Solve the equation.	$\text{\$28,675 is \$875 less than } s$ $\begin{aligned} 28{,}675 &= s - 875 \\ 28{,}675 + 875 &= s - 875 + 875 \\ 29{,}550 &= s \end{aligned}$
Step 6. Check the answer. Is $875 less than $29,550 equal to $28,675? $\begin{aligned} 29{,}550 - 875 &\stackrel{?}{=} 28{,}675 \\ 28{,}675 &= 28{,}675 ✓ \end{aligned}$	
Step 7. Answer the question with a complete sentence.	The sticker price of the car was $29,550.

Table 2.2

TRY IT 2.23 Translate into an algebraic equation and solve:

Eddie paid $19,875 for his new car. This was $1,025 less than the sticker price. What was the sticker price of the car?

TRY IT 2.24 Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

SECTION 2.1 EXERCISES

Practice Makes Perfect

Verify a Solution of an Equation

In the following exercises, determine whether the given value is a solution to the equation.

1. Is $y = \frac{5}{3}$ a solution of $6y + 10 = 12y$?

2. Is $x = \frac{9}{4}$ a solution of $4x + 9 = 8x$?

3. Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?

4. Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction and Addition Properties of Equality.

5. $x + 24 = 35$

6. $x + 17 = 22$

7. $y + 45 = -66$

8. $y + 39 = -83$

9. $b + \frac{1}{4} = \frac{3}{4}$

10. $a + \frac{2}{5} = \frac{4}{5}$

11. $p + 2.4 = -9.3$

12. $m + 7.9 = 11.6$

13. $a - 45 = 76$

14. $a - 30 = 57$

15. $m - 18 = -200$

16. $m - 12 = -12$

17. $x - \frac{1}{3} = 2$

18. $x - \frac{1}{5} = 4$

19. $y - 3.8 = 10$

20. $y - 7.2 = 5$

21. $x - 165 = -420$

22. $z - 101 = -314$

23. $z + 0.52 = -8.5$

24. $x + 0.93 = -4.1$

25. $q + \frac{3}{4} = \frac{1}{2}$

26. $p + \frac{1}{3} = \frac{5}{6}$

27. $p - \frac{2}{5} = \frac{2}{3}$

28. $y - \frac{3}{4} = \frac{3}{5}$

Solve Equations that Require Simplification

In the following exercises, solve each equation.

29. $c + 31 - 10 = 46$

30. $m + 16 - 28 = 5$

31. $9x + 5 - 8x + 14 = 20$

32. $6x + 8 - 5x + 16 = 32$

33. $-6x - 11 + 7x - 5 = -16$

34. $-8n - 17 + 9n - 4 = -41$

35. $5(y - 6) - 4y = -6$

36. $9(y - 2) - 8y = -16$

37. $8(u + 1.5) - 7u = 4.9$

38. $5(w + 2.2) - 4w = 9.3$

39. $6a - 5(a - 2) + 9 = -11$

40. $8c - 7(c - 3) + 4 = -16$

41. $6(y - 2) - 5y = 4(y + 3) - 4(y - 1)$

42. $9(x - 1) - 8x = -3(x + 5) + 3(x - 5)$

43. $3(5n - 1) - 14n + 9 = 10(n - 4) - 6n - 4(n + 1)$

44. $2(8m + 3) - 15m - 4 = 9(m + 6) - 2(m - 1) - 7m$

45. $-(j + 2) + 2j - 1 = 5$

46. $-(k + 7) + 2k + 8 = 7$

47. $-\left(\frac{1}{4}a - \frac{3}{4}\right) + \frac{5}{4}a = -2$

48. $-\left(\frac{2}{3}d - \frac{1}{3}\right) + \frac{5}{3}d = -4$

49. $8(4x + 5) - 5(6x) - x = 53 - 6(x + 1) + 3(2x + 2)$

50. $6(9y - 1) - 10(5y) - 3y = 22 - 4(2y - 12) + 8(y - 6)$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve it.

51. Nine more than x is equal to 52.

52. The sum of x and -15 is 23.

53. Ten less than m is -14.

54. Three less than y is -19.

55. The sum of y and -30 is 40.

56. Twelve more than p is equal to 67.

57. The difference of $9x$ and $8x$ is 107.

58. The difference of $5c$ and $4c$ is 602.

59. The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

60. The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

61. The sum of $-4n$ and $5n$ is -82.

62. The sum of $-9m$ and $10m$ is -95.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

63. **Distance** Avril rode her bike a total of 18 miles, from home to the library and then to the beach. The distance from Avril's house to the library is 7 miles. What is the distance from the library to the beach?

64. **Reading** Jeff read a total of 54 pages in his History and Sociology textbooks. He read 41 pages in his History textbook. How many pages did he read in his Sociology textbook?

65. **Age** Eva's daughter is 15 years younger than her son. Eva's son is 22 years old. How old is her daughter?

66. **Age** Pablo's father is 3 years older than his mother. Pablo's mother is 42 years old. How old is his father?

67. **Groceries** For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

68. **Weight** Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

69. **Health** Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

70. **Health** The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?

71. **Salary** Ron's paycheck this week was $17.43 less than his paycheck last week. His paycheck this week was $103.76. How much was Ron's paycheck last week?

72. **Textbooks** Melissa's math book cost $22.85 less than her art book cost. Her math book cost $93.75. How much did her art book cost?

Everyday Math

73. **Construction** Miguel wants to drill a hole for a $\frac{5}{8}$ inch screw. The hole should be $\frac{1}{12}$ inch smaller than the screw. Let d equal the size of the hole he should drill. Solve the equation $d + \frac{1}{12} = \frac{5}{8}$ to see what size the hole should be.

74. **Baking** Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{3}{8}$ cup of sugar and will borrow the rest from her neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{3}{8} + s = \frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Writing Exercises

75. Is -8 a solution to the equation $3x = 16 - 5x$? How do you know?

76. What is the first step in your solution to the equation $10x + 2 = 4x + 26$?

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
verify a solution of an equation.			
solve equations using the subtraction and addition properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ *If most of your checks were:*

...confidently. *Congratulations! You have achieved your goals in this section! Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific!*

...with some help. *This must be addressed quickly as topics you do not master become potholes in your road to success. Math is sequential - every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Whom can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?*

...no - I don't get it! *This is critical and you must not ignore it. You need to get help immediately or you will quickly be overwhelmed. See your instructor as soon as possible to discuss your situation. Together you can come up with a plan to get you the help you need.*

2.2 Solve Equations using the Division and Multiplication Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

BE PREPARED 2.5 Before you get started, take this readiness quiz.

Simplify: $-7\left(\frac{1}{-7}\right)$.

If you missed this problem, review Example 1.68.

BE PREPARED 2.6 Evaluate $9x + 2$ when $x = -3$.

If you missed this problem, review Example 1.57.

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in Figure 2.5.

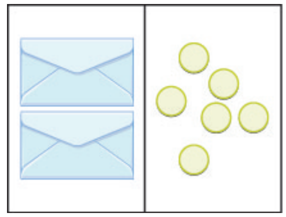


Figure 2.5 The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are “hidden” in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in Figure 2.6? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters.

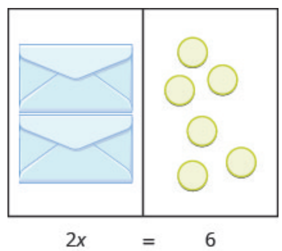


Figure 2.6 The illustration shows a model of the equation $2x = 6$.

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the **Division Property of Equality**.

The Division Property of Equality

For any numbers *a*, *b*, and *c*, and $c \neq 0$,

$$\begin{array}{lccc} \text{If} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c} \end{array}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

MANIPULATIVE MATHEMATICS

Doing the Manipulative Mathematics activity “Division Property of Equality” will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

EXAMPLE 2.13

Solve: $5x = -27$.

Solution

To isolate x, "undo" the multiplication by 5.		$5x = -27$
Divide to 'undo' the multiplication.		$\frac{5x}{5} = -\frac{27}{5}$
Simplify.		$x = -\frac{27}{5}$
Check:	$5x = -27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right) \stackrel{?}{=} -27$	
	$-27 = -27$ ✓	
		Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

> **TRY IT** 2.25 Solve: $3y = -41$.

> **TRY IT** 2.26 Solve: $4z = -55$.

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

The Multiplication Property of Equality

For any numbers *a*, *b*, and *c*,

$$\begin{array}{lrcl} \text{If} & a & = & b, \\ \text{then} & ac & = & bc \end{array}$$

If you multiply both sides of an equation by the same number, you still have equality.

EXAMPLE 2.14

Solve: $\frac{y}{-7} = -14$.

Solution

Here y is divided by -7. We must multiply by -7 to isolate y.

		$\frac{y}{-7} = -14$
Multiply both sides by -7.		$-7\left(\frac{y}{-7}\right) = -7(-14)$
Multiply.		$\frac{-7y}{7} = 98$
Simplify.		$y = 98$
Check: $\frac{y}{-7} = -14$		
Substitute $y = 98$.	$\frac{98}{-7} \stackrel{?}{=} -14$	
Divide.	$-14 = -14\checkmark$	

TRY IT 2.27 Solve: $\frac{a}{-7} = -42$.

TRY IT 2.28 Solve: $\frac{b}{-6} = -24$.

EXAMPLE 2.15

Solve: $-n = 9$.

Solution

		$-n = 9$
Remember $-n$ is equivalent to $-1n$.		$-1n = 9$
Divide both sides by -1.		$\frac{-1n}{-1} = \frac{9}{-1}$
Divide.		$n = -9$
Notice that there are two other ways to solve $-n = 9$. We can also solve this equation by multiplying both sides by -1 and also by taking the opposite of both sides.		
Check:	$-n = 9$	
Substitute $n = -9$.	$-(-9) \stackrel{?}{=} 9$	
Simplify.	$9 = 9\checkmark$	

TRY IT 2.29 Solve: $-k = 8$.

TRY IT 2.30 Solve: $-g = 3$.

EXAMPLE 2.16

Solve: $\frac{3}{4}x = 12$.

Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

		$\frac{3}{4}x = 12$
Multiply by the reciprocal of $\frac{3}{4}$.		$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$
Reciprocals multiply to 1.		$1x = \frac{4}{3} \cdot \frac{12}{1}$
Multiply.		$x = 16$
Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.		
Check:	$\frac{3}{4}x = 12$	
Substitute $x = 16$.	$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$	
	$12 = 12 ✓$	

> **TRY IT** 2.31 Solve: $\frac{2}{5}n = 14$.

> **TRY IT** 2.32 Solve: $\frac{5}{6}y = 15$.

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

EXAMPLE 2.17

Solve: $\frac{8}{15} = -\frac{4}{5}x$.

Solution

	$\frac{8}{15} = -\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.	$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right) = \left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.	$-\frac{\cancel{5} \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 3 \cdot \cancel{5}} = 1x$
Multiply.	$-\frac{2}{3} = x$

Check:	$\frac{8}{15}=-\frac{4}{5}x$	
Let $x=-\frac{2}{3}$.	$\frac{8}{15}=-\frac{4}{5}\left(-\frac{2}{3}\right)$	
	$\frac{8}{15}=\frac{8}{15}$ ✓	

TRY IT 2.33 Solve: $\frac{9}{25}=-\frac{4}{5}z$.

TRY IT 2.34 Solve: $\frac{5}{6}=-\frac{8}{3}r$.

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

EXAMPLE 2.18

Solve: $14-23=12y-4y-5y$.

Solution

Begin by simplifying each side of the equation.

		$14-23=12y-4y-5y$
Simplify each side.		$-9=3y$
Divide both sides by 3.		$-3=y$
Check:	$14-23=12y-4y-5y$	
Substitute $y=-3$.	$14-23=12(-3)-4(-3)-5(-3)$	
	$-9=-36+12+15$	
	$-9=-9$ ✓	

TRY IT 2.35 Solve: $18-27=15c-9c-3c$.

TRY IT 2.36 Solve: $18-22=12x-x-4x$.

EXAMPLE 2.19

Solve: $-4(a-3)-7=25$.

Solution

Here we will simplify each side of the equation by using the distributive property first.

		$-4(a-3)-7=25$
Distribute.		$-4a+12-7=25$
Simplify.		$-4a+5=25$
Simplify.		$-4a=20$
Divide both sides by -4 to isolate a.		$\frac{-4a}{-4}=\frac{20}{-4}$
Divide.		$a=-5$
Check:	$-4(a-3)-7=25$	
Substitute $a=-5$.	$-4(-5-3)-7\stackrel{?}{=}25$	
	$-4(-8)-7\stackrel{?}{=}25$	
	$32-7\stackrel{?}{=}25$	
	$25=25\checkmark$	

TRY IT 2.37 Solve: $-4(q-2)-8=24$.

TRY IT 2.38 Solve: $-6(r-2)-12=30$.

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

Properties of Equality

Subtraction Property of Equality

For any real numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a-c & = & b-c. \end{array}$$

Addition Property of Equality

For any real numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & a+c & = & b+c. \end{array}$$

Division Property of Equality

For any numbers a, b, and c, and $c \neq 0$,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c}. \end{array}$$

Multiplication Property of Equality

For any numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & = & b, \\ \text{then} & ac & = & bc. \end{array}$$

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

EXAMPLE 2.20

Translate and solve: The number 143 is the product of -11 and y.

Solution

Begin by translating the sentence into an equation.

Translate.	$\underbrace{\text{The number 143}}_{143}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the product of }-11\text{ and }y.}_{-11y}$
Divide by -11.	$\frac{143}{-11}=\frac{-11y}{-11}$
Simplify.	$-13=y$
Check: $\begin{aligned}143&=-11y\\143&\stackrel{?}{=}-11(-13)\\143&=143\checkmark\end{aligned}$	

TRY IT 2.39 Translate and solve: The number 132 is the product of -12 and y.

TRY IT 2.40 Translate and solve: The number 117 is the product of -13 and z.

EXAMPLE 2.21

Translate and solve: n divided by 8 is -32.

Solution

Begin by translating the sentence into an equation. Translate.		$\underbrace{n\text{ divided by 8}}_{\frac{n}{8}}\ \underbrace{\text{is}}_{=}\ \underbrace{-32.}_{-32}$
Multiple both sides by 8.		$8\cdot\frac{n}{8}=8(-32)$
Simplify.		$n=-256$
Check:	Is n divided by 8 equal to -32?	
Let $n=-256$.	Is -256 divided by 8 equal to -32?	
Translate.	$\frac{-256}{8}\stackrel{?}{=}-32$	
Simplify.	$-32=-32\checkmark$	

TRY IT 2.41 Translate and solve: n divided by 7 is equal to -21.

TRY IT 2.42 Translate and solve: n divided by 8 is equal to -56.

EXAMPLE 2.22

Translate and solve: The quotient of y and -4 is 68.

Solution

Begin by translating the sentence into an equation.

Translate.		The quotient of y and -4 is 68. $\underbrace{\frac{y}{-4}}\ \underbrace{=}\ \underbrace{68}$
Multiply both sides by -4.		$-4\left(\frac{y}{-4}\right)=-4(68)$
Simplify.		$y=-272$
Check:	Is the quotient of y and -4 equal to 68?	
Let $y=-272$.	Is the quotient of -272 and -4 equal to 68?	
Translate.	$\frac{-272}{-4}\stackrel{?}{=}68$	
Simplify.	$68=68✓$	

TRY IT 2.43 Translate and solve: The quotient of q and -8 is 72.

TRY IT 2.44 Translate and solve: The quotient of p and -9 is 81.

EXAMPLE 2.23

Translate and solve: Three-fourths of p is 18.

Solution

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.		Three-fourths of p is 18. $\underbrace{\frac{3}{4}p}\ \underbrace{=}\ \underbrace{18}$
Multiply both sides by $\frac{4}{3}$.		$\frac{4}{3}\cdot\frac{3}{4}p=\frac{4}{3}\cdot 18$
Simplify.		$p=24$
Check:	Is three-fourths of p equal to 18?	
Let $p=24$.	Is three-fourths of 24 equal to 18?	

Translate.	$\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$	
Simplify.	$18 = 18✓$	

> TRY IT 2.45 Translate and solve: Two-fifths of f is 16.

> TRY IT 2.46 Translate and solve: Three-fourths of f is 21.

EXAMPLE 2.24

Translate and solve: The sum of three-eighths and x is one-half.

Solution

Begin by translating the sentence into an equation.

Translate.	$\underbrace{\text{The sum of three – eighths and } x}_{\frac{3}{8} + x} \underbrace{\text{is}}_{=} \underbrace{\frac{1}{2}}_{\frac{1}{2}}$
Subtract $\frac{3}{8}$ from each side.	$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.	$x = \frac{4}{8} - \frac{3}{8}$
Simplify.	$x = \frac{1}{8}$

Check:	Is the sum of three-eighths and x equal to one-half?	
Let $x = \frac{1}{8}$.	Is the sum of three-eighths and one-eighth equal to one-half?	
Translate.	$\frac{3}{8} + \frac{1}{8} \stackrel{?}{=} \frac{1}{2}$	
Simplify.	$\frac{4}{8} \stackrel{?}{=} \frac{1}{2}$	
Simplify.	$\frac{1}{2} = \frac{1}{2}✓$	

> TRY IT 2.47 Translate and solve: The sum of five-eighths and x is one-fourth.

> TRY IT 2.48 Translate and solve: The sum of three-fourths and x is five-sixths.

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

EXAMPLE 2.25

Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Solution

What are you asked to find?	The cost of 1 pound of grapes
Assign a variable.	Let c = the cost of one pound.
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74.
Translate into an equation.	$6c = 10.74$
Solve.	$\frac{6c}{6} = \frac{10.74}{6}$ $c = 1.79$
	The grapes cost $1.79 per pound.
Check: If one pound costs $1.79, do 6 pounds cost #10.74? $6(1.79) \stackrel{?}{=} 10.74$ $10.74 = 10.74\checkmark$	

Table 2.3

TRY IT 2.49 Translate and solve:

Arianna bought a 24-pack of water bottles for $9.36. What was the cost of one water bottle?

TRY IT 2.50 Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $34.98. What is the cost for each person?

EXAMPLE 2.26

Andreas bought a used car for $12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Solution

What are you asked to find?	The original price of the car
Assign a variable.	Let p = the original price.
Write a sentence that gives the information to find it.	$12,000 is $\frac{3}{4}$ of the original price.
Translate into an equation.	$12{,}000 = \frac{3}{4}p$

Table 2.4

Solve.	$\frac{4}{3}(12{,}000) = \frac{4}{3} \cdot \frac{3}{4}p$ $16{,}000 = p$
	The original cost of the car was $16,000.
Check: Is $\frac{3}{4}$ of $16,000 equal to $12,000? $\frac{3}{4} \cdot 16{,}000 \stackrel{?}{=} 12{,}000$ $12{,}000 = 12{,}000 \checkmark$	

Table 2.4

> **TRY IT** 2.51 Translate and solve:
>
> The annual property tax on the Mehta's house is $1,800, calculated as $\frac{15}{1{,}000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

> **TRY IT** 2.52 Translate and solve:
>
> Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

SECTION 2.2 EXERCISES

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division and Multiplication Properties of Equality and check the solution.

77. $8x = 56$

78. $7p = 63$

79. $-5c = 55$

80. $-9x = -27$

81. $-809 = 15y$

82. $-731 = 19y$

83. $-37p = -541$

84. $-19m = -586$

85. $0.25z = 3.25$

86. $0.75a = 11.25$

87. $-13x = 0$

88. $24x = 0$

89. $\frac{x}{4} = 35$

90. $\frac{z}{2} = 54$

91. $-20 = \frac{q}{-5}$

92. $\frac{c}{-3} = -12$

93. $\frac{y}{9} = -16$

94. $\frac{q}{6} = -38$

95. $\frac{m}{-12} = 45$

96. $-24 = \frac{p}{-20}$

97. $-y = 6$

98. $-u = 15$

99. $-v = -72$

100. $-x = -39$

101. $\frac{2}{3}y = 48$

102. $\frac{3}{5}r = 75$

103. $-\frac{5}{8}w = 40$

104. $24 = -\frac{3}{4}x$

105. $-\frac{2}{5} = \frac{1}{10}a$

106. $-\frac{1}{3}q = -\frac{5}{6}$

107. $-\frac{7}{10}x = -\frac{14}{3}$

108. $\frac{3}{8}y = -\frac{1}{4}$

109. $\frac{7}{12} = -\frac{3}{4}p$

110. $\frac{11}{18} = -\frac{5}{6}q$

111. $-\frac{5}{18} = -\frac{10}{9}u$

112. $-\frac{7}{20} = -\frac{7}{4}v$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

113. $100 - 16 = 4p - 10p - p$

114. $-18 - 7 = 5t - 9t - 6t$

115. $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

116. $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$

117. $0.25d + 0.10d = 6 - 0.75$

118. $0.05p - 0.01p = 2 + 0.24$

119. $-10(q - 4) - 57 = 93$

120. $-12(d - 5) - 29 = 43$

121. $-10(x + 4) - 19 = 85$

122. $-15(z + 9) - 11 = 75$

Mixed Practice

In the following exercises, solve each equation.

123. $\frac{9}{10}x = 90$

124. $\frac{5}{12}y = 60$

125. $y + 46 = 55$

126. $x + 33 = 41$

127. $\frac{w}{-2} = 99$

128. $\frac{s}{-3} = -60$

129. $27 = 6a$

130. $-a = 7$

131. $-x = 2$

132. $z - 16 = -59$

133. $m - 41 = -14$

134. $0.04r = 52.60$

135. $63.90 = 0.03p$

136. $-15x = -120$

137. $84 = -12z$

138. $19.36 = x - 0.2x$

139. $c - 0.3c = 35.70$

140. $-y = -9$

141. $-x = -8$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

142. 187 is the product of -17 and *m*.

143. 133 is the product of -19 and *n*.

144. -184 is the product of 23 and *p*.

145. -152 is the product of 8 and *q*.

146. *u* divided by 7 is equal to -49.

147. *r* divided by 12 is equal to -48.

148. *h* divided by -13 is equal to -65.

149. *j* divided by -20 is equal to -80.

150. The quotient c and -19 is 38.

151. The quotient of *b* and -6 is 18.

152. The quotient of h and 26 is -52.

153. The quotient k and 22 is -66.

154. Five-sixths of *y* is 15.

155. Three-tenths of *x* is 15.

156. Four-thirds of *w* is 36.

157. Five-halves of *v* is 50.

158. The sum of nine-tenths and *g* is two-thirds.

159. The sum of two-fifths and *f* is one-half.

160. The difference of *p* and one-sixth is two-thirds.

161. The difference of *q* and one-eighth is three-fourths.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

162. **Kindergarten** Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. How many children will she put in each group?

163. **Balloons** Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. How many balloons did she use in each bunch?

164. **Tickets** Mollie paid $36.25 for 5 movie tickets. What was the price of each ticket?

165. **Shopping** Serena paid \$12.96 for a pack of 12 pairs of sport socks. What was the price of pair of sport socks?

166. **Sewing** Nancy used 14 yards of fabric to make flags for one-third of the drill team. How much fabric, would Nancy need to make flags for the whole team?

167. **MPG** John's SUV gets 18 miles per gallon (mpg). This is half as many mpg as his wife's hybrid car. How many miles per gallon does the hybrid car get?

168. **Height** Aiden is 27 inches tall. He is $\frac{3}{8}$ as tall as his father. How tall is his father?

169. **Real estate** Bea earned \$11,700 commission for selling a house, calculated as $\frac{6}{100}$ of the selling price. What was the selling price of the house?

Everyday Math

170. **Commission** Every week Perry gets paid \$150 plus 12% of his total sales amount over \$1,250. Solve the equation $840 = 150 + 0.12(a - 1250)$ for a, to find the total amount Perry must sell in order to be paid \$840 one week.

171. **Stamps** Travis bought \$9.45 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21 \quad (s - 5) \quad = 9.45$ for s, to find the number of 49-cent stamps Travis bought.

Writing Exercises

172. Frida started to solve the equation $-3x = 36$ by adding 3 to both sides. Explain why Frida's method will not solve the equation.

173. Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
solve equations using the Division and Multiplication Properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

ⓑ *What does this checklist tell you about your mastery of this section? What steps will you take to improve?*

2.3 Solve Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

BE PREPARED 2.7 Before you get started, take this readiness quiz.

Simplify: $4y - 9 + 9$.

If you missed this problem, review Example 1.129.

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

EXAMPLE 2.27

Solve: $7x + 8 = -13$.

Solution

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

$$\overset{\text{variable}}{7x + 8} = \overset{\text{constant}}{-13}$$

Since the left side is the "x", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

		$\overset{\text{variable}}{7x + 8} = \overset{\text{constant}}{-13}$
Use the Subtraction Property of Equality.		$7x + 8 - 8 = -13 - 8$
Simplify.		$7x = -21$
Now all the variables are on the left and the constant on the right. The equation looks like those you learned to solve earlier.		
Use the Division Property of Equality.		$\frac{7x}{7} = \frac{-21}{7}$
Simplify.		$x = -3$
Check:	$7x + 8 = -13$	
Let $x = -3$.	$7(-3) + 8 \stackrel{?}{=} -13$	
	$-21 + 8 \stackrel{?}{=} -13$ $-13 = -13\ \checkmark$	

> **TRY IT** 2.53 Solve: $3x + 4 = -8$.

> **TRY IT** 2.54 Solve: $5a + 3 = -37$.

EXAMPLE 2.28

Solve: $8y - 9 = 31$.

Solution

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the 9 is out of place. It is subtracted from the $8y$, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

	variable constant $8y - 9 = 31$
Add 9 to both sides.	$8y - 9 + 9 = 31 + 9$
Simplify.	$8y = 40$
	The variables are now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.	$\frac{8y}{8} = \frac{40}{8}$
Simplify.	$y = 5$

Check:	$8y - 9 = 31$	
Let $y = 5$.	$8 \cdot 5 - 9 \stackrel{?}{=} 31$	
	$40 - 9 \stackrel{?}{=} 31$	
	$31 = 31 ✓$	

TRY IT 2.55 Solve: $5y - 9 = 16$.

TRY IT 2.56 Solve: $3m - 8 = 19$.

Solve Equations with Variables on Both Sides

What if there are variables on both sides of the equation? For equations like this, begin as we did above—choose a "variable" side and a "constant" side, and then use the subtraction and addition properties of equality to collect all variables on one side and all constants on the other side.

EXAMPLE 2.29

Solve: $9x = 8x - 6$.

Solution

Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

		variable constant $9x = 8x - 6$
We don't want any x's on the right, so subtract the $8x$ from both sides.		$9x - 8x = 8x - 8x - 6$
Simplify.		$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.		
Check:	$9x = 8x - 6$	
Let $x = -6$.	$9(-6) \stackrel{?}{=} 8(-6) - 6$	
	$-54 \stackrel{?}{=} -48 - 6$	
	$-54 = -54 \checkmark$	

> **TRY IT** 2.57 Solve: $6n = 5n - 10$.

> **TRY IT** 2.58 Solve: $-6c = -7c - 1$.

EXAMPLE 2.30

Solve: $5y - 9 = 8y$.

Solution

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

		constant variable $5y - 9 = 8y$
Subtract $5y$ from both sides.		$5y - 5y - 9 = 8y - 5y$
Simplify.		$-9 = 3y$
We have the ys on the right and the constants on the left. Divide both sides by 3.		$\frac{-9}{3} = \frac{3y}{3}$
Simplify.		$-3 = y$
Check:	$5y - 9 = 8y$	
Let $y = -3$.	$5(-3) - 9 \stackrel{?}{=} 8(-3)$	
	$-15 - 9 \stackrel{?}{=} -24$	
	$-24 = -24 \checkmark$	

TRY IT 2.59 Solve: $3p - 14 = 5p$.

TRY IT 2.60 Solve: $8m + 9 = 5m$.

EXAMPLE 2.31

Solve: $12x = -x + 26$.

Solution

The only constant is on the right, so let the left side be the "variable" side.

	variable constant $12x = -x + 26$
Remove the $-x$ from the right side by adding x to both sides.	$12x + x = -x + x + 26$
Simplify.	$13x = 26$
All the x's are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13} = \frac{26}{13}$
Simplify.	$x = 2$

TRY IT 2.61 Solve: $12j = -4j + 32$.

TRY IT 2.62 Solve: $8h = -4h + 12$.

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

EXAMPLE 2.32

How to Solve Equations with Variables and Constants on Both Sides

Solve: $7x + 5 = 6x + 2$.

Solution

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$

Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$
Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16\ \checkmark$

TRY IT 2.63 Solve: $12x + 8 = 6x + 2$.

TRY IT 2.64 Solve: $9y + 4 = 7y + 12$.

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

HOW TO

Beginning Strategy for Solving Equations with Variables and Constants on Both Sides of the Equation.

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.
Step 2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
Step 3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
Step 4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
Step 5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

EXAMPLE 2.33

Solve: $8n - 4 = -2n + 6$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8 > -2$, make the left side the "variable" side.	variable constant $8n - 4 = -2n + 6$
We don't want variable terms on the right side—add $2n$ to both sides to leave only constants on the right.	$8n + 2n - 4 = -2n + 2n + 6$

Combine like terms.		$10n - 4 = 6$
We don't want any constants on the left side, so add 4 to both sides.		$10n - 4 + 4 = 6 + 4$
Simplify.		$10n = 10$
The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 10.		$\frac{10n}{10} = \frac{10}{10}$
Simplify.		$n = 1$
Check:	$8n - 4 = -2n + 6$	
Let $n = 1$.	$8 \cdot 1 - 4 \stackrel{?}{=} -2 \cdot 1 + 6$	
	$8 - 4 \stackrel{?}{=} -2 + 6$	
	$4 = 4$ ✓	

TRY IT 2.65 Solve: $8q - 5 = -4q + 7$.

TRY IT 2.66 Solve: $7n - 3 = n + 3$.

EXAMPLE 2.34

Solve: $7a - 3 = 13a + 7$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $13 > 7$, make the right side the "variable" side and the left side the "constant" side.

		constant variable $7a - 3 = 13a + 7$
Subtract $7a$ from both sides to remove the variable term from the left.		$7a - 7a - 3 = 13a - 7a + 7$
Combine like terms.		$-3 = 6a + 7$
Subtract 7 from both sides to remove the constant from the right.		$-3 - 7 = 6a + 7 - 7$
Simplify.		$-10 = 6a$
Divide both sides by 6 to make 1 the coefficient of a.		$\frac{-10}{6} = \frac{6a}{6}$
Simplify.		$-\frac{5}{3} = a$
Check:	$7a - 3 = 13a + 7$	

Let $a = -\frac{5}{3}$.	$7\left(-\frac{5}{3}\right) - 3 \stackrel{?}{=} 13\left(-\frac{5}{3}\right) + 7$	
	$-\frac{35}{3} - \frac{9}{3} \stackrel{?}{=} -\frac{65}{3} + \frac{21}{3}$	
	$-\frac{54}{3} = -\frac{54}{3}$ ✓	

> **TRY IT** 2.67 Solve: $2a - 2 = 6a + 18$.

> **TRY IT** 2.68 Solve: $4k - 1 = 7k + 17$.

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

EXAMPLE 2.35

Solve: $\frac{5}{4}x + 6 = \frac{1}{4}x - 2$.

Solution

Since $\frac{5}{4} > \frac{1}{4}$, make the left side the “variable” side and the right side the “constant” side.

	variable constant $\frac{5}{4}x + 6 = \frac{1}{4}x - 2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x - \frac{1}{4}x + 6 = \frac{1}{4}x - \frac{1}{4}x - 2$
Combine like terms.	$x + 6 = -2$
Subtract 6 from both sides.	$x + 6 - 6 = -2 - 6$
Simplify.	$x = -8$

$$\begin{array}{lrcl} \text{Check:} & \frac{5}{4}x + 6 & = & \frac{1}{4}x - 2 \\ \text{Let } x = -8. & \frac{5}{4}(-8) + 6 & \stackrel{?}{=} & \frac{1}{4}(-8) - 2 \\ & -10 + 6 & \stackrel{?}{=} & -2 - 2 \\ & -4 & = & -4\checkmark \end{array}$$

> **TRY IT** 2.69 Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$.

> **TRY IT** 2.70 Solve: $\frac{7}{6}y + 11 = \frac{1}{6}y + 8$.

We will use the same strategy to find the solution for an equation with decimals.

EXAMPLE 2.36

Solve: $7.8x + 4 = 5.4x - 8$.

Solution

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x + 4 = 5.4x - 8$
Subtract $5.4x$ from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Propery of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$

Check:	$7.8x + 4 = 5.4x - 5$	
Let $x = -5$.	$7.8(-5) + 4 = 5.4(-5) - 8$	
	$-39 + 4 \stackrel{?}{=} -27 - 8$	
	$-35 = -35$ ✓	

> **TRY IT** 2.71 Solve: $2.8x + 12 = -1.4x - 9$.

> **TRY IT** 2.72 Solve: $3.6y + 8 = 1.2y - 4$.

SECTION 2.3 EXERCISES

Practice Makes Perfect

Solve Equations with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

174. $9x - 3 = 60$

175. $12x - 8 = 64$

176. $14w + 5 = 117$

177. $15y + 7 = 97$

178. $2a + 8 = -28$

179. $3m + 9 = -15$

180. $-62 = 8n - 6$

181. $-77 = 9b - 5$

182. $35 = -13y + 9$

183. $60 = -21x - 24$

184. $-12p - 9 = 9$

185. $-14q - 2 = 16$

Solve Equations with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

186. $19z = 18z - 7$

187. $21k = 20k - 11$

188. $9x + 36 = 15x$

189. $8x + 27 = 11x$

190. $c = -3c - 20$

191. $b = -4b - 15$

192. $9q = 44 - 2q$

193. $5z = 39 - 8z$

194. $6y + \frac{1}{2} = 5y$

195. $4x + \frac{3}{4} = 3x$

196. $-18a - 8 = -22a$

197. $-11r - 8 = -7r$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

198. $8x - 15 = 7x + 3$

199. $6x - 17 = 5x + 2$

200. $26 + 13d = 14d + 11$

201. $21 + 18f = 19f + 14$

202. $2p - 1 = 4p - 33$

203. $12q - 5 = 9q - 20$

204. $4a + 5 = -a - 40$

205. $8c + 7 = -3c - 37$

206. $5y - 30 = -5y + 30$

207. $7x - 17 = -8x + 13$

208. $7s + 12 = 5 + 4s$

209. $9p + 14 = 6 + 4p$

210. $2z - 6 = 23 - z$

211. $3y - 4 = 12 - y$

212. $\frac{5}{3}c - 3 = \frac{2}{3}c - 16$

213. $\frac{7}{4}m - 7 = \frac{3}{4}m - 13$

214. $8 - \frac{2}{5}q = \frac{3}{5}q + 6$

215. $11 - \frac{1}{5}a = \frac{4}{5}a + 4$

216. $\frac{4}{3}n + 9 = \frac{1}{3}n - 9$

217. $\frac{5}{4}a + 15 = \frac{3}{4}a - 5$

218. $\frac{1}{4}y + 7 = \frac{3}{4}y - 3$

219. $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

220. $14n + 8.25 = 9n + 19.60$

221. $13z + 6.45 = 8z + 23.75$

222. $2.4w - 100 = 0.8w + 28$

223. $2.7w - 80 = 1.2w + 10$

224. $5.6r + 13.1 = 3.5r + 57.2$

225. $6.6x - 18.9 = 3.4x + 54.7$

Everyday Math

226. **Concert tickets** At a school concert the total value of tickets sold was $1506. Student tickets sold for $6 and adult tickets sold for $9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, *s*, by solving the equation $6s + 27s - 45 = 1506$.

227. **Making a fence** Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, *w*, by solving the equation $150 = 2w + 30 + 2w$.

Writing Exercises

228. Solve the equation $\frac{6}{5}y - 8 = \frac{1}{5}y + 7$ explaining all the steps of your solution as in the examples in this section.

229. Solve the equation $10x + 14 = -2x + 38$ explaining all the steps of your solution as in the examples in this section.

230. When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient of x to be the "variable" side?

231. Is $x = -2$ a solution to the equation $5 - 2x = -4x + 1$? How do you know?

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

ⓑ *What does this checklist tell you about your mastery of this section? What steps will you take to improve?*

2.4 Use a General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

BE PREPARED 2.8 Before you get started, take this readiness quiz.

Simplify: $-(a - 4)$.
If you missed this problem, review Example 1.137.

BE PREPARED 2.9 Multiply: $\frac{3}{2}(12x + 20)$.
If you missed this problem, review Example 1.133.

BE PREPARED 2.10 Simplify: $5 - 2(n + 1)$.
If you missed this problem, review Example 1.138.

BE PREPARED 2.11 Multiply: $3(7y + 9)$.
If you missed this problem, review Example 1.132.

BE PREPARED 2.12 Multiply: $(2.5)(6.4)$.
If you missed this problem, review Example 1.97.

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

EXAMPLE 2.37

How to Solve Linear Equations Using the General Strategy

Solve: $-6(x + 3) = 24$.

Solution

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x + 3) = 24$ $-6x - 18 = 24$

Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x - 18 + 18 = 24 + 18$ $-6x = 42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6. Simplify.	$\frac{-6x}{-6} = \frac{42}{-6}$ $x = -7$
Step 5. Check the solution.	Let $x = -7$ Simplify. Multiply.	**Check:** $-6(x + 3) = 24$ $-6(-7 + 3) \stackrel{?}{=} 24$ $-6(-4) \stackrel{?}{=} 24$ $24 = 24 \checkmark$

TRY IT 2.73 Solve: $5(x + 3) = 35$.

TRY IT 2.74 Solve: $6(y - 4) = -18$.

HOW TO

General strategy for solving linear equations.

Step 1. **Simplify each side of the equation as much as possible.**
Use the Distributive Property to remove any parentheses.
Combine like terms.

Step 2. **Collect all the variable terms on one side of the equation.**
Use the Addition or Subtraction Property of Equality.

Step 3. **Collect all the constant terms on the other side of the equation.**
Use the Addition or Subtraction Property of Equality.

Step 4. **Make the coefficient of the variable term to equal to 1.**
Use the Multiplication or Division Property of Equality.
State the solution to the equation.

Step 5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

EXAMPLE 2.38

Solve: $-(y + 9) = 8$.

Solution

	$-(y + 9) = 8$
Simplify each side of the equation as much as possible by distributing.	$-y - 9 = 8$

The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y-9+9=8+9$
Simplify.	$-y=17$
Rewrite $-y$ as $-1y$.	$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.	$\frac{-1y}{-1}=\frac{17}{-1}$
Simplify.	$y=-17$

Check:	$-(y+9)=8$	
Let $y=-17$.	$-(-17+9)\stackrel{?}{=}8$	
	$-(-8)\stackrel{?}{=}8$	
	$8=8\checkmark$	

TRY IT 2.75 Solve: $-(y+8)=-2$.

TRY IT 2.76 Solve: $-(z+4)=-12$.

EXAMPLE 2.39

Solve: $5(a-3)+5=-10$.

Solution

	$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.	
Distribute.	$5a-15+5=-10$
Combine like terms.	$5a-10=-10$
The only a term is on the left side, so all variable terms are on one side of the equation.	
Add 10 to both sides to get all constant terms on the other side of the equation.	$5a-10+10=-10+10$
Simplify.	$5a=0$
Make the coefficient of the variable term to equal to 1 by dividing both sides by 5.	$\frac{5a}{5}=\frac{0}{5}$
Simplify.	$a=0$

Check:	$5(a-3)+5=-10$	
Let $a=0$.	$5(0-3)+5\stackrel{?}{=}-10$	
	$5(-3)+5\stackrel{?}{=}-10$	
	$-15+5\stackrel{?}{=}-10$	
	$-10=-10$ ✓	

TRY IT 2.77 Solve: $2(m-4)+3=-1$.

TRY IT 2.78 Solve: $7(n-3)-8=-15$.

EXAMPLE 2.40

Solve: $\frac{2}{3}(6m-3)=8-m$.

Solution

	$\frac{2}{3}(6m-3)=8-m$
Distribute.	$4m-2=8-m$
Add m to get the variables only to the left.	$4m+m-2=8-m+m$
Simplify.	$5m-2=8$
Add 2 to get constants only on the right.	$5m-2+2=8+2$
Simplify.	$5m=10$
Divide by 5.	$\frac{5m}{5}=\frac{10}{5}$
Simplify.	$m=2$

Check:	$\frac{2}{3}(6m-3)=8-m$	
Let $m=2$.	$\frac{2}{3}(6\cdot2-3)\stackrel{?}{=}8-2$	
	$\frac{2}{3}(12-3)\stackrel{?}{=}6$	

	$\frac{2}{3}(9) \stackrel{?}{=} 6$	
	$6 = 6 \checkmark$	

> TRY IT 2.79 Solve: $\frac{1}{3}(6u + 3) = 7 - u$.

> TRY IT 2.80 Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$.

EXAMPLE 2.41

Solve: $8 - 2(3y + 5) = 0$.

Solution

	$8 - 2(3y + 5) = 0$
Simplify—use the Distributive Property.	$8 - 6y - 10 = 0$
Combine like terms.	$-6y - 2 = 0$
Add 2 to both sides to collect constants on the right.	$-6y - 2 + 2 = 0 + 2$
Simplify.	$-6y = 2$
Divide both sides by -6.	$\frac{-6y}{-6} = \frac{2}{-6}$
Simplify.	$y = -\frac{1}{3}$
Check: Let $y = -\frac{1}{3}$. $8 - 2(3y + 5) = 0$ $8 - 2\left[3\left(-\frac{1}{3}\right) + 5\right] = 0$ $8 - 2(-1 + 5) \stackrel{?}{=} 0$ $8 - 2(4) \stackrel{?}{=} 0$ $8 - 8 \stackrel{?}{=} 0$ $0 = 0 \checkmark$	

> TRY IT 2.81 Solve: $12 - 3(4j + 3) = -17$.

> TRY IT 2.82 Solve: $-6 - 8(k - 2) = -10$.

EXAMPLE 2.42

Solve: $4(x-1)-2=5(2x+3)+6$.

Solution

		$4(x-1)-2=5(2x+3)+6$
Distribute.		$4x-4-2=10x+15+6$
Combine like terms.		$4x-6=10x+21$
Subtract $4x$ to get the variables only on the right side since $10>4$.		$4x-4x-6=10x-4x+21$
Simplify.		$-6=6x+21$
Subtract 21 to get the constants on left.		$-6-21=6x+21-21$
Simplify.		$-27=6x$
Divide by 6.		$\frac{-27}{6}=\frac{6x}{6}$
Simplify.		$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$	
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$	
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$	
	$-22-2\stackrel{?}{=}5(-6)+6$	
	$-24\stackrel{?}{=}-30+6$	
	$-24=-24$ ✓	

TRY IT 2.83 Solve: $6(p-3)-7=5(4p+3)-12$.

TRY IT 2.84 Solve: $8(q+1)-5=3(2q-4)-1$.

EXAMPLE 2.43

Solve: $10[3-8(2s-5)]=15(40-5s)$.

Solution

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add $160s$ to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute $s=-2$.	$10[3-8(2(-2)-5)]\stackrel{?}{=}15(40-5(-2))$	
	$10[3-8(-4-5)]\stackrel{?}{=}15(40+10)$	
	$10[3-8(-9)]\stackrel{?}{=}15(50)$	
	$10[3+72]\stackrel{?}{=}750$	
	$10[75]\stackrel{?}{=}750$	
	$750=750$ ✓	

TRY IT 2.85 Solve: $6[4-2(7y-1)]=8(13-8y)$.

TRY IT 2.86 Solve: $12[1-5(4z-1)]=3(24+11z)$.

EXAMPLE 2.44

Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution

	$0.36(100n+5)=0.6(30n+15)$
Distribute.	$36n+1.8=18n+9$
Subtract $18n$ to get the variables to the left.	$36n-18n+1.8=18n-18n+9$
Simplify.	$18n+1.8=9$
Subtract 1.8 to get the constants to the right.	$18n+1.8-1.8=9-1.8$
Simplify.	$18n=7.2$
Divide.	$\frac{18n}{18}=\frac{7.2}{18}$
Simplify.	$n=0.4$

Check:	$0.36(100n+5)=0.6(30n+15)$	
Let $n=0.4$.	$0.36(100(0.4)+5)\stackrel{?}{=}0.6(30(0.4)+15)$	
	$0.36(40+5)\stackrel{?}{=}0.6(12+15)$	
	$0.36(45)\stackrel{?}{=}0.6(27)$	
	$16.2=16.2$ ✓	

TRY IT 2.87 Solve: $0.55(100n+8)=0.6(85n+14)$.

TRY IT 2.88 Solve: $0.15(40m-120)=0.5(60m+12)$.

Classify Equations

Consider the equation we solved at the start of the last section, $7x+8=-13$. The solution we found was $x=-3$. This means the equation $7x+8=-13$ is true when we replace the variable, x, with the value -3. We showed this when we checked the solution $x=-3$ and evaluated $7x+8=-13$ for $x=-3$.

$$7(-3)+8\stackrel{?}{=}-13$$
$$-21+8\stackrel{?}{=}-13$$
$$-13=-13\checkmark$$

If we evaluate $7x+8$ for a different value of x, the left side will not be -13.

The equation $7x+8=-13$ is true when we replace the variable, x, with the value -3, but not true when we replace x with any other value. Whether or not the equation $7x+8=-13$ is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

Conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation $2y + 6 = 2\,(y + 3)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y.

	$2y + 6 = 2(y + 3)$
Distribute.	$2y + 6 = 2y + 6$
Subtract $2y$ to get the y's to one side.	$2y - 2y + 6 = 2y - 2y + 6$
Simplify—the y's are gone!	$6 = 6$

But $6 = 6$ is true.

This means that the equation $2y + 6 = 2\,(y + 3)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an identity.

Identity

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is all real numbers.

What happens when we solve the equation $5z = 5z - 1$?

	$5z = 5z - 1$
Subtract $5z$ to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify—the z's are gone!	$0 \neq -1$

But $0 \neq -1$.

Solving the equation $5z = 5z - 1$ led to the false statement $0 = -1$. The equation $5z = 5z - 1$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

Contradiction

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

EXAMPLE 2.45

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$$

Solution

	$6(2n-1)+3=2n-8+5(2n+1)$
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract $12n$ to get the n's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

TRY IT 2.89 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

TRY IT 2.90 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$

EXAMPLE 2.46

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Solution

	$10+4(p-5)=0$
Distribute.	$10+4p-20=0$
Combine like terms.	$4p-10=0$
Add 10 to both sides.	$4p-10+10=0+10$
Simplify.	$4p=10$
Divide.	$\frac{4p}{4}=\frac{10}{4}$
Simplify.	$p=\frac{5}{2}$
The equation is true when $p=\frac{5}{2}$.	This is a conditional equation. The solution is $p=\frac{5}{2}$.

TRY IT 2.91 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q+3)-5=19$

TRY IT 2.92 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6+14(k-8)=95$

EXAMPLE 2.47

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m+3(9+3m)=2(7m-11)$

Solution

	$5m+3(9+3m)=2(7m-11)$
Distribute.	$5m+27+9m=14m-22$
Combine like terms.	$14m+27=14m-22$
Subtract $14m$ from both sides.	$14m+27-14m=14m-22-14m$
Simplify.	$27\neq-22$
But $27\neq-22$.	The equation is a contradiction. It has no solution.

TRY IT 2.93 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c+5(5+3c)=3(9c-4)$

TRY IT 2.94 Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d+18)=13(3d-2)-11d$

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Table 2.5

SECTION 2.4 EXERCISES

Practice Makes Perfect

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

232. $15(y-9)=-60$

233. $21(y-5)=-42$

234. $-9(2n+1)=36$

235. $-16(3n+4)=32$

236. $8(22+11r)=0$

237. $5(8+6p)=0$

238. $-(w-12)=30$

239. $-(t-19)=28$

240. $9(6a+8)+9=81$

241. $8(9b-4)-12=100$

242. $32+3(z+4)=41$

243. $21+2(m-4)=25$

244. $51+5(4-q)=56$

245. $-6+6(5-k)=15$

246. $2(9s-6)-62=16$

247. $8(6t-5)-35=-27$

248. $3(10-2x)+54=0$

249. $-2(11-7x)+54=4$

250. $\frac{2}{3}(9c-3)=22$

251. $\frac{3}{5}(10x-5)=27$

252. $\frac{1}{5}(15c+10)=c+7$

253. $\frac{1}{4}(20d+12)=d+7$

254. $18-(9r+7)=-16$

255. $15-(3r+8)=28$

256. $5-(n-1)=19$

257. $-3-(m-1)=13$

258. $11-4(y-8)=43$

259. $18-2(y-3)=32$

260. $24-8(3v+6)=0$

261. $35-5(2w+8)=-10$

262. $4(a-12)=3(a+5)$

263. $-2(a-6)=4(a-3)$

264. $2(5-u)=-3(2u+6)$

265. $5(8-r)=-2(2r-16)$

266. $3(4n-1)-2=8n+3$

267. $9(2m-3)-8=4m+7$

268. $12+2(5-3y)=-9(y-1)-2$

269. $-15+4(2-5y)=-7(y-4)+4$

270. $8(x-4)-7x=14$

271. $5(x-4)-4x=14$

272. $5+6(3s-5)=-3+2(8s-1)$

273. $-12+8(x-5)=-4+3(5x-2)$

274. $4(u-1)-8=6(3u-2)-7$

275. $7(2n-5)=8(4n-1)-9$

276. $4(p-4)-(p+7)=5(p-3)$

277. $3(a-2)-(a+6)=4(a-1)$

278. $-(9y+5)-(3y-7)=16-(4y-2)$

279. $-(7m+4)-(2m-5)=14-(5m-3)$

280. $4[5-8(4c-3)]=12(1-13c)-8$

281. $5[9-2(6d-1)]=11(4-10d)-139$

282. $3[-9+8(4h-3)]=2(5-12h)-19$

283. $3[-14+2(15k-6)]=8(3-5k)-24$

284. $5[2(m+4)+8(m-7)]=2[3(5+m)-(21-3m)]$

285. $10[5(n+1)+4(n-1)]=11[7(5+n)-(25-3n)]$

286. $5(1.2u-4.8)=-12$

287. $4(2.5v-0.6)=7.6$

288. $0.25(q-6)=0.1(q+18)$

289. $0.2(p-6)=0.4(p+14)$

290. $0.2(30n+50)=28$

291. $0.5(16m+34)=-15$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

292. $23z+19=3(5z-9)+8z+46$

293. $15y+32=2(10y-7)-5y+46$

294. $5(b-9)+4(3b+9)=6(4b-5)-7b+21$

295. $9(a-4)+3(2a+5)=7(3a-4)-6a+7$

296. $18(5j-1)+29=47$

297. $24(3d-4)+100=52$

298. $22(3m-4)=8(2m+9)$

299. $30(2n-1)=5(10n+8)$

300. $7v+42=11(3v+8)-2(13v-1)$

301. $18u-51=9(4u+5)-6(3u-10)$

302. $3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$

303. $5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$

304. $12(6h-1)=8(8h+5)-4$

305. $9(4k-7)=11(3k+1)+4$

306. $45(3y-2)=9(15y-6)$

307. $60(2x-1)=15(8x+5)$

308. $16(6n + 15) = 48\,(2n + 5)$

309. $36(4m + 5) = 12\,(12m + 15)$

310. $9(14d + 9) + 4d = 13\,(10d + 6) + 3$

311. $11(8c + 5) - 8c = 2\,(40c + 25) + 5$

Everyday Math

312. **Fencing** Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, *L*, by solving the equation $2L + 2(L - 2.5) = 44$.

313. **Coins** Rhonda has $1.90 in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, *n*, by solving the equation $0.05n + 0.10(2n - 1) = 1.90$.

Writing Exercises

314. Using your own words, list the steps in the general strategy for solving linear equations.

315. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

316. What is the first step you take when solving the equation $3 - 7(y - 4) = 38$? Why is this your first step?

317. Solve the equation $\frac{1}{4}(8x + 20) = 3x - 4$ explaining all the steps of your solution as in the examples in this section.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objective of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the general strategy for solving linear equations.			
classify equations.			

ⓑ On a scale of 1-10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

2.5 Solve Equations with Fractions or Decimals

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

BE PREPARED 2.13 Before you get started, take this readiness quiz.

Multiply: $8 \cdot \frac{3}{8}$.
If you missed this problem, review Example 1.69.

BE PREPARED 2.14 Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.
If you missed this problem, review Example 1.82.

BE PREPARED 2.15 Multiply 4.78 by 100.
If you missed this problem, review Example 1.98.

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x + \frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x = \frac{1}{4} - \frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x = \frac{1}{4} - \frac{2}{4}$
Subtract.	$\frac{1}{8}x = -\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1} \cdot \frac{1}{8}x = \frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x = -2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

EXAMPLE 2.48

How to Solve Equations with Fraction Coefficients

Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$.

Solution

Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y - \frac{1}{3} = \frac{5}{6}$ LCD $= 6$
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6. Use the Distributive Property. Simplify – and notice, no more fractions!	$6\left(\frac{1}{6}y - \frac{1}{3}\right) = 6\left(\frac{5}{6}\right)$ $6 \cdot \frac{1}{6}y - 6 \cdot \frac{1}{3} = 6 \cdot \frac{5}{6}$ $y - 2 = 5$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "y" term, add 2. Simplify.	$y - 2 + 2 = 5 + 2$ $y = 7$

TRY IT 2.95 Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

TRY IT 2.96 Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Notice in Example 2.48, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

HOW TO

Strategy to solve equations with fraction coefficients.

Step 1. Find the least common denominator of *all* the fractions in the equation.
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.
Step 3. Solve using the General Strategy for Solving Linear Equations.

EXAMPLE 2.49

Solve: $6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$.

Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.		$6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$
The LCD is 20.		
Multiply both sides of the equation by 20.		$20(6) = 20 \cdot \left(\frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v\right)$
Distribute.		$20(6) = 20 \cdot \frac{1}{2}v + 20 \cdot \frac{2}{5}v - 20 \cdot \frac{3}{4}v$
Simplify—notice, no more fractions!		$120 = 10v + 8v - 15v$
Combine like terms.		$120 = 3v$
Divide by 3.		$\frac{120}{3} = \frac{3v}{3}$
Simplify.		$40 = v$
Check:	$6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$	
Let $v = 40$.	$6 \stackrel{?}{=} \frac{1}{2}(40) + \frac{2}{5}(40) - \frac{3}{4}(40)$	
	$6 \stackrel{?}{=} 20 + 16 - 30$	
	$6 = 6 \checkmark$	

TRY IT 2.97 Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

TRY IT 2.98 Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

In the next example, we again have variables on both sides of the equation.

EXAMPLE 2.50

Solve: $a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$.

Solution

		$a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.		
Multiply both sides by the LCD.		$8\left(a + \frac{3}{4}\right) = 8\left(\frac{3}{8}a - \frac{1}{2}\right)$
Distribute.		$8 \cdot a + 8 \cdot \frac{3}{4} = 8 \cdot \frac{3}{8}a - 8 \cdot \frac{1}{2}$
Simplify—no more fractions.		$8a + 6 = 3a - 4$
Subtract $3a$ from both sides.		$8a - 3a + 6 = 3a - 3a - 4$
Simplify.		$5a + 6 = -4$
Subtract 6 from both sides.		$5a + 6 - 6 = -4 - 6$
Simplify.		$5a = -10$
Divide by 5.		$\frac{5a}{5} = \frac{-10}{5}$
Simplify.		$a = -2$
Check:	$a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$	
Let $a = -2$.	$-2 + \frac{3}{4} \stackrel{?}{=} \frac{3}{8}(-2) - \frac{1}{2}$	
	$-\frac{8}{4} + \frac{3}{4} \stackrel{?}{=} -\frac{16}{8} - \frac{4}{8}$	
	$-\frac{5}{4} = -\frac{10}{8}$	
	$-\frac{5}{4} = -\frac{5}{4}$ ✓	

TRY IT 2.99 Solve: $x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}$.

TRY IT 2.100 Solve: $c + \frac{3}{4} = \frac{1}{2}c - \frac{1}{4}$.

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

EXAMPLE 2.51

Solve: $-5 = \frac{1}{4}(8x + 4)$.

Solution

	$-5 = \frac{1}{4}(8x + 4)$
Distribute.	$-5 = \frac{1}{4} \cdot 8x + \frac{1}{4} \cdot 4$
Simplify. Now there are no fractions.	$-5 = 2x + 1$
Subtract 1 from both sides.	$-5 - 1 = 2x + 1 - 1$
Simplify.	$-6 = 2x$
Divide by 2.	$\frac{-6}{2} = \frac{2x}{2}$
Simplify.	$-3 = x$

Check:	$-5 = \frac{1}{4}(8x + 4)$	
Let $x = -3$.	$-5 \stackrel{?}{=} \frac{1}{2}(4(-3) + 2)$	
	$-5 \stackrel{?}{=} \frac{1}{2}(-12 + 2)$	
	$-5 \stackrel{?}{=} \frac{1}{2}(-10)$	
	$-5 = -5 \checkmark$	

TRY IT 2.101 Solve: $-11 = \frac{1}{2}(6p + 2)$.

TRY IT 2.102 Solve: $8 = \frac{1}{3}(9q + 6)$.

In the next example, even after distributing, we still have fractions to clear.

EXAMPLE 2.52

Solve: $\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$.

Solution

	$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$
Distribute.	$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$

Simplify.	$\frac{1}{2}y - \frac{5}{2} = \frac{1}{4}y - \frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y - \frac{5}{2}\right) = 4\left(\frac{1}{4}y - \frac{1}{4}\right)$
Distribute.	$4 \cdot \frac{1}{2}y - 4 \cdot \frac{5}{2} = 4 \cdot \frac{1}{4}y - 4 \cdot \frac{1}{4}$
Simplify.	$2y - 10 = y - 1$
Collect the variables to the left.	$2y - y - 10 = y - y - 1$
Simplify.	$y - 10 = -1$
Collect the constants to the right.	$y - 10 + 10 = -1 + 10$
Simplify.	$y = 9$

Check:	$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$
Let $y = 9$.	$\frac{1}{2}(9 - 5) \stackrel{?}{=} \frac{1}{4}(9 - 1)$
Finish the check on your own.	

> **TRY IT** 2.103 Solve: $\frac{1}{5}(n + 3) = \frac{1}{4}(n + 2)$.

> **TRY IT** 2.104 Solve: $\frac{1}{2}(m - 3) = \frac{1}{4}(m - 7)$.

EXAMPLE 2.53

Solve: $\frac{5x-3}{4} = \frac{x}{2}$.

Solution

	$\frac{5x - 3}{4} = \frac{x}{2}$
Multiply by the LCD, 4.	$4\left(\frac{5x - 3}{4}\right) = 4\left(\frac{x}{2}\right)$
Simplify.	$5x - 3 = 2x$
Collect the variables to the right.	$5x - 5x - 3 = 2x - 5x$
Simplify.	$-3 = -3x$
Divide.	$\frac{-3}{-3} = \frac{-3x}{-3}$

Simplify.		$1 = x$
Check:	$\frac{5x-3}{4} = \frac{x}{2}$	
Let $x = 1$.	$\frac{5(1)-3}{4} \stackrel{?}{=} \frac{1}{2}$	
	$\frac{2}{4} \stackrel{?}{=} \frac{1}{2}$	
	$\frac{1}{2} = \frac{1}{2}$ ✓	

TRY IT 2.105 Solve: $\frac{4y-7}{3} = \frac{y}{6}$.

TRY IT 2.106 Solve: $\frac{-2z-5}{4} = \frac{z}{8}$.

EXAMPLE 2.54

Solve: $\frac{a}{6} + 2 = \frac{a}{4} + 3$.

Solution

		$\frac{a}{6} + 2 = \frac{a}{4} + 3$
Multiply by the LCD, 12.		$12\left(\frac{a}{6} + 2\right) = 12\left(\frac{a}{4} + 3\right)$
Distribute.		$12 \cdot \frac{a}{6} + 12 \cdot 2 = 12 \cdot \frac{a}{4} + 12 \cdot 3$
Simplify.		$2a + 24 = 3a + 36$
Collect the variables to the right.		$2a - 2a + 24 = 3a - 2a + 36$
Simplify.		$24 = a + 36$
Collect the constants to the left.		$24 - 36 = a + 36 - 36$
Simplify.		$a = -12$
Check:	$\frac{a}{6} + 2 = \frac{a}{4} + 3$	
Let $a = -12$.	$\frac{-12}{6} + 2 \stackrel{?}{=} \frac{-12}{4} + 3$	
	$-2 + 2 \stackrel{?}{=} -3 + 3$	
	$0 = 0$ ✓	

TRY IT 2.107 Solve: $\frac{b}{10} + 2 = \frac{b}{4} + 5$.

TRY IT 2.108 Solve: $\frac{c}{6} + 3 = \frac{c}{3} + 4$.

EXAMPLE 2.55

Solve: $\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$.

Solution

	$\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$
Multiply by the LCD, 4.	$4\left(\frac{4q+3}{2} + 6\right) = 4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\frac{4q+3}{2}\right) + 4 \cdot 6 = 4 \cdot \left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3) + 24 = 3q + 5$ $8q + 6 + 24 = 3q + 5$ $8q + 30 = 3q + 5$
Collect the variables to the left.	$8q - 3q + 30 = 3q - 3q + 5$
Simplify.	$5q + 30 = 5$
Collect the constants to the right.	$5q + 30 - 30 = 5 - 30$
Simplify.	$5q = -25$
Divide by 5.	$\frac{5q}{5} = \frac{-25}{5}$
Simplify.	$q = -5$

Check:	$\frac{4q+3}{2} + 6 = \frac{3q+5}{4}$
Let $q = -5$.	$\frac{4(-5)+3}{2} + 6 \stackrel{?}{=} \frac{3(-5)+5}{4}$
Finish the check on your own.	

TRY IT 2.109 Solve: $\frac{3r+5}{6} + 1 = \frac{4r+3}{3}$.

TRY IT 2.110 Solve: $\frac{2s+3}{2} + 1 = \frac{3s+2}{4}$.

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3 = \frac{3}{10}$ and $0.17 = \frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

EXAMPLE 2.56

Solve: $0.06x + 0.02 = 0.25x - 1.5$.

Solution

Look at the decimals and think of the equivalent fractions.

$$0.06 = \frac{6}{100} \qquad 0.02 = \frac{2}{100} \qquad 0.25 = \frac{25}{100} \qquad 1.5 = 1\frac{5}{10}$$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both sides by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the constants to the left.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check:	Let $x = 8$. $0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$ $0.50 = 0.50$ ✓

TRY IT 2.111 Solve: $0.14h + 0.12 = 0.35h - 2.4$.

TRY IT 2.112 Solve: $0.65k - 0.1 = 0.4k - 0.35$.

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

EXAMPLE 2.57

Solve: $0.25x + 0.05(x + 3) = 2.85$.

Solution

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting $x = 9$ into the original equation.	

TRY IT 2.113 Solve: $0.25n + 0.05(n + 5) = 2.95$.

TRY IT 2.114 Solve: $0.10d + 0.05(d - 5) = 2.15$.

SECTION 2.5 EXERCISES

Practice Makes Perfect

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

318. $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

319. $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

320. $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

321. $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

322. $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

323. $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

324. $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

325. $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

326. $\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$

327. $\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

328. $x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$

329. $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

330. $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

331. $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

332. $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

333. $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

334. $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

335. $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

336. $1 = \frac{1}{6}(12x - 6)$

337. $1 = \frac{1}{5}(15x - 10)$

338. $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$

339. $\frac{1}{5}(q + 3) = \frac{1}{2}(q - 3)$

340. $\frac{1}{2}(x + 4) = \frac{3}{4}$

341. $\frac{1}{3}(x + 5) = \frac{5}{6}$

342. $\frac{5q-8}{5} = \frac{2q}{10}$

343. $\frac{4m+2}{6} = \frac{m}{3}$

344. $\frac{4n+8}{4} = \frac{n}{3}$

345. $\frac{3p+6}{3} = \frac{p}{2}$

346. $\frac{u}{3} - 4 = \frac{u}{2} - 3$

347. $\frac{v}{10} + 1 = \frac{v}{4} - 2$

348. $\frac{c}{15}+1=\frac{c}{10}-1$

349. $\frac{d}{6}+3=\frac{d}{8}+2$

350. $\frac{3x+4}{2}+1=\frac{5x+10}{8}$

351. $\frac{10y-2}{3}+3=\frac{10y+1}{9}$

352. $\frac{7u-1}{4}-1=\frac{4u+8}{5}$

353. $\frac{3v-6}{2}+5=\frac{11v-4}{5}$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

354. $0.6y+3=9$

355. $0.4y-4=2$

356. $3.6j-2=5.2$

357. $2.1k+3=7.2$

358. $0.4x+0.6=0.5x-1.2$

359. $0.7x+0.4=0.6x+2.4$

360. $0.23x+1.47=0.37x-1.05$

361. $0.48x+1.56=0.58x-0.64$

362. $0.9x-1.25=0.75x+1.75$

363. $1.2x-0.91=0.8x+2.29$

364. $0.05n+0.10\,(n+8)=2.15$

365. $0.05n+0.10\,(n+7)=3.55$

366. $0.10d+0.25\,(d+5)=4.05$

367. $0.10d+0.25\,(d+7)=5.25$

368. $0.05\,(q-5)+0.25q=3.05$

369. $0.05\,(q-8)+0.25q=4.10$

Everyday Math

370. **Coins** Taylor has $2.00 in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d+0.01(d+2)=2$ for d, the number of dimes.

371. **Stamps** Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 8 less than the number of 49-cent stamps. Solve the equation $0.49s+0.21(s-8)=22.82$ for s, to find the number of 49-cent stamps Paula bought.

Writing Exercises

372. Explain how you find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$.

373. If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

374. If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

375. In the equation $0.35x+2.1=3.85$ what is the LCD? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can…	Confidently	With some help	No-I don't get it!
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

2.6 Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

BE PREPARED 2.16 Before you get started, take this readiness quiz.

Solve: $15t=120$.
If you missed this problem, review Example 2.13.

BE PREPARED 2.17 Solve: $6x + 24 = 96$.
If you missed this problem, review Example 2.27.

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for “speed.” The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car’s cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

Distance, Rate, and Time

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \qquad \text{where} \qquad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

HOW TO

Solve an application (with a formula).

Step 1. **Read** the problem. Make sure all the words and ideas are understood.
Step 2. **Identify** what we are looking for.
Step 3. **Name** what we are looking for. Choose a variable to represent that quantity.
Step 4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
Step 5. **Solve** the equation using good algebra techniques.
Step 6. **Check** the answer in the problem and make sure it makes sense.
Step 7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

EXAMPLE 2.58

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$

	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

TRY IT 2.115 Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

TRY IT 2.116 Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

EXAMPLE 2.59

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$

Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$$\begin{aligned} d &= rt \\ 520 &\stackrel{?}{=} 65 \cdot 8 \\ 520 &= 520✓ \end{aligned}$$	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

> **TRY IT** 2.117 Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

> **TRY IT** 2.118 Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Example 2.58 and Example 2.59, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Example 2.59, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for t. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

EXAMPLE 2.60

Solve the formula $d = rt$ for t:

ⓐ when $d = 520$ and $r = 65$ ⓑ in general

Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

ⓐ when $d = 520$ and $r = 65$			ⓑ in general	
Write the formula.	$d = rt$		Write the formula.	$d = rt$
Substitute.	$520 = 65t$			

Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$			Divide, to isolate t.	$\frac{d}{r} = \frac{rt}{r}$
Simplify.	$8 = t$			Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

TRY IT 2.119 Solve the formula $d = rt$ for r:

ⓐ when $d = 180$ and $t = 4$ ⓑ in general

TRY IT 2.120 Solve the formula $d = rt$ for r:

ⓐ when $d = 780$ and $t = 12$ ⓑ in general

EXAMPLE 2.61

Solve the formula $A = \frac{1}{2}bh$ for h:

ⓐ when $A = 90$ and $b = 15$ ⓑ in general

Solution

ⓐ when $A = 90$ and $b = 15$				ⓑ in general	
Write the formula.	$A = \frac{1}{2}bh$			Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$				
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2} 15h$			Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$			Simplify.	$2A = bh$
Solve for h.	$12 = h$			Solve for h.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$.

TRY IT 2.121 Use the formula $A = \frac{1}{2}bh$ to solve for h:

ⓐ when $A = 170$ and $b = 17$ ⓑ in general

TRY IT 2.122 Use the formula $A = \frac{1}{2}bh$ to solve for b:

ⓐ when $A = 62$ and $h = 31$ ⓑ in general

The formula $I = Prt$ is used to calculate simple interest, I, for a principal, P, invested at rate, r, for t years.

EXAMPLE 2.62

Solve the formula $I = Prt$ to find the principal, P:

ⓐ when $I = \$5{,}600, r = 4\%, t = 7$ years ⓑ in general

Solution

ⓐ $I = \$5{,}600$, $r = 4\%$, $t = 7$ years ⓑ in general

Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate P.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate P.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20{,}000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000		$P = \frac{I}{rt}$

TRY IT 2.123 Use the formula $I = Prt$ to find the principal, P:

ⓐ when $I = \$2{,}160, r = 6\%, t = 3$ years ⓑ in general

TRY IT 2.124 Use the formula $I = Prt$ to find the principal, P:

ⓐ when $I = \$5{,}400, r = 12\%, t = 5$ years ⓑ in general

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we're given an equation with both *x* and *y* on the same side and we'll solve it for *y*.

EXAMPLE 2.63

Solve the formula $3x + 2y = 18$ for *y*:

ⓐ when $x = 4$ ⓑ in general

Solution

ⓐ when $x = 4$ ⓑ in general

	$3x + 2y = 18$		$3x + 2y = 18$
Substitute.	$3(4) + 2y = 18$		
Subtract to isolate the y-term.	$12 - 12 + 2y = 18 - 12$	Subtract to isolate the y-term.	$3x - 3x + 2y = 18 - 3x$

Divide.	$\frac{2y}{2}=\frac{6}{2}$	Divide.	$\frac{2y}{2}=\frac{18}{2}-\frac{3x}{2}$
Simplify.	$y=3$	Simplify.	$y=-\frac{3x}{2}+9$

TRY IT 2.125 Solve the formula $3x+4y=10$ for y:

ⓐ when $x=\frac{14}{3}$ ⓑ in general

TRY IT 2.126 Solve the formula $5x+2y=18$ for y:

ⓐ when $x=4$ ⓑ in general

In Examples 1.60 through 1.64 we used the numbers in part ⓐ as a guide to solving in general in part ⓑ. Now we will solve a formula in general without using numbers as a guide.

EXAMPLE 2.64

Solve the formula $P=a+b+c$ for a.

Solution

We will isolate a on one side of the equation.	$P=a+b+c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P-b-c=a+b+c-b-c$
Simplify.	$P-b-c=a$ $a=P-b-c$

TRY IT 2.127 Solve the formula $P=a+b+c$ for b.

TRY IT 2.128 Solve the formula $P=a+b+c$ for c.

EXAMPLE 2.65

Solve the formula $6x+5y=13$ for y.

Solution

	$6x+5y=13$
Subtract $6x$ from both sides to isolate the term with y.	$6x-6x+5y=13-6x$
Simplify.	$5y=13-6x$

Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13-6x}{5}$
Simplify.	$y = \frac{13-6x}{5}$

The fraction is simplified. We cannot divide $13 - 6x$ by 5.

TRY IT 2.129 Solve the formula $4x + 7y = 9$ for y.

TRY IT 2.130 Solve the formula $5x + 8y = 1$ for y.

SECTION 2.6 EXERCISES

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

376. Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?

377. Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

378. Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

379. Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

380. Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

381. Megan is taking the bus from New York City to Montreal. The distance is 380 miles and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

382. Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

383. Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

384. Javier is driving to Bangor, 240 miles away. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

385. Alejandra is driving to Cincinnati, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

386. Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

387. Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Solve a Formula for a Specific Variable

In the following exercises, use the formula $d = rt$.

388. Solve for t ⓐ when $d = 350$ and $r = 70$ ⓑ in general

389. Solve for t ⓐ when $d = 240$ and $r = 60$ ⓑ in general

390. Solve for t ⓐ when $d = 510$ and $r = 60$ ⓑ in general

391. Solve for t
ⓐ when $d = 175$ and $r = 50$
ⓑ in general

392. Solve for r
ⓐ when $d = 204$ and $t = 3$ ⓑ in general

393. Solve for r ⓐ when $d = 420$ and $t = 6$ ⓑ in general

394. Solve for r ⓐ when $d = 160$ and $t = 2.5$ ⓑ in general

395. Solve for r ⓐ when $d = 180$ and $t = 4.5$ ⓑ in general

In the following exercises, use the formula $A = \frac{1}{2}bh$.

396. Solve for b ⓐ when $A = 126$ and $h = 18$ ⓑ in general

397. Solve for h
ⓐ when $A = 176$ and $b = 22$ ⓑ in general

398. Solve for h ⓐ when $A = 375$ and $b = 25$ ⓑ in general

399. Solve for b ⓐ when $A = 65$ and $h = 13$ ⓑ in general

In the following exercises, use the formula I = Prt.

400. Solve for the principal, *P* for ⓐ $I = \$5{,}480, r = 4\%, t = 7$ years ⓑ in general

401. Solve for the principal, *P* for
ⓐ $I = \$3{,}950, r = 6\%, t = 5$ years ⓑ in general

402. Solve for the time, *t* for ⓐ $I = \$2{,}376, P = \$9{,}000, r = 4.4\%$ ⓑ in general

403. Solve for the time, *t* for
ⓐ $I = \$624, P = \$6{,}000, r = 5.2\%$ ⓑ in general

In the following exercises, solve.

404. Solve the formula $2x + 3y = 12$ for y ⓐ when $x = 3$ ⓑ in general

405. Solve the formula $5x + 2y = 10$ for y ⓐ when $x = 4$ ⓑ in general

406. Solve the formula $3x - y = 7$ for y ⓐ when $x = -2$ ⓑ in general

407. Solve the formula $4x + y = 5$ for y ⓐ when $x = -3$ ⓑ in general

408. Solve $a + b = 90$ for b.

409. Solve $a + b = 90$ for a.

410. Solve $180 = a + b + c$ for a.

411. Solve $180 = a + b + c$ for c.

412. Solve the formula $8x + y = 15$ for y.

413. Solve the formula $9x + y = 13$ for y.

414. Solve the formula $-4x + y = -6$ for y.

415. Solve the formula $-5x + y = -1$ for y.

416. Solve the formula $4x + 3y = 7$ for y.

417. Solve the formula $3x + 2y = 11$ for y.

418. Solve the formula $x - y = -4$ for y.

419. Solve the formula $x - y = -3$ for y.

420. Solve the formula $P = 2L + 2W$ for L.

421. Solve the formula $P = 2L + 2W$ for W.

422. Solve the formula $C = \pi d$ for d.

423. Solve the formula $C = \pi d$ for π.

424. Solve the formula $V = LWH$ for L.

425. Solve the formula $V = LWH$ for H.

Everyday Math

426. **Converting temperature** While on a tour in Greece, Tatyana saw that the temperature was 40^{o} Celsius. Solve for F in the formula $C=\frac{5}{9}(F-32)$ to find the Fahrenheit temperature.

427. **Converting temperature** Yon was visiting the United States and he saw that the temperature in Seattle one day was 50^{o} Fahrenheit. Solve for C in the formula $F=\frac{9}{5}C+32$ to find the Celsius temperature.

Writing Exercises

428. Solve the equation $2x+3y=6$ for y ⓐ when $x=-3$ ⓑ in general ⓒ Which solution is easier for you, ⓐ or ⓑ ? Why?

429. Solve the equation $5x-2y=10$ for x ⓐ when $y=10$ ⓑ in general ⓒ Which solution is easier for you, ⓐ or ⓑ ? Why?

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

ⓑ *What does this checklist tell you about your mastery of this section? What steps will you take to improve?*

2.7 Solve Linear Inequalities

Learning Objectives

By the end of this section, you will be able to:

- Graph inequalities on the number line
- Solve inequalities using the Subtraction and Addition Properties of inequality
- Solve inequalities using the Division and Multiplication Properties of inequality
- Solve inequalities that require simplification
- Translate to an inequality and solve

BE PREPARED 2.18 Before you get started, take this readiness quiz.

Translate from algebra to English: $15>x$.
If you missed this problem, review Example 1.12.

BE PREPARED 2.19 Solve: $n-9=-42$.
If you missed this problem, review Example 2.3.

BE PREPARED 2.20 Solve: $-5p=-23$.
If you missed this problem, review Example 2.13.

BE PREPARED 2.21 Solve: $3a-12=7a-20$.
If you missed this problem, review Example 2.34.

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

What about the solution of an inequality? What number would make the inequality $x>3$ true? Are you thinking, '*x* could be 4'? That's correct, but *x* could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality

$x > 3$.

We show the solutions to the inequality $x > 3$ on the number line by shading in all the numbers to the right of 3, to show that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown in Figure 2.7. Please note that the following convention is used: light blue arrows point in the positive direction and dark blue arrows point in the negative direction.

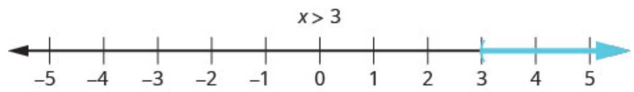


Figure 2.7 The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We do that by putting a bracket at $x = 3$, as shown in Figure 2.8.

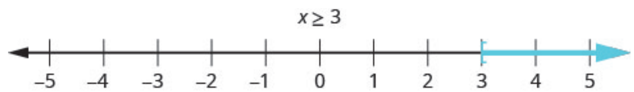


Figure 2.8 The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

EXAMPLE 2.66

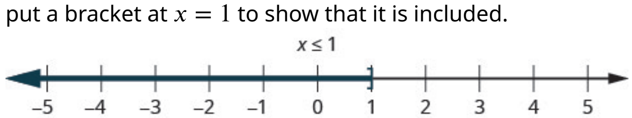
Graph on the number line:

ⓐ $x \leq 1$ ⓑ $x < 5$ ⓒ $x > -1$

Solution

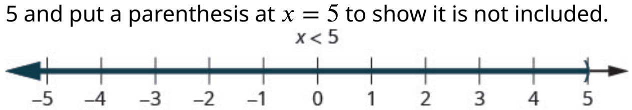
ⓐ $x \leq 1$

This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at $x = 1$ to show that it is included.



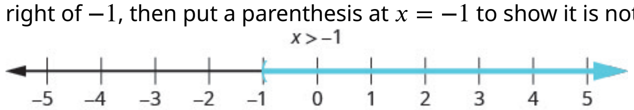
ⓑ $x < 5$

This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put a parenthesis at $x = 5$ to show it is not included.



ⓒ $x > -1$

This means all numbers greater than -1, but not including -1. We shade in all the numbers on the number line to the right of -1, then put a parenthesis at $x = -1$ to show it is not included.



TRY IT 2.131 Graph on the number line: ⓐ $x \leq -1$ ⓑ $x > 2$ ⓒ $x < 3$

TRY IT 2.132 Graph on the number line: ⓐ $x > -2$ ⓑ $x < -3$ ⓒ $x \geq -1$

We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In interval notation, we express $x > 3$ as $\left(3, \infty\right)$.

The symbol ∞ is read as 'infinity'. It is not an actual number. Figure 2.9 shows both the number line and the interval notation.

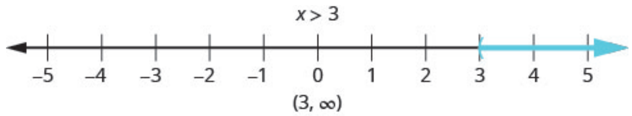


Figure 2.9 The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $\left(-\infty, 1\right]$. The symbol $-\infty$ is read as 'negative infinity'. Figure 2.10 shows both the number line and interval notation.

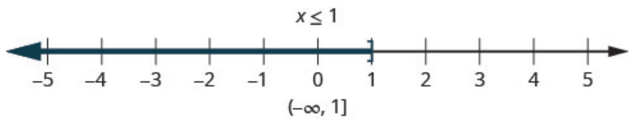


Figure 2.10 The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

Inequalities, Number Lines, and Interval Notation

Did you notice how the parenthesis or bracket in the interval notation matches the symbol at the endpoint of the arrow? These relationships are shown in Figure 2.11.

Figure 2.11 The notation for inequalities on a number line and in interval notation use similar symbols to express the endpoints of intervals.

EXAMPLE 2.67

Graph on the number line and write in interval notation.

ⓐ $x \geq -3$ ⓑ $x < 2.5$ ⓒ $x \leq -\frac{3}{5}$

Solution

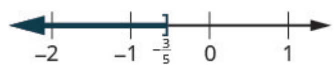
ⓐ

	$x \geq -3$
Shade to the right of -3, and put a bracket at -3.	
Write in interval notation.	$[-3, \infty)$

ⓑ

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

ⓒ

	$x \leq -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

TRY IT 2.133 Graph on the number line and write in interval notation:

ⓐ $x > 2$ ⓑ $x \leq -1.5$ ⓒ $x \geq \frac{3}{4}$

TRY IT 2.134 Graph on the number line and write in interval notation:

ⓐ $x \leq -4$ ⓑ $x \geq 0.5$ ⓒ $x < -\frac{2}{3}$

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

Properties of Equality

Subtraction Property of Equality	Addition Property of Equality
For any numbers a, b, and c,	For any numbers a, b, and c,
if $\quad a = b,$	if $\quad a = b,$
then $\quad a - c = b - c.$	then $\quad a + c = b + c.$

Similar properties hold true for inequalities.

For example, we know that −4 is less than 2.	$-4 < 2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4 - 5 \; ? \; 2 - 5$
We get −9 on the left and −3 on the right.	$-9 \; ? \; -3$

And we know −9 is less than −3.	$-9 < -3$
	The inequality sign stayed the same.

Similarly we could show that the inequality also stays the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

Properties of Inequality

Subtraction Property of Inequality

For any numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a-c & < & b-c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a-c & > & b-c. \end{array}$$

Addition Property of Inequality

For any numbers a, b, and c,

$$\begin{array}{lrcl} \text{if} & a & < & b \\ \text{then} & a+c & < & b+c. \end{array}$$

$$\begin{array}{lrcl} \text{if} & a & > & b \\ \text{then} & a+c & > & b+c. \end{array}$$

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x + 5 > 9$, the steps would look like this:

	$x + 5 > 9$
Subtract 5 from both sides to isolate x.	$x + 5 - 5 > 9 - 5$
Simplify.	$x > 4$

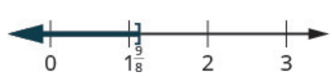
Table 2.6

Any number greater than 4 is a solution to this inequality.

EXAMPLE 2.68

Solve the inequality $n - \frac{1}{2} \le \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$n - \frac{1}{2} \le \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n - \frac{1}{2} + \frac{1}{2} \le \frac{5}{8} + \frac{1}{2}$
Simplify.	$n \le \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left(-\infty, \frac{9}{8}\right]$

TRY IT 2.135 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$p - \frac{3}{4} \geq \frac{1}{6}$

TRY IT 2.136 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$r - \frac{1}{3} \leq \frac{7}{12}$

Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don't divide by 0).

Properties of Equality

Division Property of Equality

For any numbers a, b, c, and $c \neq 0$,

$$\begin{array}{lccc} \text{if} & a & = & b, \\ \text{then} & \frac{a}{c} & = & \frac{b}{c}. \end{array}$$

Multiplication Property of Equality

For any real numbers a, b, c,

$$\begin{array}{lccc} \text{if} & a & = & b, \\ \text{then} & ac & = & bc. \end{array}$$

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples.

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} \ ? \ \frac{15}{5}$	Multiply both sides by 5.	$10(5) \ ? \ 15(5)$
Simplify.	$2 \ ? \ 3$		$50 \ ? \ 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

	$10 < 15$		$10 < 15$
Divide both sides by −5.	$\frac{10}{-5} \ ? \ \frac{15}{-5}$	Multiply both sides by −5.	$10(-5) \ ? \ 15(-5)$
Simplify.	$-2 \ ? \ -3$		$-50 \ ? \ -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

Division and Multiplication Properties of Inequality

For any real numbers a, b, c

$$\begin{array}{ll} \text{if } a < b \text{ and } c > 0, \text{ then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc. \\ \text{if } a > b \text{ and } c > 0, \text{ then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc. \\ \text{if } a < b \text{ and } c < 0, \text{ then} & \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc. \\ \text{if } a > b \text{ and } c < 0, \text{ then} & \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc. \end{array}$$

When we **divide or multiply** an inequality by a:

- **positive** number, the inequality stays the **same**.
- **negative** number, the inequality **reverses**.

EXAMPLE 2.69

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

> **TRY IT** 2.137 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $c - 8 > 0$

> **TRY IT** 2.138 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $12d \leq 60$

EXAMPLE 2.70

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$-10a \geq 50$
Divide both sides of the inequality by −10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

TRY IT 2.139 Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-8q < 32$

TRY IT 2.140 Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq \quad -70$

Solving Inequalities

Sometimes when solving an inequality, the variable ends up on the right. We can rewrite the inequality in reverse to get the variable to the left.

$$x > a \text{ has the same meaning as } a < x$$

Think about it as "If Xavier is taller than Alex, then Alex is shorter than Xavier."

EXAMPLE 2.71

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$

Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

> **TRY IT** 2.141 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $24 \le \frac{3}{8}m$

> **TRY IT** 2.142 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $-24 < \frac{4}{3}n$

EXAMPLE 2.72

Solve the inequality $\frac{t}{-2} \ge 8$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$\frac{t}{-2} \ge 8$
Multiply both sides of the inequality by -2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \le -2(8)$
Simplify.	$t \le -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

> **TRY IT** 2.143 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $\frac{k}{-12} \le 15$

> **TRY IT** 2.144 Solve the inequality, graph the solution on the number line, and write the solution in interval notation.
>
> $\frac{u}{-4} \ge -16$

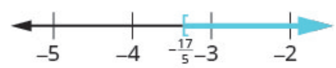
Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

EXAMPLE 2.73

Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$4m \leq 9m + 17$
Subtract $9m$ from both sides to collect the variables on the left.	$4m - 9m \leq 9m - 9m + 17$
Simplify.	$-5m \leq 17$
Divide both sides of the inequality by −5, and reverse the inequality.	$\frac{-5m}{-5} \geq \frac{17}{-5}$
Simplify.	$m \geq -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

TRY IT 2.145 Solve the inequality $3q \geq 7q - 23$, graph the solution on the number line, and write the solution in interval notation.

TRY IT 2.146 Solve the inequality $6x < 10x + 19$, graph the solution on the number line, and write the solution in interval notation.

EXAMPLE 2.74

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

Solution

Simplify each side as much as possible.	$8p + 3(p - 12) > 7p - 28$
Distribute.	$8p + 3p - 36 > 7p - 28$
Combine like terms.	$11p - 36 > 7p - 28$
Subtract $7p$ from both sides to collect the variables on the left.	$11p - 36 - 7p > 7p - 28 - 7p$
Simplify.	$4p - 36 > -28$
Add 36 to both sides to collect the constants on the right.	$4p - 36 + 36 > -28 + 36$
Simplify.	$4p > 8$

Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > \frac{8}{4}$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interal notation.	$(2, \infty)$

TRY IT 2.147 Solve the inequality $9y + 2(y + 6) > 5y - 24$, graph the solution on the number line, and write the solution in interval notation.

TRY IT 2.148 Solve the inequality $6u + 8(u - 1) > 10u + 32$, graph the solution on the number line, and write the solution in interval notation.

Just like some equations are identities and some are contradictions, inequalities may be identities or contradictions, too. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an identity. If the result is a false statement, we have a contradiction.

EXAMPLE 2.75

Solve the inequality $8x - 2\,(5 - x) < 4\,(x + 9) + 6x$, graph the solution on the number line, and write the solution in interval notation.

Solution

Simplify each side as much as possible.	$8x - 2(5 - x) < 4(x + 9) + 6x$
Distribute.	$8x - 10 + 2x < 4x + 36 + 6x$
Combine like terms.	$10x - 10 < 10x + 36$
Subtract $10x$ from both sides to collect the variables on the left.	$10x - 10 - 10x < 10x + 36 - 10x$
Simplify.	$-10 < 36$
The x's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \infty)$

TRY IT 2.149 Solve the inequality $4b - 3(3 - b) > 5(b - 6) + 2b$, graph the solution on the number line, and write the solution in interval notation.

TRY IT 2.150 Solve the inequality $9h - 7\,(2 - h) < 8\,(h + 11) + 8h$, graph the solution on the number line, and write the solution in interval notation.

EXAMPLE 2.76

Solve the inequality $\frac{1}{3}a - \frac{1}{8}a > \frac{5}{24}a + \frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$\frac{1}{3}a - \frac{1}{8}a > \frac{5}{24}a + \frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a - \frac{1}{8}a\right) > 24\left(\frac{5}{24}a + \frac{3}{4}\right)$
Simplify.	$8a - 3a > 5a + 18$
Combine like terms.	$5a > 5a + 18$
Subtract $5a$ from both sides to collect the variables on the left.	$5a - 5a > 5a - 5a + 18$
Simplify.	$0 > 18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

TRY IT 2.151 Solve the inequality $\frac{1}{4}x - \frac{1}{12}x > \frac{1}{6}x + \frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

TRY IT 2.152 Solve the inequality $\frac{2}{5}z - \frac{1}{3}z < \frac{1}{15}z - \frac{3}{5}$, graph the solution on the number line, and write the solution in interval notation.

Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like 'more than' and 'less than'. But others are not as obvious.

Think about the phrase 'at least' – what does it mean to be 'at least 21 years old'? It means 21 or more. The phrase 'at least' is the same as 'greater than or equal to'.

Table 2.7 shows some common phrases that indicate inequalities.

$>$	$\geq$	$<$	$\leq$
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most

Table 2.7

$>$	$\geq$	$<$	$\leq$
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

Table 2.7

EXAMPLE 2.77

Translate and solve. Then write the solution in interval notation and graph on the number line.

Twelve times *c* is no more than 96.

Solution

Translate.	Twelve times c $\underbrace{\text{is no more than}}$ 96 $12c \leq 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \leq \frac{96}{12}$
Simplify.	$c \leq 8$
Write in interval notation.	$(-\infty, 8]$
Graph on the number line.	

TRY IT 2.153 Translate and solve. Then write the solution in interval notation and graph on the number line.

Twenty times *y* is at most 100

TRY IT 2.154 Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times *z* is no less than 135

EXAMPLE 2.78

Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than *x* is at least 45.

Solution

Translate.	Thirty less than x $\underbrace{\text{is at least}}$ 45. $x - 30 \geq 45$
Solve—add 30 to both sides.	$x - 30 + 30 \geq 45 + 30$
Simplify.	$x \geq 75$

Write in interval notation.	[75, ∞)
Graph on the number line.	

TRY IT 2.155 Translate and solve. Then write the solution in interval notation and graph on the number line.

Nineteen less than p is no less than 47

TRY IT 2.156 Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than a is at most 15.

SECTION 2.7 EXERCISES

Practice Makes Perfect

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

430. (a) $x \le -2$ (b) $x > -1$ (c) $x < 0$

431. (a) $x > 1$ (b) $x < -2$ (c) $x \ge -3$

432. (a) $x \ge -3$ (b) $x < 4$ (c) $x \le -2$

433. (a) $x \le 0$ (b) $x > -4$ (c) $x \ge -1$

In the following exercises, graph each inequality on the number line and write in interval notation.

434. (a) $x < -2$ (b) $x \ge -3.5$ (c) $x \le \frac{2}{3}$

435. (a) $x > 3$ (b) $x \le -0.5$ (c) $x \ge \frac{1}{3}$

436. (a) $x \ge -4$ (b) $x < 2.5$ (c) $x > -\frac{3}{2}$

437. (a) $x \le 5$ (b) $x \ge -1.5$ (c) $x < -\frac{7}{3}$

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

438. $n - 11 < 33$

439. $m - 45 \le 62$

440. $u + 25 > 21$

441. $v + 12 > 3$

442. $a + \frac{3}{4} \ge \frac{7}{10}$

443. $b + \frac{7}{8} \ge \frac{1}{6}$

444. $f - \frac{13}{20} < -\frac{5}{12}$

445. $g - \frac{11}{12} < -\frac{5}{18}$

Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

446. $8x > 72$

447. $6y < 48$

448. $7r \le 56$

449. $9s \geq 81$

450. $-5u \geq 65$

451. $-8v \leq 96$

452. $-9c < 126$

453. $-7d > 105$

454. $20 > \frac{2}{5}h$

455. $40 < \frac{5}{8}k$

456. $\frac{7}{6}j \geq 42$

457. $\frac{9}{4}g \leq 36$

458. $\frac{a}{-3} \leq 9$

459. $\frac{b}{-10} \geq 30$

460. $-25 < \frac{p}{-5}$

461. $-18 > \frac{q}{-6}$

462. $9t \geq -27$

463. $7s < -28$

464. $\frac{2}{3}y > -36$

465. $\frac{3}{5}x \leq -45$

Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

466. $4v \geq 9v - 40$

467. $5u \leq 8u - 21$

468. $13q < 7q - 29$

469. $9p > 14p - 18$

470. $12x + 3(x + 7) > 10x - 24$

471. $9y + 5(y + 3) < 4y - 35$

472. $6h - 4(h - 1) \leq 7h - 11$

473. $4k - (k - 2) \geq 7k - 26$

474. $8m - 2(14 - m) \geq 7(m - 4) + 3m$

475. $6n - 12(3 - n) \leq 9(n - 4) + 9n$

476. $\frac{3}{4}b - \frac{1}{3}b < \frac{5}{12}b - \frac{1}{2}$

477. $9u + 5(2u - 5) \geq 12(u - 1) + 7u$

478. $\frac{2}{3}g - \frac{1}{2}(g - 14) \leq \frac{1}{6}(g + 42)$

479. $\frac{5}{6}a - \frac{1}{4}a > \frac{7}{12}a + \frac{2}{3}$

480. $\frac{4}{5}h - \frac{2}{3}(h - 9) \geq \frac{1}{15}(2h + 90)$

481. $12v + 3(4v - 1) \leq 19(v - 2) + 5v$

Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

482. $15k \leq -40$

483. $35k \geq -77$

484. $23p - 2(6 - 5p) > 3(11p - 4)$

485. $18q - 4(10 - 3q) < 5(6q - 8)$

486. $-\frac{9}{4}x \geq -\frac{5}{12}$

487. $-\frac{21}{8}y \leq -\frac{15}{28}$

488. $c + 34 < -99$

489. $d + 29 > -61$

490. $\frac{m}{18} \geq -4$

491. $\frac{n}{13} \leq -6$

Translate to an Inequality and Solve

In the following exercises, translate and solve .Then write the solution in interval notation and graph on the number line.

492. Fourteen times *d* is greater than 56.

493. Ninety times *c* is less than 450.

494. Eight times *z* is smaller than -40.

495. Ten times *y* is at most -110.

496. Three more than *h* is no less than 25.

497. Six more than *k* exceeds 25.

498. Ten less than *w* is at least 39.

499. Twelve less than *x* is no less than 21.

500. Negative five times *r* is no more than 95.

501. Negative two times *s* is lower than 56.

502. Nineteen less than *b* is at most -22.

503. Fifteen less than *a* is at least -7.

Everyday Math

504. **Safety** A child's height, *h*, must be at least 57 inches for the child to safely ride in the front seat of a car. Write this as an inequality.

505. **Fighter pilots** The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

506. **Elevators** The total weight, *w*, of an elevator's passengers can be no more than 1,200 pounds. Write this as an inequality.

507. **Shopping** The number of items, *n*, a shopper can have in the express check-out lane is at most 8. Write this as an inequality.

Writing Exercises

508. Give an example from your life using the phrase 'at least'.

509. Give an example from your life using the phrase 'at most'.

510. Explain why it is necessary to reverse the inequality when solving $-5x > 10$.

511. Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$.

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

ⓑ *What does this checklist tell you about your mastery of this section? What steps will you take to improve?*

Chapter Review

Key Terms

conditional equation An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

solution of an equation A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Key Concepts

2.1 Solve Equations Using the Subtraction and Addition Properties of Equality

- **To Determine Whether a Number is a Solution to an Equation**
  Step 1. **Substitute the number in for the variable in the equation.**
  Step 2. **Simplify the expressions on both sides of the equation.**
  Step 3. **Determine whether the resulting statement is true.**
    - If it is true, the number is a solution.
    - If it is not true, the number is not a solution.

- **Addition Property of Equality**
  - For any numbers *a*, *b*, and *c*, if $a = b$, then $a + c = b + c$.

- **Subtraction Property of Equality**
  - For any numbers *a*, *b*, and *c*, if $a = b$, then $a - c = b - c$.

- **To Translate a Sentence to an Equation**
  Step 1. Locate the “equals” word(s). Translate to an equal sign (=).
  Step 2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
  Step 3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

- **To Solve an Application**
  Step 1. Read the problem. Make sure all the words and ideas are understood.
  Step 2. Identify what we are looking for.
  Step 3. Name what we are looking for. Choose a variable to represent that quantity.
  Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
  Step 5. Solve the equation using good algebra techniques.
  Step 6. Check the answer in the problem and make sure it makes sense.
  Step 7. Answer the question with a complete sentence.

2.2 Solve Equations using the Division and Multiplication Properties of Equality

- **The Division Property of Equality**—For any numbers *a*, *b*, and *c*, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
  When you divide both sides of an equation by any non-zero number, you still have equality.
- **The Multiplication Property of Equality**—For any numbers *a*, *b*, and *c*, if $a = b$, then $ac = bc$.
  If you multiply both sides of an equation by the same number, you still have equality.

2.3 Solve Equations with Variables and Constants on Both Sides

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**
  Step 1. Choose which side will be the “variable” side—the other side will be the “constant” side.
  Step 2. Collect the variable terms to the “variable” side of the equation, using the Addition or Subtraction Property of Equality.
  Step 3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  Step 4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  Step 5. Check the solution by substituting it into the original equation.

2.4 Use a General Strategy to Solve Linear Equations

- **General Strategy for Solving Linear Equations**

Step 1. Simplify each side of the equation as much as possible.
Use the Distributive Property to remove any parentheses.
Combine like terms.
Step 2. Collect all the variable terms on one side of the equation.
Use the Addition or Subtraction Property of Equality.
Step 3. Collect all the constant terms on the other side of the equation.
Use the Addition or Subtraction Property of Equality.
Step 4. Make the coefficient of the variable term to equal to 1.
Use the Multiplication or Division Property of Equality.
State the solution to the equation.
Step 5. Check the solution.
Substitute the solution into the original equation.

2.5 Solve Equations with Fractions or Decimals

- **Strategy to Solve an Equation with Fraction Coefficients**
  Step 1. Find the least common denominator of all the fractions in the equation.
  Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.
  Step 3. Solve using the General Strategy for Solving Linear Equations.

2.6 Solve a Formula for a Specific Variable

- **To Solve an Application (with a formula)**
  Step 1. **Read** the problem. Make sure all the words and ideas are understood.
  Step 2. **Identify** what we are looking for.
  Step 3. **Name** what we are looking for. Choose a variable to represent that quantity.
  Step 4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  Step 5. **Solve** the equation using good algebra techniques.
  Step 6. **Check** the answer in the problem and make sure it makes sense.
  Step 7. **Answer** the question with a complete sentence.

- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: $d = rt$ where *d* = distance, *r* = rate, *t* = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

2.7 Solve Linear Inequalities

- **Subtraction Property of Inequality**
  For any numbers a, b, and c,
  if $a < b$ then $a - c < b - c$ and
  if $a > b$ then $a - c > b - c$.
- **Addition Property of Inequality**
  For any numbers a, b, and c,
  if $a < b$ then $a + c < b + c$ and
  if $a > b$ then $a + c > b + c$.
- **Division and Multiplication Properties of Inequality**
  For any numbers a, b, and c,
  if $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac > bc$.
  if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$.
  if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.
- When we **divide or multiply** an inequality by a:
  - **positive** number, the inequality stays the **same**.
  - **negative** number, the inequality **reverses**.

Exercises

Review Exercises

Solve Equations using the Subtraction and Addition Properties of Equality

Verify a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

512. $10x - 1 = 5x; x = \frac{1}{5}$

513. $w + 2 = \frac{5}{8}; w = \frac{3}{8}$

514. $-12n + 5 = 8n; n = -\frac{5}{4}$

515. $6a - 3 = -7a, a = \frac{3}{13}$

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction Property of Equality.

516. $x + 7 = 19$

517. $y + 2 = -6$

518. $a + \frac{1}{3} = \frac{5}{3}$

519. $n + 3.6 = 5.1$

In the following exercises, solve each equation using the Addition Property of Equality.

520. $u - 7 = 10$

521. $x - 9 = -4$

522. $c - \frac{3}{11} = \frac{9}{11}$

523. $p - 4.8 = 14$

In the following exercises, solve each equation.

524. $n - 12 = 32$

525. $y + 16 = -9$

526. $f + \frac{2}{3} = 4$

527. $d - 3.9 = 8.2$

Solve Equations That Require Simplification

In the following exercises, solve each equation.

528. $y + 8 - 15 = -3$

529. $7x + 10 - 6x + 3 = 5$

530. $6(n - 1) - 5n = -14$

531. $8(3p + 5) - 23(p - 1) = 35$

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

532. The sum of -6 and m is 25.

533. Four less than n is 13.

Translate and Solve Applications

In the following exercises, translate into an algebraic equation and solve.

534. Rochelle’s daughter is 11 years old. Her son is 3 years younger. How old is her son?

535. Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

536. Peter paid \$9.75 to go to the movies, which was \$46.25 less than he paid to go to a concert. How much did he pay for the concert?

537. Elissa earned \$152.84 this week, which was \$21.65 more than she earned last week. How much did she earn last week?

Solve Equations using the Division and Multiplication Properties of Equality

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the division and multiplication properties of equality and check the solution.

538. $8x = 72$

539. $13a = -65$

540. $0.25p = 5.25$

541. $-y = 4$

542. $\frac{n}{6} = 18$

543. $\frac{y}{-10} = 30$

544. $36 = \frac{3}{4}x$

545. $\frac{5}{8}u = \frac{15}{16}$

546. $-18m = -72$

547. $\frac{c}{9} = 36$

548. $0.45x = 6.75$

549. $\frac{11}{12} = \frac{2}{3}y$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

550. $5r - 3r + 9r = 35 - 2$

551. $24x + 8x - 11x = -7 - 14$

552. $\frac{11}{12}n - \frac{5}{6}n = 9 - 5$

553. $-9(d - 2) - 15 = -24$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

554. 143 is the product of -11 and *y*.

555. The quotient of *b* and 9 is -27.

556. The sum of *q* and one-fourth is one.

557. The difference of *s* and one-twelfth is one fourth.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

558. Ray paid $21 for 12 tickets at the county fair. What was the price of each ticket?

559. Janet gets paid $24 per hour. She heard that this is $\frac{3}{4}$ of what Adam is paid. How much is Adam paid per hour?

Solve Equations with Variables and Constants on Both Sides

Solve an Equation with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

560. $8p + 7 = 47$

561. $10w - 5 = 65$

562. $3x + 19 = -47$

563. $32 = -4 - 9n$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

564. $7y = 6y - 13$

565. $5a + 21 = 2a$

566. $k = -6k - 35$

567. $4x - \frac{3}{8} = 3x$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

568. $12x - 9 = 3x + 45$

569. $5n - 20 = -7n - 80$

570. $4u + 16 = -19 - u$

571. $\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

Use a General Strategy for Solving Linear Equations

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

572. $6(x + 6) = 24$

573. $9(2p - 5) = 72$

574. $-(s + 4) = 18$

575. $8 + 3(n - 9) = 17$

576. $23 - 3(y - 7) = 8$

577. $\frac{1}{3}(6m + 21) = m - 7$

578. $4(3.5y + 0.25) = 365$

579. $0.25(q - 8) = 0.1(q + 7)$

580. $8(r - 2) = 6(r + 10)$

581. $5 + 7(2 - 5x) = 2(9x + 1) - (13x - 57)$

582. $(9n + 5) - (3n - 7) = 20 - (4n - 2)$

583. $2[-16 + 5(8k - 6)] = 8(3 - 4k) - 32$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

584. $17y - 3(4 - 2y) = 11(y - 1) + 12y - 1$

585. $9u + 32 = 15(u - 4) - 3(2u + 21)$

586. $-8(7m + 4) = -6(8m + 9)$

587. $21(c - 1) - 19(c + 1) = 2(c - 20)$

Solve Equations with Fractions and Decimals

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

588. $\frac{2}{5}n - \frac{1}{10} = \frac{7}{10}$

589. $\frac{1}{3}x + \frac{1}{5}x = 8$

590. $\frac{3}{4}a - \frac{1}{3} = \frac{1}{2}a - \frac{5}{6}$

591. $\frac{1}{2}(k - 3) = \frac{1}{3}(k + 16)$

592. $\frac{3x-2}{5} = \frac{3x+4}{8}$

593. $\frac{5y-1}{3} + 4 = \frac{-8y+4}{6}$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

594. $0.8x - 0.3 = 0.7x + 0.2$

595. $0.36u + 2.55 = 0.41u + 6.8$

596. $0.6p - 1.9 = 0.78p + 1.7$

597. $0.7y + 2.5 = 0.95y - 9.25$

Solve a Formula for a Specific Variable

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

598. Natalie drove for $7\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

599. Mallory is taking the bus from St. Louis to Chicago. The distance is 300 miles and the bus travels at a steady rate of 60 miles per hour. How long will the bus ride be?

600. Aaron's friend drove him from Buffalo to Cleveland. The distance is 187 miles and the trip took 2.75 hours. How fast was Aaron's friend driving?

601. Link rode his bike at a steady rate of 15 miles per hour for $2\frac{1}{2}$ hours. How much distance did he travel?

Solve a Formula for a Specific Variable

In the following exercises, solve.

602. Use the formula. $d = rt$ to solve for t
ⓐ when $d = 510$ and $r = 60$
ⓑ in general

603. Use the formula. $d = rt$ to solve for r
ⓐ when when $d = 451$ and $t = 5.5$
ⓑ in general

604. Use the formula $A = \frac{1}{2}bh$ to solve for b
ⓐ when $A = 390$ and $h = 26$
ⓑ in general

605. Use the formula $A = \frac{1}{2}bh$ to solve for h
ⓐ when $A = 153$ and $b = 18$
ⓑ in general

606. Use the formula $I = Prt$ to solve for the principal, *P* for
ⓐ $I = \$2,501, r = 4.1\%,$ $t = 5$ years
ⓑ in general

607. Solve the formula $4x + 3y = 6$ for *y*
ⓐ when $x = -2$
ⓑ in general

608. Solve $180 = a + b + c$ for c.

609. Solve the formula $V = LWH$ for H.

Solve Linear Inequalities

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line.

610.
ⓐ $x \leq 4$
ⓑ $x > -2$
ⓒ $x < 1$

611.
ⓐ $x > 0$
ⓑ $x < -3$
ⓒ $x \geq -1$

In the following exercises, graph each inequality on the number line and write in interval notation.

612.
ⓐ $x < -1$
ⓑ $x \geq -2.5$
ⓒ $x \leq \frac{5}{4}$

613.
ⓐ $x > 2$
ⓑ $x \leq -1.5$
ⓒ $x \geq \frac{5}{3}$

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

614. $n - 12 \leq 23$

615. $m + 14 \leq 56$

616. $a + \frac{2}{3} \geq \frac{7}{12}$

617. $b - \frac{7}{8} \geq -\frac{1}{2}$

Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

618. $9x > 54$

619. $-12d \leq 108$

620. $\frac{5}{6}j < -60$

621. $\frac{q}{-2} \geq -24$

Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

622. $6p > 15p - 30$

623. $9h - 7(h - 1) \leq 4h - 23$

624. $5n - 15(4 - n) < 10(n - 6) + 10n$

625. $\frac{3}{8}a - \frac{1}{12}a > \frac{5}{12}a + \frac{3}{4}$

Translate to an Inequality and Solve

In the following exercises, translate and solve. Then write the solution in interval notation and graph on the number line.

626. Five more than *z* is at most 19.

627. Three less than *c* is at least 360.

628. Nine times *n* exceeds 42.

629. Negative two times *a* is no more than 8.

Everyday Math

630. Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Practice Test

631. Determine whether each number is a solution to the equation $6x - 3 = x + 20$.
ⓐ 5 ⓑ $\frac{23}{5}$

In the following exercises, solve each equation.

632. $n - \frac{2}{3} = \frac{1}{4}$

633. $\frac{9}{2}c = 144$

634. $4y - 8 = 16$

635. $-8x - 15 + 9x - 1 = -21$

636. $-15a = 120$

637. $\frac{2}{3}x = 6$

638. $x - 3.8 = 8.2$

639. $10y = -5y - 60$

640. $8n - 2 = 6n - 12$

641. $9m - 2 - 4m - m = 42 - 8$

642. $-5(2x - 1) = 45$

643. $-(d - 9) = 23$

644. $\frac{1}{4}(12m - 28) = 6 - 2(3m - 1)$

645. $2(6x - 5) - 8 = -22$

646. $8(3a - 5) - 7(4a - 3) = 20 - 3a$

647. $\frac{1}{4}p - \frac{1}{3} = \frac{1}{2}$

648. $0.1d + 0.25(d + 8) = 4.1$

649. $14n - 3(4n + 5) = -9 + 2(n - 8)$

650. $9(3u - 2) - 4[6 - 8(u - 1)] = 3(u - 2)$

651. Solve the formula $x - 2y = 5$ for y
ⓐ when $x = -3$
ⓑ in general

In the following exercises, graph on the number line and write in interval notation.

652. $x \geq -3.5$

653. $x < \frac{11}{4}$

In the following exercises,, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

654. $8k \geq 5k - 120$

655. $3c - 10(c - 2) < 5c + 16$

In the following exercises, translate to an equation or inequality and solve.

656. 4 less than twice *x* is 16.

657. Fifteen more than *n* is at least 48.

658. Samuel paid \$25.82 for gas this week, which was \$3.47 less than he paid last week. How much had he paid last week?

659. Jenna bought a coat on sale for \$120, which was $\frac{2}{3}$ of the original price. What was the original price of the coat?

660. Sean took the bus from Seattle to Boise, a distance of 506 miles. If the trip took $7\frac{2}{3}$ hours, what was the speed of the bus?