TRY IT 2.2 Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in Figure 2.2. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

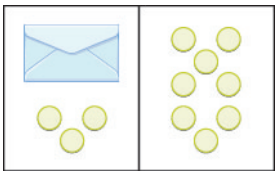

Figure 2.2 The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure 2.3 for an illustration of this process.

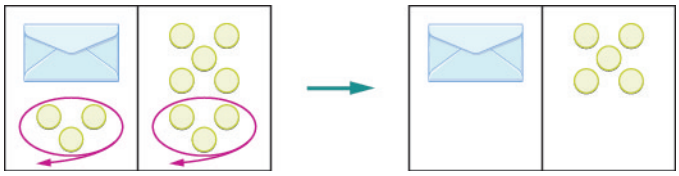

Figure 2.3 The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure 2.4 each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.

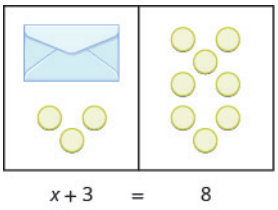

Figure 2.4 The illustration shows a model for the equation $x + 3 = 8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

$$5 + 3 = 8$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

Subtraction Property of Equality

For any numbers *a*, *b*, and *c*,

$$\begin{aligned} &\text{If} && a &&= b,\\ &\text{then} && a-c &&= b-c \end{aligned}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

MANIPULATIVE MATHEMATICS

Doing the Manipulative Mathematics activity "Subtraction Property of Equality" will help you develop a better understanding of how to solve equations by using the Subtraction Property of Equality.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

EXAMPLE 2.2

Solve: $y+37=-13$.

Solution

To get *y* by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

	$y+37=-13$
Subtract 37 from each side to 'undo' the addition.	$y+37-37=-13-37$
Simplify.	$y=-50$
Check:	$y+37=-13$
Substitute $y=-50$	$-50+37=-13$
	$-13\stackrel{?}{=}-13$ ✓

Since $y=-50$ makes $y+37=-13$ a true statement, we have the solution to this equation.

TRY IT 2.3 Solve: $x+19=-27$.

TRY IT 2.4 Solve: $x+16=-34$.

What happens when an equation has a number subtracted from the variable, as in the equation $x-5=8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

Addition Property of Equality

For any numbers *a*, *b*, and *c*,