An Improved Multiobjective Evolutionary Approach for Community Detection in Multilayer Networks

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***Abstract*—The detection of shared community structure in multilayer network is an interesting and important issue that has attracted many researches. Traditional methods for community detection of single layer networks are not suitable for that of multilayer networks. In a previous work, the authors modeled the community discovery problem in multilayer network as a multiobjective one and devised a genetic algorithm to carry out it. In this paper, based on their model, we propose an improved multiobjective evolutionary approach MOEA-MultiNet for community detection in multilayer networks. The proposed MOEA-MultiNet is based on the framework of NSGA-II which employs the string-based representation scheme and synthesizes the genetic operation and local search to perform individual refinement. Experimental results on two real-world networks both demonstrate the ability and efficiency of the proposed MOEA-MultiNet in detecting community structure in multilayer networks.**

I. Introduction

Community structure has been proved to be a vital property to discover the functionality and construction of complex networks and systems. In academic domains, vertices represent objects in the systems and links are those connections among them. Communities are actually some tightly connected sets of nodes in the network. Connections among communities are much sparser than those within communities [1], [2]. Many researchers have focused on community discovery in single-layer networks, involving weighted, directed, overlapping and dynamic networks [3], [4], [1], [5], [6]. However, in many real-world networks and systems, the relationships and interactions among objects are always multiple. For instance, people may play diverse roles in various social medias. Such networks or systems can be modeled as multilayer networks with multiple layers. The connections among vertices in different layers are mutual independent and each layer network is regarded as an aspect of the objects activity.

Multilayer network [7], which is also known as multidimensional networks [8], [9] and multiplex networks [10], [11], means vertices in different layers are with different connections. Absolutely, traditional community detection methods for single-layer networks can detect only one layer of multilayer networks. Thus the detection result of only one layer is probably not the real community structure of multilayer networks. To exploit the community structure of multilayer networks, the objective is to find a common or shared community structure that combined the connection informations of all layers networks [12].

As we known, detecting shared community structure in a multilayer network is a hard optimization problem. Evolutionary approaches are a kind of population-based stochastic methods which can combine the ability of global exploitation and local refinement. Multiobjective evolutionary approaches, which are proved to be a kind of simple and efficient methods to deal with many various hard optimization problems [13], [14], [15], have received more and more interest in recent years. Among them the most well-known method is NSGA-II [16]. NSGA-II is an effective algorithm which incorporate an elitism preservation strategy in evolutionary algorithm. The main idea of NSGA-II is to reproduce a population by genetic operator and then sort them based on the non-domination rank and crowding distance [16].

In this paper, an improved multiobjective evolutionary algorithm, denoted as MOEA-MultiNet, is proposed to deal with the problem of community detection in multilayer network. The proposed MOEA-MultiNet is based on the framework of the multiobjective algorithm NSGA-II. The proposed MOEA-MultiNet use the string-based representation scheme, and a two-way crossover and a newly k-nearest neighbor based mutation operator is incorporated in our algorithm. Besides that, a hill-climbing strategy based local search operator is employed to accelerate the convergence process of the population. Then we try to carry out the multiobjective model introduced by Amelio and Pizzuti in [12]. The authors modeled the detection of community structure as a multiobjective optimization problem. One objective is to maximize the facet quality (FQ), which can guarantee the detection quality of current layer network. The second objective is to minimize the sharing cost (SC), which reduce the difference between the current layer clustering and the previous layer one. In their paper, the maximization of FQ was realized by optimizing the modularity Q proposed by Girvan and Newman [2]. The minimization of SC was fulfilled by maximizing the well-known measure NMI between the clustering derived from the current layer with that derived from the previous layer.

This paper is organized as follows: Section II gives an introduction of the related work of community detection in multilayer networks. The proposed MOEA-MultiNet is de-

978-1-5090-4601-0/17/$31.00

scribed in detail in section III. In section IV, the comparision results of two real-world multilayer networks was used to demonstrate the effects of the proposed MOEA-MultiNet. The summary is given in the last section.

II. PROBLEM DEFINITION

In academic domain, a single layer network generally can be modeled as an graph $g = \{V, E\}$, where V is the set of nodes and E is the set of links in the network. The network g also can be represented as an adjacency matrix A. Considering the multilayer networks, a multilayer network can be denoted as $G = \{g_1, g_2, \ldots, g_m\}$, where $g_i = \{V_i, E_i\}$ is the ith layer network of the multilayer networks, and $\beta = \{1, 2, \ldots, m\}$ is the set of layers. A partition or clustering of a multilayer network is defined as $P = \{C_1, C_2, \ldots, C_k\}$.

A toy network $G = \{g_1, g_2, g_3\}$ with 8 nodes and 3 layers is shown in fig. 1. As we can see, in the toy network G, the connections among vertices in different layer network represents different types of entities interactions. In g_1, the network is a tightly connected component. In g_2 and g_3, the networks are divided into two and four isolated parts, respectively.

The partition of each layer network can be easily obtained by optimizing the modularity function Q. For example, both the partition of g_1 and g_2 are $P = \{C_1, C_2\}$, where $C_1 = \{1, 2, 3, 4\}$ and $C_2 = \{5, 6, 7, 8\}$. The partition of g_3 is given by $P = \{C_1, C_2, C_3, C_4\}$, where $C_1 = \{1, 2, 3\}$, $C_2 = \{4, 5\}$, $C_3 = \{7\}$ and $C_4 = \{6, 8\}$. The problem of community detection in multilayer networks is to find a shared community structure that considers the vertices connection informations of all layers. Obviously, the detection results of any single layer can not represent the shared community structure.

III. RELATED WORK

The main methods for community detection in single layer networks is to optimize modularity Q and some other extend metrics. But these methods can not be used to deal with the problem of community detection in multilayer networks. In fact, with the development of social media networking, people connect with each other by many different ways like Wechat, QQ and Facebook. People may have different interactions and relations in different social media.

In last years, the community detection on multiplex networks have received more and more interests. Mucha et al. [10] proposed a new index Q_m to evaluate the quality of community structure of multilayer networks and designed a heuristic approach GenLouvain to optimize this index. The Genlovain synthesizes the capacity of the community detection and extraction method, and can detect good quality community structure of multilayer networks. In [17], a new approach called mux-LICOD was proposed to carry out community detection based on seed-centric algorithm. The proposed mux-LICOD firstly find certain seed-centric nodes in the network, and then detect the communities around those seed nodes on the basis of a local computation approach.

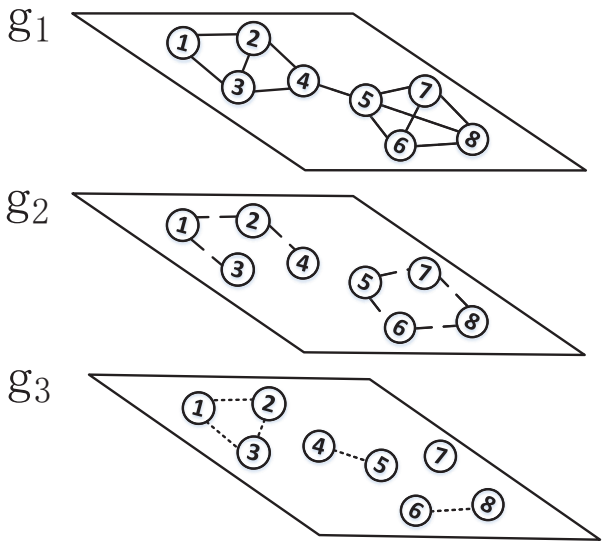


Fig. 1. A schematic illstration of a simple toy multilayer network G. The multilayer network G consists of 8 nodes and 3 layers. Different types of lines represent the connections in different layer network.

Besides optimizing Q_m, in [18], [19], these approaches firstly combined different layer networks into a single weighted network, and then adopted the classical community detection methods to detect shared community structure. The detected community structure of the weighted network is considered as a shared community structure of the multilayer network. Another way to detect community structure of a multilayer is also divided into two step. In the first step, a community detection approach is performed on each single layer of a multilayer network. In the second step, the partitions of all layer are emerged by ensemble clustering approaches. In [20], each layer network was firstly detected by the well-know BGLL algorithm. Then, a consensus community matrix was generated and BGLL was used to obtain the community structure of the multilayer network. Both these two types of approaches ignore global analysis of multilayer networks, thus the results obtained by them cannot well reflect the true community structure. In [12], Amelio and Pizzuti modeled the optimization problem of multilayer networks as a multiobjective optimization one. The idea of facet quality FQ and dimensional sharing SC was used as the two opposite objectives. Then, they proposed a multiobjective optimization method called MultiMOGA to search community structures.

IV. THE PROPOSED ALGORITHM

In this paper, we employ the multiobjective model proposed in [12] and present an improved evolutionary algorithm to solve it. That is, given a multilayer networks $G =$

$\{g_1, g_2, \ldots, g_m\}$, an optimal community structure is obtained by iteratively optimizing the partition quality of the current layer and the similarity between the partition obtained for the previous $i-1$ layer network and the partition of the current layer.

As to the first objective that maximize the partition quality, the most popular way to evaluate the quality of community structures is the modularity Q. The modularity is defined as follow.

$$Q = \frac{1}{2M}\sum_{i,j}^{N}(A_{ij} - \frac{d_i \times d_j}{2M})\delta(C_i, C_j) \tag{1}$$

where N and M are the number of nodes and edges of the network, A is the correspoding adjcency matrix of the network, d_i (d_j) denotes the degree of node $i(j)$, $C_i(C_j)$ is the community that node $i(j)$ belongs to. $\delta(C_i, C_j) = 1$, if $C_i = C_j$, otherwise, 0. Higher values of Q approaching 1 show better partitions.

Considering the second objective, the Normalized Mutual Information (NMI) [21] is used for evaluate the similarity between the current clustering and the previous one. Suppose A and B are two network partitions. Thus, $NMI(A, B)$ is written as:

$$NMI(A,B) = \frac{-2\sum_{i=1}^{l_A}\sum_{j=1}^{l_B}F_{ij}\log(F_{ij}N/F_{i\cdot}F_{\cdot j})}{\sum_{i=1}^{l_A}F_{i\cdot}\log(F_{i\cdot}/N) + \sum_{j=1}^{l_B}F_{\cdot j}\log(F_{\cdot j}/N)} \tag{2}$$

where F is a confusion matrix. $F_i(F_j)$ is the number of elements of F in row i (or column j). $l_A(l_B)$ equals to the sum of clusters in the partition $A(B)$. The value of NMI lies in the range $[0, 1]$. Obviously, the more similar the original and optimized network partitions are, the larger the value of NMI is.

Considering the modularity Q and NMI as our two objectives, for each pair of layer networks S_i and S_{i-1}, the optimization problem can be formulated as follow

$$max\ F(S_i) = max\ (Q(S_i), NMI(S_i, S_{i-1})) \tag{3}$$

We give a detail description of the proposed MOEA-MultiNet in this section. Initially, given a multilayer network after ordering, the BGLL [22] is performed in the first layer network g_1 and obtain its community structure S_1. Then, for each pair of layer networks g_i and g_{i-1} the multiobjective evolutionary algorithm based NSGA-II is performed to optimize Eq. (3). Finally, the community structure with the highest modularity in the last layer network is regarded as the optimal shared community structure. This is same as the solution selection strategy introduced in [12]. The framework of MOEA-MultiNet is shown in **Algorithm 1**.

In **Algorithm 1**, the function Initialization() is used to generate a initial population of chromosomes. The function NonDominationSort() is responsible for sorting the individuals based on non-domination rank and crowding distance. The function GeneticOperator() is used to generate offspring chromosomes. The function LocalSearch() is used to find individuals by local refinement.

Algorithm 1 Algorithm framework of MOEA-MultiNet

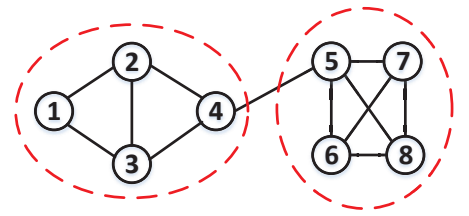
Input: A multilayer network after ordering $G = \{g_1, g_2, ..., g_m\}$; The population size n_{pop}; The mating pool size n_{pool}; The tournament size n_{tour}; the probability of crossover operator p_c and the probability of mutation operator p_m.
Perform BGLL on the first layer network g_1 and obtain the community structure S_1;
for $i = 2 : m$ **do**
    // Perform the multiobjective evolutionary algorithm with the two objectives
    $\mathbf{P} \leftarrow$ Initialization(n_{pop}, g_i);
    $\mathbf{P} \leftarrow$ NonDominationSort($\mathbf{P}$);
    repeat
        $\mathbf{P}_{parent} \leftarrow$ TournamentSelection($\mathbf{P}$, n_{pool}, n_{tour});
        $\mathbf{P}_{child} \leftarrow$ GeneticOperator($\mathbf{P}_{parent}$, p_c, p_m);
        $\mathbf{P}_{new} \leftarrow$ LocalSearch($\mathbf{P}_{child}$);
        $\mathbf{P} \leftarrow \mathbf{P} \bigcup \mathbf{P}_{new}$;
        $\mathbf{P} \leftarrow$ NonDominationSort($\mathbf{P}$);
    until Maximum generation is reached
    Select the chromosome with the highest modularity S_i as the optimal community structure of current layer network;
end for
Output: The shared community structure S_m.

It should be pointed that the performance of the proposed MOEA-MultiNet can be greatly influenced by the ordering of networks. In [12], the authors introduced a clustering coefficient based ordering and obtained the best experimental results. In this paper, we compare the performance between using the random ordering and the clustering coefficient based ordering and also find that the latter one can get the best results. We thus use the clustering coefficient based ordering in this paper. The details of the experimental results is shown in section V.

A. Representation and initialization

The MultiMOGA proposed in [12] used the locus-based representation to encode individuals, which may cause some nodes without allocating any label. Thus, a label assignment procedure is devised to assign labels to those nodes. However, this procedure may assign inappropriate labels to these isolated nodes and increase the computation complexity. In our approach, we employ a string-based representation. This representation has two advantages. First, each node in individuals will be assigned an integer label. Thus, there is no need to implement the label assignment process. Second, string-based representation scheme can give benefit to individual reproduction operations. In the encoding step of MOEA-MultiNet, a chromosome is represented by a integer string $S = \{s^1, s^2, \ldots, s^N\}$, where N is the total number of vertices in the network, and s^i corresponds the cluster identifier of

node i and its value lies in the range $[1, N]$. Nodes with the same cluster identifier will be assigned to the same cluster. An extremely case in which each node of an individual have uniform values, all the nodes in the network will be assigned to the same cluster, the other particular situation when values of all the nodes are extremely different, the network will be divided into n totally different clusters. Fig. 2 gives an example of the string-based representation. Fig. 2(a) is a toy network with 8 nodes and can be divided into two cluster. Fig. 2(b) shows a possible encode representation.



(a)

nodes	1	2	3	4	5	6	7	8
encode	3	3	3	3	6	6	6	6

(b)

Fig. 2. An illustration of the string-based representation.(a) the true partition of a single toy network, which can be divided into two clusters. (b) An possible individual encoding of the network. Red dashed circles represent clusters

In this paper, we employ a heuristic strategy based initializaiton operation proposed in [23]. The heuristic strategy is introduce in [24] In the initialization process, randomly select a node in the individual and find all of its neighbors. Then allocate the cluster identifier of this node to all of its neighbors. This initialization population has better diversity and quality than a randomly initialized population. The algorithm is shown in **Algorithm 2**.

Algorithm 2 Initialization procedure

1: **Input**: The ith layer network g_i, population size n_{pop}.
2: **for** $k = 1 : n_{pop}$ **do**
3: chromosome $S_k$is set to$\{1, 2, \ldots, n\}$;
4: Randomly select a node x_i ;
5: $V^i_{nei} \leftarrow$ all the neighbors of x_i;
6: $s^j_k \leftarrow s^i_k$, $v_j \in V^i_{nei}$;
7: **end for**
8: **Output**: Population $\mathbf{P}$.

B. Genetic operations

1) Crossover: Traditional crossover operators like uniform crossover lacks of heuristic knowledges which may slow down the convergence. Genetic operations used in this paper is a two-way crossover operation [23]. The two-way crossover operator, which is inspired by the one-way crossover operator [24], has been been demonstrated to be an efficient operator in evolutionary algorithms (EA). The crossover operator is work as follows: Let two parental chromosomes are S_a and S_b, respectively. Randomly select a node v_i and find all the other nodes that have the same cluster identifier in S_a. Then assign this cluster identifier to the same nodes in S_b. The similar operations is performed in S_a. Two child chromosomes S_c and S_d are generated after this procedure. An simple illustration of the two-way crossover operator is shown in fig. 3.

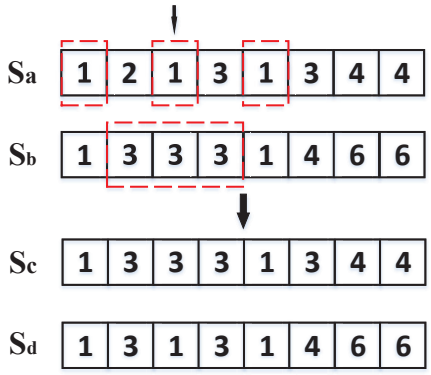


Fig. 3. An illustration of the two-way crossover operation.

2) mutation: We design a k-nearest neighbors based mutation operator in our algorithm. In this mutation procedure, A node is randomly selected to be mutated. Then, find all the neighbors of the node in the network, and its cluster is reassigned by the highest appearance frequency of cluster identifier of its neighbors. Fig. 4 gives an simple illustration of the proposed mutation operator. In fig. 4, v_4 is selected to be the mutation position. Among its neighbors $\{v_2, v_3, \ldots, v_5\}$ the highest appearance frequency of cluster identifier is 3. Thus, the cluster identifier of node v_4 is assigned to 3.

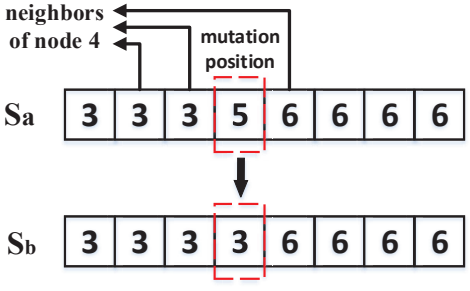


Fig. 4. An illustration of the proposed mutation operation. The 4th node is selected to be mutated

C. Local search operation

In this work, we employ a hill-climbing based local search operator proposed in [23]. First, the neighbors of a chro-

mosome is defined as follow: Given a chromosome $S_c = \{s_c^1, s_c^2, \ldots, s_c^m\}$, randomly chose a node in S_c and get its cluster identifier s_c^i. Then, reassigned this node into another cluster s_c^j, where $j \neq i$. The new generated partition is denoted a neighbor of S_c. In the local search process, randomly select a chromosome to get refinement, all possible neighbors will be found and the neighbor with best fitness will be selected to replace this chromosome. The function FindBestNeighbor() is used for finding the best neighbor of the current chromosome.

The details of the local search procedure is given in **Algorithm 3**.

Algorithm 3 Local search procedure

Input: chromosome S, the local search probability p_l.
$flag \leftarrow$ FALSE;
repeat
  $S^* \leftarrow FindBestNeighbor(S)$;
  if $Eval(S^*) > Eval(S)$ **then**
    $S \leftarrow S^*$;
  else
    $flag \leftarrow$ TRUE;
  end if
until $flag$ is TRUE
Output: Chromosome after performing the local search S.

V. EXPERIMENTS

In this section, we employ two real-world multiplex networks and two evaluation index to demonstrate the effect of the proposed MOEA-MultiNet. The two small-scale multiplex networks are KAPFERER TAILOR SHOP and CS-AARHUS. This two networks can be downloaded from http://deim.urv.cat/manlio.dedomenico/data.php. (i) The KAPFERER TAILOR SHOP network collects the interactions information of a tailor shop. The network is composed of 39 vertices and 1018 links and contains four layers which records the work-related, assistance-related, friendship and socioemotional interactions. (ii) The CS-AARHUS social network consists of 61 nodes and 620 links in five different layers. It is collected from five types of social relationships, which are online relation Facebook and offline relations like leisure, work, co-authorship and lunch. Both the two networks considered are unweighted and undirected.

The parameters used in the proposed MOEA-MultiNet and their values are given in Table I.

TABLE I
PARAMETERS SETTING IN MOEA-MULTINET.

Parameter	Meaning	Value
n_{pop}	Population size	100
n_{pool}	Size of mating pool	50
n_{tour}	Tournament size	2
P_c	Crossover probability	0.9
P_m	Mutation probability	0.15

A. Evaluation metrics

Absolutely, traditional supervised metrics for single network are not suitable for this problem domain. In this study, we adopt two evaluation metrics to evaluate the quality of detected community structures in multilayer networks.

Few metrics have proposed to directly evaluate the shared community structure of all layers of multilayer networks. Among them the first metric is a multilayer modularity index Q_m. Q_m is defined as:

$$Q_m = \frac{1}{2\mu} \sum_{C} \sum_{i,j \in C, l \in \beta} \left(A_{ij}^{[l]} - \lambda \frac{d_i^{[l]} d_j^{[l]}}{2M^{[l]}} \right) \tag{4}$$

where the normalization factor $\mu = \sum_{l \in \beta} M^{[l]}$ denotes the number of links in multilayer networks. $A^{[l]}$ is the adjacency matrix of the lth layer network. $d_i^{[l]}(d_j^{[l]})$ is the degree of node $i(j)$ in the lth layer network. λ is a resolution parameter and the value is set to 1 for most multilayer community detection problem [10]. High values of Q_m denotes high quality of detected community structures.

The redundancy index R_c [17], [25] figures out the fraction of redundant links in a multilayer network. The intuition of this indicator is that the shared community should have connections in different layers. R_c is denoted as:

$$R_c = \frac{1}{\|P\|} \sum_{C} \sum_{(p,q) \in V_C^*} \frac{\|l | \exists A_{pq}^{[l]} \neq 0\|}{\beta \times \|V_C\|} \tag{5}$$

where $\|P\|$ is the number of communities in multilayer networks. V_C is the couples (p, q) in the community C that have connections in no less than one layer. V_C^* denotes the couples (p, q) in the community C that have connections in no less than two layers. The larger the value of R_c is, the better quality the partition is.

B. Experiments on two real-world networks

The comparison results between the proposed MOEA-MultiNet and the BGLL that uses only one layer network on two real-world networks are given in Table II and Table III. The result in Table II shows that, by running the BGLL in only one layer network, the best value of Q_m can reach 0.2179 and the best value of R_c can reach 0.4717. By running the proposed MOEA-MultiNet, the average value of Q_m can reach 0.2094 and the average value of R_c can reach 0.4735. In Table III, by running the BGLL in only one layer network, the best value of Q_m can reach 0.4685 and the best value of R_c can reach 0.2852. By running the proposed MOEA-MultiNet, the average value of Q_m can reach 0.4010 and the average value of R_c can reach 0.3186. This means that the shared community structure uncovered by the proposed MOEA-MultiNet is superior than most single layer based method. The results also show that the community structure obtained by only one layer can not be regarded as optimal shared community structure.

In Table IV, we compare the performance between using the random ordering and the clustering coefficient based ordering.

TABLE II
THE COMPARISON RESULTS BETWEEN THE PROPOSED MOEA-MULTINET AND THE BGLL THAT USES ONLY ONE LAYER NETWORK ON KAPFERER TAILOR SHOP NETWORK: THE MULTILAYER METRIC Q_m, THE REDUNDANCY INDEX R_c.

strategies	Algorithms	Q_m	R_c
One-layer	L_1	0.2179	0.3964
	L_2	0.2006	0.4717
	L_3	0.1380	0.2657
	L_4	0.0932	0.4094
Multi-layer	MOEA-MultiNet	0.2094	0.4735

TABLE III
THE COMPARISON RESULTS BETWEEN THE PROPOSED MOEA-MULTINET AND THE BGLL THAT USES ONLY ONE LAYER NETWORK ON CS-AARHUS NETWORK: THE MULTILAYER METRIC Q_m, THE REDUNDANCY INDEX R_c.

strategies	Algorithms	Q_m	R_c
One-layer	L_1	0.4685	0.2852
	L_2	0.1672	0.0472
	L_3	0.0832	0.1205
	L_4	0.2893	0.1611
	L_5	0.4115	0.2715
Multi-layer	MOEA-MultiNet	0.4010	0.3186

Table IV clearly show that on both two networks, the cluster coefficient based ordering can obtain better community structure of multilayer networks than the random ordering. This indicates that the ordering of networks play an important role in the proposed MOEA-MultiNet, the clustering coefficient based ordering is thus employed in our paper.

TABLE IV
THE COMPARISON RESULTS BETWEEN USING RANDOM ORDERING AND CLUSTER COEFFICIENT BASED ORDERING ON TWO REAL-WORLD NETWORKS: THE MULTILAYER METRIC Q_m, THE REDUNDANCY INDEX R_c.

networks	ordering	Q_m	R_c
KAPFERER	random	0.1741	0.3987
	cluster coefficient	0.2094	0.4735
CS-AARHUS	random	0.2463	0.1740
	cluster coefficient	0.4010	0.3186

VI. CONCLUSION

In this paper, we propose an improved multiobjective evolutionary approach to discover community structure in multilayer networks. The proposed MOEA-MultiNet use the string-based representation scheme to encode individuals, and a two-way crossover and k-nearest neighbor based mutation is employed in our method. To improve the quality of population, a local search is incorporated in our method. Experimental results on two real-world networks show the good shared community structure obtained by the proposed MOEA-MultiNet. Moreover, the network ordering can have a important effect on our approach. To test the influence of the ordering way, a comparison experiment on the ordering way of multilayer networks demonstrate that using a cluster coefficient based ordering in our method can get better quality community structure than a random ordering.

VII. ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (Grant no. 61422209), the National Program for Support of Top-notch Young Professionals of China, and the Specialized Research Fund for the Doctoral Program of Higher Education (Grant no. 20130203110011).

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